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In this note the nominal parameters for deuterons and gold ions in Booster, AGS, and RHIC are given for the FY2016 running period.

The setup parameters are summarized in **Sections 23 through 33.**

1 Mass

A gold ion with charge eQ has $N = 118$ neutrons, $Z = 79$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the positive elementary charge. The mass number is

$$A = N + Z = 197. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_u c^2 - Qm_e c^2 + E_Q \tag{2}$$

where [1, 2]

$$a = 196.9665687(6) \tag{3}$$

is the relative atomic mass of the neutral gold atom,

$$m_u c^2 = 931.494061(21) \text{ MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_e c^2 = 0.510998928(11) \text{ MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral gold atom. This amounts to [3, 4] 0.3324 MeV for the helium-like gold ion ($Q = 77$) and 0.5170 MeV for the fully stripped ion. For $Q = 32$ we have $E_Q = 14.5$ KeV.

Thus the mass energy equivalents for the Au32+, Au77+, and Au79+ ions are

$$mc^2(\text{Au32+}) = 183.456851494 \text{ GeV} \tag{6}$$

$$mc^2(\text{Au77+}) = 183.434174442 \text{ GeV} \tag{7}$$

and

$$mc^2(\text{Au79+}) = 183.433337044 \text{ GeV.} \tag{8}$$

The mass energy equivalent of the deuteron is [2]

$$m_d c^2 = 1875.612859(41) \text{ MeV.} \tag{9}$$

2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{10}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{11}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{12}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = k\,(cp/Q) \tag{13}$$

where $k = 10^9/299792458$ and cp is given in units of GeV. The angular frequency is

$$\omega = 2\pi f. \tag{14}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{15}$$

where γ_t is the transition gamma. Note that as defined here, η is negative below transition and positive above transition.

3 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{16}$$

   where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S\,|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s|^{1/2} \tag{17}$$

   where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{18}$$

   where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{19}$$

   where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{20}$$

5. The area of a bucket is

$$A_{\rm bk} = \alpha(\phi_s)A_S \tag{21}$$

   where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\,d\phi. \tag{22}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The function $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [5]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{23}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s} \left\{ \frac{-h\eta_s e Q V_g \cos\phi_s}{2\pi E_s} \right\}^{1/2} \tag{24}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g \cos\phi_s$, while measurement of dB/dt gives a value for $V_g \sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundaries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{25}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{26}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\} \tag{27}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\}. \tag{28}$$

If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{29}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{30}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \, |\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2} \,. \tag{31}$$

9. The area of a bunch matched to a bucket is

$$A_{\rm b} = F(\phi_s, \Delta\phi)\, A_S \tag{32}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2}\, d\phi. \tag{33}$$

The function $F(\phi_s, \Delta\phi)$ must be evaluated numerically. In **Section 35** it is given in terms of elliptic integrals for the case of a stationary bucket.

If $\Delta\phi$ is small, (33) reduces to

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2\, |\cos\phi_s|^{1/2} \,. \tag{34}$$

4 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{35}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [6, 7]. The radius of curvature ρ in the Booster and AGS main dipoles is given in Refs. [6, 7, 8]. The RHIC ring parameters are taken from Ref. [9] and from MAD runs by Steve Tepikian.

5 Initial Conditions and Assumptions

1. The revolution frequency of the Au32+ ion (from EBIS) at Booster injection is 96.640 kHz. The radius is taken to be the nominal radius $C_b/(2\pi)$.

2. The magnetic rigidity of deuterons (from Tandem) at Booster injection is 0.79 Tm.

3. The revolution frequency of the Au32+ ion at Booster extraction is 658.91 KHz [10]. The radius is taken to be one fourth the nominal AGS radius $C_a/(2\pi)$. The corresponding magnetic rigidity is 9.46202773202 Tm. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is $B\rho = 9.5$ Tm [11].

4. The revolution frequency of the deuteron at Booster extraction is 1044 kHz. The corresponding magnetic rigidity is 6.17834053492 Tm.

5. The set revolution frequency of the Au77+ ion at AGS injection is 163.125 KHz. This gives an energy loss of 2.453 MeV per nucleon in the BTA stripper.

6. The magnetic rigidity of the Au79+ ion at RHIC injection is taken to be 81.11378003 Tm.

7. The magnetic rigidity of the deuteron at RHIC injection is taken to be 65.5299887938 Tm. This gives a deuteron-gold center-of-mass (CM) energy of 19.6631971889 GeV at injection [12].

8. The circumference shift in the Au79+ (yellow) ring at RHIC injection is −2.01505451732 mm [12].

9. The circumference shift in the deuteron (blue) ring at RHIC injection is +2.24571872963 mm. This gives equal revolution frequencies (77842.2731561 Hz) for deuterons and gold ions at injection [12].

10. The circumference shifts in the Au79+ (yellow) ring are −2.01187169836, −2.00524603531, and −1.90877361938 mm, respectively, for deuteron-gold CM energies of 39.0, 62.4, and 200.7 GeV per nucleon. The circumference shifts in the deuteron (blue) ring for these energies are 2.24176368488, 2.23353521142, and 2.11443269604 mm respectively. The corresponding magnetic

rigidities are given in **Section 31.** These give equal revolution frequencies for deuterons and gold ions at each energy [12].

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be set or measured; this is done for computational convenience.

6 Gold Bunch Merging

The desired number of ions per bunch in RHIC is achieved by merging bunches in both Booster and AGS. There are two setups for doing this called here **Setup 1** and **Setup 2**. Setup 1 was developed in 2012 and is documented in [13, 14, 15, 16]. Setup 2 was developed this year (2016) and is documented in [16, 17].

The basic operation performed in **Setup 1** is a 2 to 1 merge in which two adjacent bunches are merged into one. In both Booster and AGS this operation is done twice. For each merge the RF harmonic number is reduced by a factor of two. The final result in both Booster and AGS is that 4 adjacent bunches are merged into one.

In **Setup 2**, two 2 to 1 merges are again done in Booster, but in AGS the 2 to 1 merge is followed by a 3 to 1 merge (in which three adjacent bunches are merged into one). In this case the RF harmonic number is reduced by a factor of two and then a factor of three. The final result in AGS is that 6 adjacent bunches are merged into one.

In both setups Au32+ ions from EBIS are captured in Booster at RF harmonic number $h = 4$ and then accelerated to a merging porch where the 4 bunches are merged into 2 followed by a merge of the 2 into 1. The RF harmonic numbers used are $h = 4$, 2, and 1. This gives a single bunch at Booster extraction.

In **Setup 1**, eight Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 16 buckets on the AGS injection porch. The filling pattern is illustrated in **Section 19**. The 8 bunches are then accelerated to a merging porch where they are merged into 4 followed by a merge of the 4 into 2. This puts 4 Booster loads into each merged bunch. The RF harmonic numbers used are $h = 16$, 8, and 4. Doing the merges on a porch that sits above injection energy helps reduce losses that are believed to be due to the space-charge force acting on the bunched particles [16, 18].

In **Setup 2**, twelve Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 24 buckets on the AGS injection porch. The filling pattern is illustrated in **Section 22**. The 12 bunches are merged into 6 on the injection porch; the 6 are then merged into 2 on the merging porch that sits above injection energy. This puts 6 Booster loads into each merged bunch. The RF harmonic numbers used are $h = 24$, 12, 8, and 4.

Simulations [15, 17] have shown that the growth in longitudinal emittance during the AGS merges can be kept very small. This is consistent with measurements made in 2014 [13]. Measurements made in 2016 [16] show that there is some growth during the 2 to 1 merge on the AGS injection porch, but little growth during the 3 to 1 merge. In Booster, measurements and simulation show growth during the merges. This is due to the limited time available for merging during the Booster magnetic cycle.

7 Deuteron Bunch Merging

In Booster, deuterons (from Tandem) are captured at RF harmonic number $h = 2$ and accelerated to a merging porch where the 2 bunches are merged into 1. The bunch is then accelerated to extraction energy.

In AGS, 8 Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 12 buckets on the AGS injection porch. The filling pattern is illustrated in **Section 20**. The merges take place on the injection porch. The 8 bunches are merged into 4 followed by a merge of the 4 into 2. This puts 4 Booster loads into each merged bunch. The RF harmonic numbers used are $h = 12$, 6, and 3. If just one merge is desired, 6 Booster loads are transferred to AGS with the filling pattern illustrated in **Section 21**.

8 Frequencies for AGS Merges

The revolution frequency, f, required for bunch merges in AGS is dictated by the lowest frequency, F, available from the RF system. This is provided by the L10 cavity, which is presently set up to oscillate at

$$F = 783\ \text{kHz}. \tag{36}$$

The RF harmonic numbers required for **two 2 to 1 merges** are h, $2h$, and $4h$, where h is a positive integer. The RF harmonic numbers required for the squeezing and subsequent acceleration of the merged bunch are h, $2h$, and $3h$. The RF harmonic numbers required for a **3 to 1 merge** are also h, $2h$, and $3h$. The revolution frequency must satisfy

$$hf = F. \tag{37}$$

The RF system therefore must provide frequencies

$$hf = F = \underline{783\ \text{kHz}} \tag{38}$$

$$2hf = 2F = \underline{1566\ \text{kHz}} \tag{39}$$

$$3hf = 3F = \underline{2349\ \text{kHz}} \tag{40}$$

and

$$4hf = 4F = \underline{3132\ \text{kHz}}. \tag{41}$$

The lower frequencies, F and $2F$, are provided by the L10 and KL cavities, respectively. The higher frequencies, $3F$ and $4F$, are provided by the standard AGS RF cavities. For Au77+ ions we take $h = 4$. For deuterons we take $h = 3$.

9 Gold Longitudinal Emittance

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$\mathcal{E} = \frac{2}{A}\Delta E \Delta T \tag{42}$$

where ΔE is the energy half-width of the beam,

$$\Delta T = \frac{1}{f} = \frac{2\pi R}{c\beta} \tag{43}$$

is the revolution period, and A is the number of nucleons. Using the differential relation

$$\Delta E = \beta^2 \frac{\Delta p}{p} mc^2\gamma \tag{44}$$

we have

$$\mathcal{E} = \frac{2\beta^2\gamma}{f} \frac{mc^2}{A} \frac{\Delta p}{p} \tag{45}$$

where Δp is the momentum half-width of the unbunched beam. Taking

$$f = 96.640 \text{ kHz} \tag{46}$$

gives

$$\Delta T = 10.3476821192\ \mu\text{s} \tag{47}$$

$$\beta = 0.0650450626079, \quad \gamma = 1.00212216641 \tag{48}$$

and

$$\frac{2\beta^2\gamma}{f} = 87.7450074295 \text{ ns}. \tag{49}$$

For Au32+ ions we have

$$\frac{mc^2}{A} = 0.931253053269 \text{ GeV} \tag{50}$$

which gives

$$\frac{2\beta^2\gamma}{f} \frac{mc^2}{A} = 81.7128060778 \text{ eV s}. \tag{51}$$

Using this in (45) then gives the longitudinal emittance (per nucleon) for any given fractional momentum half-width.

For fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{52}$$

we have longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.0817128060778 \text{ eV s}. \tag{53}$$

The most recent fractional momentum half-width obtained from measurements [16] is

$$\frac{\Delta p}{p} = 0.00049 \tag{54}$$

which gives longitudinal emittance

$$\mathcal{E} = 0.040 \text{ eV s} \tag{55}$$

for unbunched Au32+ beam circulating in Booster at injection.

A detailed account of the longitudinal emittance evolution in Booster and AGS is given in [13, 14] and [16].

10 Deuteron Longitudinal Emittance

For deuterons at Booster injection we have

$$B\rho = 0.79 \text{ Tm} \tag{56}$$

which gives

$$f = 186.128209518 \text{ kHz} \tag{57}$$

$$\Delta T = 5.37264073291 \ \mu\text{s} \tag{58}$$

$$\beta = 0.125276500840, \quad \gamma = 1.00794069115 \tag{59}$$

and

$$\frac{2\beta^2\gamma}{f} = 169.977721399 \text{ ns}. \tag{60}$$

Note that the value of β for deuterons (from Tandem) is nearly twice as large as that for Au32+ ions (from EBIS).

For deuterons we have

$$\frac{mc^2}{A} = 0.937806429500 \text{ GeV} \tag{61}$$

which gives

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A} = 159.406200000 \text{ eV s}. \tag{62}$$

Comparing with (51) we see that for the same fractional momentum half-width, the longitudinal emittance of unbunched deuteron beam circulating in Booster at injection will be nearly twice that of Au32+ ions.

For a fractional momentum half-width of

$$\frac{\Delta p}{p} = 0.001, \tag{63}$$

equation (45) gives longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.159406200000 \text{ eV s}. \tag{64}$$

11 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into h buckets we must have RF voltage V_g (i.e. total gap voltage per turn) such that

$$\mathcal{E} \leq \frac{hA_S}{A} \tag{65}$$

where A_S is given by (16). Thus we must have

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{cA}\left\{\frac{2eQV_gE}{\pi h|\eta|}\right\}^{1/2} \tag{66}$$

which gives

$$2\beta^2\gamma\left(\frac{2\pi R}{c\beta}\right)\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{c}\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\frac{mc^2}{A}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{67}$$

$$\beta^2\gamma\left(\frac{\pi}{\beta}\right)\frac{\Delta p}{p} \leq 2\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{68}$$

$$\beta^2\gamma^2\pi^2\left(\frac{\Delta p}{p}\right)^2 \leq \left(\frac{8\gamma}{\pi h|\eta|}\right)\left(\frac{Q}{mc^2}\right)eV_g \tag{69}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta|\left(\frac{mc^2}{Q}\right)\left(\frac{\Delta p}{p}\right)^2 \leq eV_g. \tag{70}$$

For Au32+ ions at Booster injection we have

$$h = 4 \tag{71}$$

$$f = 96.640 \text{ kHz} \tag{72}$$

$$\eta = -0.952939329734 \tag{73}$$

$$\frac{1}{8}\, h\pi^3\beta^2\gamma\, |\eta| = 0.0626374709945 \tag{74}$$

and

$$\frac{mc^2}{Q} = 5.73302660918 \text{ GeV}. \tag{75}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{76}$$

then gives

$$359.102287944 \text{ volts} \leq V_g. \tag{77}$$

For <u>deuterons</u> at Booster injection we have

$$h = 2 \tag{78}$$

$$f = 186.128209518 \text{ kHz} \tag{79}$$

$$\eta = -0.941475988241 \tag{80}$$

$$\frac{1}{8}\, h\pi^3\beta^2\gamma\, |\eta| = 0.115444456081 \tag{81}$$

and

$$\frac{mc^2}{Q} = 1.87561285900 \text{ GeV}. \tag{82}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{83}$$

then gives

$$216.529106325 \text{ volts} \leq V_g. \tag{84}$$

12 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma. \tag{85}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the values of β, γ, and mc^2/Q given by (48) and (75), we obtain

$$V_I = 58.396 \text{ kV} \tag{86}$$

for Au32+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 59.740 \text{ kV}. \tag{87}$$

For deuterons from Tandem we obtain

$$V_I = 71.2794 \text{ kV}. \tag{88}$$

The required setpoint is

$$V_I(\text{setpoint}) = 72.76 \text{ kV}. \tag{89}$$

13 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{90}$$

where $B\rho$ is given by (13) and ρ is the nominal radius of curvature. Writing

$$B\rho = \frac{10^9}{c}\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{91}$$

and using the values of β, γ, and mc^2/Q given by (48) and (75), we obtain

$$B\rho = 1.24651715338 \text{ Tm} \tag{92}$$

for Au32+ ions from EBIS. Here we have used the mass energy equivalent mc^2 in units of GeV and the velocity of light in units of m/s. Using

$$\rho = 13.8656 \text{ m} \tag{93}$$

we then obtain

$$B = 898.999793284 \text{ Gauss.} \tag{94}$$

For deuterons from Tandem we have

$$B = 569.755365797 \text{ Gauss.} \tag{95}$$

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock. For Au32+ ions from EBIS the measured field at injection is

$$B(\text{measured}) = 894.0 \text{ Gauss.} \tag{96}$$

14 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is 454.96 Gauss for Au77+ ions. The measured magnetic field is 482.0 gauss [19].

For deuterons the nominal magnetic field is 723.6425 Gauss.

15 BTA Stripper

The stripper [20, 21] used to strip gold ions in the BTA (Booster-To-AGS) transfer line consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 carbon foil. In **Section 34** we use these surface densities to calculate the energy loss of Au77+ ions in the foils.

16 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{97}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [23] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{98}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [23] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For Au77+ ions at injection, the magnetic rigidity is $B\rho = 3.88434088$ Tm. This gives $B = 0.208555$ T and $I = 7750$ A.

For deuterons at injection, the magnetic rigidity is $B\rho = 6.17834053492$ Tm. This gives $B = 0.331723$ T and $I = 12328$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

17 AGS Injection Kicker Current

The current required in the A5 kicker is [22, 23]

$$I = \frac{B\rho}{K} \sin\phi \tag{99}$$

where

$$K = 1.8718 \times 10^{-5} \text{ Tm/A} \tag{100}$$

and

$$\phi = 3.35 \text{ milliradians} \tag{101}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 695.2 A for Au77+ ions and 1106 A for deuterons. The maximum available current is 1100 A.

18 AGS Injection Kicker Short Pulse Waveforms

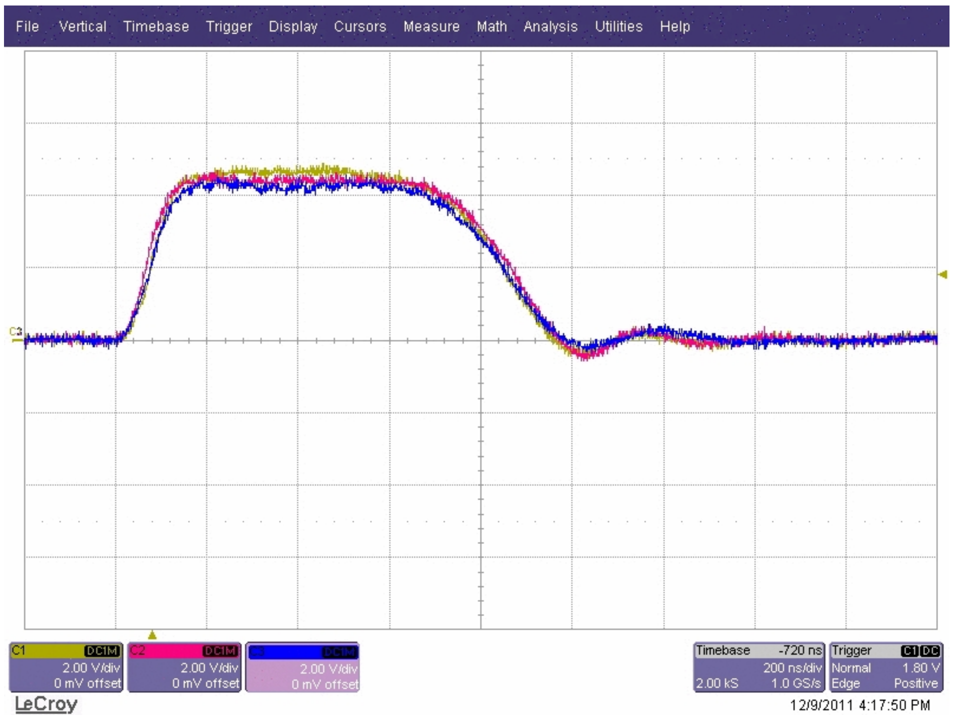

Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions in harmonic 16 buckets. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $B - W$, where B is the bucket width and W the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 283 ns for Au77+ bunches. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns. With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. One workable filling pattern is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This is illustrated in **Section 19.**

19 Kicker timing for 8 transfers of gold ions to AGS followed by two 2 to 1 merges

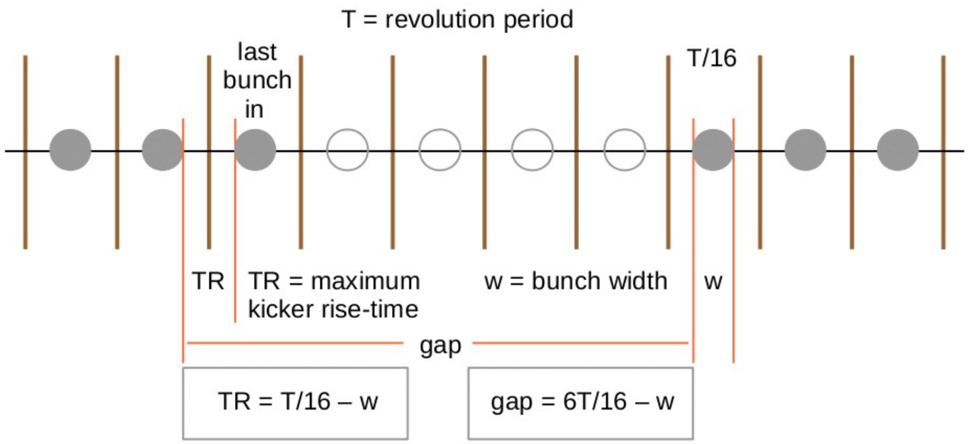

Figure 2: Here T is the revolution period on the AGS injection porch and $T/16 = 383$ ns is the harmonic 16 RF bucket width for Au77+ ions. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/16 - 100 = 283$ ns. The filling pattern in this case is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This allows each group of four adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G = 6T/16 - W = 2298 - W$ ns.

20 Kicker timing for 8 transfers of deuterons to AGS followed by two 2 to 1 merges

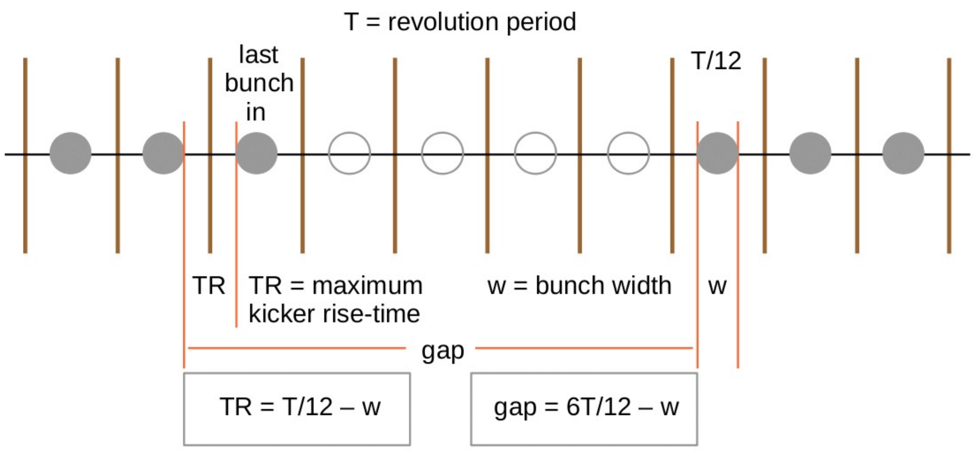

Figure 3: Here T is the revolution period on the AGS injection porch and $T/12 = 319$ ns is the harmonic 12 RF bucket width for deuterons. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/12 - 100 = 219$ ns. The filling pattern is again four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This allows each group of four adjacent bunches to be merged into a single bunch. One ends up with 2 merged bunches, each one sitting in a harmonic 3 bucket. The total gap available for the kicker pulse is $G = 6T/12 - W = 1914 - W$ ns.

21 Kicker timing for 6 transfers of deuterons to AGS followed by a single 2 to 1 merge

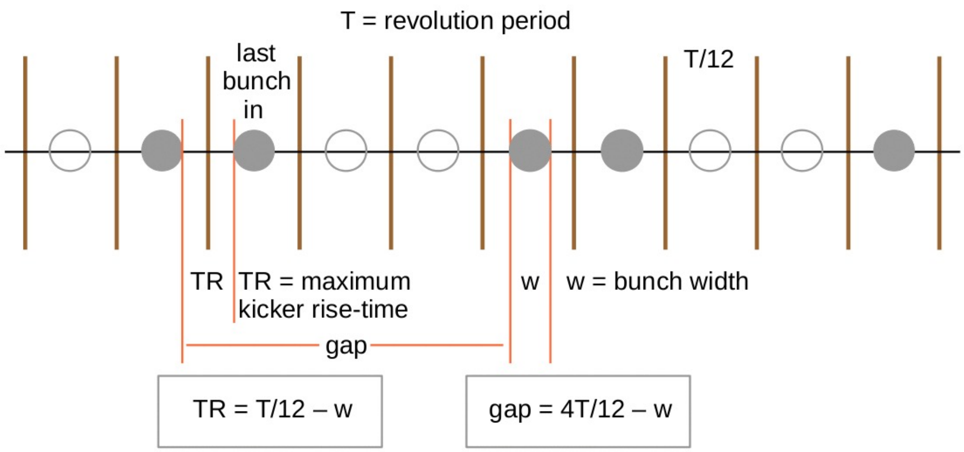

Figure 4: Here, again, T is the revolution period on the AGS injection porch and $T/12 = 319$ ns is the harmonic 12 RF bucket width for deuterons. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/12 - 100 = 219$ ns. The filling pattern in this case is two adjacent filled buckets followed by two adjacent empty buckets, followed by another two adjacent filled buckets, and so on. This allows each group of two adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 6 bucket. The total gap available for the kicker pulse is $G = 4T/12 - W = 1276 - W$ ns.

22 Kicker timing for 12 transfers to AGS followed by a 2 to 1 merge and a 3 to 1 merge

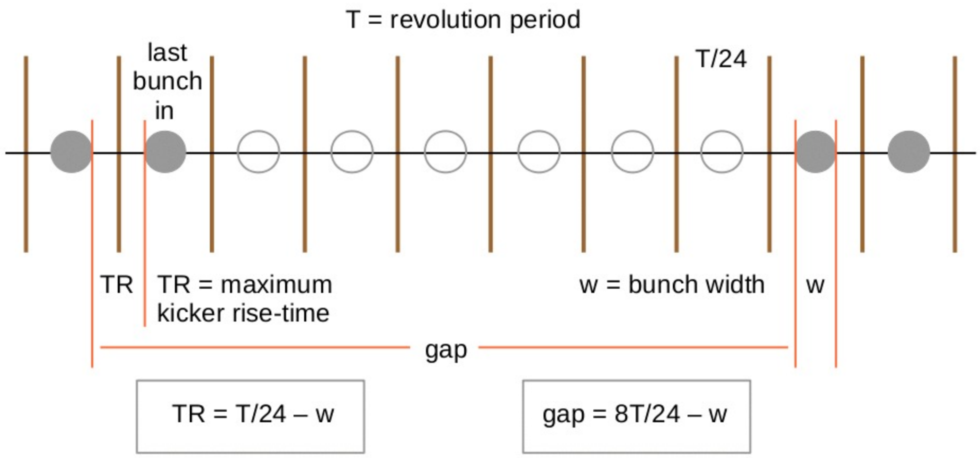

Figure 5: Here $T/24 = 255$ ns is the harmonic 24 RF bucket width for Au77+ ions. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/24 - 100 = 155$ ns. The filling pattern in this case is six adjacent filled buckets followed by six adjacent empty buckets, followed by another six adjacent filled buckets. This allows each group of six adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G = 8T/24 - W = 2040 - W$ ns.

23 Gold in Booster for 8 transfers to AGS

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	49.259795	107.75879	MeV
cp/A	60.701960016	306.87652	460.77475	MeV
E/A	0.9332293272	0.98051285	1.0390118	GeV
$B\rho$	1.24651715338	6.3017214	9.46202773202	Tm
β	0.065045062608	0.31297552	0.44347401	
$\gamma - 1$	0.002122166406	0.052896251	0.11571376	
η	-0.953	-0.859	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	**386.560**	**465.000**	**658.910**	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Ext	Ext	Ext	Unit
V_g	5.730	25.2	25.2	25.2	kV
A_S	16.076	318.54	318.54	318.54	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	50.999	22.866	0	deg
F_s	1.1557	0.8139	0.9849	1.0260	kHz
A_{bk}	16.076	36.294	140.88	318.54	eV s
A_b	1.82	13.987	13.987	13.987	eV s
Δt	635.1	264.6	235.1	229.8	ns
ΔE	1.836	34.11	38.00	38.84	MeV

Parameter	Injection	Extraction	Unit
No. Bunches	4	1	
Bucket Width	2586.92053	1517.65795	ns
Ions/Bunch	1.25/4	1.04	10^9 [16]
Bunch Area	0.037/4 [16]	0.071 [16]	eV s/A

24 Gold in Booster for 12 transfers to AGS

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	72.089750	107.75879	MeV
cp/A	60.701960016	373.44950	460.77475	MeV
E/A	0.9332293272	1.0033428	1.0390118	GeV
$B\rho$	1.24651715338	7.6688003	9.46202773202	Tm
β	0.065045062608	0.37220529	0.44347401	
$\gamma - 1$	0.002122166406	0.07741156	0.11571376	
η	-0.953	-0.8186	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	**386.560**	**553.000**	**658.910**	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Ext	Ext	Ext	Unit
V_g	5.730	25.2	25.2	25.2	kV
A_S	16.076	318.54	318.54	318.54	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	50.999	22.866	0	deg
F_s	1.1557	0.8139	0.9849	1.0260	kHz
$A_{\rm bk}$	16.076	36.294	140.88	318.54	eV s
A_b	1.82	13.987	13.987	13.987	eV s
Δt	635.1	264.6	235.1	229.8	ns
ΔE	1.836	34.11	38.00	38.84	MeV

Parameter	Injection	Extraction	Unit
No. Bunches	4	1	
Bucket Width	2586.92053	1517.65795	ns
Ions/Bunch	1.25/4	1.04	10^9 [16]
Bunch Area	0.037/4 [16]	0.071 [16]	eV s/A

25 Gold in AGS with 4 to 1 Merge on Porch

Parameter	Injection	Porch	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	0.16448553	8.86486804142	GeV
cp/A	0.45515837	0.57738456	9.75165192920	GeV
E/A	1.0364299	1.09562347	9.79600598275	GeV
$B\rho$	3.88434088	4.9274243	83.2210113809	Tm
β	0.43915981	0.52699177	0.995472230864	
γ	1.1130788	1.1766500	10.5204669984	
η	-0.793	-0.708	0.00481	
h	16	4	12	
hf	**2.610000**	**0.783**	**4.43700956990**	MHz
R	128.4526	128.4526	128.457913874	m

26 Gold in AGS with 3 to 1 Merge on Porch

Parameter	Injection	Porch	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	0.16448553	8.86486804142	GeV
cp/A	0.45515837	0.57738456	9.75165192920	GeV
E/A	1.0364299	1.09562347	9.79600598275	GeV
$B\rho$	3.88434088	4.9274243	83.2210113809	Tm
β	0.43915981	0.52699177	0.995472230864	
γ	1.1130788	1.1766500	10.5204669984	
η	-0.793	-0.708	0.00481	
h	24	4	12	
hf	**3.915000**	**0.783**	**4.43700956990**	MHz
R	128.4526	128.4526	128.457913874	m

27 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
mc^2	183.433337044	183.433337044	183.433337044	GeV
W/A	8.86482757134	20.3825164868	98.4580745246	GeV
cp/A	9.75160741084	21.2933011516	99.3848464289	GeV
E/A	9.79596126192	21.3136501774	99.3892082152	GeV
$B\rho$	**81.11378003**	177.117481555	**826.682231135**	Tm
β	0.995472230863	0.999045258528	0.999956114085	
γ	10.5204669974	**22.8900**	106.739997941	
η	−0.00713	0.0	0.00182	
f	77.8422731561	78.1216704928	78.1928941920	kHz
h	360	360	2520	
hf	28.0232183362	28.1238013774	197.046093364	MHz
δC	−2.01505451732	−2.0	−1.90877361938	mm

Here the deuteron-gold CM energies at injection and store are **19.6631971889** and **200.7** GeV per nucleon, respectively.

Parameter	Injection	Store	Unit
h	360	2520	
V_g	405.5	3000	kV
A_S	177.2	163.9	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.204	0.232	kHz
A_{bk}	177.2	163.9	eV s
A_b	147.75	147.75	eV s
A_b	0.75	0.75	eV s/A [24]
Δt	28.0	4.28	ns
ΔE	3678	24604	MeV

28 Deuterons in Booster

Parameter	Injection	Merge porch	Extraction	Unit
W/A	7.44683121259	182.630896514	380.207430204	MeV
cp/A	118.418020910	613.105947054	926.109947663	MeV
E/A	945.253260713	1.12043732601	1318.01385970	MeV
$B\rho$	**0.79000**	4.09020260979	6.17834053492	Tm
β	0.125276500840	0.547202358239	0.702655697316	
$\gamma - 1$	0.00794069114727	0.194742636400	0.405422076714	
η	-0.941	-0.658	-0.463	
ϵ_H (95%)	23.4π	23.4π	23.4π	mm mrad
ϵ_V (95%)	11.0π	11.0π	11.0π	mm mrad
h	2	1	1	
hf	372.256419036	**813.000**	**1044.000**	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (unnormalized) respectively.

Parm	Injection	Ext	Ext	Ext	Unit
V_g	4.000	25	25	25	kV
A_S	0.6851	8.153	8.153	8.153	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	51.57	23.06	0	deg
F_s	1.183	0.980	1.192	1.243	kHz
A_{bk}	0.6851	0.896	3.580	8.153	eV s
A_b	0.090	0.180	0.180	0.180	eV s
Δt	712	117	105	103	ns
ΔE	0.0810	0.987	1.095	1.119	MeV

Parameter	Injection	Extraction	Unit
No. Bunches	2	1	
Bucket Width	2686.3204	957.8544	ns
Ions/Bunch	1.29/2	0.588	10^{11} [25, 26]
Area/Bunch	0.045	0.090	eV s/A

29 Deuterons in AGS

Parameter	Injection	Transition	Extraction	Unit
W/A	0.380207430204	7.03354822125	8.92955800875	GeV
cp/A	0.926109947663	7.91599735149	9.82269820660	GeV
E/A	1.31801385970	7.97135465075	9.86736443825	GeV
$B\rho$	6.17834053492	52.8098498828	**65.5299887938**	Tm
β	0.702655697316	0.993055471537	0.995473337189	
γ	1.40542207671	**8.5000**	10.5217496147	
η	-0.492	0.0	0.00481	
h	12	9	9	
hf	**3.132000**	3.31981553918	3.32775717742	MHz
R	128.4526	128.4526	128.458056636	m

Parameter	Injection	Injection	Extraction	Unit
h	12	3	9	
V_g	50.69	8.0	151.7	kV
A_S	1.084	3.444	79.94	eV s
dB/dt	0	0	0	G/ms
ϕ_s	0	0	180	degrees
F_s	1.580	0.3138	0.08546	kHz
A_{bk}	1.084	3.444	79.94	eV s
A_b	0.207	0.828	0.828	eV s
Δt	103	465.8	22	ns
ΔE	1.294	1.148	23.97	MeV

Parameter	Injection	Injection	Extraction	Unit
h	12	3	9	
Bucket Width	319.285	1277.139	300.503	ns
No. of Bunches	8	2	2	
Ions/Bunch	4.5/8	2.25	2.25	10^{11} [25, 26]
Area/Bunch	0.828/8	0.414	0.414	eV s/A [27]

30 Deuterons in RHIC

Parameter	Injection	Transition	Store	Unit
W/A	8.92955800875	20.5285827418	100.382273503	GeV
cp/A	9.82269820660	21.4458943193	101.315739728	GeV
E/A	9.86736443825	21.4663891713	101.320079933	GeV
$B\rho$	**65.5299887938**	143.071606686	**675.905861034**	Tm
β	0.995473337189	0.999045258528	0.999957163432	
γ	10.5217496147	**22.8900**	108.039438359	
η	-0.00712	0.0	0.00182	
f	77.8422731561	78.1215849101	78.1928941920	kHz
h	360	360	2520	
hf	28.0232183362	28.1237705676	197.046093364	MHz
δC	$+2.24571872963$	$+2.2$	$+2.11443269604$	mm

Here the deuteron-gold CM energies at injection and store are **19.6631971889** and **200.7** GeV per nucleon, respectively.

Parameter	Injection	Store	Unit
h	360	2520	
V_g	188.0	3000	kV
A_S	1.373	1.875	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.1542	0.2573	kHz
$A_{\rm bk}$	1.373	1.875	eV s
A_b	0.82	0.82	eV s
A_b	0.41	0.41	eV s/A [24]
Δt	22	2.58	ns
ΔE	24.9	208	MeV

31 Center-of-Mass Energy for Deuteron-Ion Collisions in RHIC

Let E_1 and P_1 be the energy and momentum of an ion circulating in RHIC, and let E_2 and $-P_2$ be the energy and momentum of the counter-circulating ion. The counter-circulating ion may be identical to the circulating one or it may be some other kind of ion.

The center-of-mass (CM) mass-energy equivalent, Mc^2, is given by the Lorentz invariant

$$M^2c^4 = (E_1 + E_2)^2 - (cP_1 - cP_2)^2 \tag{102}$$

where

$$E_1 = m_1c^2\gamma_1, \quad cP_1 = m_1c^2\beta_1\gamma_1 \tag{103}$$

and

$$E_2 = m_2c^2\gamma_2, \quad cP_2 = m_2c^2\beta_2\gamma_2. \tag{104}$$

Thus we have

$$M^2 = (m_1\gamma_1 + m_2\gamma_2)^2 - (m_1\beta_1\gamma_1 - m_2\beta_2\gamma_2)^2 \tag{105}$$

$$M^2 = m_1^2\left(\gamma_1^2 - \beta_1^2\gamma_1^2\right) + m_2^2\left(\gamma_2^2 - \beta_2^2\gamma_2^2\right) + 2m_1m_2\gamma_1\gamma_2\left(1 + \beta_1\beta_2\right) \tag{106}$$

and

$$M^2 = m_1^2 + m_2^2 + 2m_1m_2\gamma_1\gamma_2\left(1 + \beta_1\beta_2\right). \tag{107}$$

Here we have used the identities

$$\gamma_1^2 - \beta_1^2\gamma_1^2 = 1, \quad \gamma_2^2 - \beta_2^2\gamma_2^2 = 1. \tag{108}$$

For Deuteron-Ion collisions we take

$$\beta_1 = \beta_d, \quad \gamma_1 = \gamma_d \tag{109}$$

$$\beta_2 = \beta_I, \quad \gamma_2 = \gamma_I \tag{110}$$

and

$$m_1 = m_d/A_d, \quad m_2 = m_I/A_I \tag{111}$$

where the subscripts d and I refer to the deuteron and ion respectively. Thus (107) becomes

$$M^2 = \left(\frac{m_d}{A_d}\right)^2 + \left(\frac{m_I}{A_I}\right)^2 + 2\left(\frac{m_d}{A_d}\right)\left(\frac{m_I}{A_I}\right)\gamma_d\gamma_I\left(1 + \beta_d\beta_I\right) \tag{112}$$

where

$$\beta_d \gamma_d = \left\{ \gamma_d^2 - 1 \right\}^{1/2}, \quad \beta_I \gamma_I = \left\{ \gamma_I^2 - 1 \right\}^{1/2}. \tag{113}$$

For the Au79+ ion, the mass number is

$$A_I = 197 \tag{114}$$

and the mass-energy equivalent is

$$m_I c^2 = 183.433337044 \text{ GeV}. \tag{115}$$

For the deuteron, the mass number is

$$A_d = 2 \tag{116}$$

and the mass-energy equivalent is

$$m_d c^2 = 1875.612859(41) \text{ MeV}. \tag{117}$$

Thus, for Au79+ ions with rigidities

81.11378003, **161.398057131**, **258.321126382**, and **826.682231135** Tm,

and for deuterons with corresponding rigidities

65.5299887938, **130.436804843**, **208.923466783**, and **675.905861034** Tm,

the CM energies, Mc^2, are

19.6631971889, **39.0**, **62.4**, and **200.7** GeV per nucleon,

respectively.

32 Gold in RHIC at CM Energies 39 and 62.4 GeV

Parameter	Injection	Energy 39	Energy 62.4	Unit
Mc^2	**19.6631971889**	**39.0**	**62.4**	GeV
W/A	8.86482757134	18.4946859262	30.1385343474	GeV
cp/A	9.75160741084	19.4034908672	31.0557122288	GeV
E/A	9.79596126192	19.4258196167	31.0696680380	GeV
$B\rho$	**81.11378003**	**161.398057131**	**258.321126382**	Tm
β	0.995472230863	0.998850563322	0.999550822071	
γ	10.5204669974	20.8625461771	33.3675693966	
η	-0.00713	-0.000389	0.00101	
f	77.8422731561	78.1064462846	78.1612038123	kHz
h	360	360	360	
hf	28.0232183362	28.1183206624	28.1380333724	MHz
δC	-2.01505451732	-2.01187169836	-2.00524603531	mm

33 Deuterons in RHIC at CM Energies 39 and 62.4 GeV

Parameter	Injection	Energy 39	Energy 62.4	Unit
Mc^2	**20**	**39.0**	**62.4**	GeV
W/A	9.09859906949	18.6366566523	30.3930719054	GeV
cp/A	9.99249490572	19.5519851688	31.3168398204	GeV
E/A	10.0364054990	19.5744630818	31.3308783349	GeV
$B\rho$	**66.66275044**	**130.436804843**	**208.923466783**	Tm
β	0.995624868558	0.998851671543	0.999551927196	
γ	10.7020011628	20.8726049065	33.4086836572	
η	-0.00682	-0.000387	0.00101	
f	77.8541212636	78.1064462846	78.1612038160	kHz
h	360	360	360	
hf	28.0274836549	28.1183206624	28.1380333738	MHz
δC	$+2.2987$	$+2.24176368488$	$+2.23353521142$	mm

34 Au77+ Energy Loss in the BTA Stripper Foils

The stripper used to strip gold ions consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 "glassy" carbon foil [20, 21]. We can estimate the energy loss in the foils as follows:

The kinetic energy of a proton that has the same velocity as the Au77+ ion just upstream of the aluminum foil is

$$W_p = 108.6 \text{ MeV}. \tag{118}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is [28]

$$-\frac{dE_p}{dx} = 5.348 \text{ MeV cm}^2/\text{g}. \tag{119}$$

The rate of energy loss of the Au77+ ion is obtained by scaling the Bethe-Bloch result for protons [29]. Thus

$$-\frac{dE}{dx} = -Z^2 \frac{dE_p}{dx} \text{ cm}^2/\text{g} \tag{120}$$

where $Z = 77$. Multiplying this by the surface density of the aluminum foil (6.45 mg/cm^2) gives

$$\Delta E_a = 1.038 \text{ MeV per nucleon}. \tag{121}$$

This is the energy lost by the Au77+ ion upon passing through the aluminum foil. The kinetic energy of a proton that has the same velocity as the Au77+ ion just downstream of the aluminum foil is then

$$W_p = 107.5 \text{ MeV}. \tag{122}$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [28]

$$-\frac{dE_p}{dx} = 6.180 \text{ MeV cm}^2/\text{g}. \tag{123}$$

Using this result in (120) with $Z = 77$, and multiplying by the surface density of the carbon foil (8.39 mg/cm^2) gives

$$\Delta E_c = 1.561 \text{ MeV per nucleon}. \tag{124}$$

The total energy lost upon passing through both foils is then

$$\Delta E = \Delta E_a + \Delta E_c = 2.599 \text{ MeV per nucleon}. \tag{125}$$

This agrees reasonably well with the value 2.453 MeV per nucleon obtained in **Section 5.**

35 Area of Bunch Matched to Stationary Bucket

For the case of a bunch matched to a stationary bucket, the function (33) becomes

$$F(0,\Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |\cos\phi_L - \cos\phi|^{1/2}\, d\phi \tag{126}$$

where

$$\phi_R = -\phi_L = \Delta\phi/2. \tag{127}$$

Defining

$$\mathcal{F}(Z) = \frac{2\sqrt{2}}{8}\int_0^Z \sqrt{\cos\phi - \cos Z}\, d\phi \tag{128}$$

we have

$$F(0,\Delta\phi) = \mathcal{F}(\Delta\phi/2). \tag{129}$$

The integral in (128) can be expressed in terns of the indefinite integral

$$I(a,b,x) = \int \sqrt{a + b\cos x}\, dx. \tag{130}$$

This is given in Gradsheteyn and Ryzhick [30] as

$$I(a,b,x) = \sqrt{\frac{2}{b}}\left\{(a-b)F\left(\gamma, \frac{1}{r}\right) + 2bE\left(\gamma, \frac{1}{r}\right)\right\} \tag{131}$$

where

$$F(\gamma,k) = \int_0^\gamma \frac{d\alpha}{\sqrt{1 - k^2\sin^2\alpha}} \tag{132}$$

$$E(\gamma,k) = \int_0^\gamma \sqrt{1 - k^2\sin^2\alpha}\, d\alpha \tag{133}$$

$$\gamma = \arcsin\sqrt{\frac{b\,(1-\cos x)}{a+b}}, \quad r = \sqrt{\frac{2b}{a+b}} \tag{134}$$

and

$$0 \le x < \arccos\left(-\frac{a}{b}\right), \quad 0 < |a| \le b. \tag{135}$$

Here $F(\gamma,k)$ and $E(\gamma,k)$ are the elliptic integrals of the first and second kind [31].

Thus, setting

$$a = -\cos Z, \quad b = 1 \tag{136}$$

we have

$$\mathcal{F}(Z) = \frac{\sqrt{2}}{4}\left\{I(-\cos Z, 1, Z) - I(-\cos Z, 1, 0)\right\} \tag{137}$$

and, for $x = Z$ and $x = 0$, we see that $\gamma = \pi/2$ and $\gamma = 0$ respectively. This gives

$$I(-\cos Z, 1, Z) = \sqrt{2}\left\{(-1 - \cos Z)F\left(\frac{\pi}{2}, \frac{1}{r}\right) + 2E\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{138}$$

and

$$I(-\cos Z, 1, 0) = 0 \tag{139}$$

where

$$\frac{1}{r} = \frac{1}{\sqrt{2}}\sqrt{1 - \cos Z}. \tag{140}$$

We therefore have

$$\mathcal{F}(Z) = \frac{1}{2}\left\{2E\left(\frac{\pi}{2}, \frac{1}{r}\right) - (1 + \cos Z)F\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{141}$$

which we can write as

$$\mathcal{F}(Z) = E(X) - \frac{1}{2}(1 + \cos Z)K(X) \tag{142}$$

where

$$E(X) = E\left(\frac{\pi}{2}, X\right) \tag{143}$$

$$K(X) = F\left(\frac{\pi}{2}, X\right) \tag{144}$$

and

$$X = \frac{1}{r} = \frac{1}{\sqrt{2}}\sqrt{1 - \cos Z}. \tag{145}$$

Using (132) and (133) we have

$$K(X) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - X^2 \sin^2 \alpha}} \tag{146}$$

and

$$E(X) = \int_0^{\pi/2} \sqrt{1 - X^2 \sin^2 \alpha}\, d\alpha \tag{147}$$

which are the **complete** elliptic integrals of the first and second kind respectively. These can be evaluated numerically using routines **ellf**, **elle**, **rd**, and **rf** given in Numerical Recipes [32].

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