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RHIC Spin flipper, fast-sweep efficiency simulations

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Collider Accelerator Department
Brookhaven National Laboratory

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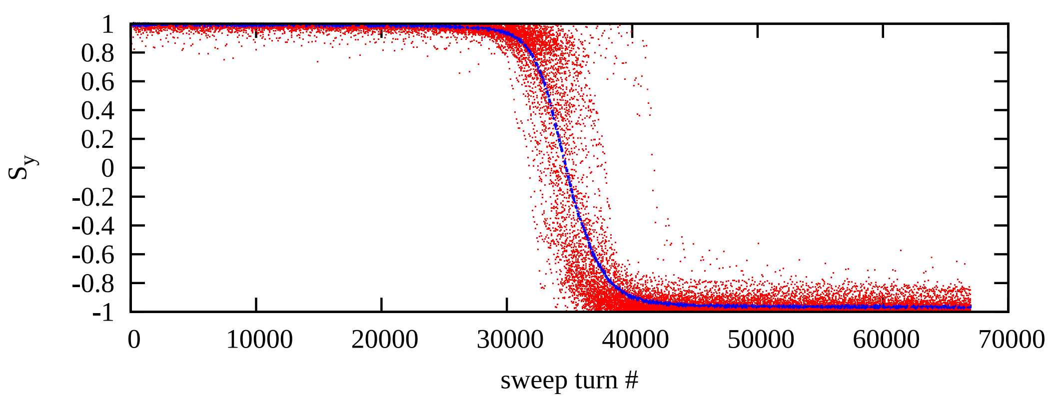
August 8, 2017

Abstract

Polarization flip simulations have been performed during Run 17 in parallel to resuming spin flipper experiments in RHIC. They are reported here, including some guidance they brought.



Tech. Note CA/AP/589
BNL C-AD

Contents

1 Introduction

Spin flip simulations in RHIC were performed in the May-June period during Run 17, in parallel with RHIC spin flipper experiments which were resumed during this very RHIC run.

These simulations have had two significant outcomes :

(i) increasing the AC dipole frequency sweep rate, by a substantial amount compared to the value initially used during the experiments, appears to improve the spin flip efficiency,

(ii) in the conditions of the simulations (*e.g.*, defect-free ring ; ideal snakes : 180° spin rotation at $\pm 45^{\mathrm{o}}$ from the longitudinal axis ; 180^{o} orbit angle between the two snakes opposing in RHIC diameter ; perfect spin flipper setting including absence of image resonance, etc.), the difference in the dispersion derivative at the two snakes as a source of spin tune spread via the chromatic orbit angles at the snakes [1], subject to dedicated RHIC optics settings during Run 17 spin flipper experiment, does not seem of very stringent importance.

These two observations from the simulations were used as a guidance : the first one was confirmed experimentally, and there are indications from the experiments that the second observation holds.

Section 2 below recalls the principles of the spin flipper system in RHIC, and introduces the resulting input data to the simulations for the sake of clarity. Section 3 presents the main outcomes of these simulations, which Section 4 comments on.

2 Spin flipper in RHIC, simulation conditions

The spin flipper in RHIC is an interleave of four horizontal dipoles ("DC rotator" in Fig. 1) and five vertical AC dipoles (AC dipole#1-5). It is located in the RHIC Blue ring, between "BI9_TQ4" and "BI9_QF3" quadrupoles.

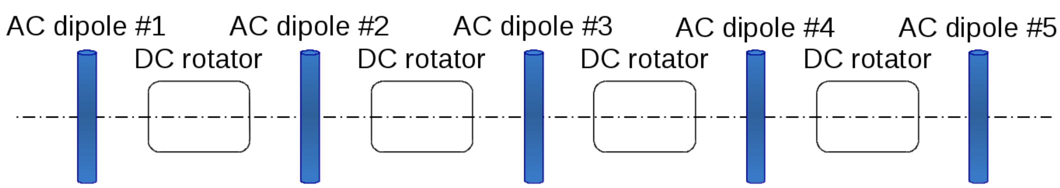


Figure 1: Layout of the spin flipper [1].

The principles of this spin flipper and its installation in RHIC are detailed and discussed in Refs. [1]-[5]. The 4 "spin rotator" dipoles are DC, they yield spin y-rotation angles $+/-/-/+\times\phi_0$ respectively, with

$$\phi_0 = (1+\mathrm{G}\gamma)\frac{\mathrm{BL}}{\mathrm{B}\rho} \tag{1}$$

Orbit-wise this defines a closed local horizontal bump and, spin-wise, it leaves the spin tune $\mathrm{Q_s}$ unchanged.

The 5 AC dipoles are operated at frequency $\mathrm{Q_{osc}} \approx \mathrm{f_{rev}}/2$, with fractional part in the vicinity of $\mathrm{Q_s}$. ACD1-3 and ACD3-5 ensure the same $+1/-2/+1\times\psi_{\mathrm{osc}}$ spin x-rotation sequence,

$$\psi_{\mathrm{osc}} = (1+\mathrm{G}\gamma)\frac{\mathrm{B_{osc}L}}{\mathrm{B}\rho} \tag{2}$$

Orbit-wise, each triplet ensures a local vertical orbit bump. Fields in ACD 2 and 4 are $180^{\mathrm{o}}+\phi_0$ distant in phase.

This configuration induces spin resonance at $\mathrm{Q_{osc}} = \mathrm{Q_s}$ while eliminating the image resonance at $1-\mathrm{Q_s}$, therefore ensuring *single resonance crossing* during $\mathrm{Q_{osc}}$ sweep through $\mathrm{Q_s} \approx \frac{1}{2}$ and allowing full spin flip (isolated resonance crossing in the presence of image resonance instead, would otherwise require $\mathrm{Q_s}$ to be away enough from $\frac{1}{2}$ and $\mathrm{Q_{osc}}$ excursion accordingly small).

The strength of this spin-resonance system is

$$|\epsilon| = \frac{\psi_{\mathrm{osc}}}{\pi}\sin\phi_0 \sin\frac{\phi_0}{2} \tag{3}$$

The crossing speed (rate of sweep of $\mathrm{Q_{osc}}$ through $\mathrm{Q_s} \approx \frac{1}{2}$) is

$$\alpha = \frac{\Delta \mathrm{Q_{osc}}}{\mathrm{d}\theta}, \quad \mathrm{d}\theta = 2\pi\mathrm{N} \tag{4}$$

with $\Delta\mathrm{Q_{osc}}$ the AC dipole frequency span and N the number of turns of the sweep. α has to be small enough (N large enough) for spins to follow the flip of the spin precession axis so that the polarization states $\mathrm{P_f}$ (final) and $\mathrm{P_i}$ (initial) satisfy

$$\frac{\mathrm{P_f}}{\mathrm{P_i}} = 2\exp^{-\frac{\pi}{2}\frac{|\epsilon|^2}{\alpha}} - 1 \approx -1 \tag{5}$$

D' lattice : Momentum dispersion induces spin tune spread, via

$$\delta \mathrm{Q_s} = \frac{\mathrm{G}\gamma}{\pi} \Delta \mathrm{D}' \frac{\Delta \mathrm{p}}{\mathrm{p}} \quad (6)$$

with $\Delta \mathrm{D}'$ the difference in D' at the two snakes [6]. This effect is considered a cause of loss of polarization during the frequency sweep, in particular by inducing multiple resonance crossing under the effect of synchrotron motion - following the sketch in page 7.

"D' lattice" optics are aimed at overcoming that effect, by ensuring minimized $\Delta \mathrm{D}'$ during the ACD frequency sweep. They require dedicated, possibly lengthy, tunings based on the use of gamma transition quads [1].

Typically: $\delta \mathrm{Q_s}/\frac{\Delta \mathrm{p}}{\mathrm{p}} = 150 \times \Delta \mathrm{D}'$ at 250 GeV, about 10 times less at injection. Thus, assuming $\delta \mathrm{p}/\mathrm{p} < 10^{-3}$, reducing $\delta \mathrm{Q_s}$ at $< 10^{-3}$ level (namely, small enough by comparison with $\Delta \mathrm{Q_{osc}}$, see sketch p. 7) requires a typical $\Delta \mathrm{D}' < 70$ mrad at injection, $\Delta \mathrm{D}' < 7$ mrad at store.

DC rotator and AC dipole settings in these simulations are consistent with (although not strictly identical to) Run 17 APEX conditions in RHIC, Table below.

Numerical simulation data (actual APEX conditions in column 5)

		injection	store	
$\mathrm{G}\gamma$		45.5	487	
$\mathrm{B}\rho$	(T.m)	79.37	850.128	
ACD strength	(G.m)	102	102	APEX : 100 G.m
ACD ψ_{osc}	(deg)	0.342	0.336	
DC rotator ϕ_0	(deg)	29.9	44.9	APEX : 48.8$^{\mathrm{o}}$ at store
$\lvert\epsilon\rvert$	(10^{-4})	2.45	4.32	
$\mathrm{Q_{osc}}$ range		0.49 - 0.51	0.49 - 0.51	APEX : $\Delta \mathrm{Q_{osc}} = 0.005$
RF systems	(MHz)	9 or 28	9 + 197	APEX : 9 + 197 at injection and store

Details of RF conditions during APEX :

Blue 9MHz cavity is the main one for injection and acceleration. It is set at 22 kV at injection and 30 kV at store. The 28 MHz (BA1 and BA2) are considered off. However 28 MHz has been investigated in the simulations (not reported here - with a trend to loss of spin flip efficiency due to multiple crossing, sketch in page 7, also discussed in [7]).

The Landau cavities (BS1 and BS2) are 197 MHz ones and voltage is around 10 kV at injection, 15 kV at store. These are the conditions as well for the double-RF simulations (see p. 11).

Detailed data for the rotator and AC dipoles in Zgoubi [8] are given in page 5. Details of RF conditions are given in due place together with the tracking outcomes (Sec. 3).

Zgoubi input/output files used in these studies can be found in

$$/\mathrm{rap/lattice\_tools/zgoubi/RHICZgoubiModel/spinFlipper}$$

C-AD server folders.

- ZGOUBI INPUT -

• DC rotators and AC dipoles input data, injection :

AC dipole sweep command :

```
MULTIPOL ACD1
-88
0.521678913421  0.49 0.51 1.  C Q1 Q2 P
50 3000 20000 3000
MULTIPOL ACD2
-88
0.0 0.49 0.51 1.    C Q1 Q2 P
50 3000 20000 3000
```

Spin flipper sequence :

```
 'MULTIPOL' ACD1      BI9_DAC3.5
0  .kicker
0.100000E+03 10. -0.0494 0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_87
36.464200
 'MULTIPOL' DCRot      BI9_TH3.4
0  .kicker
0.183000E+03 10. -0.691857780671  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
1 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_88
109.414800
 'MULTIPOL' ACD1      BI9_DAC3.4
0  .kicker
0.100000E+03 10. 0.101857839   0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
```

```
 'DRIFT'    DRIF      DRIFT_89
82.463600
 'MULTIPOL' DCRot      BI9_TH3.3
0  .kicker
0.183000E+03 10. 0.691857780671  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
1 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_90
63.415400
 'MULTIPOL' ACD1      BI9_DAC3.3_1
0  .kicker
50. 10. -0.104915681  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
 'MULTIPOL' ACD2      BI9_DAC3.3_2
0  .kicker
50. 10. -0.104915681  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_87
36.464200
 'MULTIPOL' DCRot      BI9_TH3.2
0  .kicker
0.183000E+03 10. 0.691857780671  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
1 0. 0. 0.
```

```
 'DRIFT'    DRIF      DRIFT_88
109.414800
 'MULTIPOL' ACD2      BI9_DAC3.2
0  .kicker
0.100000E+03 10. 0.101857834  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_89
82.463600
 'MULTIPOL' DCRot      BI9_TH3.1
0  .kicker
0.183000E+03 10. -0.691857780671  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
1 0. 0. 0.
 'DRIFT'    DRIF      DRIFT_90
63.415400
 'MULTIPOL' ACD2      BI9_DAC3.1
0  .kicker
0.100000E+03 10. -0.0494  0. 0. 0. 0. 0. 0. 0. 0. 0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
.0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
1.57079632679 0. 0. 0. 0. 0. 0. 0. 0. 0.
#20|4|20  Kick
3 0. 0. 0.
```

• DC rotators and AC dipoles input data, store :

AC dipole sweep command :

```
BEND ACD1
-88
0.785398163 0.49 0.51 1.
50 3000 28000 3000
BEND ACD2
-88
0.0 0.49 0.51 1.
50 3000 28000 3000
```

Spin flipper sequence :

```
 'BEND'     ACD1
0
100.0000000     1.5707964   -0.0494
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
 'DRIFT'    DRIF      O3B9L9
2.1336
 'DRIFT'    DRIF      O3B9L10
17.3228
 'DRIFT'    DRIF      O3B9L11
17.0078
 'BEND'     DCRot
0  .Bend
183.0000000     0.0000000   -7.47658814
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|100|10
3 0. 0. 0.
 'DRIFT'    DRIF      O3B9L12
23.4360
 'DRIFT'    DRIF      O3B9L13
23.9268
 'DRIFT'    DRIF      O3B9L14
17.2720
 'DRIFT'    DRIF      O3B9L15
17.7800
 'DRIFT'    DRIF      O3B9L15X
27.0000
 'BEND'     ACD1
0
100.0000000     1.5707964   0.101857839
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
 'DRIFT'    DRIF      O3B9L16
17.7800
```

```
 'DRIFT'    DRIF      O3B9L17
19.4818
 'DRIFT'    DRIF      O3B9L18
45.2018
 'BEND'     DCRot
0  .Bend
183.0000000     0.0000000   7.47658814
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|100|10
3 0. 0. 0.
 'DRIFT'    DRIF      O3B9L19
16.9590
 'DRIFT'    DRIF      O3B9L20
17.3228
 'DRIFT'    DRIF      O3B9L21
2.1336
 'DRIFT'    DRIF      O3B9L21X
27.0000
 'BEND'     ACD1
0
50.0000000     1.5707964   -0.104915681
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
 'BEND'     ACD2
0
50.0000000     1.5707964   -0.104915681
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
 'DRIFT'    DRIF      O3B9L22
2.1336
 'DRIFT'    DRIF      O3B9L23
17.3228
 'DRIFT'    DRIF      O3B9L24
17.0078
 'BEND'     DCRot
0  .Bend
183.0000000     0.0000000   7.47658814
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|100|10
3 0. 0. 0.
 'DRIFT'    DRIF      O3B9L25
45.1530
```

```
 'DRIFT'    DRIF      O3B9L26
19.4818
 'DRIFT'    DRIF      O3B9L27
17.7800
 'DRIFT'    DRIF      O3B9L27X
27.0000
 'BEND'     ACD2
0
100.0000000     1.5707964   0.101857834
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
 'DRIFT'    DRIF      O3B9L28
17.7800
 'DRIFT'    DRIF      O3B9L29
19.4818
 'DRIFT'    DRIF      O3B9L30
45.2018
 'BEND'     DCRot
0  .Bend
183.0000000     0.0000000   -7.47658814
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|100|10
3 0. 0. 0.
 'DRIFT'    DRIF      O3B9L31
16.9590
 'DRIFT'    DRIF      O3B9L32
17.3228
 'DRIFT'    DRIF      O3B9L33
2.1336
 'DRIFT'    DRIF      O3B9L33X
27.0000
 'BEND'     ACD2
0
100.0000000     1.5707964   -0.0494
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
0.00  0.00  0.00
4 .2401  1.8639  -.5572  .3904 0. 0. 0.
#10|200|10
1 0. 0. 0.
```

3 Numerical outcomes

3.1 Injection ($G\gamma = 45.5$)

Fig. 2 shows the two optics tracked, regular Run 17 lattice (left plot), and D' lattice (right) using the gamma transition patch translated from RHIC MADX model. $\Delta D'$ can easily be changed by tweaking a sensitive one of the γ_{tr} quadrupole families [1] - this is done here regardless of possible $\delta\beta/\beta$ betatron modulation however that tweak is small. Separation bumps at all IPs have been zeroed in the simulations, the orbit is flat. Snakes are simulated using "SPINR", pure spin rotation, no local orbit bump.

D' at snakes 1 and 2 takes the following values :

◇ **without transition gamma quadrupole patch :**

```
SNAKE 1   : +13.713194E-003 rd
SNAKE 2   : -49.502845E-003
```

$|D'_1 - D'_2| = 63.2$ **mrad.**

◇ **with transition gamma quadrupole patch, typical :**

```
SNAKE 1   : -10.379716E-003
SNAKE 2   : -16.113063E-003
```

$|D'_1 - D'_2| = 5.7$ **mrad.**

Spin flip has been tracked with several different $\Delta D'$ values. Note that in the non-D' lattice case $\Delta D' = 63$ mrad < 70 mrad, so that in the presence of $dp/p < 1.5 \times 10^{-3}$ it ensures, as seen earlier, $\delta Q_s < 1.5 \times 10^{-3}$, smaller than the $\Delta Q_{osc} = 0.02$ span considered here, so partial flip can be expected.

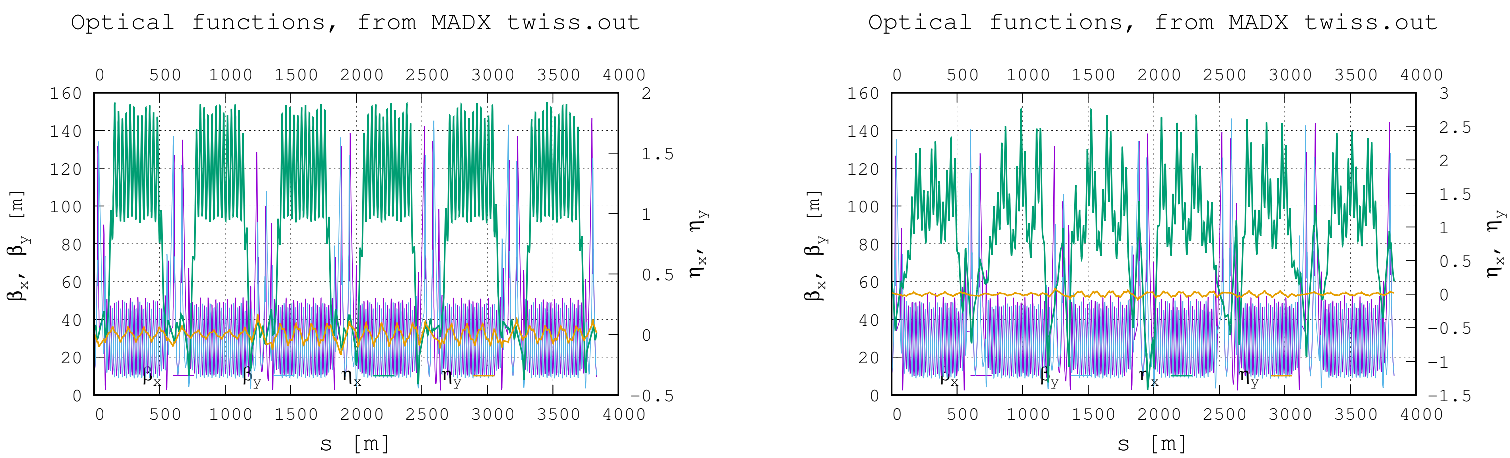
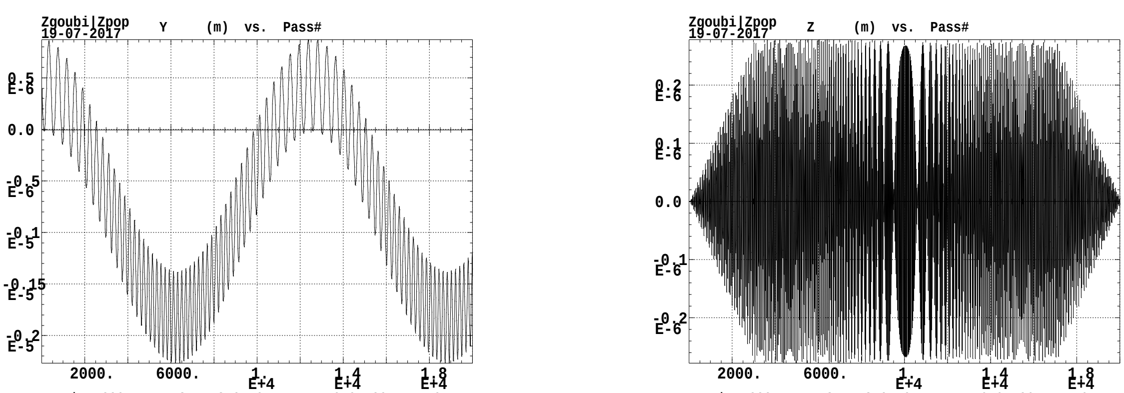


Figure 2: Blue injection optics, without (left) and with (right) D' lattice transition gamma quadrupole patch. In both cases, vertical separation bumps at IPs have been zeroed, here as well as in the flip simulations.

In addition :
- RF frequency is 9 MHz,
- 960 particles are tracked, Gaussian-distributed (truncated) in a 6-D bunch, polarization is computed as an average over that ensemble,
- particles possibly lost in some cases (due to purposely large/long bunch, in small number anyway) are not counted in the average polarization.

Orbit : possible residual orbit induced by the spin flipper (case of DC or AC bumps not strictly locally closed) may have a deleterious effect on the resonance crossing [3]. The two figures below show the horizontal and vertical orbits in the course of a sweep in the present simulations, namely, negligible orbital motion (at a few micro-meter level at IP6, about two orders of magnitude smaller than the betatron excursions of the particles).



Orbital motion observed at IP6 (left : horizontal, right : vertical), in the conditions of the "regular optics" ΔQ_{osc} sweep of pp. 8, 9.
The 9 MHz RF 6 Hz synchrotron oscillation (13,000 turns about) is apparent on the left graph.
In both cases the residual orbit is marginal, well below 1 μm.

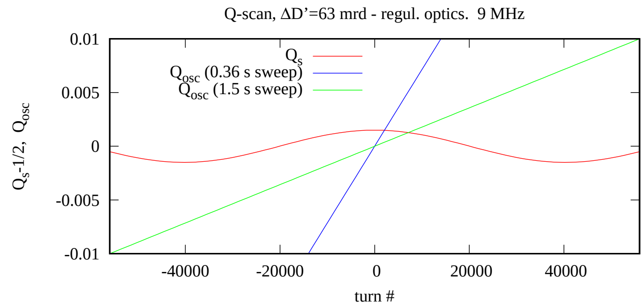
Multiple crossing has been identified as a possible cause for polarization loss during the $\mathrm{Q_{osc}}$ sweep, this is sketched in the figures on the right (with simulation conditions of page 8).

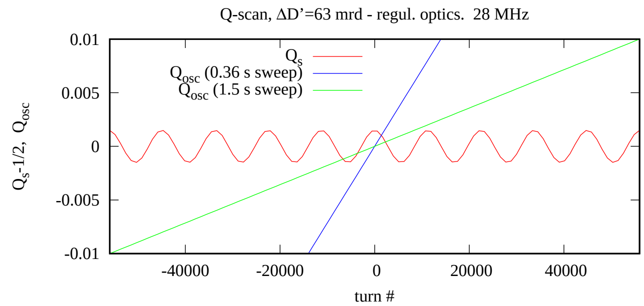
However, a $\Delta\mathrm{Q_{osc}} = 0.02$ sweep in $\approx$ 15,000 turns, with 9 MHz RF, is about one synchrotron period :

$$\Omega_\mathrm{s} = \frac{\mathrm{c}}{\mathrm{R}}\sqrt{\frac{\mathrm{h}\eta\cos(\phi_\mathrm{s})\mathrm{q}\hat{\mathrm{V}}}{2\pi\mathrm{E_s}}}$$

Here, R = 3833.8 m, $\mathrm{h} = 120$ (9 MHz), $\eta = 25.4^{-2} - 0.002 = -4.5\times10^{-4}$, $\cos(\phi_\mathrm{s}) = -1$, $\mathrm{q}\hat{\mathrm{V}} = 20$ keV, $\mathrm{E_s} = 23.8$ GeV, so that $\Omega_\mathrm{s}/2\pi \approx 6$ Hz, $2\pi/\Omega_\mathrm{s} \approx 12,000$ turns. As a consequence, a $10 - 20,000$ turn $\mathrm{Q_{osc}}$ sweep as considered in the simulations, $1\sim 2$ synchrotron oscillations, limits the possible number of crossings during the sweep (top figure on the right).

Instead, 28 MHz, 200 kV RF, $\Omega_\mathrm{s}/2\pi \approx 36$ Hz, $2\pi/\Omega_\mathrm{s} \approx 2,200$ turns, yields conditions more prone to multiple crossing, bottom figure on the right. This is confirmed by simulations (see Ref. [7] for instance).





The next pages 8-9 give sample spin flip simulation results (simulations performed on NERSC [9]), for various $\Delta\mathrm{D}'$ values and sweep durations. The graphics displayed include spin motion across the resonance for a 50-particle sample (red dots), and the average polarization over 960 particles (blue curves), as well as orbital motion monitoring of the 6-D bunch tracked.

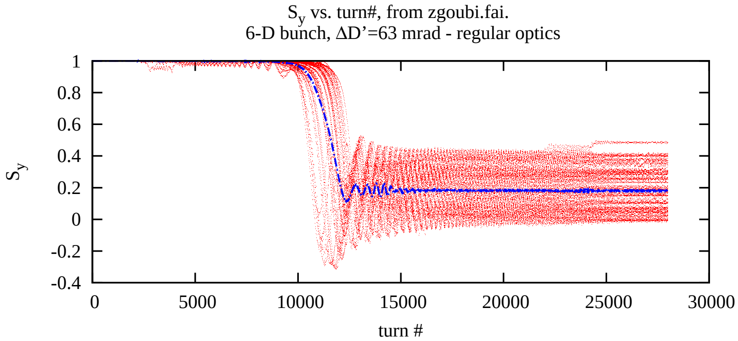
Beyond showing the efficiency improvement with increased sweep rate, the tracking results in page 9 are an indication that spin flip can be obtained with $\Delta D'$ substantially different from zero - at least in the defect free, perfect flipper setting conditions of these simulations.

- REGULAR OPTICS ($\Delta D' = 63$ mrad), TWO DIFFERENT SWEEP DURATIONS. RF 9 MHz -

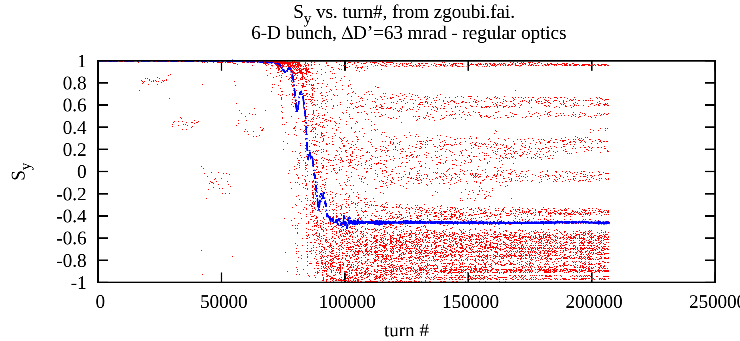
(Red dots : sample trajectories. Blue curves : average over ~ 960 particles)

The two plots below show that the simulations yield substantial spin flip efficiency, in spite of a large $\Delta D'$ value (regular Run 17 lattice).

• REGULAR OPTICS, SWEEP 20,000 TURNS



• REGULAR OPTICS, SWEEP 200,000 TURNS

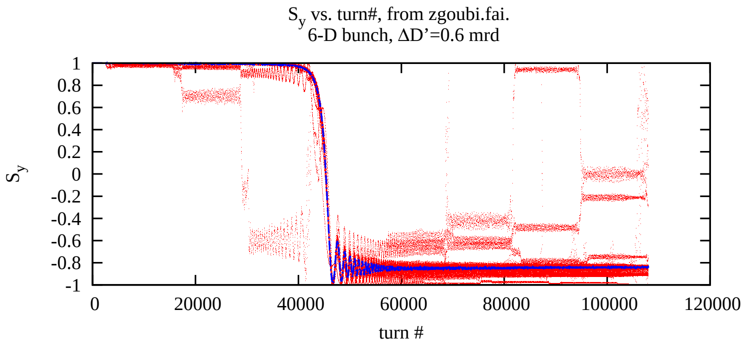


- D' OPTICS, TWO DIFFERENT $\Delta \mathrm{D}'$ VALUES. 9 MHz RF, 100,000 TURN SWEEP -

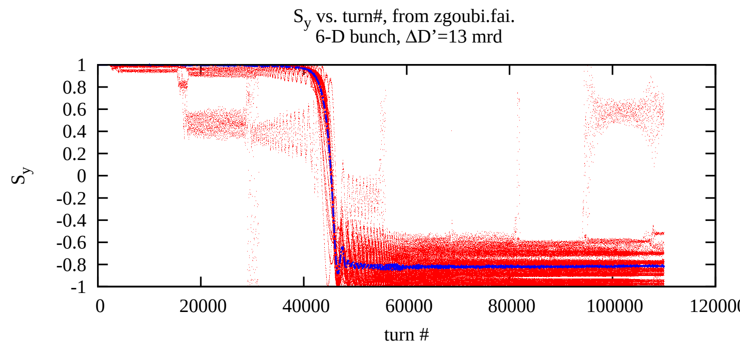
(Red dots : sample trajectories. Blue curves : average over ~ 960 particles).

$\Delta \mathrm{D}'$ is varied over a wide range here, it can be seen that the effect on the spin flip efficiency is small, with -0.837 at $\Delta \mathrm{D}' = 0.6$ mrad and -0.814 at $\Delta \mathrm{D}' = 13$ mrad. Note that in both cases, 200,000 turns sweep ensures close to 100% spin flip efficiency (see $\Delta D' = 13$ mrad case above and next page ; more results in Tab. 1, p. 12).

• $\Delta \mathrm{D}' = 0.6$ mrad



• $\Delta \mathrm{D}' = 13$ mrad,



◇ TYPICAL BUNCH AND RF CONDITIONS IN ZGOUBI :

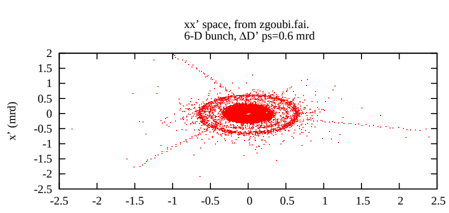
Bunch dimensions in the tracking simulations, above and next page, are about 1 μm normalized *rms* emittance, transverse, 30 ns length and $\mathrm{dp/p} = \pm 1.5 \times 10^{-3}$ momentum extent. Coordinates are taken Gaussian, truncated at 2σ transverse. The three figures below show phase space motion of a few particles from the 960 particle bunch, during a sweep.

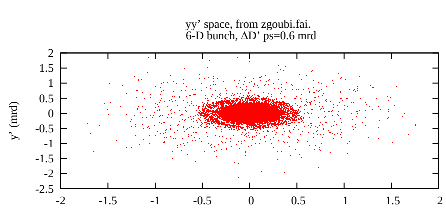
Some particles may be lost (this is apparent in the phase spaces below) depending on RF conditions such as voltage, however in a reduced number (a few percent), and in this case they are not taken into account in computing the average polarization. It has been observed (not shown here) that, taking a smaller bunch, half the transverse excursions displayed in the figures below, same longitudinal phase space, has marginal effect on final polarization.

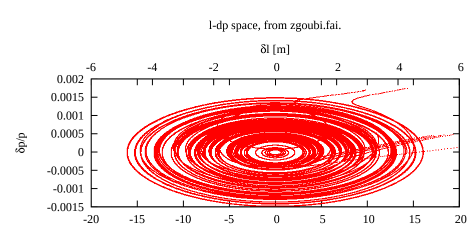
```
 'MCOBJET'
79.366774592048841e3
3
1
2 2 2 2 2 2
0.  0.   0.  0.  0. 1.  'o'
0. 10.   2.5e-8 4    ! rms emittance ~0.6pi micro-m norm.,
0.  9.8 2.5e-8 4    ! cut-off 4-sigma.
0. 5.e4 5e-4  1
123456 234567 345678
```

```
 'CAVITE'
2
3833.84668  120.
20.d3   3.14159265359
 'REBELOTE'
28000  0.4 99
```

Typical transverse and longitudinal orbital motion, in these simulations :



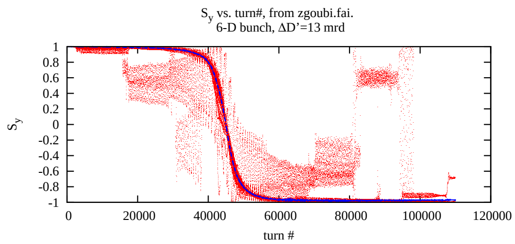




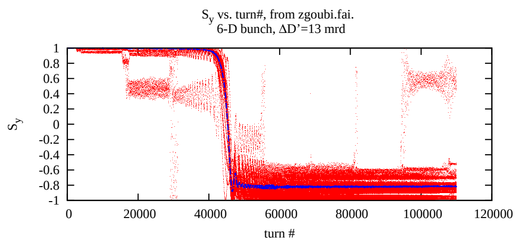
- INJECTION, D'=13 mrad, 9 MHz RF. FROM TOP TO BOTTOM : DECREASING SWEEP RATE TOWARDS $\mathrm{P_f/P_i} \gg -1$ -

A systematic scan of the sweep rate is illustrated, showing the limits of slow sweep (multiple crossing, top plot) and fast sweep ($\mathrm{P_f/P_i} \gg -1$). With $\Delta D' = 13$ mrad, a 200,000 turn sweep, 2.6 seconds, case of $\Delta Q_{\rm osc} = 0.02$, ensures spin flip, equivalent to 50,000 turns, 0.6 second about in the $\Delta Q_{\rm osc} = 0.005$ APEX conditions.

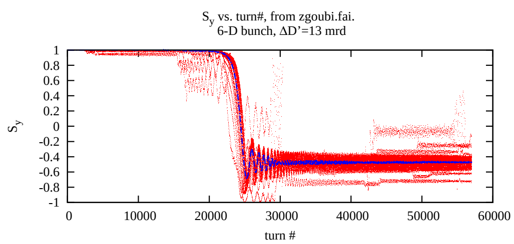
• SWEEP 200,000 TURNS



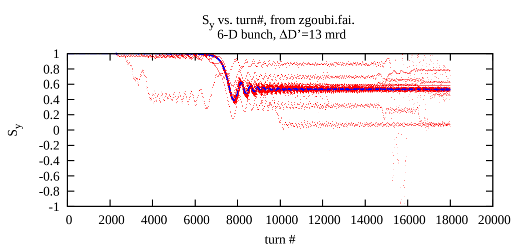
• SWEEP 100,000 TURNS



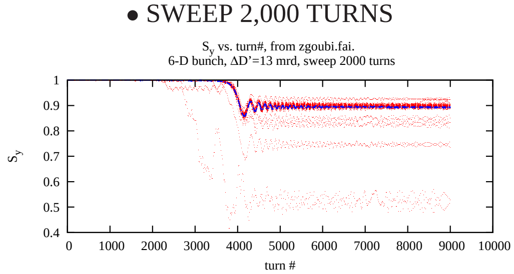
• SWEEP 50,000 TURNS



• SWEEP 10,000 TURNS



• SWEEP 2,000 TURNS

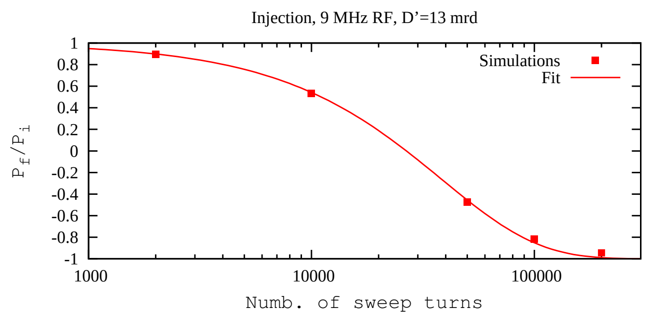


• A match (Fig. below) of $(\mathrm{P_f}, \mathrm{P_i})$ data out of the simulations on the left, using

$$\frac{\mathrm{P_f}}{\mathrm{P_i}} = 2\exp^{-\frac{\pi}{2}\frac{|\epsilon|^2}{\alpha}} - 1$$

with $\alpha = \Delta\mathrm{Q_{osc}}/(\mathrm{N} \times 2\pi)$ and $\Delta\mathrm{Q_{osc}} = 0.51 - 0.49 = 0.02$, yields

$$|\epsilon| = 2.35402 \times 10^{-4}$$



• On the other hand, the numerical simulation hypotheses, namely,

$$\psi_{\rm osc} = (1 + \mathrm{G}\gamma)\frac{\mathrm{B_{osc}L}}{\mathrm{B}\rho} = 5.97 \times 10^{-3}$$

given $\mathrm{G}\gamma = 45.5$, $\mathrm{B}\rho = 79.4$ T.m, $\mathrm{B_{osc}L} = 0.0102$ T.m, and

$$\phi_0 = (1 + \mathrm{G}\gamma)\frac{\mathrm{BL}}{\mathrm{B}\rho} = 0.5215 \text{ rd}, \ 29.9 \text{ deg}$$

given $\mathrm{BL} = 0.4866 \times 1.83$ T.m, yield a resonance strength value within 4% of the matched value above, namely

$$\frac{\psi_{\rm osc}}{\pi}\sin\phi_0 \sin\frac{\phi_0}{2} = 2.445 \times 10^{-4}$$

• The Table below compares the present simulation conditions for 99% spin flip efficiency, and Run 17 APEX conditions :

	$\mathrm{B_{osc}L}$ (G.m)	ϕ_0 (deg)	ϵ (10^{-4})	$\Delta\mathrm{Q_{osc}}$	minimal N for $>$ 99% flip (turns)	duration of the sweep (s)
simulations	102	29.9	2.35	0.02	194,000	2.48
APEX	100	29.9	2.40	0.005	47,000	0.60

3.2 Store ($G\gamma = 487$)

Fig. 3 shows typical D' optics conditions for these simulations, namely, taken from C-A/AP-478 [1]. Snakes are simulated using "SPINR", pure spin rotation, no local orbit bump.

D' values at snakes 1 and 2 in the original lattice are :

```
SNAKE 1   : -21.041930E-003
SNAKE 2   : -18.937173E-003
```

$|\mathbf{D}'_1 - \mathbf{D}'_2| = 2.1$ **mrad.**

Note that $\Delta D' = 2.1$ mrad < 7 mrad, so that $\delta Q_s < dp/p$ (see page 4), well within $\Delta Q_{osc} = 0.02$, a small amplitude which favors absence of multiple crossing.

Spin flipping has also been simulated for the following D' values :

```
SNAKE 1   : -7.0955449E-004
SNAKE 2   : -2.3282297E-002
```

$|\mathbf{D}'_1 - \mathbf{D}'_2| = 22.5$ **mrad.**

and with $\Delta p/p \in [-3 \times 10^{-4}, +3 \times 10^{-4}]$. In that case $\delta Q_s = \frac{G\gamma}{\pi}\Delta D' \frac{\Delta p}{p} \approx \pm 10^{-3}$, well within the sweep $\Delta Q_{osc} = 0.02$.

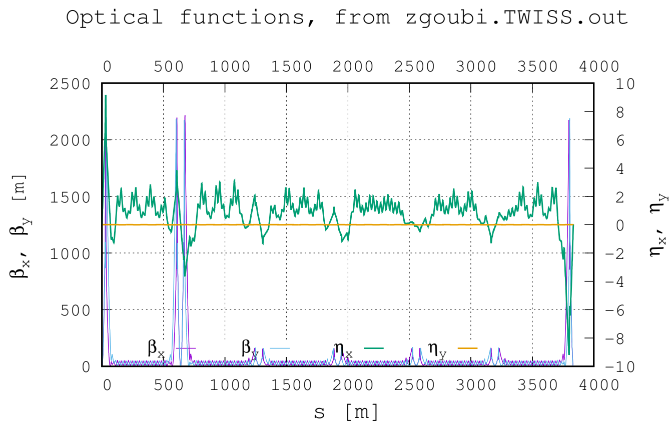


Figure 3: D' lattice optics used in C-A/AP-478.

The DC rotator and AC dipole settings in these store simulations are detailed in page 5.

In particular,

◇ AC dipole strength is set to 102 G.m (100 G.m in Run 17 APEX),
◇ DC rotator strength is set to 45° (48.8° in RHIC APEX) for appropriate resonance strength ϵ for full spin flipping,
◇ the ACD#2-ACD#4 $180^o + \phi_0$ phase relationship is set accordingly.

In addition :

- double RF frequency is considered here : 9 + 197 MHz, respectively 30 kV and 15 kV (data and figure below),
- 960 particles are tracked, polarization is the average over that ensemble,
- particles possibly lost (due to excessive bunch size) are not counted in the average polarization.

The next page gives spin flip simulation results (simulations performed on NERSC [9]), for various sweep durations.

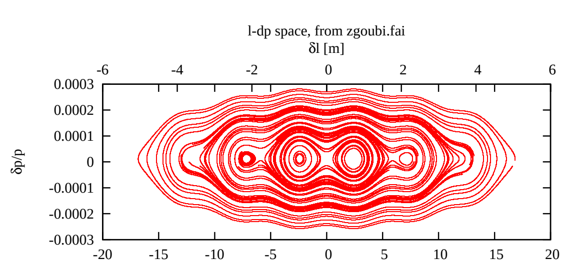
The graphics displayed include spin motion across the resonance for a 50-particle sample (red dots), and the average polarization over 960 particles (blue).

◇ TYPICAL BUNCH AND RF CONDITIONS IN ZGOUBI :

Bunch dimensions in the tracking simulations, next page, are about 2.5 μm normalized *rms* emittance, transverse, 30 ns length and $dp/p = \pm 3 \times 10^{-4}$ momentum extent. Coordinates are taken Gaussian, transverse are truncated at 2σ. The figure below shows longitudinal phase space motion of a few particles from the 960 particle bunch, during a sweep.

```
 'MCOBJET'
850128.3284              ! RIGIDITY
3
1
2 2 2 2 2 2
0.  0.   0.  0.  0. 1.  'o'
0. .6   1e-8  4    ! rms emittance ~2.7pi micro-m norm.,
0. .6   1e-8  4    ! cut-off 4-sigma.
0. 10e4 2.5e-4  1
123456 234567 345678

 'CAVITE'
2.1
3833.8459 { 120. , 2520. }          ! double RF, 9 + 197 MHz
{ 30.d3, 15.d3 } 3.14159265359      ! voltages
```

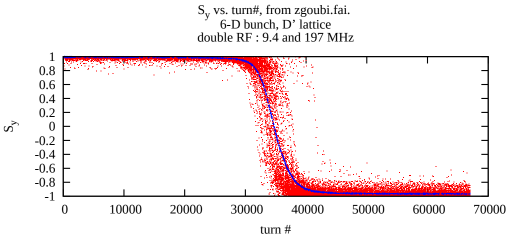


Longitudinal motion, typical.

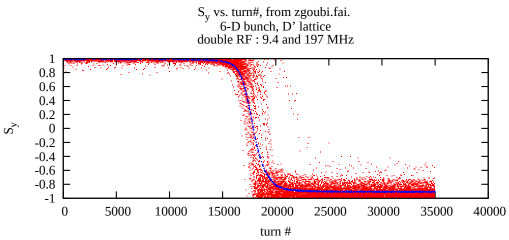
- STORE, D'=22.5 mrad, DOUBLE-RF 9 + 197 MHz (30 + 15 kV) -
FROM TOP TO BOTTOM : INCREASING SWEEP RATE, UP TO CAUSING $\mathrm{P_f/P_i} \gg -1$ -

With $\Delta D' = 22.5$ mrad, a 40,000 turn sweep, 0.5 seconds, ensures spin flip, equivalent to about 8,000 turns, 0.11 second, in the $\Delta Q_{\rm osc} = 0.005$ APEX conditions.

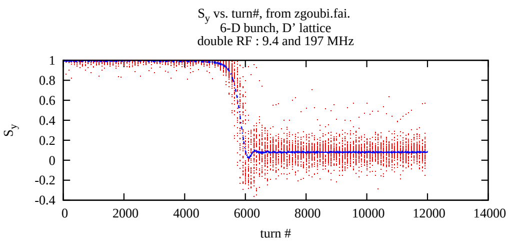
• SWEEP 60,000 TURNS



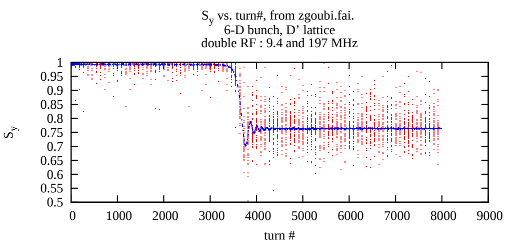
• SWEEP 28,000 TURNS



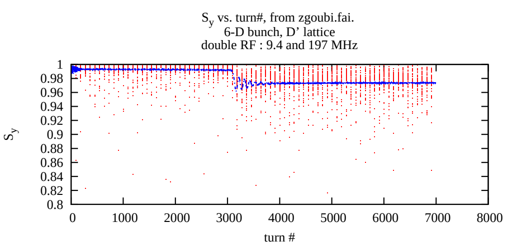
• SWEEP 5,000 TURNS



• SWEEP 1,000 TURNS



• SWEEP 500 TURNS

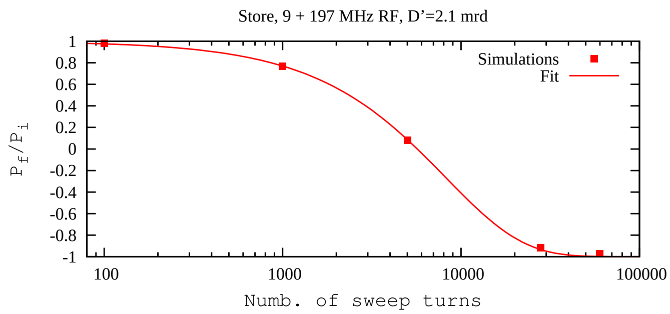


• A match (Fig. below) of $(\mathrm{P_f}, \mathrm{P_i})$ data out of the simulations on the left, using

$$\frac{\mathrm{P_f}}{\mathrm{P_i}} = 2\exp^{-\frac{\pi}{2}\frac{|\epsilon|^2}{\alpha}} - 1$$

with $\alpha = \Delta\mathrm{Q_{osc}}/(\mathrm{N}\times 2\pi)$ and $\Delta\mathrm{Q_{osc}} = 0.51 - 0.49 = 0.02$, yields

$$|\epsilon| = 5.15757 \times 10^{-4},$$



• On the other hand, the numerical simulation hypotheses, namely,

$$\psi_{\rm osc} = (1+\mathrm{G}\gamma)\frac{\mathrm{B_{osc}L}}{\mathrm{B}\rho} = 5.86 \times 10^{-3}$$

given $\mathrm{G}\gamma = 487$, $\mathrm{B}\rho = 850$ T.m, $\mathrm{B_{osc}L} = 0.0102$ T.m, and

$$\phi_0 = (1+\mathrm{G}\gamma)\frac{\mathrm{BL}}{\mathrm{B}\rho} = 0.5215 \text{ rd},\ 45.1 \text{ deg}$$

given $\mathrm{BL} = 0.7476 \times 1.83$ T.m, yield a resonance strength value within 2% of the matched value above, namely

$$\frac{\psi_{\rm osc}}{\pi}\sin\phi_0 \sin\frac{\phi_0}{2} = 5.06345 \times 10^{-4}$$

• The Table below compares the present simulation conditions for 99% spin flip efficiency, and Run 17 APEX conditions :

	$\mathrm{B_{osc}L}$ (G.m)	ϕ_0 (deg)	ϵ (10^{-4})	$\Delta\mathrm{Q_{osc}}$	minimal N for $>$ 99% flip (turns)	duration of the sweep (s)
simulations	102	45.1	5.16	0.02	40,400	0.52
APEX	100	48.8	5.68	0.005	8,300	0.11

3.3 A summary table

Table 1 below summarizes results obtained for typical cases amongst the various spin flip simulations performed.

The quantities varied here are $\Delta \mathrm{D}'$ and the number of turns of the sweep. Other parameters : RF frequency and voltage, $\Delta \mathrm{Q}_{\mathrm{osc}}$, bunch size, etc., are as specified earlier at either injection, with in particular momentum spread $\Delta \mathrm{p}/\mathrm{p} \in [-1.5 \times 10^{-3}, +1.5 \times 10^{-3}]$ (Sec. 3.1) or store with momentum spread $\Delta \mathrm{p}/\mathrm{p} \in [-3 \times 10^{-4}, +3 \times 10^{-4}]$ (Sec. 3.2).

Note that due to the large beam size, in particular large momentum spread, up to 10% of the beam is lost in the 200,000 tracking cases. Large momentum spread could be the cause for the maximum efficiency of about 95% found in the 0.6-13 mrad range, this is to be investigated further.

Table 1: A summary of $\mathrm{P_f}/\mathrm{P_i}$ values in some of the spin flip simulations performed. Initial polarization $\mathrm{P_i}$ is taken at the end of the 3000 turn AC-dipole up ramp, final polarization $\mathrm{P_f}$ is the asymptotic value after the $\Delta \mathrm{Q}_{\mathrm{osc}}$ sweep.

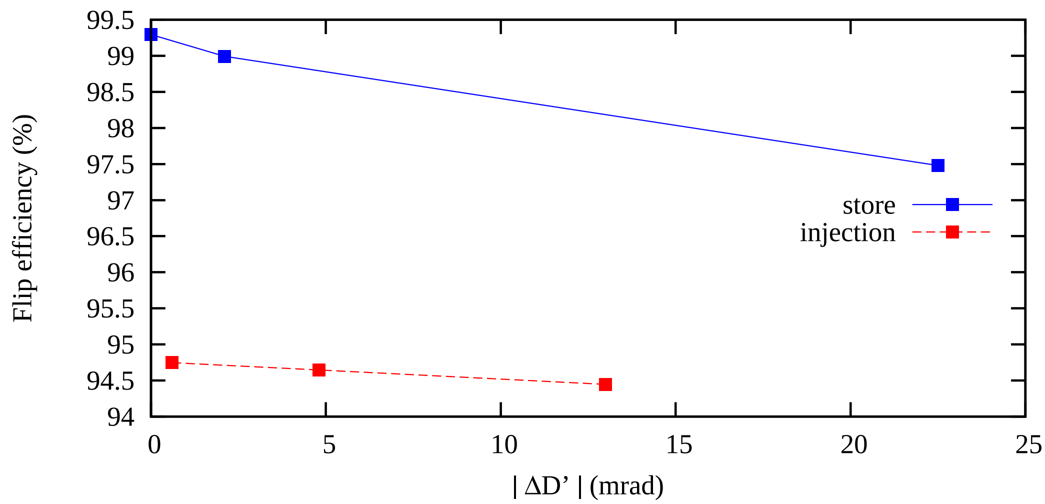
Injection					
D' (mrad)	Number of turns of the ACD sweep				
	200,000	100,000	50,000	20,000	10,000
0.6	-0.947 / 0.9995	-0.837 / 0.9995			
4.8	-0.946 / 0.9995				
13	-0.944 / 0.9995	-0.814 / 0.9995	-0.472 / 0.9995		0.53 / 0.9995
63	-0.449 / 0.9995			+0.180 / 0.9995	

Store				
D' (mrad)	Number of turns of the ACD sweep			
	60,000	28,000	5,000	1,000
0.0	-0.985 / 0.992			
2.1	-0.982 / 0.992	-0.924 / 0.992		
22.5	-0.967 / 0.992	-0.908 / 0.992	+0.080 / 0.992	+0.763 / 0.992

As summarized in Tab. 1, the simulations show weak dependence of spin flip efficiency on $\Delta \mathrm{D}'$, namely (figure below),
- at injection, efficiency $\approx 94 \sim 95$% over the range explored $0.6 \leq |\Delta \mathrm{D}'| \leq 13$ mrad,
- at store, efficiency $\approx 97 \sim 99$% over the range explored $0 \leq |\Delta \mathrm{D}'| \leq 22$ mrad.

For the record, the number of turns of the "fast sweep", respectively 200.000 and 60,000, has been determined by its yielding, in theory, 99% spin flip, see bottom right tables in pp. 9 and 11 respectively.

These results seem in contradiction with APEX measurements : polarization is lost during the sweep if $\Delta D'$ is not at mrad level or less [10, 11]. This needs further investigation.



$\Delta \mathrm{D}'$ dependence of spin flip efficiency.

4 Conclusions

These simulations show the following :

(i) All the simulations performed (much more than summarized in this note), covering various $\Delta \mathrm{D}'$ values including non-D' lattice, various RF frequencies including double RF, various transverse and longitudinal beam sizes up to such sizes as to generate dynamical beam loss, show improved flip efficiency with resonance crossing speed $\alpha = \Delta \mathrm{Q_{osc}}/2\pi \mathrm{N}$ on the high side.

This has been confirmed experimentally,

- in a first trial at injection, $\mathrm{G}\gamma = 45.5$, with sweep time decreased from 3 second as in usage (about 235,000 turns for a frequency span of 0.005, *i.e.*, $\Delta \mathrm{Q_{osc}}/\Delta \mathrm{t} = 0.005/3 = 0.0017\,\mathrm{s}^{-1}$), 92 % flip efficiency, down to 1 sec (0.005 s^{-1}) and further to 0.5 sec (0.01 s^{-1}) both yielding 97.5 % flip instead [10],

- in a second trial at store, $\mathrm{G}\gamma = 487$, with a flip efficiency of 90 % for a 1 second sweep, and 97 % for 0.5 second sweep [11] with no indication that the latter is a lower limit.

(ii) APEX measurements at injection [10] show an optimal sweep between 0.5 and 1 second, in accord with simulations which yield ≈ 0.6 second for 99% flip efficiency given the APEX $\Delta \mathrm{Q_{osc}}$ value (page 9). APEX measurements at store [11] indicate that the sweep duration could be decreased further below 0.5 second, in accord with simulation data that yield an optimal ≈ 0.11 second sweep for 99% flip efficiency given the APEX $\Delta \mathrm{Q_{osc}}$ and ϕ_0 values (page 11).

The optimum is between two limits : number of turns N small enough (greater α) to avoid multiple crossing, large enough (smaller α) to ensure $2\exp(-\frac{\pi}{2}\frac{|\epsilon|^2}{\alpha}) - 1 \approx -1$.

(iii) The numerical simulations show significant tolerance on $\Delta \mathrm{D}'$, in the limit that $\delta \mathrm{Q_s}$ (Eq. 6) is maintained well within $\Delta \mathrm{Q_{osc}}$ sweep range. This is illustrated in the injection case, $\Delta \mathrm{D}' \leq 13$ mrad (p. 8 and Tab. 1, p. 12), which satisfies $\Delta \mathrm{D}' < 70$ mrad so maintaining $\delta \mathrm{Q_s} < \delta \mathrm{p/p} \approx 1.5 \times 10^{-3}$ (bottom right plot in p. 8). In a similar manner at store, simulations with $\Delta \mathrm{D}'$ up to 22 mrad ($\delta \mathrm{Q_s} \approx 3.4 \times \Delta \mathrm{p/p} \approx 0.001$) yield high efficiency spin flip (p. 11 and Tab. 1).

The numerical simulations indicate one thing in that respect :

- a non-small $\Delta \mathrm{D}'$ value may not be in itself an overriding cause of the poor flip efficiency which seems correlatively obtained in the experiments : the mechanism of multiple crossing that large D' contributes inducing may be overcome for instance by faster sweep as discussed in (i) and (ii). Tests were planned at 255 GeV during the latest APEX, based on the fact that a 40 % flip efficiency had been obtained with 3 second sweep (and $|\Delta \mathrm{D}'| = 3$ mrad) [11], a value expected to be substantially improved with faster sweep, however APEX time allotted, time spent on tuning $\Delta \mathrm{D}'$ optics, and other machine downtime did not allow verifying that.

Understanding the details of these spin flip dynamics simulations, and answering the various questions they raise, will require more investigations. This includes, as part of the plans, more simulations in the very spin flipper and bunch conditions of Run 17 APEX.

References

[1] J. Kewisch et al., Correction of the spin chromaticity in RHIC, C-A/AP/478 (2013).

[2] M. Bai and T. Roser, Full Spin Flipping in the Presence of Full Siberian Snake, Phys. Rev. ST Accel. Beams 11, 091001 (2008).

[3] S. R. Mane, Comment on ”Full spin flipping in the presence of full Siberian Snake”, Phys. Rev. ST Accel. Beams 12, 099001 (2009).

[4] M. Bai, W. W. MacKay, and T. Roser, Comment on “Spin manipulation of 1:94 GeV=c polarized protons stored in the COSY cooler synchrotron”, Phys. Rev. ST Accel. Beams 12, 099001 (2009).

[5] Rhic Spin Flipper Project Status, M. Bai, C. Dawson, J. Kewisch, A. A. Poblaguev, P. Oddo, Brookhaven National Laboratory, Upton, NY (2015)

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