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C. Liu

November 2016

Collider Accelerator Department  
**Brookhaven National Laboratory**

**U.S. Department of Energy**

USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

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C-A/AP/577  
November 2016

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**Collider-Accelerator Department  
Brookhaven National Laboratory  
Upton, NY 11973**

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# IBS SIMULATION WITH DIFFERENT RF CONFIGURATIONS IN RHIC

C. Liu, A. Fedotov, M. Minty, V. Ptitsyn, Collider Accelerator Department, Brookhaven National Lab, Upton, NY, USA.

## Abstract

It is a crucial task to understand the beam emittance growth during RHIC cycle and the underlying causes. One would benefit not just for the current operation of RHIC, also for the design of eRHIC. This report focuses on the Intra-Beam Scattering (IBS) contribution to the emittance growth of the proton beam with two different configurations of RF system. The answers to these questions will be given in the end of the report; can IBS explain the emittance growth all alone? What's the difference of IBS growth rates for different RF configurations?

## IBS OVERVIEW

The IBS are small angle coulomb scattering between particles, which would cause beam dimensions in phase space to grow. Many theories of IBS have been developed in the past [1-5]. When beam energy is below transition energy, an equilibrium beam distribution exists so that particles only exchange energy between transverse and longitudinal dimensions. With energy above transition, beam dimensions expand simultaneously. The energy of proton beam (24 GeV) at injection in RHIC is above transition energy. One could use the high energy approximation as guidance and here we quote the rates expressed in reference [4]:

$$\begin{bmatrix} \frac{1}{\sigma_p} \frac{d\sigma_p}{dt} \\ \frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \\ \frac{1}{\sigma_y} \frac{d\sigma_y}{dt} \\ \sigma_y \end{bmatrix} = \frac{A_0}{2} \int e^{-Dz} \ln(1 + C^4 z^2) \begin{bmatrix} n_b(1-d^2)g_1 \\ a^2g_2 + (d^2 + \bar{d}^2)g_1 \\ b^2g_3 \end{bmatrix} \sin \theta \, d\theta d\phi dz$$

where  $A_0 = \frac{c^2 N^2 Z^4 \beta_x \beta_y}{32\pi^2 A^2 \sigma_x^2 \sigma_y^2 \sigma_p \sigma_s \beta^3 \gamma^4}$ ,  $d = \frac{D_p \sigma_p}{(\sigma_x^2 + D_p^2 \sigma_p^2)^{1/2}}$ ,  $\bar{d} = \frac{\bar{D}_p d}{D_p}$ ,  $\bar{D}_p = \alpha_x D_p + \beta_x D'_p$ ,  $a = \frac{\beta_x d}{D_p \gamma}$ ,  $b = \frac{\beta_y \sigma_x}{\beta_x \sigma_y} a$ ,  $D = \cos^2 \theta + b^2 \sin^2 \theta \sin^2 \phi + (a \sin \theta \cos \phi - \bar{d} \cos \theta)^2$ ,  $C = 2\beta \sigma_p [\sigma_y (1-d^2)/r_0]^{1/2}$ ,  $g_1 = 1 - 3 \cos^2 \theta$ ,  $g_2 = \cos^2 \theta - 2 \sin^2 \theta \cos^2 \phi + \sin^2 \theta \sin^2 \phi + 6 \bar{d} \cos \theta \sin \theta \cos \phi / a$ ,  $g_3 = \cos^2 \theta + \sin^2 \theta \cos^2 \phi - 2 \sin^2 \theta \sin^2 \phi$ . The growth rates increase with the beam phase space density, charge number, and decrease with beam energy. In addition, the energy spread itself also effect the growth rates.

## SIMULATION CODE AND INPUTS

The simulation was performed with code BETACOOOL [6]. Of many simulation functions provided by BETACOOOL, only IBS calculation is performed for our case.

The accelerating RF in RHIC was switched from 28 MHz to 9 MHz cavity in 2010 for better matching of the longitudinal emittance of the incoming beam from AGS and the RF bucket in RHIC. For eRHIC, the ring-ring design requires 360 ion bunches. Therefore, the ion beam needs to be accelerated by the 28 MHz cavities and then hopefully can be re-bucketed in higher harmonic cavities. Under this circumstance, it is interesting to compare the IBS growth rates and emittance evolution with these two RF configurations.

The particle distribution in 3D space are assumed Gaussian. The transverse emittance was adjusted to match the values measured by the IPMs (Ionization Profile Monitors). The longitudinal energy spread is needed with the RF configurations as inputs to define the longitudinal emittance. The RF configuration was set according to the operational conditions. The beam energy spread is given by [7]

$$\frac{\sigma_p}{p} = \left( -\frac{heV \cos \phi_s S^2 f_0^2}{18\pi\eta E_s^3 \beta^2} \right)^{1/4}$$

where  $h$  is the harmonic number,  $V$  is the cavity voltage,  $\phi_s$  is the synchronous phase,  $S$  is the 95% longitudinal emittance,  $f_0$  is the revolution frequency,  $\eta$  is the slip factor,  $E_s$  is the beam energy,  $\beta$  is the Lorentz factor.

The 95% longitudinal emittance is estimated to be  $\sim 1 \text{ eV} \cdot \text{s}$  from AGS. With 9 MHz cavity at 19 kV,  $\frac{\sigma_p}{p} = \left( \frac{120 \cdot 19000 \cdot 1^2 \cdot (78 \cdot 10^4)^2}{18\pi \cdot 4.4 \cdot 10^{-4} \cdot (24 \cdot 10^9)^3} \right)^{1/4} = 4.5 \cdot 10^{-4}$ . With 28 MHz cavity at 120 kV,  $\frac{\sigma_p}{p} = \left( \frac{360 \cdot 120000 \cdot 1^2 \cdot (78 \cdot 10^4)^2}{18\pi \cdot 4.4 \cdot 10^{-4} \cdot (24 \cdot 10^9)^3} \right)^{1/4} = 9.3 \cdot 10^{-4}$ .

The Martini model was chosen for the IBS rates calculation. The integral can be done numerically first, then one can adjust the coulomb logarithm to match the previously calculated growth rates. Thereafter, the fast calculation with coulomb logarithm can be done for the emittance evolution calculation. The Mad-X Twiss output of pp15-e0 lattice [8] was imported in BETACOOOL for simulation.

## SIMULATION VS MEASUREMENT

For proton beam at injection with 9 MHz, the settings of the BETACOOOL simulation are shown below in Fig. 1. RHIC was assumed as a well-decoupled machine in this case of simulation. The un-normalized transverse emittances were adjusted so that the normalized 95% emittances match with the measured emittances by IPMs shown in Fig. 5. The energy spread was also adjusted started from the theoretical value from the previous section so that the bunch length matches the rms value measured by Wall Current Monitor (WCM) [9]. The 28 MHz cavity was on with 100 kV voltage for Landau damping. The measurement precision of IPMs has been improved over the years. Therefore, measurement data was chosen from 100 GeV polarized proton run in 2015 for comparison with simulations.

The screenshot shows the BETACOOOL simulation interface with the following settings:

- Emittance** tab selected.
- Ion beam**: Bunched
- rms un-normalized and normalized**: 95 % = 5.991 rms
- Horizontal**: 0.0728 (input), 11.06161705 (output) [pi-mm-mrad]
- Vertical**: 0.0608 (input), 9.238273584 (output) [pi-mm-mrad]
- momentum and kinetic energy spread**
- Longitudinal**: 0.00059 (input), 0.01393765991 (output) GeV/c
- Particle number**: 2.5E11 (input), 8000 (output)
- Emittance definition**: Enclosed Percents
- Enclosed Percents**:
  - Transverse, %: 38
  - Longitudinal, %: 95
- Mean Longitudinal Invariant
- RF system** tab selected.
- Harmonic number**: 120
- RF voltage, kV**: 19
- Separatrix size, %**: 100
- Separatrix length, m**: 31.94870833
- Synchrotron tune**: 8.166491094E-5
- Non-linear RF
- Low non-linear RF**:
  - Harmonic number**: 360
  - RF voltage, kV**: 100

Fig. 1 Inputs for BETACOOOL program for the case with 9 MHz RF system, includes beam emittance in three dimensions and RF configuration.

From the simulation, the horizontal growth rate is  $3.35E-5$  [ $1/sec$ ], the vertical growth rate is  $-3.48E-6$  [ $1/sec$ ], and the longitudinal growth rate is  $1.47E-4$  [ $1/sec$ ] for the decoupled case.

The longitudinal bunch length evolution from simulation and measurement are shown both in Fig. 2 for comparison. The relative difference of bunch length between measurement and simulation is about 4% in half an hour.

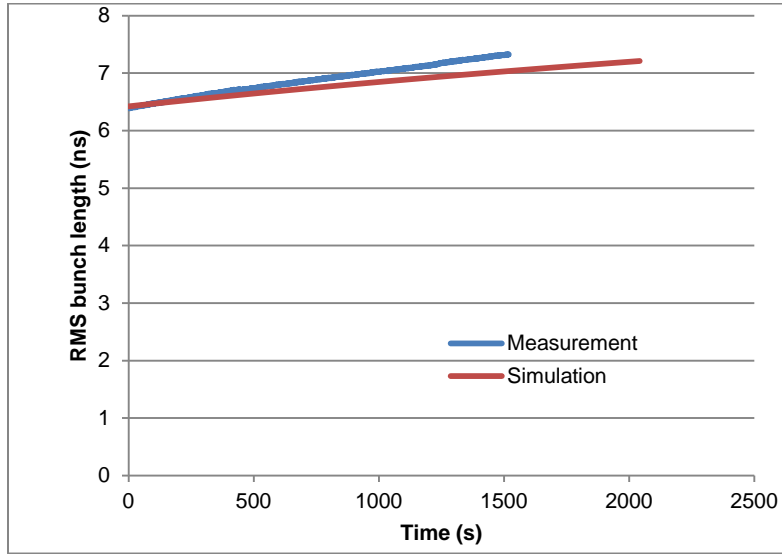


Fig. 2 Comparison between measured (fill # 18899) and simulated bunch length evolution over ~30 minutes.

The transverse emittance evolution is shown below.

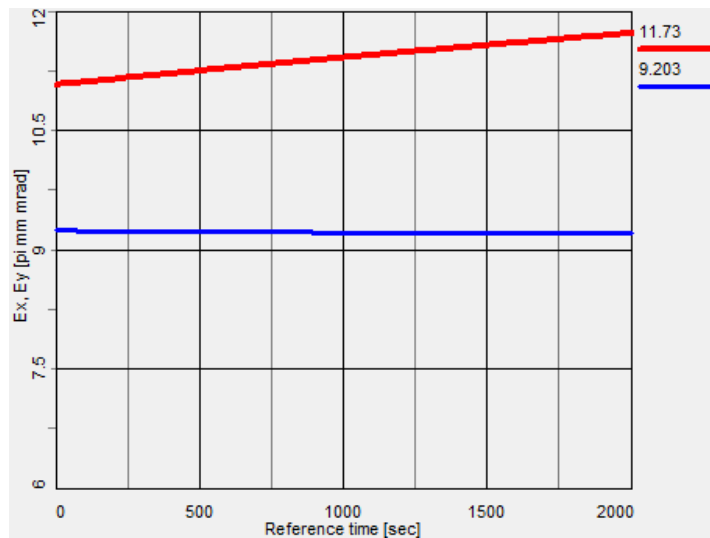


Fig. 3 Transverse emittance evolution simulated by BETACOOOL for the decoupled case with the 9 MHz RF system; the horizontal emittance is in red and vertical emittance in blue.

If one assumes full coupling between horizontal and vertical planes in the simulation, the horizontal growth rate is  $1.53\text{E-}5$  [ $1/\text{sec}$ ], the vertical growth rate is  $1.83\text{E-}5$  [ $1/\text{sec}$ ], and the longitudinal growth rate is  $1.47\text{E-}4$  [ $1/\text{sec}$ ]. With full coupled machine, the transverse growth rates are average of the rates in both planes in the case of no coupling. The evolution of transverse emittances is shown in below. The evolutions of bunch length are the same for coupled and decoupled cases.

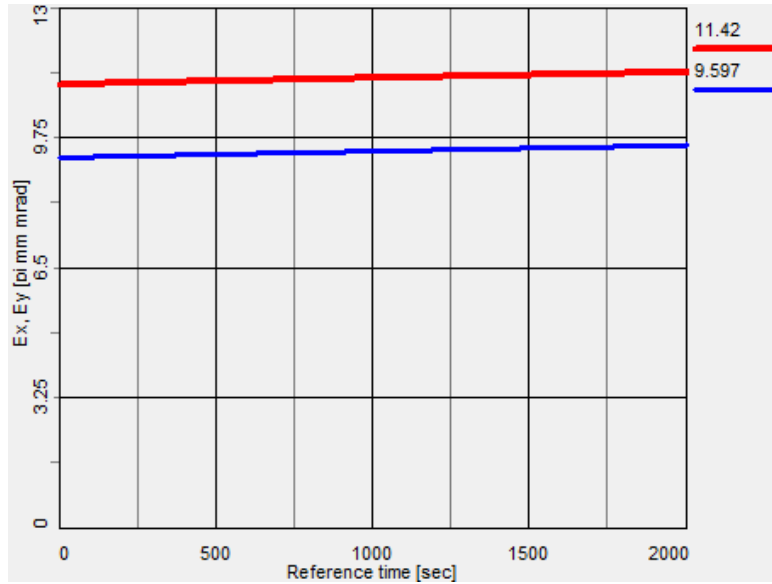


Fig. 4 Transverse emittance evolution simulated by BETACOOOL for the coupled case with 9 MHz RF system

The proton beams were sitting at injection about half an hour during the fill 18899, Run-15 100 GeV polarized proton program. The measured transverse emittances by IPMs are shown below in the upper plot of Figure 5, with the fit error in the lower plot. The IPM profile measurement and corresponding Gaussian fit are in a good agreement with a fit error around and below 0.1 as shown. In both coupled and decoupled cases, we observed some discrepancy of transverse emittance evolution between simulation and measurement.

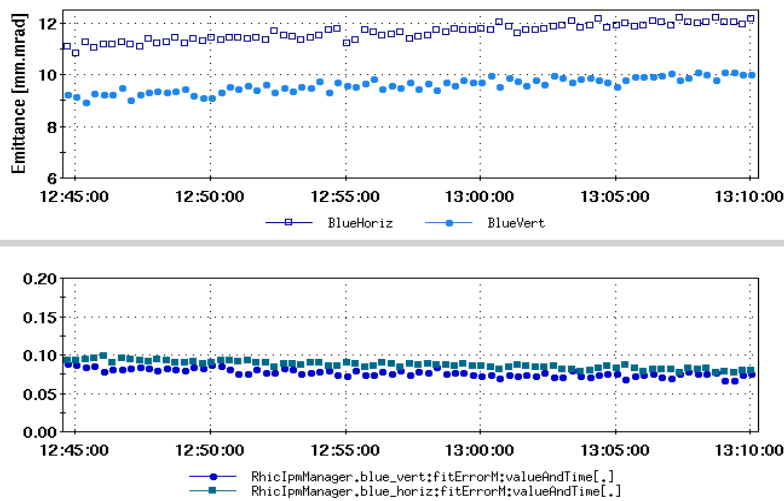


Fig. 5 Transverse emittances measured by IPMs shown in upper plot and the fit error in the lower plot, during fill 18899.

For proton beam at injection with 28 MHz RF system, the settings of the BETACOOOL simulation are shown below. Transverse emittances are kept the same as for the case with 9 MHz cavity. The energy spread was set twice as much as that for 9 MHz case based on the theoretical calculation in the previous section. Only the coupled case with 28 MHz cavity is shown here for comparison with the coupled case with 9 MHz cavity.

Emittance	Injection	Stability	Bunch	Characteristics	Ion kind	Lattice	Mean params	RF system	Reference point
Ion beam	Bunched							Harmonic number	360
				rms un-normalized and normalized 95 % = 5.9914 rms				RF voltage, kV	120
Horizontal	0.0728			11.06161705 [pi-mm-mrad]				Separatrix size, %	100
Vertical	0.0608			9.238273584 [pi-mm-mrad]				Separatrix length, m	10.64956944
				momentum and kinetic energy spread				Synchrotron tune	0.0003554757201
Longitudinal	0.00118			0.02787531981 GeV/c				<input type="checkbox"/> Non-linear RF	
				ion beam model particles				Low non-linear RF	
Particle number	2.5E11			8000				Harmonic number	360
								RF voltage, kV	100
Emittance definition	<input type="checkbox"/> (use for IBS kick)			Enclosed Percents					
				Enclosed Percents					
				Transverse, %	38				
				Longitudinal, %	95				
				<input type="checkbox"/> Mean Longitudinal Invariant					

Fig. 6 Inputs for BETACOOOL program for the case with 28 MHz RF system, includes beam emittance in three dimensions and RF configuration.

The transverse emittance evolution is shown in Fig. 7. The bunch length evolution is shown in Fig. 8.

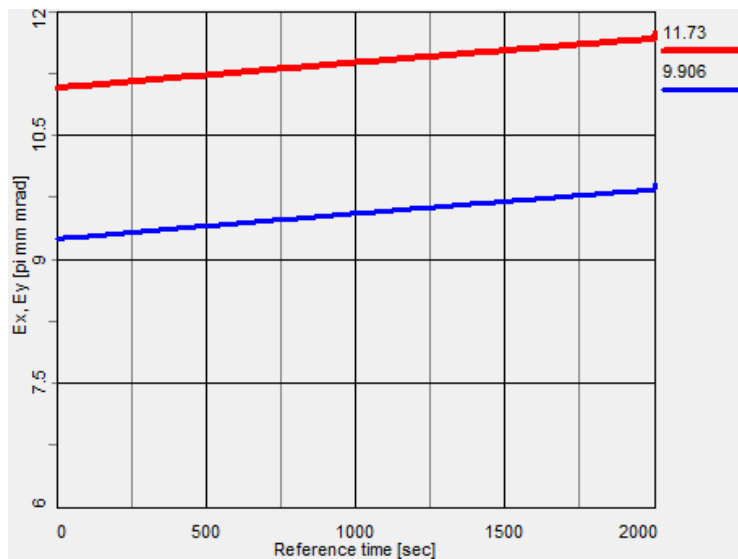


Fig. 7 Transverse emittance evolution simulated by BETACOOOL for the coupled case with 28 MHz RF system



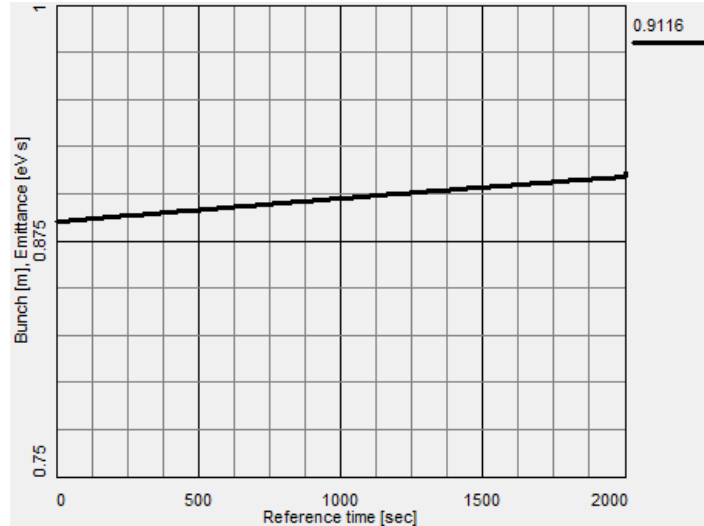


Fig. 5 The bunch length evolution simulated by BETACOOOL for the coupled case with 28 MHz RF system

From the simulation, the horizontal growth rate is  $2.85E-5 [1/sec]$ , the vertical growth rate is  $3.42E-5 [1/sec]$ , and the longitudinal growth rate is  $2.89E-5 [1/sec]$ . Compared with the growth rates with 9 MHz RF system coupled case, the transverse rates with 28 MHz RF system are higher while the longitudinal rate is lower.

## DISCUSSION

The agreement between simulated and measured emittance with IBS is worse for proton beam than for gold beam [10]. The growth rate is proportional to  $\frac{NZ^4}{A^2}$ , therefore the rates for gold beam is about 10 times higher than that of proton beam. With comparable noise contribution to the emittance evolution, IBS contribution is dominating for Au beam, but not for proton beam.

Compared with previous efforts [11,12] to bench-mark the simulation with measurements, the measurements of transverse emittance and bunch length are more robust and precise.

Experimental studies [13] on beam emittance in RHIC has been carried out in the past and will resume in the future.

## SUMMARY

With 9 MHz cavity, the simulated longitudinal growth rate is higher, the transverse growth rate is lower than those with 28 MHz cavity due to the smaller energy spread. The coupling of RHIC is visible from the evolution of transverse emittance measured by IPMs. The growth rates from simulation are generally lower than those from measurements, especially so in transverse planes.

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