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C. J. Gardner

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C.J. Gardner

Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

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In this note the nominal parameters for gold ions in Booster, AGS, and RHIC are given for the FY2016 running period.

The parameters for the setup in which there are 8 transfers to AGS (per AGS cycle) are summarized in **Sections 18, 20, and 22.**

The parameters for the setup in which there are 12 transfers to AGS are summarized in **Sections 19, 21, 23, and 24.**

1 Mass

A gold ion with charge eQ has $N = 118$ neutrons, $Z = 79$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the positive elementary charge. The mass number is

$$A = N + Z = 197. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{2}$$

where [1, 2]

$$a = 196.9665687(6) \tag{3}$$

is the relative atomic mass of the neutral gold atom,

$$m_uc^2 = 931.494061(21)\ \text{MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_ec^2 = 0.510998928(11)\ \text{MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral gold atom. This amounts to [3, 4] 0.3324 MeV for the helium-like gold ion ($Q = 77$) and 0.5170 MeV for the fully stripped ion. For $Q = 32$ we have $E_Q = 14.5$ KeV.

Thus the mass energy equivalents for the Au32+, Au77+, and Au79+ ions are

$$mc^2(\text{Au32+}) = 183.456851494 \text{ GeV} \tag{6}$$

$$mc^2(\text{Au77+}) = 183.434174442 \text{ GeV} \tag{7}$$

and

$$mc^2(\text{Au79+}) = 183.433337044 \text{ GeV}. \tag{8}$$

2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{9}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{10}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1 - \beta^2}. \tag{11}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = k\,(cp/Q) \tag{12}$$

where $k = 10^9/299792458$ and cp is given in units of GeV. The angular frequency is

$$\omega = 2\pi f. \tag{13}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{14}$$

where γ_t is the transition gamma. Note that as defined here, η is negative below transition and positive above transition.

3 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{15}$$

   where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S\,|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s|^{1/2} \tag{16}$$

   where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{17}$$

   where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{18}$$

   where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{19}$$

5. The area of a bucket is

$$A_{\text{bk}} = \alpha(\phi_s) A_S \tag{20}$$

   where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\,d\phi. \tag{21}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The function $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [5]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{22}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s} \left\{ \frac{-h\eta_s eQV_g \cos\phi_s}{2\pi E_s} \right\}^{1/2} \tag{23}$$

   and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g \cos\phi_s$, while measurement of dB/dt gives a value for $V_g \sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundaries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{24}$$

   and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{25}$$

   In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\} \tag{26}$$

   and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\}. \tag{27}$$

   If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{28}$$

   and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{29}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \, |\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2} \,. \tag{30}$$

9. The area of a bunch matched to a bucket is

$$A_{\mathrm{b}} = F(\phi_s, \Delta\phi)\, A_S \tag{31}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8} \int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2}\, d\phi. \tag{32}$$

The function $F(\phi_s, \Delta\phi)$ must be evaluated numerically. In **Section 26** it is given in terms of elliptic integrals for the case of a stationary bucket.

If $\Delta\phi$ is small, (32) reduces to

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64} (\Delta\phi)^2 \, |\cos\phi_s|^{1/2} \,. \tag{33}$$

4 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
γ_{tr}	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{34}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [6, 7]. The radius of curvature ρ in the Booster and AGS main dipoles is given in Refs. [6, 7, 8]. The RHIC ring parameters are taken from Ref. [9] and from MAD runs by Steve Tepikian.

5 Initial Conditions and Assumptions

1. The revolution frequency of the Au32+ ion (from EBIS) at Booster injection is 96.640 kHz. The radius is taken to be the nominal radius $C_b/(2\pi)$.
2. The revolution frequency of the Au32+ ion at Booster extraction is 658.91 KHz [10]. The radius is taken to be one fourth the nominal AGS radius $C_a/(2\pi)$. The corresponding magnetic rigidity is 9.46202773202 Tm. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is 9.5 Tm [11].
3. The set revolution frequency of the Au77+ ion at AGS injection is 163.125 KHz. This gives an energy loss of 2.453 MeV per nucleon in the BTA stripper.
4. The magnetic rigidity of the Au79+ ion at RHIC injection is taken to be 81.11378003 Tm.
5. The circumference at RHIC injection is C_r.
6. The circumference at Store is taken to be C_r.
7. The energy of the Au79+ ion at Store is 100 GeV per nucleon.

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be set or measured; this is done for computational convenience.

6 Bunch Merging

The desired number of ions per bunch in RHIC is achieved by merging bunches in both Booster and AGS. There are two setups for doing this called here **Setup 1** and **Setup 2**. Setup 1 was developed in 2012 and is documented in [12, 13, 14, 15]. Setup 2 was developed this year (2016) and is documented in [15, 16].

The basic operation performed in **Setup 1** is a 2 to 1 merge in which two adjacent bunches are merged into one. In both Booster and AGS this operation is done twice. For each merge the RF harmonic number is reduced by a factor of two. The final result in both Booster and AGS is that 4 adjacent bunches are merged into one.

In **Setup 2**, two 2 to 1 merges are again done in Booster, but in AGS the 2 to 1 merge is followed by a 3 to 1 merge (in which three adjacent bunches are merged into one). In this case the RF harmonic number is reduced by a factor of two and then a factor of three. The final result in AGS is that 6 adjacent bunches are merged into one.

In both setups Au32+ ions from EBIS are captured in Booster at RF harmonic number $h = 4$ and then accelerated to a merging porch where the 4 bunches are merged into 2 followed by a merge of the 2 into 1. The RF harmonic numbers used are $h = 4$, 2, and 1. This gives a single bunch at Booster extraction.

In **Setup 1**, eight Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 16 buckets on the AGS injection porch. The filling pattern is illustrated in **Section 18**. The 8 bunches are then accelerated to a merging porch where they are merged into 4 followed by a merge of the 4 into 2. This puts 4 Booster loads into each merged bunch. The RF harmonic numbers used are $h = 16$, 8, and 4. Doing the merges on a porch that sits above injection energy helps reduce losses that are believed to be due to the space-charge force acting on the bunched particles [15, 17].

In **Setup 2**, twelve Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 24 buckets on the AGS injection porch. The filling pattern is illustrated in **Section 19**. The 12 bunches are merged into 6 on the injection porch; the 6 are then merged into 2 on the merging porch that sits above injection energy. This puts 6 Booster loads into each merged bunch. The RF harmonic numbers used are $h = 24$, 12, 8, and 4.

Simulations [14, 16] have shown that the growth in longitudinal emittance during the AGS merges can be kept very small. This is consistent with measurements made in 2014 [12]. Measurements made in 2016 [15] show that there is some growth during the 2 to 1 merge on the AGS injection porch, but little growth during the 3 to 1 merge. In Booster, measurements and simulation show growth during the merges. This is due to the limited time available for merging during the Booster magnetic cycle.

7 Frequencies for AGS Merges

The revolution frequency, f, required for bunch merges in AGS is dictated by the lowest frequency, F, available from the RF system. This is provided by the L10 cavity, which is presently set up to oscillate at

$$F = 783\ \text{kHz}. \tag{35}$$

The RF harmonic numbers required for **two 2 to 1 merges** are h, $2h$, and $4h$, where h is a positive integer. The RF harmonic numbers required for the squeezing and subsequent acceleration of the merged bunch are h, $2h$, and $3h$. The RF harmonic numbers required for a **3 to 1 merge** are also h, $2h$, and $3h$. The revolution frequency must satisfy

$$hf = F. \tag{36}$$

The RF system therefore must provide frequencies

$$hf = F = \underline{783\ \text{kHz}} \tag{37}$$

$$2hf = 2F = \underline{1566\ \text{kHz}} \tag{38}$$

$$3hf = 3F = \underline{2349\ \text{kHz}} \tag{39}$$

and

$$4hf = 4F = \underline{3132\ \text{kHz}}. \tag{40}$$

The lower frequencies, F and $2F$, are provided by the L10 and KL cavities, respectively. The higher frequencies, $3F$ and $4F$, are provided by the standard AGS RF cavities. For Au77+ ions we take $h = 4$.

8 Longitudinal Emittance

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$\mathcal{E} = \frac{2}{A}\Delta E \Delta T \tag{41}$$

where ΔE is the energy half-width of the beam,

$$\Delta T = \frac{1}{f} = \frac{2\pi R}{c\beta} \tag{42}$$

is the revolution period, and A is the number of nucleons. Using the differential relation

$$\Delta E = \beta^2 \frac{\Delta p}{p} mc^2\gamma \tag{43}$$

we have

$$\mathcal{E} = \frac{2\beta^2\gamma}{f} \frac{mc^2}{A} \frac{\Delta p}{p} \tag{44}$$

where Δp is the momentum half-width of the unbunched beam. Taking

$$f = 96.640 \text{ kHz} \tag{45}$$

gives

$$\Delta T = 10.3476821192 \ \mu\text{s} \tag{46}$$

$$\beta = 0.0650450626079, \quad \gamma = 1.00212216641 \tag{47}$$

and

$$\frac{2\beta^2\gamma}{f} = 87.7450074295 \text{ ns}. \tag{48}$$

For Au32+ ions we have

$$\frac{mc^2}{A} = 0.931253053269 \text{ GeV} \tag{49}$$

which gives

$$\frac{2\beta^2\gamma}{f} \frac{mc^2}{A} = 81.7128060778 \text{ eV s}. \tag{50}$$

Using this in (44) then gives the longitudinal emittance (per nucleon) for any given fractional momentum half-width.

For fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{51}$$

we have longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.0817128060778 \text{ eV s}. \tag{52}$$

The most recent fractional momentum half-width obtained from measurements [15] is

$$\frac{\Delta p}{p} = 0.00049 \tag{53}$$

which gives longitudinal emittance

$$\mathcal{E} = 0.040 \text{ eV s} \tag{54}$$

for unbunched Au32+ beam circulating in Booster at injection.

A detailed account of the longitudinal emittance evolution in Booster and AGS is given in [12, 13] and [15].

9 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into h buckets we must have RF voltage V_g (i.e. total gap voltage per turn) such that

$$\mathcal{E} \le \frac{hA_S}{A} \tag{55}$$

where A_S is given by (15). Thus we must have

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \le \frac{8R}{cA}\left\{\frac{2eQV_gE}{\pi h|\eta|}\right\}^{1/2} \tag{56}$$

which gives

$$2\beta^2\gamma\left(\frac{2\pi R}{c\beta}\right)\frac{mc^2}{A}\,\frac{\Delta p}{p} \le \frac{8R}{c}\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\frac{mc^2}{A}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{57}$$

$$\beta^2\gamma\left(\frac{\pi}{\beta}\right)\frac{\Delta p}{p} \le 2\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{58}$$

$$\beta^2\gamma^2\pi^2\left(\frac{\Delta p}{p}\right)^2 \le \left(\frac{8\gamma}{\pi h|\eta|}\right)\left(\frac{Q}{mc^2}\right)eV_g \tag{59}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta|\left(\frac{mc^2}{Q}\right)\left(\frac{\Delta p}{p}\right)^2 \le eV_g. \tag{60}$$

Here

$$h = 4 \tag{61}$$

and taking revolution frequency

$$f = 96.640 \text{ kHz} \tag{62}$$

we have

$$\eta = -0.952939329734 \tag{63}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta| = 0.0626374709945. \tag{64}$$

For the Au32+ ion we have mass energy equivalent per unit charge

$$\frac{mc^2}{Q} = 5.73302660918 \text{ GeV}. \tag{65}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{66}$$

then gives

$$359.102287944 \text{ volts} \leq V_g. \tag{67}$$

Taking

$$\frac{\Delta p}{p} = 0.00049 \tag{68}$$

gives

$$86.2204593353 \text{ volts} \leq V_g. \tag{69}$$

10 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma. \tag{70}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the values of β, γ, and mc^2/Q given by (47) and (65), we obtain

$$V_I = 58.396 \text{ kV} \tag{71}$$

for Au32+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 59.740 \text{ kV}. \tag{72}$$

11 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{73}$$

where $B\rho$ is given by (12) and ρ is the nominal radius of curvature. Writing

$$B\rho = \frac{10^9}{c}\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{74}$$

and using the values of β, γ, and mc^2/Q given by (47) and (65), we obtain

$$B\rho = 1.24651715338 \text{ Tm.} \tag{75}$$

Here we have used the mass energy equivalent mc^2 in units of GeV and the velocity of light in units of m/s. Using

$$\rho = 13.8656 \text{ m} \tag{76}$$

we then obtain

$$B = 898.999793284 \text{ Gauss} \tag{77}$$

for Au32+ ions from EBIS.

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock. For Au32+ ions from EBIS the measured field at injection is

$$B(\text{measured}) = 894.0 \text{ Gauss.} \tag{78}$$

12 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B = 454.96$ Gauss for the Au77+ ions. The measured magnetic field is 482.0 gauss [18].

13 BTA Stripper

The stripper [19, 20] used to strip gold ions in the BTA (Booster-To-AGS) transfer line consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 carbon foil. In **Section 25** we use these surface densities to calculate the energy loss of Au77+ ions in the foils.

14 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{79}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [22] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{80}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [22] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For Au77+ ions at injection, the magnetic rigidity is $B\rho = 3.88434088$ Tm. This gives $B = 0.208555$ T and $I = 7750$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

15 AGS Injection Kicker Current

The current required in the A5 kicker is [21, 22]

$$I = \frac{B\rho}{K} \sin\phi \tag{81}$$

where

$$K = 1.8718 \times 10^{-5} \text{ Tm/A} \tag{82}$$

and

$$\phi = 3.35 \text{ milliradians} \tag{83}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 695.2 A for Au77+ ions. The maximum available current is 1100 A.

16 AGS Injection Kicker Short Pulse Waveforms

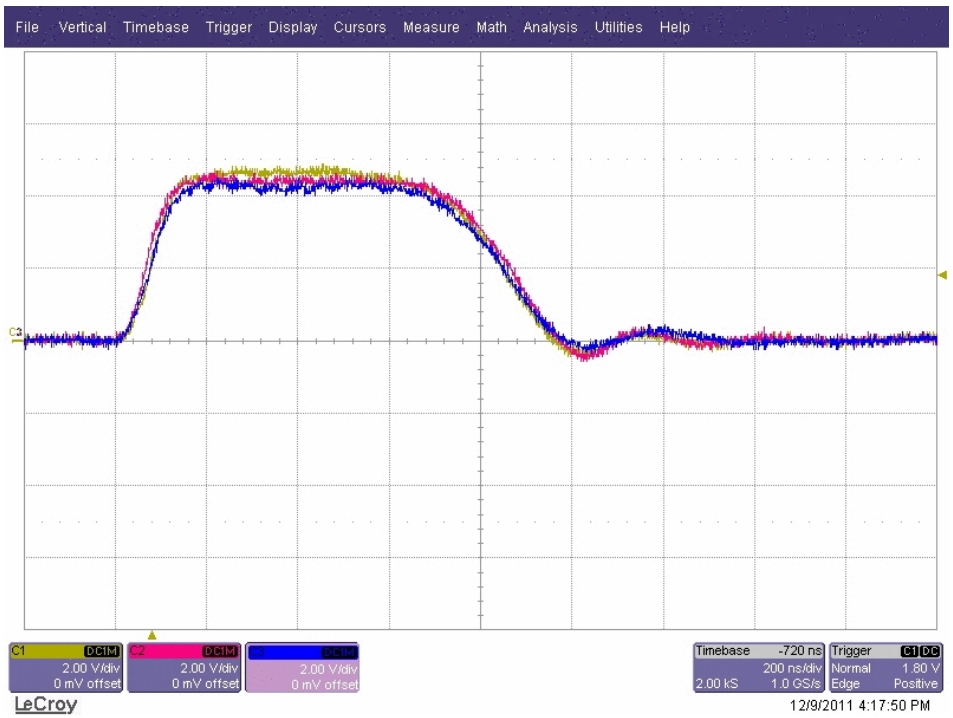

Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions in harmonic 16 buckets. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $B - W$, where B is the bucket width and W the bunch width. The rise time is approximately 100 ns, which implies that the bunch width must be less than or equal to 283 ns for Au77+ bunches. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns. With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. One workable filling pattern is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This is illustrated in **Section 18.**

17 AGS Injection Kicker Long Pulse Waveforms

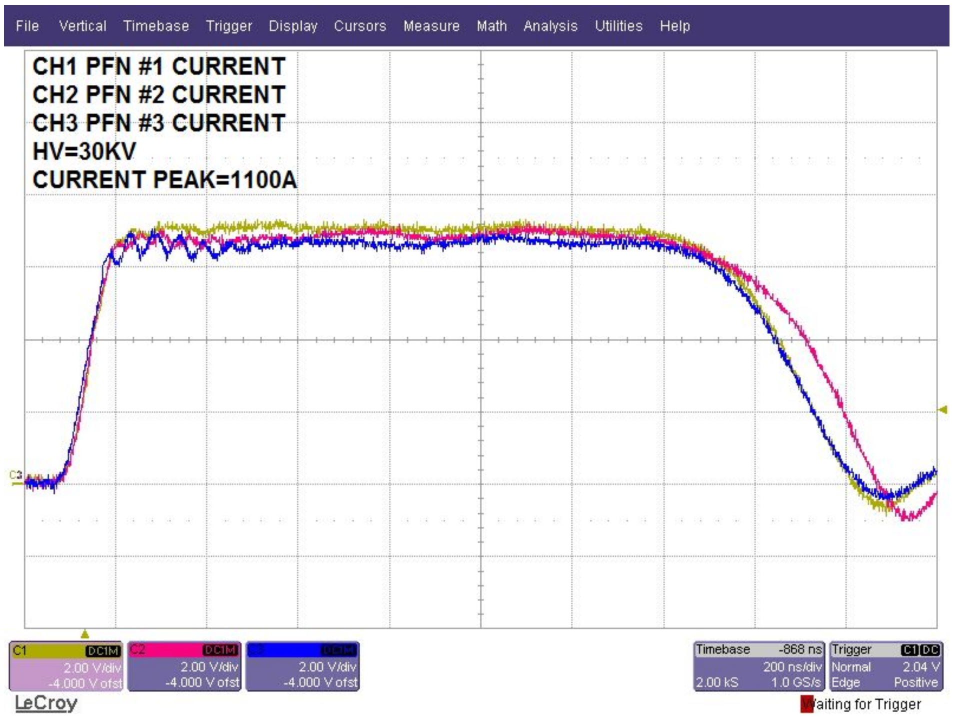

Figure 2: AGS injection kicker waveforms in the long pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan in Oct 2010. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions in harmonic 16 buckets. Here the flattop portion of the pulse is some 1300 ns long. The total pulse width is approximately 2000 ns. In principle this kicker pulse could be used to fill 8 buckets with a filling pattern of four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This is illustrated in **Section 18.**

18 Kicker timing for 8 transfers to AGS followed by two 2 to 1 merges

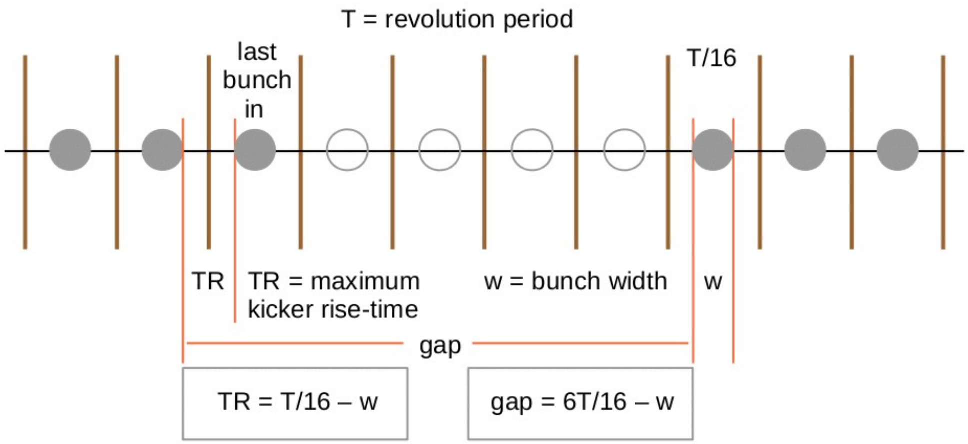

Figure 3: Here T is the revolution period on the AGS injection porch and $T/16 = 383$ ns is the harmonic 16 RF bucket width. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/16-100 = 283$ ns. The filling pattern in this case is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This allows each group of four adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G = 6T/16 - W = 2298 - W$ ns.

19 Kicker timing for 12 transfers to AGS followed by a 2 to 1 merge and a 3 to 1 merge

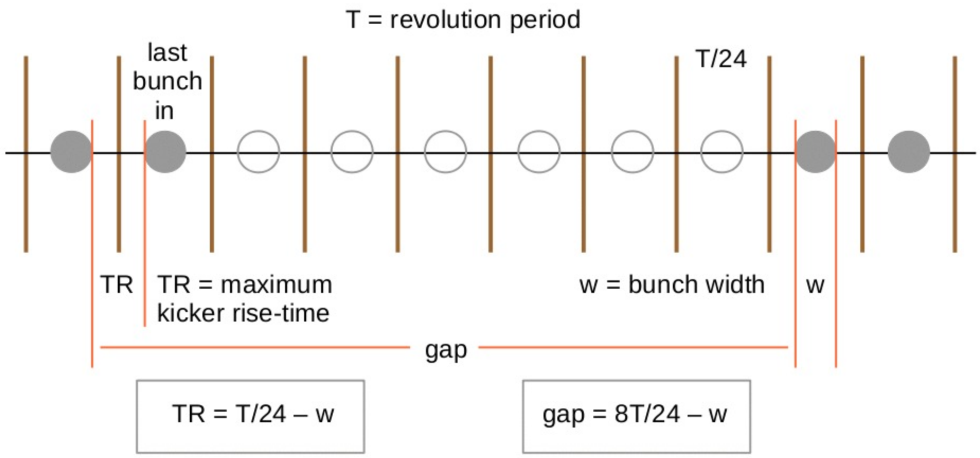

Figure 4: Here $T/24 = 255$ ns is the harmonic 24 RF bucket width. The kicker rise time is 100 ns. This means that the bunch width, W, must be less than $T/24 - 100 = 155$ ns. The filling pattern in this case is six adjacent filled buckets followed by six adjacent empty buckets, followed by another six adjacent filled buckets. This allows each group of six adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G = 8T/24 - W = 2040 - W$ ns.

20 Gold in Booster for 8 transfers to AGS

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	49.259795	107.75879	MeV
cp/A	60.701960016	306.87652	460.77475	MeV
E/A	0.9332293272	0.98051285	1.0390118	GeV
$B\rho$	1.24651715338	6.3017214	9.46202773202	Tm
β	0.065045062608	0.31297552	0.44347401	
$\gamma - 1$	0.002122166406	0.052896251	0.11571376	
η	-0.953	-0.859	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	**386.560**	**465.000**	**658.910**	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Ext	Ext	Ext	Unit
V_g	5.730	25.2	25.2	25.2	kV
A_S	16.076	318.54	318.54	318.54	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	50.999	22.866	0	deg
F_s	1.1557	0.8139	0.9849	1.0260	kHz
$A_{\rm bk}$	16.076	36.294	140.88	318.54	eV s
A_b	1.82	13.987	13.987	13.987	eV s
Δt	635.1	264.6	235.1	229.8	ns
ΔE	1.836	34.11	38.00	38.84	MeV

Parameter	Injection	Extraction	Unit
No. Bunches	4	1	
Bucket Width	2586.92053	1517.65795	ns
Ions/Bunch	1.25/4	1.04	10^9 [15]
Bunch Area	0.037/4 [15]	0.071 [15]	eV s/A

21 Gold in Booster for 12 transfers to AGS

Parameter	Injection	Merge porch	Extraction	Unit
Q	32	32	32	
mc^2	183.456851	183.456851	183.456851	GeV
W/A	1.9762739452	72.089750	107.75879	MeV
cp/A	60.701960016	373.44950	460.77475	MeV
E/A	0.9332293272	1.0033428	1.0390118	GeV
$B\rho$	1.24651715338	7.6688003	9.46202773202	Tm
β	0.065045062608	0.37220529	0.44347401	
$\gamma - 1$	0.002122166406	0.07741156	0.11571376	
η	-0.953	-0.8186	-0.7605	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	**386.560**	**553.000**	**658.910**	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parm	Injection	Ext	Ext	Ext	Unit
V_g	5.730	25.2	25.2	25.2	kV
A_S	16.076	318.54	318.54	318.54	eV s
dB/dt	0	**70**	**35**	**0**	G/ms
ϕ_s	0	50.999	22.866	0	deg
F_s	1.1557	0.8139	0.9849	1.0260	kHz
$A_{\rm bk}$	16.076	36.294	140.88	318.54	eV s
A_b	1.82	13.987	13.987	13.987	eV s
Δt	635.1	264.6	235.1	229.8	ns
ΔE	1.836	34.11	38.00	38.84	MeV

Parameter	Injection	Extraction	Unit
No. Bunches	4	1	
Bucket Width	2586.92053	1517.65795	ns
Ions/Bunch	1.25/4	1.04	10^9 [15]
Bunch Area	0.037/4 [15]	0.071 [15]	eV s/A

22 Gold in AGS with 4 to 1 Merge on Porch

Parameter	Injection	Porch	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	0.16448553	8.86486804031	GeV
cp/A	0.45515837	0.57738456	9.75165192809	GeV
E/A	1.0364299	1.09562347	9.79600598164	GeV
$B\rho$	3.88434088	4.9274243	83.2210113714	Tm
β	0.43915981	0.52699177	0.995472230863	
γ	1.1130788	1.1766500	10.5204669972	
η	-0.793	-0.708	0.00481	
$\beta\gamma$	0.48881949	0.62008488	10.472833	
$\beta\gamma^2$	0.54409463	0.72962288	110.17909	
h	16	4	12	
hf	**2.610000**	**0.783**	**4.43700723782**	MHz
R	128.4526	128.4526	128.457981391	m

Parameter	Inj	Porch	Porch	Ext	Unit
h	16	16	4	12	
V_g	40.0	20.0	15.0	192.0	kV
A_S	38.0	29.27	202.8	5070	eV s
dB/dt	0	0	0	0	G/ms
ϕ_s	0	0	0	180	degrees
F_s	2.050	1.333	0.577	0.0985	kHz
$A_{\rm bk}$	38.0	29.27	202.8	5070	eV s
A_b	19.7	19.7	84.316	118.2	eV s
Δt	216	213.1	631.7	24.8	ns
ΔE	60.4	62.4	87.4	3038	MeV

Parameter	Inj	Porch	Porch	Ext	Unit
h	16	16	4	12	
Bucket Width	383.142	319.285	1277.139	225.377	ns
No. of Bunches	8	8	2	2	
Ions/Bunch	0.56	0.56	2.24	2.14 [15]	10^9
Bunch Area	0.10	0.10	0.428	0.60 [15]	eV s/A

23 Gold in AGS with 3 to 1 Merge on Porch

Parameter	Injection	Porch	Extraction	Unit
Q	77	77	77	
mc^2	183.434174	183.434174	183.434174	GeV
W/A	0.10529199	0.16448553	8.86486804031	GeV
cp/A	0.45515837	0.57738456	9.75165192809	GeV
E/A	1.0364299	1.09562347	9.79600598164	GeV
$B\rho$	3.88434088	4.9274243	83.2210113714	Tm
β	0.43915981	0.52699177	0.995472230863	
γ	1.1130788	1.1766500	10.5204669972	
η	-0.793	-0.708	0.00481	
$\beta\gamma$	0.48881949	0.62008488	10.472833	
$\beta\gamma^2$	0.54409463	0.72962288	110.17909	
h	24	4	12	
hf	**3.915000**	**0.783**	**4.43700723782**	MHz
R	128.4526	128.4526	128.457981391	m

Parameter	Inj	Porch	Porch	Ext	Unit
h	24	12	4	12	
V_g	119.8	100	15.0	185.2	kV
A_S	35.84	100.8	202.8	4979	eV s
dB/dt	0	0	0	0	G/ms
ϕ_s	0	0	0	180	degrees
F_s	4.346	2.581	0.577	0.0967	kHz
A_{bk}	35.84	100.8	202.8	4979	eV s
A_b	17.73	39.4	118.2	147.75	eV s
Δt	140	203	775	28.0	ns
ΔE	83.6	127	102	3365	MeV

Parameter	Inj	Porch	Porch	Ext	Unit
h	24	12	4	12	
Bucket Width	255.428	425.713	1277.139	225.377	ns
No. of Bunches	12	6	2	2	
Ions/Bunch	0.53	1.06	3.18	3.0	10^9 [15]
Bunch Area	0.09	0.20	0.60	0.75 [15]	eV s/A

24 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
mc^2	183.433337	183.433337	183.433337	GeV
W/A	8.86482757134	20.3825164868	99.0688663094	GeV
cp/A	9.75160741084	21.2933011516	99.9956648563	GeV
E/A	9.79596126192	21.3136501774	**100.000**	GeV
$B\rho$	**81.11378003**	177.117481555	831.763013151	Tm
β	0.995472230863	0.999045258528	0.999956648563	
γ	10.5204669974	**22.8900**	107.395963664	
η	-0.00713	0.0	0.00182	
f	77.8422322425	78.1216297391	78.1928970559	kHz
h	360	360	2520	
hf	28.0232036073	28.1237867061	197.046100581	MHz
δC	0	0	0	mm

Parameter	Injection	Store	Unit
h	360	2520	
V_g	405.5	3000	kV
A_S	177.2	164.4	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.204	0.232	kHz
A_{bk}	177.2	164.4	eV s
A_b	147.75	147.75	eV s
A_b	0.75	0.75	eV s/A [23]
Δt	28.0	4.27	ns
ΔE	3678	24649	MeV

25 Au77+ Energy Loss in the BTA Stripper Foils

The stripper used to strip gold ions consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 "glassy" carbon foil [19, 20]. We can estimate the energy loss in the foils as follows:

The kinetic energy of a proton that has the same velocity as the Au77+ ion just upstream of the aluminum foil is

$$W_p = 108.6 \text{ MeV}. \tag{84}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is [24]

$$-\frac{dE_p}{dx} = 5.348 \text{ MeV cm}^2/\text{g}. \tag{85}$$

The rate of energy loss of the Au77+ ion is obtained by scaling the Bethe-Bloch result for protons [25]. Thus

$$-\frac{dE}{dx} = -Z^2 \frac{dE_p}{dx} \text{ cm}^2/\text{g} \tag{86}$$

where $Z = 77$. Multiplying this by the surface density of the aluminum foil (6.45 mg/cm^2) gives

$$\Delta E_a = 1.038 \text{ MeV per nucleon}. \tag{87}$$

This is the energy lost by the Au77+ ion upon passing through the aluminum foil. The kinetic energy of a proton that has the same velocity as the Au77+ ion just downstream of the aluminum foil is then

$$W_p = 107.5 \text{ MeV}. \tag{88}$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [24]

$$-\frac{dE_p}{dx} = 6.180 \text{ MeV cm}^2/\text{g}. \tag{89}$$

Using this result in (86) with $Z = 77$, and multiplying by the surface density of the carbon foil (8.39 mg/cm^2) gives

$$\Delta E_c = 1.561 \text{ MeV per nucleon}. \tag{90}$$

The total energy lost upon passing through both foils is then

$$\Delta E = \Delta E_a + \Delta E_c = 2.599 \text{ MeV per nucleon}. \tag{91}$$

This agrees reasonably well with the value 2.453 MeV per nucleon obtained in **Section 5.**

26 Area of Bunch Matched to Stationary Bucket

For the case of a bunch matched to a stationary bucket, the function (32) becomes

$$F(0,\Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |\cos\phi_L - \cos\phi|^{1/2}\,d\phi \tag{92}$$

where

$$\phi_R = -\phi_L = \Delta\phi/2. \tag{93}$$

Defining

$$\mathcal{F}(Z) = \frac{2\sqrt{2}}{8}\int_0^Z \sqrt{\cos\phi - \cos Z}\,d\phi \tag{94}$$

we have

$$F(0,\Delta\phi) = \mathcal{F}(\Delta\phi/2). \tag{95}$$

The integral in (94) can be expressed in terns of the indefinite integral

$$I(a,b,x) = \int \sqrt{a + b\cos x}\,dx. \tag{96}$$

This is given in Gradsheteyn and Ryzhick [26] as

$$I(a,b,x) = \sqrt{\frac{2}{b}}\left\{(a-b)F\left(\gamma,\frac{1}{r}\right) + 2bE\left(\gamma,\frac{1}{r}\right)\right\} \tag{97}$$

where

$$F(\gamma,k) = \int_0^\gamma \frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}} \tag{98}$$

$$E(\gamma,k) = \int_0^\gamma \sqrt{1-k^2\sin^2\alpha}\,d\alpha \tag{99}$$

$$\gamma = \arcsin\sqrt{\frac{b\,(1-\cos x)}{a+b}}, \quad r = \sqrt{\frac{2b}{a+b}} \tag{100}$$

and

$$0 \le x < \arccos\left(-\frac{a}{b}\right), \quad 0 < |a| \le b. \tag{101}$$

Here $F(\gamma,k)$ and $E(\gamma,k)$ are the elliptic integrals of the first and second kind [27].

Thus, setting

$$a = -\cos Z, \quad b = 1 \tag{102}$$

we have

$$\mathcal{F}(Z) = \frac{\sqrt{2}}{4}\left\{I(-\cos Z, 1, Z) - I(-\cos Z, 1, 0)\right\} \tag{103}$$

and, for $x = Z$ and $x = 0$, we see that $\gamma = \pi/2$ and $\gamma = 0$ respectively. This gives

$$I(-\cos Z, 1, Z) = \sqrt{2}\left\{(-1-\cos Z)F\left(\frac{\pi}{2}, \frac{1}{r}\right) + 2E\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{104}$$

and

$$I(-\cos Z, 1, 0) = 0 \tag{105}$$

where

$$\frac{1}{r} = \frac{1}{\sqrt{2}}\sqrt{1-\cos Z}. \tag{106}$$

We therefore have

$$\mathcal{F}(Z) = \frac{1}{2}\left\{2E\left(\frac{\pi}{2}, \frac{1}{r}\right) - (1+\cos Z)F\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{107}$$

which we can write as

$$\mathcal{F}(Z) = E(X) - \frac{1}{2}(1+\cos Z)K(X) \tag{108}$$

where

$$E(X) = E\left(\frac{\pi}{2}, X\right) \tag{109}$$

$$K(X) = F\left(\frac{\pi}{2}, X\right) \tag{110}$$

and

$$X = \frac{1}{r} = \frac{1}{\sqrt{2}}\sqrt{1-\cos Z}. \tag{111}$$

Using (98) and (99) we have

$$K(X) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - X^2\sin^2\alpha}} \tag{112}$$

and

$$E(X) = \int_0^{\pi/2} \sqrt{1 - X^2\sin^2\alpha}\, d\alpha \tag{113}$$

which are the **complete** elliptic integrals of the first and second kind respectively. These can be evaluated numerically using routines **ellf**, **elle**, **rd**, and **rf** given in Numerical Recipes [28].

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