# FY2016 Parameters for gold ions in Booster, AGS, and RHIC 

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# FY2016 Parameters for Gold Ions in Booster, AGS, and RHIC 

C.J. Gardner

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In this note the nominal parameters for gold ions in Booster, AGS, and RHIC are given for the FY2016 running period.
The parameters for the setup in which there are 8 transfers to AGS (per AGS cycle) are summarized in Sections 18, 20, and 22.
The parameters for the setup in which there are 12 transfers to AGS are summarized in Sections 19, 21, 23, and 24.

## 1 Mass

A gold ion with charge $e Q$ has $N=118$ neutrons, $Z=79$ protons, and $(Z-Q)$ electrons. Here $Q$ is an integer and $e$ is the positive elementary charge. The mass number is

$$
\begin{equation*}
A=N+Z=197 . \tag{1}
\end{equation*}
$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$
\begin{equation*}
m c^{2}=a m_{u} c^{2}-Q m_{e} c^{2}+E_{Q} \tag{2}
\end{equation*}
$$

where $[1,2]$

$$
\begin{equation*}
a=196.9665687(6) \tag{3}
\end{equation*}
$$

is the relative atomic mass of the neutral gold atom,

$$
\begin{equation*}
m_{u} c^{2}=931.494061(21) \mathrm{MeV} \tag{4}
\end{equation*}
$$

is the mass energy equivalent of the atomic mass constant, and

$$
\begin{equation*}
m_{e} c^{2}=0.510998928(11) \mathrm{MeV} \tag{5}
\end{equation*}
$$

is the electron mass energy equivalent. The binding energy $E_{Q}$ is the energy required to remove $Q$ electrons from the neutral gold atom. This amounts to $[3,4] 0.3324 \mathrm{MeV}$ for the helium-like gold ion $(Q=77)$ and 0.5170 MeV for the fully stripped ion. For $Q=32$ we have $E_{Q}=14.5 \mathrm{KeV}$. Thus the mass energy equivalents for the Au32+, Au77+, and Au79+ ions are

$$
\begin{align*}
& m c^{2}(\mathrm{Au} 32+)=183.456851494 \mathrm{GeV}  \tag{6}\\
& m c^{2}(\mathrm{Au} 77+)=183.434174442 \mathrm{GeV} \tag{7}
\end{align*}
$$

and

$$
\begin{equation*}
m c^{2}(\mathrm{Au} 79+)=183.433337044 \mathrm{GeV} \tag{8}
\end{equation*}
$$

## 2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference $C$ with revolution frequency $f$. The radius of the orbit is defined to be $R=C /(2 \pi)$. The velocity of the ion is then

$$
\begin{equation*}
v=2 \pi R f . \tag{9}
\end{equation*}
$$

This gives momentum, energy, and kinetic energy

$$
\begin{equation*}
p=m c \beta \gamma, \quad E=m c^{2} \gamma, \quad W=m c^{2}(\gamma-1) \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta=v / c, \quad \gamma=1 / \sqrt{1-\beta^{2}} \tag{11}
\end{equation*}
$$

The magnetic rigidity of the ion in units of Tm is

$$
\begin{equation*}
B \rho=k(c p / Q) \tag{12}
\end{equation*}
$$

where $k=10^{9} / 299792458$ and $c p$ is given in units of GeV . The angular frequency is

$$
\begin{equation*}
\omega=2 \pi f . \tag{13}
\end{equation*}
$$

We also define the phase-slip factor

$$
\begin{equation*}
\eta=\frac{1}{\gamma_{t}^{2}}-\frac{1}{\gamma^{2}} \tag{14}
\end{equation*}
$$

where $\gamma_{t}$ is the transition gamma. Note that as defined here, $\eta$ is negative below transition and positive above transition.

## 3 RF Parameters

1. The stationary bucket area is

$$
\begin{equation*}
A_{S}=8 \frac{R_{s}}{h c}\left\{\frac{2 e Q V_{g} E_{s}}{\pi h\left|\eta_{s}\right|}\right\}^{1 / 2} \tag{15}
\end{equation*}
$$

where $h$ is the RF harmonic number, $V_{g}$ is the total RF gap voltage per turn, and the subscript " $s$ " denotes parameter values for the synchronous particle.
2. The half-height of a bucket is

$$
\begin{equation*}
\Delta E=\left(\frac{h \omega_{s}}{8 \sqrt{2}}\right) A_{S}\left|\left(\pi-2 \phi_{s}\right) \sin \phi_{s}-2 \cos \phi_{s}\right|^{1 / 2} \tag{16}
\end{equation*}
$$

where $\phi_{s}$ is the synchronous phase.
3. The synchronous phase is given by

$$
\begin{equation*}
V_{g} \sin \phi_{s}=2 \pi R_{s} \rho_{s} \dot{B} / c \tag{17}
\end{equation*}
$$

where $\rho_{s}$ is the radius of curvature, $B$ is the magnetic field and $\dot{B}=d B / d t$. Employing Gaussian units ( $R_{s}$ and $\rho_{s}$ in cm , $c=2.99792458 \times 10^{10} \mathrm{~cm} / \mathrm{s}$, and $\dot{B}$ in $\mathrm{G} / \mathrm{s}$ ) gives $V_{g} \sin \phi_{s}$ in Statvolts. Multiplying by 299.792458 then gives $V_{g} \sin \phi_{s}$ in Volts.
4. The width of a bucket is

$$
\begin{equation*}
\Delta t=\frac{\left|\pi-\phi_{s}-\phi_{e}\right|}{h \omega_{s}} \tag{18}
\end{equation*}
$$

where the phase $\phi_{e}$ satisfies

$$
\begin{equation*}
\cos \left(\pi-\phi_{s}\right)-\cos \phi_{e}=-\left(\pi-\phi_{s}-\phi_{e}\right) \sin \phi_{s} . \tag{19}
\end{equation*}
$$

5. The area of a bucket is

$$
\begin{equation*}
A_{\mathrm{bk}}=\alpha\left(\phi_{s}\right) A_{S} \tag{20}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha\left(\phi_{s}\right)=\frac{\sqrt{2}}{8} \int_{\phi_{L}}^{\phi_{R}}\left|\left(\pi-\phi_{s}-\phi\right) \sin \phi_{s}-\cos \phi_{s}-\cos \phi\right|^{1 / 2} d \phi \tag{21}
\end{equation*}
$$

Below transition we have $\phi_{e}<\pi-\phi_{s}$ and the limits of integration are $\phi_{L}=\phi_{e}$ and $\phi_{R}=\pi-\phi_{s}$. Above transition we have $\pi-\phi_{s}<\phi_{e}$ and the limits of integration are $\phi_{L}=\pi-\phi_{s}$ and $\phi_{R}=\phi_{e}$. The function $\alpha\left(\phi_{s}\right)$ must be evaluated numerically. An approximate expression is [5]

$$
\begin{equation*}
\alpha\left(\phi_{s}\right) \approx \frac{1-\sin \phi_{s}}{1+\sin \phi_{s}} \tag{22}
\end{equation*}
$$

6. The synchrotron frequency for small-amplitude oscillations about $\phi_{s}$ is

$$
\begin{equation*}
F_{s}=\frac{c}{2 \pi R_{s}}\left\{\frac{-h \eta_{s} e Q V_{g} \cos \phi_{s}}{2 \pi E_{s}}\right\}^{1 / 2} \tag{23}
\end{equation*}
$$

and the corresponding synchrotron tune is $Q_{s}=2 \pi F_{s} / \omega_{s}$. Note that measurement of $F_{s}$ gives a value for $V_{g} \cos \phi_{s}$, while measurement of $d B / d t$ gives a value for $V_{g} \sin \phi_{s}$. These two can be used to obtain $V_{g}$ and $\phi_{s}$.
7. Let $\phi_{l}$ and $\phi_{r}$ be the phases at the left and right boundaries of a bunch matched to a bucket. We have

$$
\begin{equation*}
\phi_{l}<\phi_{s}<\phi_{r} \tag{24}
\end{equation*}
$$

and the width of the bunch is

$$
\begin{equation*}
\Delta t=\frac{\Delta \phi}{h \omega_{s}}, \quad \Delta \phi=\phi_{r}-\phi_{l} \tag{25}
\end{equation*}
$$

In terms of $\Delta \phi$ and $\phi_{s}$ we have

$$
\begin{equation*}
\phi_{r}=\frac{\Delta \phi}{2}+\arcsin \left\{\frac{\Delta \phi \sin \phi_{s}}{2 \sin (\Delta \phi / 2)}\right\} \tag{26}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi_{l}=-\frac{\Delta \phi}{2}+\arcsin \left\{\frac{\Delta \phi \sin \phi_{s}}{2 \sin (\Delta \phi / 2)}\right\} . \tag{27}
\end{equation*}
$$

If $\Delta \phi$ is small we have

$$
\begin{equation*}
\sin (\Delta \phi / 2) \approx \frac{\Delta \phi}{2}, \quad \frac{\Delta \phi \sin \phi_{s}}{2 \sin (\Delta \phi / 2)} \approx \sin \phi_{s} \tag{28}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi_{l} \approx \phi_{s}-\frac{\Delta \phi}{2}, \quad \phi_{r} \approx \phi_{s}+\frac{\Delta \phi}{2} \tag{29}
\end{equation*}
$$

8. The half-height of a bunch matched to a bucket is

$$
\begin{equation*}
\Delta E=\left(\frac{h \omega_{s}}{8 \sqrt{2}}\right) A_{S}\left|\cos \phi_{r}-\cos \phi_{s}+\left(\phi_{r}-\phi_{s}\right) \sin \phi_{s}\right|^{1 / 2} . \tag{30}
\end{equation*}
$$

9. The area of a bunch matched to a bucket is

$$
\begin{equation*}
A_{\mathrm{b}}=F\left(\phi_{s}, \Delta \phi\right) A_{S} \tag{31}
\end{equation*}
$$

where

$$
\begin{equation*}
F\left(\phi_{s}, \Delta \phi\right)=\frac{\sqrt{2}}{8} \int_{\phi_{l}}^{\phi_{r}}\left|\cos \phi_{l}-\cos \phi+\left(\phi_{l}-\phi\right) \sin \phi_{s}\right|^{1 / 2} d \phi . \tag{32}
\end{equation*}
$$

The function $F\left(\phi_{s}, \Delta \phi\right)$ must be evaluated numerically. In
Section 26 it is given in terms of elliptic integrals for the case of a stationary bucket.

If $\Delta \phi$ is small, (32) reduces to

$$
\begin{equation*}
F\left(\phi_{s}, \Delta \phi\right) \approx \frac{\pi}{64}(\Delta \phi)^{2}\left|\cos \phi_{s}\right|^{1 / 2} \tag{33}
\end{equation*}
$$

## 4 Ring Parameters

| Parameter | Booster | AGS | RHIC | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $C_{I}$ | $C_{b}$ | $C_{a}$ | $C_{r}+\delta C$ | m |
| $C_{E}$ | $C_{a} / 4$ | $4\left(C_{r}+\delta C\right) / 19$ | $C_{r}+\delta C$ | m |
| $\rho$ | 13.8656 | 85.378351 | 242.7806 | m |
| $\gamma_{\text {tr }}$ | 4.832 | 8.5 | 22.89 |  |

Here $C_{I}$ and $C_{E}$ are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. $C_{b}, C_{a}$, and $C_{r}$ are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$
\begin{equation*}
C_{b}=201.780, \quad C_{a}=2 \pi(128.4526), \quad C_{r}=3833.845181 \tag{34}
\end{equation*}
$$

meters. $\delta C$ is the shift (if any) of the RHIC orbit circumference from the design value $C_{r}$. Note that $4\left(C_{r} / 19\right)=2 \pi(128.4580) \mathrm{m}$ which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius ( 128.4526 m ) reported by Bleser [6, 7]. The radius of curvature $\rho$ in the Booster and AGS main dipoles is given in Refs. $[6,7,8]$. The RHIC ring parameters are taken from Ref. [9] and from MAD runs by Steve Tepikian.

## 5 Initial Conditions and Assumptions

1. The revolution frequency of the Au32+ ion (from EBIS) at Booster injection is 96.640 kHz . The radius is taken to be the nominal radius $C_{b} /(2 \pi)$.
2. The revolution frequency of the Au32+ ion at Booster extraction is 658.91 KHz [10]. The radius is taken to be one fourth the nominal AGS radius $C_{a} /(2 \pi)$. The corresponding magnetic rigidity is 9.46202773202 Tm . The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is 9.5 Tm [11].
3. The set revolution frequency of the Au77+ ion at AGS injection is 163.125 KHz. This gives an energy loss of 2.453 MeV per nucleon in the BTA stripper.
4. The magnetic rigidity of the Au79+ ion at RHIC injection is taken to be 81.11378003 Tm .
5. The circumference at RHIC injection is $C_{r}$.
6. The circumference at Store is taken to be $C_{r}$.
7. The energy of the Au79+ ion at Store is 100 GeV per nucleon.

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be set or measured; this is done for computational convenience.

## 6 Bunch Merging

The desired number of ions per bunch in RHIC is achieved by merging bunches in both Booster and AGS. There are two setups for doing this called here Setup 1 and Setup 2. Setup 1 was developed in 2012 and is documented in $[12,13,14,15]$. Setup 2 was developed this year (2016) and is documented in $[15,16]$.
The basic operation performed in Setup 1 is a 2 to 1 merge in which two adjacent bunches are merged into one. In both Booster and AGS this
operation is done twice. For each merge the RF harmonic number is reduced by a factor of two. The final result in both Booster and AGS is that 4 adjacent bunches are merged into one.
In Setup 2, two 2 to 1 merges are again done in Booster, but in AGS the 2 to 1 merge is followed by a 3 to 1 merge (in which three adjacent bunches are merged into one). In this case the RF harmonic number is reduced by a factor of two and then a factor of three. The final result in AGS is that 6 adjacent bunches are merged into one.

In both setups Au32+ ions from EBIS are captured in Booster at RF harmonic number $h=4$ and then accelerated to a merging porch where the 4 bunches are merged into 2 followed by a merge of the 2 into 1 . The RF harmonic numbers used are $h=4,2$, and 1. This gives a single bunch at Booster extraction.

In Setup 1, eight Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 16 buckets on the AGS injection porch. The filling pattern is illustrated in Section 18. The 8 bunches are then accelerated to a merging porch where they are merged into 4 followed by a merge of the 4 into 2 . This puts 4 Booster loads into each merged bunch. The RF harmonic numbers used are $h=16,8$, and 4 . Doing the merges on a porch that sits above injection energy helps reduce losses that are believed to be due to the space-charge force acting on the bunched particles $[15,17]$.
In Setup 2, twelve Booster loads (each consisting of a single bunch) are transferred to waiting harmonic 24 buckets on the AGS injection porch. The filling pattern is illustrated in Section 19. The 12 bunches are merged into 6 on the injection porch; the 6 are then merged into 2 on the merging porch that sits above injection energy. This puts 6 Booster loads into each merged bunch. The RF harmonic numbers used are $h=24,12,8$, and 4 .
Simulations $[14,16]$ have shown that the growth in longitudinal emittance during the AGS merges can be kept very small. This is consistent with measurements made in 2014 [12]. Measurements made in 2016 [15] show that there is some growth during the 2 to 1 merge on the AGS injection porch, but little growth during the 3 to 1 merge. In Booster, measurements and simulation show growth during the merges. This is due to the limited time available for merging during the Booster magnetic cycle.

## 7 Frequencies for AGS Merges

The revolution frequency, $f$, required for bunch merges in AGS is dictated by the lowest frequency, $F$, available from the RF system. This is provided by the L10 cavity, which is presently set up to oscillate at

$$
\begin{equation*}
F=783 \mathrm{kHz} . \tag{35}
\end{equation*}
$$

The RF harmonic numbers required for two 2 to 1 merges are $h, 2 h$, and $4 h$, where $h$ is a positive integer. The RF harmonic numbers required for the squeezing and subsequent acceleration of the merged bunch are $h$, $2 h$, and $3 h$. The RF harmonic numbers required for a $\mathbf{3}$ to 1 merge are also $h, 2 h$, and $3 h$. The revolution frequency must satisfy

$$
\begin{equation*}
h f=F . \tag{36}
\end{equation*}
$$

The RF system therefore must provide frequencies

$$
\begin{gather*}
h f=F=\underline{783 \mathrm{kHz}} \\
2 h f=2 F=\underline{1566 \mathrm{kHz}}  \tag{38}\\
3 h f=3 F=\underline{2349 \mathrm{kHz}} \tag{39}
\end{gather*}
$$

and

$$
\begin{equation*}
4 h f=4 F=\underline{3132 \mathrm{kHz}} . \tag{40}
\end{equation*}
$$

The lower frequencies, $F$ and $2 F$, are provided by the L10 and KL cavities, respectively. The higher frequencies, $3 F$ and $4 F$, are provided by the standard AGS RF cavities. For Au77+ ions we take $h=4$.

## 8 Longitudinal Emittance

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$
\begin{equation*}
\mathcal{E}=\frac{2}{A} \Delta E \Delta T \tag{41}
\end{equation*}
$$

where $\Delta E$ is the energy half-width of the beam,

$$
\begin{equation*}
\Delta T=\frac{1}{f}=\frac{2 \pi R}{c \beta} \tag{42}
\end{equation*}
$$

is the revolution period, and $A$ is the number of nucleons. Using the differential relation

$$
\begin{equation*}
\Delta E=\beta^{2} \frac{\Delta p}{p} m c^{2} \gamma \tag{43}
\end{equation*}
$$

we have

$$
\begin{equation*}
\mathcal{E}=\frac{2 \beta^{2} \gamma}{f} \frac{m c^{2}}{A} \frac{\Delta p}{p} \tag{44}
\end{equation*}
$$

where $\Delta p$ is the momentum half-width of the unbunched beam. Taking

$$
\begin{equation*}
f=96.640 \mathrm{kHz} \tag{45}
\end{equation*}
$$

gives

$$
\begin{gather*}
\Delta T=10.3476821192 \mu \mathrm{~s}  \tag{46}\\
\beta=0.0650450626079, \quad \gamma=1.00212216641
\end{gather*}
$$

and

$$
\begin{equation*}
\frac{2 \beta^{2} \gamma}{f}=87.7450074295 \mathrm{~ns} . \tag{48}
\end{equation*}
$$

For Au32+ ions we have

$$
\begin{equation*}
\frac{m c^{2}}{A}=0.931253053269 \mathrm{GeV} \tag{49}
\end{equation*}
$$

which gives

$$
\begin{equation*}
\frac{2 \beta^{2} \gamma}{f} \frac{m c^{2}}{A}=81.7128060778 \mathrm{eV} \mathrm{s.} \tag{50}
\end{equation*}
$$

Using this in (44) then gives the longitudinal emittance (per nucleon) for any given fractional momentum half-width.
For fractional momentum half-width

$$
\begin{equation*}
\frac{\Delta p}{p}=0.001 \tag{51}
\end{equation*}
$$

we have longitudinal emittance (per nucleon)

$$
\begin{equation*}
\mathcal{E}=0.0817128060778 \mathrm{eV} \mathrm{~s} \tag{52}
\end{equation*}
$$

The most recent fractional momentum half-width obtained from measurements [15] is

$$
\begin{equation*}
\frac{\Delta p}{p}=0.00049 \tag{53}
\end{equation*}
$$

which gives longitudinal emittance

$$
\begin{equation*}
\mathcal{E}=0.040 \mathrm{eV} \mathrm{~s} \tag{54}
\end{equation*}
$$

for unbunched Au32+ beam circulating in Booster at injection.
A detailed account of the longitudinal emittance evolution in Booster and AGS is given in $[12,13]$ and [15].

## 9 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into $h$ buckets we must have RF voltage $V_{g}$ (i.e. total gap voltage per turn) such that

$$
\begin{equation*}
\mathcal{E} \leq \frac{h A_{S}}{A} \tag{55}
\end{equation*}
$$

where $A_{S}$ is given by (15). Thus we must have

$$
\begin{equation*}
\frac{2 \beta^{2} \gamma}{f} \frac{m c^{2}}{A} \frac{\Delta p}{p} \leq \frac{8 R}{c A}\left\{\frac{2 e Q V_{g} E}{\pi h|\eta|}\right\}^{1 / 2} \tag{56}
\end{equation*}
$$

which gives

$$
\begin{gather*}
2 \beta^{2} \gamma\left(\frac{2 \pi R}{c \beta}\right) \frac{m c^{2}}{A} \frac{\Delta p}{p} \leq \frac{8 R}{c}\left(\frac{2 \gamma}{\pi h|\eta|}\right)^{1 / 2} \frac{m c^{2}}{A}\left(\frac{e Q V_{g}}{m c^{2}}\right)^{1 / 2}  \tag{57}\\
\beta^{2} \gamma\left(\frac{\pi}{\beta}\right) \frac{\Delta p}{p} \leq 2\left(\frac{2 \gamma}{\pi h|\eta|}\right)^{1 / 2}\left(\frac{e Q V_{g}}{m c^{2}}\right)^{1 / 2}  \tag{58}\\
\beta^{2} \gamma^{2} \pi^{2}\left(\frac{\Delta p}{p}\right)^{2} \leq\left(\frac{8 \gamma}{\pi h|\eta|}\right)\left(\frac{Q}{m c^{2}}\right) e V_{g} \tag{59}
\end{gather*}
$$

and

$$
\begin{equation*}
\frac{1}{8} h \pi^{3} \beta^{2} \gamma|\eta|\left(\frac{m c^{2}}{Q}\right)\left(\frac{\Delta p}{p}\right)^{2} \leq e V_{g} \tag{60}
\end{equation*}
$$

Here

$$
\begin{equation*}
h=4 \tag{61}
\end{equation*}
$$

and taking revolution frequency

$$
\begin{equation*}
f=96.640 \mathrm{kHz} \tag{62}
\end{equation*}
$$

we have

$$
\begin{equation*}
\eta=-0.952939329734 \tag{63}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{1}{8} h \pi^{3} \beta^{2} \gamma|\eta|=0.0626374709945 \tag{64}
\end{equation*}
$$

For the Au32+ ion we have mass energy equivalent per unit charge

$$
\begin{equation*}
\frac{m c^{2}}{Q}=5.73302660918 \mathrm{GeV} \tag{65}
\end{equation*}
$$

Taking fractional momentum half-width

$$
\begin{equation*}
\frac{\Delta p}{p}=0.001 \tag{66}
\end{equation*}
$$

then gives

$$
\begin{equation*}
359.102287944 \text { volts } \leq V_{g} . \tag{67}
\end{equation*}
$$

Taking

$$
\begin{equation*}
\frac{\Delta p}{p}=0.00049 \tag{68}
\end{equation*}
$$

gives

$$
\begin{equation*}
86.2204593353 \text { volts } \leq V_{g} . \tag{69}
\end{equation*}
$$

## 10 Inflector Voltage

At Booster injection, the voltage $V_{I}$ required for particles with mass $m$, velocity $c \beta$, and charge $e Q$ to follow the nominal trajectory through the inflector is given by

$$
\begin{equation*}
e V_{I}=\frac{G}{R_{I}}\left(\frac{m c^{2}}{Q}\right) \beta^{2} \gamma \tag{70}
\end{equation*}
$$

Here $G=0.021 \mathrm{~m}$ is the gap between the cathode and septum of the inflector and $R_{I}=8.74123 \mathrm{~m}$ is the radius of curvature along the nominal trajectory. Using the values of $\beta, \gamma$, and $m c^{2} / Q$ given by (47) and (65), we obtain

$$
\begin{equation*}
V_{I}=58.396 \mathrm{kV} \tag{71}
\end{equation*}
$$

for Au32+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$
\begin{equation*}
V_{I}(\text { setpoint })=59.740 \mathrm{kV} \tag{72}
\end{equation*}
$$

## 11 Booster Injection Field

The nominal magnetic field in the Booster dipoles at injection is

$$
\begin{equation*}
B=(B \rho) / \rho \tag{73}
\end{equation*}
$$

where $B \rho$ is given by (12) and $\rho$ is the nominal radius of curvature. Writing

$$
\begin{equation*}
B \rho=\frac{10^{9}}{c}\left(\frac{m c^{2}}{Q}\right) \beta \gamma \tag{74}
\end{equation*}
$$

and using the values of $\beta, \gamma$, and $m c^{2} / Q$ given by (47) and (65), we obtain

$$
\begin{equation*}
B \rho=1.24651715338 \mathrm{Tm} . \tag{75}
\end{equation*}
$$

Here we have used the mass energy equivalent $m c^{2}$ in units of GeV and the velocity of light in units of $\mathrm{m} / \mathrm{s}$. Using

$$
\begin{equation*}
\rho=13.8656 \mathrm{~m} \tag{76}
\end{equation*}
$$

we then obtain

$$
\begin{equation*}
B=898.999793284 \text { Gauss } \tag{77}
\end{equation*}
$$

for Au32+ ions from EBIS.
The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock. For Au32+ ions from EBIS the measured field at injection is

$$
\begin{equation*}
B(\text { measured })=894.0 \text { Gauss. } \tag{78}
\end{equation*}
$$

## 12 AGS Injection Field

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B=454.96$ Gauss for the Au77+ ions. The measured magnetic field is 482.0 gauss [18].

## 13 BTA Stripper

The stripper $[19,20]$ used to strip gold ions in the BTA (Booster-To-AGS) transfer line consists of a $6.45 \mathrm{mg} / \mathrm{cm}^{2}$ aluminum foil followed by a 8.39 $\mathrm{mg} / \mathrm{cm}^{2}$ carbon foil. In Section 25 we use these surface densities to calculate the energy loss of Au77+ ions in the foils.

## 14 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$
\begin{equation*}
B=(B \rho) / \rho \tag{79}
\end{equation*}
$$

where $B \rho$ is the magnetic rigidity of the beam and $\rho=18.625 \mathrm{~m}[22]$ is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$
\begin{equation*}
N I=g B / \mu_{0} \tag{80}
\end{equation*}
$$

where $N=1$ is the number of conductor turns; $g=0.0467 \mathrm{~m}[22]$ is the magnet gap; and $\mu_{0}=4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}$.
For Au77+ ions at injection, the magnetic rigidity is $B \rho=3.88434088 \mathrm{Tm}$. This gives $B=0.208555 \mathrm{~T}$ and $I=7750 \mathrm{~A}$.
For comparison, the magnetic rigidity of polarized protons at AGS injection is $B \rho=7.205178 \mathrm{Tm}$. This gives $B=0.3869 \mathrm{~T}$ and $I=14380 \mathrm{~A}$.

## 15 AGS Injection Kicker Current

The current required in the A5 kicker is [21, 22]

$$
\begin{equation*}
I=\frac{B \rho}{K} \sin \phi \tag{81}
\end{equation*}
$$

where

$$
\begin{equation*}
K=1.8718 \times 10^{-5} \mathrm{Tm} / \mathrm{A} \tag{82}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi=3.35 \text { milliradians } \tag{83}
\end{equation*}
$$

is the desired kick angle. Using the calculated values of $B \rho$ at AGS injection we obtain a current of 695.2 A for Au77+ ions. The maximum available current is 1100 A .

## 16 AGS Injection Kicker Short Pulse Waveforms



Figure 1: AGS injection kicker waveforms in the short pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan on 9 Dec 2011. The time per division is 200 ns . The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions in harmonic 16 buckets. In order to put beam into adjacent buckets, the rise time of the kicker must be less than or equal to $B-W$, where $B$ is the bucket width and $W$ the bunch width. The rise time is approximately 100 ns , which implies that the bunch width must be less than or equal to 283 ns for Au77+ bunches. A single bunch of this width easily fits on the flattop portion of the pulse which is some 600 ns long. The total width of the pulse is approximately 1000 ns . With this kicker pulse one could in principle fill 14 of the 16 RF buckets on the AGS injection porch. The pulse is too wide to fill the remaining buckets without interfering with beam in the adjacent buckets. This is not an issue as only 8 of the buckets need to be filled. One workable filling pattern is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This is illustrated in Section 18.

## 17 AGS Injection Kicker Long Pulse Waveforms



Figure 2: AGS injection kicker waveforms in the long pulse mode. The three traces are from the three modules of the kicker. They were taken by Yugang Tan in Oct 2010. The time per division is 200 ns. The RF bucket width on the AGS injection porch is 383 ns for Au77+ ions in harmonic 16 buckets. Here the flattop portion of the pulse is some 1300 ns long. The total pulse width is approximately 2000 ns. In principle this kicker pulse could be used to fill 8 buckets with a filling pattern of four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This is illustrated in Section 18.

## 18 Kicker timing for 8 transfers to AGS followed by two 2 to 1 merges



Figure 3: Here $T$ is the revolution period on the AGS injection porch and $T / 16=383 \mathrm{~ns}$ is the harmonic 16 RF bucket width. The kicker rise time is 100 ns. This means that the bunch width, $W$, must be less than $T / 16-100=$ 283 ns . The filling pattern in this case is four adjacent filled buckets followed by four adjacent empty buckets, followed by another four adjacent filled buckets. This allows each group of four adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G=6 T / 16-W=2298-W$ ns.

## 19 Kicker timing for 12 transfers to AGS

 followed by a 2 to 1 merge and a 3 to 1 merge

Figure 4: Here $T / 24=255 \mathrm{~ns}$ is the harmonic 24 RF bucket width. The kicker rise time is 100 ns. This means that the bunch width, $W$, must be less than $T / 24-100=155 \mathrm{~ns}$. The filling pattern in this case is six adjacent filled buckets followed by six adjacent empty buckets, followed by another six adjacent filled buckets. This allows each group of six adjacent bunches to be merged into a single bunch. One ends up with a merged bunch sitting in every other harmonic 4 bucket. The total gap available for the kicker pulse is $G=8 T / 24-W=2040-W$ ns.

## 20 Gold in Booster for 8 transfers to AGS

| Parameter | Injection | Merge porch | Extraction | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $Q$ | 32 | 32 | 32 |  |
| $m c^{2}$ | 183.456851 | 183.456851 | 183.456851 | GeV |
| $W / A$ | 1.9762739452 | 49.259795 | 107.75879 | MeV |
| $c p / A$ | 60.701960016 | 306.87652 | 460.77475 | MeV |
| $E / A$ | 0.9332293272 | 0.98051285 | 1.0390118 | GeV |
| $B \rho$ | 1.24651715338 | 6.3017214 | 9.46202773202 | Tm |
| $\beta$ | 0.065045062608 | 0.31297552 | 0.44347401 |  |
| $\gamma-1$ | 0.002122166406 | 0.052896251 | 0.11571376 |  |
| $\eta$ | -0.953 | -0.859 | -0.7605 |  |
| $\epsilon_{H}(95 \%)$ | $12.1 \pi$ | $12.1 \pi$ | $12.1 \pi$ | mm mrad |
| $\epsilon_{V}(95 \%)$ | $5.68 \pi$ | $5.68 \pi$ | $5.68 \pi$ | mm mrad |
| $h$ | 4 | 1 | 1 |  |
| $h f$ | $\mathbf{3 8 6 . 5 6 0}$ | $\mathbf{4 6 5 . 0 0 0}$ | $\mathbf{6 5 8 . 9 1 0}$ | KHz |
| $R$ | $201.780 /(2 \pi)$ | $201.780 /(2 \pi)$ | $128.4526 / 4$ | m |

Here $\epsilon_{H}$ and $\epsilon_{V}$ are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are $185 \pi$ and $87 \pi \mathrm{~mm} \mathrm{mrad}$ (un-normalized) respectively.

| Parm | Injection | Ext | Ext | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{g}$ | 5.730 | 25.2 | 25.2 | 25.2 | kV |
| $A_{S}$ | 16.076 | 318.54 | 318.54 | 318.54 | eV s |
| $d B / d t$ | 0 | $\mathbf{7 0}$ | $\mathbf{3 5}$ | $\mathbf{0}$ | $\mathrm{G} / \mathrm{ms}$ |
| $\phi_{s}$ | 0 | 50.999 | 22.866 | 0 | deg |
| $F_{s}$ | 1.1557 | 0.8139 | 0.9849 | 1.0260 | kHz |
| $A_{\mathrm{bk}}$ | 16.076 | 36.294 | 140.88 | 318.54 | eV s |
| $A_{b}$ | 1.82 | 13.987 | 13.987 | 13.987 | eV s |
| $\Delta t$ | 635.1 | 264.6 | 235.1 | 229.8 | ns |
| $\Delta E$ | 1.836 | 34.11 | 38.00 | 38.84 | MeV |


| Parameter | Injection | Extraction | Unit |
| :---: | :---: | :---: | :---: |
| No. Bunches | 4 | 1 |  |
| Bucket Width | 2586.92053 | 1517.65795 | ns |
| Ions/Bunch | $1.25 / 4$ | 1.04 | $10^{9}[15]$ |
| Bunch Area | $0.037 / 4[15]$ | $0.071[15]$ | $\mathrm{eV} \mathrm{s} / A$ |

## 21 Gold in Booster for 12 transfers to AGS

| Parameter | Injection | Merge porch | Extraction | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $Q$ | 32 | 32 | 32 |  |
| $m c^{2}$ | 183.456851 | 183.456851 | 183.456851 | GeV |
| $W / A$ | 1.9762739452 | 72.089750 | 107.75879 | MeV |
| $c p / A$ | 60.701960016 | 373.44950 | 460.77475 | MeV |
| $E / A$ | 0.9332293272 | 1.0033428 | 1.0390118 | GeV |
| $B \rho$ | 1.24651715338 | 7.6688003 | 9.46202773202 | Tm |
| $\beta$ | 0.065045062608 | 0.37220529 | 0.44347401 |  |
| $\gamma-1$ | 0.002122166406 | 0.07741156 | 0.11571376 |  |
| $\eta$ | -0.953 | -0.8186 | -0.7605 |  |
| $\epsilon_{H}(95 \%)$ | $12.1 \pi$ | $12.1 \pi$ | $12.1 \pi$ | mm mrad |
| $\epsilon_{V}(95 \%)$ | $5.68 \pi$ | $5.68 \pi$ | $5.68 \pi$ | mm mrad |
| $h$ | 4 | 1 | 1 |  |
| $h f$ | $\mathbf{3 8 6 . 5 6 0}$ | $\mathbf{5 5 3 . 0 0 0}$ | $\mathbf{6 5 8 . 9 1 0}$ | KHz |
| $R$ | $201.780 /(2 \pi)$ | $201.780 /(2 \pi)$ | $128.4526 / 4$ | m |

Here $\epsilon_{H}$ and $\epsilon_{V}$ are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are $185 \pi$ and $87 \pi \mathrm{~mm} \mathrm{mrad}$ (un-normalized) respectively.

| Parm | Injection | Ext | Ext | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{g}$ | 5.730 | 25.2 | 25.2 | 25.2 | kV |
| $A_{S}$ | 16.076 | 318.54 | 318.54 | 318.54 | eV s |
| $d B / d t$ | 0 | $\mathbf{7 0}$ | $\mathbf{3 5}$ | $\mathbf{0}$ | $\mathrm{G} / \mathrm{ms}$ |
| $\phi_{s}$ | 0 | 50.999 | 22.866 | 0 | deg |
| $F_{s}$ | 1.1557 | 0.8139 | 0.9849 | 1.0260 | kHz |
| $A_{\mathrm{bk}}$ | 16.076 | 36.294 | 140.88 | 318.54 | eV s |
| $A_{b}$ | 1.82 | 13.987 | 13.987 | 13.987 | eV s |
| $\Delta t$ | 635.1 | 264.6 | 235.1 | 229.8 | ns |
| $\Delta E$ | 1.836 | 34.11 | 38.00 | 38.84 | MeV |


| Parameter | Injection | Extraction | Unit |
| :---: | :---: | :---: | :---: |
| No. Bunches | 4 | 1 |  |
| Bucket Width | 2586.92053 | 1517.65795 | ns |
| Ions/Bunch | $1.25 / 4$ | 1.04 | $10^{9}[15]$ |
| Bunch Area | $0.037 / 4[15]$ | $0.071[15]$ | $\mathrm{eV} \mathrm{s} / A$ |

## 22 Gold in AGS with 4 to 1 Merge on Porch

| Parameter | Injection | Porch | Extraction | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $Q$ | 77 | 77 | 77 |  |
| $m c^{2}$ | 183.434174 | 183.434174 | 183.434174 | GeV |
| $W / A$ | 0.10529199 | 0.16448553 | 8.86486804031 | GeV |
| $c p / A$ | 0.45515837 | 0.57738456 | 9.75165192809 | GeV |
| $E / A$ | 1.0364299 | 1.09562347 | 9.79600598164 | GeV |
| $B \rho$ | 3.88434088 | 4.9274243 | 83.2210113714 | Tm |
| $\beta$ | 0.43915981 | 0.52699177 | 0.995472230863 |  |
| $\gamma$ | 1.1130788 | 1.1766500 | 10.5204669972 |  |
| $\eta$ | -0.793 | -0.708 | 0.00481 |  |
| $\beta \gamma$ | 0.48881949 | 0.62008488 | 10.472833 |  |
| $\beta \gamma^{2}$ | 0.54409463 | 0.72962288 | 110.17909 |  |
| $h$ | 16 | 4 | 12 |  |
| $h f$ | $\mathbf{2 . 6 1 0 0 0 0}$ | $\mathbf{0 . 7 8 3}$ | $\mathbf{4 . 4 3 7 0 0 7 2 3 7 8 2}$ | MHz |
| $R$ | 128.4526 | 128.4526 | 128.457981391 | m |


| Parameter | Inj | Porch | Porch | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 16 | 16 | 4 | 12 |  |
| $V_{g}$ | 40.0 | 20.0 | 15.0 | 192.0 | kV |
| $A_{S}$ | 38.0 | 29.27 | 202.8 | 5070 | eV s |
| $d B / d t$ | 0 | 0 | 0 | 0 | $\mathrm{G} / \mathrm{ms}$ |
| $\phi_{s}$ | 0 | 0 | 0 | 180 | degrees |
| $F_{s}$ | 2.050 | 1.333 | 0.577 | 0.0985 | kHz |
| $A_{\mathrm{bk}}$ | 38.0 | 29.27 | 202.8 | 5070 | eV s |
| $A_{b}$ | 19.7 | 19.7 | 84.316 | 118.2 | eV s |
| $\Delta t$ | 216 | 213.1 | 631.7 | 24.8 | ns |
| $\Delta E$ | 60.4 | 62.4 | 87.4 | 3038 | MeV |


| Parameter | Inj | Porch | Porch | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 16 | 16 | 4 | 12 |  |
| Bucket Width | 383.142 | 319.285 | 1277.139 | 225.377 | ns |
| No. of Bunches | 8 | 8 | 2 | 2 |  |
| Ions/Bunch | 0.56 | 0.56 | 2.24 | $2.14[15]$ | $10^{9}$ |
| Bunch Area | 0.10 | 0.10 | 0.428 | $0.60[15]$ | $\mathrm{eV} \mathrm{s} / A$ |

## 23 Gold in AGS with 3 to 1 Merge on Porch

| Parameter | Injection | Porch | Extraction | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $Q$ | 77 | 77 | 77 |  |
| $m c^{2}$ | 183.434174 | 183.434174 | 183.434174 | GeV |
| $W / A$ | 0.10529199 | 0.16448553 | 8.86486804031 | GeV |
| $c p / A$ | 0.45515837 | 0.57738456 | 9.75165192809 | GeV |
| $E / A$ | 1.0364299 | 1.09562347 | 9.79600598164 | GeV |
| $B \rho$ | 3.88434088 | 4.9274243 | 83.2210113714 | Tm |
| $\beta$ | 0.43915981 | 0.52699177 | 0.995472230863 |  |
| $\gamma$ | 1.1130788 | 1.1766500 | 10.5204669972 |  |
| $\eta$ | -0.793 | -0.708 | 0.00481 |  |
| $\beta \gamma$ | 0.48881949 | 0.62008488 | 10.472833 |  |
| $\beta \gamma^{2}$ | 0.54409463 | 0.72962288 | 110.17909 |  |
| $h$ | 24 | 4 | 12 |  |
| $h f$ | $\mathbf{3 . 9 1 5 0 0 0}$ | $\mathbf{0 . 7 8 3}$ | $\mathbf{4 . 4 3 7 0 0 7 2 3 7 8 2}$ | MHz |
| $R$ | 128.4526 | 128.4526 | 128.457981391 | m |


| Parameter | Inj | Porch | Porch | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 24 | 12 | 4 | 12 |  |
| $V_{g}$ | 119.8 | 100 | 15.0 | 185.2 | kV |
| $A_{S}$ | 35.84 | 100.8 | 202.8 | 4979 | eV s |
| $d B / d t$ | 0 | 0 | 0 | 0 | $\mathrm{G} / \mathrm{ms}$ |
| $\phi_{s}$ | 0 | 0 | 0 | 180 | degrees |
| $F_{s}$ | 4.346 | 2.581 | 0.577 | 0.0967 | kHz |
| $A_{\mathrm{bk}}$ | 35.84 | 100.8 | 202.8 | 4979 | eV s |
| $A_{b}$ | 17.73 | 39.4 | 118.2 | 147.75 | eV s |
| $\Delta t$ | 140 | 203 | 775 | 28.0 | ns |
| $\Delta E$ | 83.6 | 127 | 102 | 3365 | MeV |


| Parameter | Inj | Porch | Porch | Ext | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 24 | 12 | 4 | 12 |  |
| Bucket Width | 255.428 | 425.713 | 1277.139 | 225.377 | ns |
| No. of Bunches | 12 | 6 | 2 | 2 |  |
| Ions/Bunch | 0.53 | 1.06 | 3.18 | 3.0 | $10^{9}[15]$ |
| Bunch Area | 0.09 | 0.20 | 0.60 | $0.75[15]$ | $\mathrm{eV} \mathrm{s} / A$ |

## 24 Gold in RHIC

| Parameter | Injection | Transition | Store | Unit |
| :---: | :---: | :---: | :---: | :---: |
| $Q$ | 79 | 79 | 79 |  |
| $m c^{2}$ | 183.433337 | 183.433337 | 183.433337 | GeV |
| $W / A$ | 8.86482757134 | 20.3825164868 | 99.0688663094 | GeV |
| $c p / A$ | 9.75160741084 | 21.2933011516 | 99.9956648563 | GeV |
| $E / A$ | 9.79596126192 | 21.3136501774 | $\mathbf{1 0 0 . 0 0 0}$ | GeV |
| $B \rho$ | $\mathbf{8 1 . 1 1 3 7 8 0 0 3}$ | 177.117481555 | 831.763013151 | Tm |
| $\beta$ | 0.995472230863 | 0.999045258528 | 0.999956648563 |  |
| $\gamma$ | 10.5204669974 | $\mathbf{2 2 . 8 9 0 0}$ | 107.395963664 |  |
| $\eta$ | -0.00713 | 0.0 | 0.00182 |  |
| $f$ | 77.8422322425 | 78.1216297391 | 78.1928970559 | kHz |
| $h$ | 360 | 360 | 2520 |  |
| $h f$ | 28.0232036073 | 28.1237867061 | 197.046100581 | MHz |
| $\delta C$ | 0 | 0 | 0 | mm |


| Parameter | Injection | Store | Unit |
| :---: | :---: | :---: | :---: |
| $h$ | 360 | 2520 |  |
| $V_{g}$ | 405.5 | 3000 | kV |
| $A_{S}$ | 177.2 | 164.4 | eV s |
| $d B / d t$ | 0 | 0 | $\mathrm{G} / \mathrm{ms}$ |
| $\phi_{s}$ | 0 | 180 | degrees |
| $F_{s}$ | 0.204 | 0.232 | kHz |
| $A_{\mathrm{bk}}$ | 177.2 | 164.4 | eV s |
| $A_{b}$ | 147.75 | 147.75 | eV s |
| $A_{b}$ | 0.75 | 0.75 | $\mathrm{eV} \mathrm{s} / \mathrm{A}[23]$ |
| $\Delta t$ | 28.0 | 4.27 | ns |
| $\Delta E$ | 3678 | 24649 | MeV |

## 25 Au77+ Energy Loss in the BTA Stripper Foils

The stripper used to strip gold ions consists of a $6.45 \mathrm{mg} / \mathrm{cm}^{2}$ aluminum foil followed by a $8.39 \mathrm{mg} / \mathrm{cm}^{2}$ "glassy" carbon foil $[19,20]$. We can estimate the energy loss in the foils as follows:
The kinetic energy of a proton that has the same velocity as the Au77+ ion just upstream of the aluminum foil is

$$
\begin{equation*}
W_{p}=108.6 \mathrm{MeV} \tag{84}
\end{equation*}
$$

The rate of energy loss of a proton passing through the foil with kinetic energy $W_{p}$ is [24]

$$
\begin{equation*}
-\frac{d E_{p}}{d x}=5.348 \mathrm{MeV} \mathrm{~cm}{ }^{2} / \mathrm{g} \tag{85}
\end{equation*}
$$

The rate of energy loss of the Au77+ ion is obtained by scaling the Bethe-Bloch result for protons [25]. Thus

$$
\begin{equation*}
-\frac{d E}{d x}=-Z^{2} \frac{d E_{p}}{d x} \mathrm{~cm}^{2} / \mathrm{g} \tag{86}
\end{equation*}
$$

where $Z=77$. Multiplying this by the surface density of the aluminum foil ( $6.45 \mathrm{mg} / \mathrm{cm}^{2}$ ) gives

$$
\begin{equation*}
\Delta E_{a}=1.038 \mathrm{MeV} \text { per nucleon. } \tag{87}
\end{equation*}
$$

This is the energy lost by the Au77+ ion upon passing through the aluminum foil. The kinetic energy of a proton that has the same velocity as the Au77+ ion just downstream of the aluminum foil is then

$$
\begin{equation*}
W_{p}=107.5 \mathrm{MeV} \tag{88}
\end{equation*}
$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [24]

$$
\begin{equation*}
-\frac{d E_{p}}{d x}=6.180 \mathrm{MeV} \mathrm{~cm}{ }^{2} / \mathrm{g} \tag{89}
\end{equation*}
$$

Using this result in (86) with $Z=77$, and multiplying by the surface density of the carbon foil ( $8.39 \mathrm{mg} / \mathrm{cm}^{2}$ ) gives

$$
\begin{equation*}
\Delta E_{c}=1.561 \mathrm{MeV} \text { per nucleon } \tag{90}
\end{equation*}
$$

The total energy lost upon passing through both foils is then

$$
\begin{equation*}
\Delta E=\Delta E_{a}+\Delta E_{c}=2.599 \mathrm{MeV} \text { per nucleon. } \tag{91}
\end{equation*}
$$

This agrees reasonably well with the value 2.453 MeV per nucleon obtained in Section 5.

## 26 Area of Bunch Matched to Stationary Bucket

For the case of a bunch matched to a stationary bucket, the function (32) becomes

$$
\begin{equation*}
F(0, \Delta \phi)=\frac{\sqrt{2}}{8} \int_{\phi_{L}}^{\phi_{R}}\left|\cos \phi_{L}-\cos \phi\right|^{1 / 2} d \phi \tag{92}
\end{equation*}
$$

where

$$
\begin{equation*}
\phi_{R}=-\phi_{L}=\Delta \phi / 2 \tag{93}
\end{equation*}
$$

Defining

$$
\begin{equation*}
\mathcal{F}(Z)=\frac{2 \sqrt{2}}{8} \int_{0}^{Z} \sqrt{\cos \phi-\cos Z} d \phi \tag{94}
\end{equation*}
$$

we have

$$
\begin{equation*}
F(0, \Delta \phi)=\mathcal{F}(\Delta \phi / 2) \tag{95}
\end{equation*}
$$

The integral in (94) can be expressed in terns of the indefinite integral

$$
\begin{equation*}
I(a, b, x)=\int \sqrt{a+b \cos x} d x \tag{96}
\end{equation*}
$$

This is given in Gradsheteyn and Ryzhick [26] as

$$
\begin{equation*}
I(a, b, x)=\sqrt{\frac{2}{b}}\left\{(a-b) F\left(\gamma, \frac{1}{r}\right)+2 b E\left(\gamma, \frac{1}{r}\right)\right\} \tag{97}
\end{equation*}
$$

where

$$
\begin{gather*}
F(\gamma, k)=\int_{0}^{\gamma} \frac{d \alpha}{\sqrt{1-k^{2} \sin ^{2} \alpha}}  \tag{98}\\
E(\gamma, k)=\int_{0}^{\gamma} \sqrt{1-k^{2} \sin ^{2} \alpha} d \alpha  \tag{99}\\
\gamma=\arcsin \sqrt{\frac{b(1-\cos x)}{a+b}}, \quad r=\sqrt{\frac{2 b}{a+b}} \tag{100}
\end{gather*}
$$

and

$$
\begin{equation*}
0 \leq x<\arccos \left(-\frac{a}{b}\right), \quad 0<|a| \leq b \tag{101}
\end{equation*}
$$

Here $F(\gamma, k)$ and $E(\gamma, k)$ are the elliptic integrals of the first and second kind [27].
Thus, setting

$$
\begin{equation*}
a=-\cos Z, \quad b=1 \tag{102}
\end{equation*}
$$

we have

$$
\begin{equation*}
\mathcal{F}(Z)=\frac{\sqrt{2}}{4}\{I(-\cos Z, 1, Z)-I(-\cos Z, 1,0)\} \tag{103}
\end{equation*}
$$

and, for $x=Z$ and $x=0$, we see that $\gamma=\pi / 2$ and $\gamma=0$ respectively. This gives

$$
\begin{equation*}
I(-\cos Z, 1, Z)=\sqrt{2}\left\{(-1-\cos Z) F\left(\frac{\pi}{2}, \frac{1}{r}\right)+2 E\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{104}
\end{equation*}
$$

and

$$
\begin{equation*}
I(-\cos Z, 1,0)=0 \tag{105}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{1}{r}=\frac{1}{\sqrt{2}} \sqrt{1-\cos Z} \tag{106}
\end{equation*}
$$

We therefore have

$$
\begin{equation*}
\mathcal{F}(Z)=\frac{1}{2}\left\{2 E\left(\frac{\pi}{2}, \frac{1}{r}\right)-(1+\cos Z) F\left(\frac{\pi}{2}, \frac{1}{r}\right)\right\} \tag{107}
\end{equation*}
$$

which we can write as

$$
\begin{equation*}
\mathcal{F}(Z)=E(X)-\frac{1}{2}(1+\cos Z) K(X) \tag{108}
\end{equation*}
$$

where

$$
\begin{align*}
& E(X)=E\left(\frac{\pi}{2}, X\right)  \tag{109}\\
& K(X)=F\left(\frac{\pi}{2}, X\right) \tag{110}
\end{align*}
$$

and

$$
\begin{equation*}
X=\frac{1}{r}=\frac{1}{\sqrt{2}} \sqrt{1-\cos Z} \tag{111}
\end{equation*}
$$

Using (98) and (99) we have

$$
\begin{equation*}
K(X)=\int_{0}^{\pi / 2} \frac{d \alpha}{\sqrt{1-X^{2} \sin ^{2} \alpha}} \tag{112}
\end{equation*}
$$

and

$$
\begin{equation*}
E(X)=\int_{0}^{\pi / 2} \sqrt{1-X^{2} \sin ^{2} \alpha} d \alpha \tag{113}
\end{equation*}
$$

which are the complete elliptic integrals of the first and second kind respectively. These can be evaluated numerically using routines ellf, elle, rd, and rf given in Numerical Recipes [28].

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