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In this note the nominal parameters for aluminum ions in Booster, AGS, and RHIC are given for the PP-on-Al setup in RHIC.

The setup parameters are summarized in Sections 13, 14, 15.

1 Mass

An aluminum ion with charge eQ has $N = 14$ neutrons, $Z = 13$ protons, and $(Z - Q)$ electrons. Here Q is an integer and e is the charge of a single proton. The mass number is

$$A = N + Z = 27. \tag{1}$$

This is also called the number of nucleons. The mass energy equivalent of the ion is

$$mc^2 = am_uc^2 - Qm_ec^2 + E_Q \tag{2}$$

where [1, 2]

$$a = 26.98153863(12) \tag{3}$$

is the relative atomic mass of the neutral aluminum atom,

$$m_uc^2 = 931.494061(21) \text{ MeV} \tag{4}$$

is the mass energy equivalent of the atomic mass constant, and

$$m_ec^2 = 0.510998928(11) \text{ MeV} \tag{5}$$

is the electron mass energy equivalent. The binding energy E_Q is the energy required to remove Q electrons from the neutral atom. As shown in

Section 18 this amounts to 0.322 KeV for $Q = 5$ and 6.592 KeV for the fully stripped ion ($Q = 13$).

Thus the mass energy equivalents for the Al5+ and Al13+ ions are

$$mc^2(\text{Al5+}) = 25.1305883178 \text{ GeV} \tag{6}$$

and

$$mc^2(\text{Al13+}) = 25.1265065964 \text{ GeV}. \tag{7}$$

2 Kinetic Parameters

In a circular accelerator the ion moves along an orbit of circumference C with revolution frequency f. The radius of the orbit is defined to be $R = C/(2\pi)$. The velocity of the ion is then

$$v = 2\pi R f. \tag{8}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma, \quad W = mc^2(\gamma - 1) \tag{9}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1-\beta^2}. \tag{10}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kcp/Q \tag{11}$$

where $k = 10^9/299792458$ and cp is given in units of GeV. The angular frequency is

$$\omega = 2\pi f. \tag{12}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{13}$$

where γ_t is the transition gamma. Note that as defined here, η is negative below transition and positive above transition.

3 RF Parameters

1. The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{14}$$

where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

2. The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \left|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s\right|^{1/2} \tag{15}$$

where ϕ_s is the synchronous phase.

3. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{16}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

4. The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{17}$$

where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{18}$$

5. The area of a bucket is

$$A_{\mathrm{bk}} = \alpha(\phi_s) A_S \tag{19}$$

where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} \left|(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi\right|^{1/2} d\phi. \tag{20}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [3]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{21}$$

6. The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s} \left\{ \frac{-h\eta_s eQV_g \cos\phi_s}{2\pi E_s} \right\}^{1/2} \tag{22}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g \cos\phi_s$, while measurement of dB/dt gives a value for $V_g \sin\phi_s$. These two can be used to obtain V_g and ϕ_s.

7. Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{23}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{24}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\} \tag{25}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{ \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \right\}. \tag{26}$$

If $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{27}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{28}$$

8. The half-height of a bunch matched to a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \, |\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2} . \tag{29}$$

9. The area of a bunch matched to a bucket is

$$A_{\rm b} = F(\phi_s, \Delta\phi)\, A_S \tag{30}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8}\int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2}\, d\phi. \tag{31}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2\, |\cos\phi_s|^{1/2} . \tag{32}$$

4 Ring Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \delta C$	m
C_E	$C_a/4$	$4(C_r + \delta C)/19$	$C_r + \delta C$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.832	8.5	22.89	

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{33}$$

meters. δC is the shift (if any) of the RHIC orbit circumference from the design value C_r. Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [4, 5]. The radius of curvature ρ in the Booster and AGS main dipoles is given in Refs. [4, 5, 6]. The RHIC ring parameters are taken from Ref. [7] and from MAD runs by Steve Tepikian.

5 Initial Conditions and Assumptions

1. The revolution frequency of the Al5+ ion (from EBIS) at Booster injection is 96.640 kHz. The radius is taken to be the nominal radius $C_b/(2\pi)$.

2. The revolution frequency of the Al5+ ion at Booster extraction is $f = 654.157$ KHz. The radius is taken to be one fourth the nominal AGS radius $C_a/(2\pi)$. The corresponding magnetic rigidity is $B\rho = 8.22102484908$ Tm. The rigidity that can be extracted from Booster into the BTA line is limited by the F3 extraction kicker. The advertised limit is $B\rho = 9.5$ Tm [8].

3. The set revolution frequency of the Al13+ ion at AGS injection is $f = 163.125$ KHz. This gives an energy loss of 0.63 MeV per nucleon in the BTA stripper. (This number was obtained by Peter Thieberger using the SRIM code.)

4. The magnetic rigidity of the Al13+ ion at RHIC injection is taken to be $B\rho = 81.11378003$ Tm.

5. The circumference shift in RHIC yellow ring at Al13+ injection is $\delta C = -1.9045$ mm [9].

6. The circumference shift in RHIC yellow ring at PP injection is $\delta C = -5.1767$ mm [9].

7. The circumference shift in RHIC yellow ring at Store is $\delta C = -5.120372$ mm [9].

8. The magnetic rigidity of the Al13+ ion in RHIC at PP injection is $B\rho = 162.954345233$ Tm [9].

9. The magnetic rigidity of the Al13+ ion at RHIC store is $B\rho = 683.161277225$ Tm [9].

The parameter values given in the following sections are calculated with these initial conditions and assumptions. For many of the parameters more digits are given than would be warranted by the precision with which the parameter could be measured; this is done for computational convenience.

6 Longitudinal Emittance of Unbunched Beam in Booster at Injection

The longitudinal emittance per nucleon of unbunched beam in Booster at injection is

$$\mathcal{E} = \frac{2}{A}\,\Delta E \Delta T \tag{34}$$

where ΔE is the energy half-width of the beam,

$$\Delta T = \frac{1}{f} = \frac{2\pi R}{c\beta} \tag{35}$$

is the revolution period, and A is the number of nucleons. Using the differential relation

$$\Delta E = \beta^2 \frac{\Delta p}{p}\, mc^2\gamma \tag{36}$$

we have

$$\mathcal{E} = \frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \tag{37}$$

where Δp is the momentum half-width of the unbunched beam. Taking

$$f = 96.640 \text{ kHz} \tag{38}$$

gives

$$\Delta T = 10.3476821192\ \mu\text{s} \tag{39}$$

$$\beta = 0.0650450626079, \quad \gamma = 1.00212216641 \tag{40}$$

and

$$\frac{2\beta^2\gamma}{f} = 87.7450074295 \text{ ns.} \tag{41}$$

For Al5+ ions we have

$$\frac{mc^2}{A} = 0.930762530291 \text{ GeV} \tag{42}$$

which gives

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A} = 81.6697651354 \text{ eV s} \tag{43}$$

Taking the fractional momentum half-width to be at most

$$\frac{\Delta p}{p} = 0.001 \tag{44}$$

then gives longitudinal emittance (per nucleon)

$$\mathcal{E} = 0.0816697651354 \text{ eV s}. \tag{45}$$

The bunch merging schemes used in Booster and AGS put 4 Booster loads into each AGS bunch. The minimum longitudinal emittance of that bunch would then be

$$4\mathcal{E} = 0.32668 \text{ eV s}. \tag{46}$$

7 Minimum RF Voltage Required to Capture the Unbunched Beam

In order to capture the unbunched beam into h buckets we must have RF voltage V_g (i.e. total gap voltage per turn) such that

$$\mathcal{E} \leq \frac{hA_S}{A} \tag{47}$$

where A_S is given by (14). Thus we must have

$$\frac{2\beta^2\gamma}{f}\,\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{cA}\left\{\frac{2eQV_gE}{\pi h|\eta|}\right\}^{1/2} \tag{48}$$

which gives

$$2\beta^2\gamma\left(\frac{2\pi R}{c\beta}\right)\frac{mc^2}{A}\,\frac{\Delta p}{p} \leq \frac{8R}{c}\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\frac{mc^2}{A}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{49}$$

$$\beta^2\gamma\left(\frac{\pi}{\beta}\right)\frac{\Delta p}{p} \leq 2\left(\frac{2\gamma}{\pi h|\eta|}\right)^{1/2}\left(\frac{eQV_g}{mc^2}\right)^{1/2} \tag{50}$$

$$\beta^2\gamma^2\pi^2\left(\frac{\Delta p}{p}\right)^2 \leq \left(\frac{8\gamma}{\pi h|\eta|}\right)\left(\frac{Q}{mc^2}\right)eV_g \tag{51}$$

and

$$\frac{1}{8}\,h\pi^3\beta^2\gamma\,|\eta|\left(\frac{mc^2}{Q}\right)\left(\frac{\Delta p}{p}\right)^2 \leq eV_g. \tag{52}$$

Here

$$h = 4 \tag{53}$$

and taking revolution frequency

$$f = 96.640 \text{ kHz} \tag{54}$$

we have

$$\eta = -0.952939329734 \tag{55}$$

and

$$\frac{1}{8}\, h\pi^3\beta^2\gamma\, |\eta| = 0.0626374709945. \tag{56}$$

For the Al5+ ion we have mass energy equivalent per unit charge

$$\frac{mc^2}{Q} = 5.02611766357 \text{ GeV}. \tag{57}$$

Taking fractional momentum half-width

$$\frac{\Delta p}{p} = 0.001 \tag{58}$$

then gives

$$314.823299367 \text{ volts} \le V_g. \tag{59}$$

8 Inflector Voltage

At Booster injection, the voltage V_I required for particles with mass m, velocity $c\beta$, and charge eQ to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{mc^2}{Q}\right)\beta^2\gamma. \tag{60}$$

Here $G = 0.021$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the values of β, γ, and mc^2/Q given by (40) and (57), we obtain

$$V_I = 51.195 \text{ kV} \tag{61}$$

for Al5+ ions from EBIS. Because of an unresolved calibration problem, the actual setpoint for the inflector voltage needs to be

$$V_I(\text{setpoint}) = 51.968 \text{ kV}. \tag{62}$$

9 Booster and AGS Injection Fields

The nominal magnetic field in the Booster dipoles at injection is

$$B = (B\rho)/\rho \tag{63}$$

where $B\rho$ is given by (11) and ρ is the nominal radius of curvature. Writing

$$B\rho = \frac{10^9}{c}\left(\frac{mc^2}{Q}\right)\beta\gamma \tag{64}$$

and using the values of β, γ, and mc^2/Q given by (40) and (57), we obtain

$$B\rho = 1.09281576898 \text{ Tm}. \tag{65}$$

Here we have used the mass energy equivalent mc^2 in units of GeV and the velocity of light in units of m/s. Using

$$\rho = 13.8656 \text{ m} \tag{66}$$

we then obtain

$$B = 788.148921781 \text{ Gauss} \tag{67}$$

for Al5+ ions from EBIS.

The magnetic field is measured with a Hall probe and the Booster Gauss Clock. The Hall probe sits in the reference dipole and gives the value of the field at BT0. The Gauss Clock gives the change in field between BT0 and the time of measurement. The measured field is defined to be the field at BT0 plus the field change given by the Gauss Clock.

Similarly, the nominal magnetic field in the AGS dipoles at injection is $B = 369.12$ Gauss for the Al13+ ions.

10 BTA Stripper

The stripper used to strip aluminum ions in the BTA (Booster-To-AGS) transfer line consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 carbon foil. In Section 17 we use these surface densities to calculate the energy loss of Al13+ ions in the foils.

11 AGS Injection Septum Magnet Current

The field required in the L20 septum magnet is

$$B = (B\rho)/\rho \tag{68}$$

where $B\rho$ is the magnetic rigidity of the beam and $\rho = 18.625$ m [11] is the radius of curvature of the nominal trajectory through the magnet. The required current is given by

$$NI = gB/\mu_0 \tag{69}$$

where $N = 1$ is the number of conductor turns; $g = 0.0467$ m [11] is the magnet gap; and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

For Al13+ ions at injection, the magnetic rigidity is $B\rho = 3.15149462610$ Tm. This gives $B = 0.169208$ T and $I = 6288$ A.

For comparison, the magnetic rigidity of polarized protons at AGS injection is $B\rho = 7.205178$ Tm. This gives $B = 0.3869$ T and $I = 14380$ A.

12 AGS Injection Kicker Current

The current required in the A5 kicker is [10, 11]

$$I = \frac{B\rho}{K} \sin\phi \tag{70}$$

where

$$K = 1.8718 \times 10^{-5} \text{ Tm/A} \tag{71}$$

and

$$\phi = 3.35 \text{ milliradians} \tag{72}$$

is the desired kick angle. Using the calculated values of $B\rho$ at AGS injection we obtain a current of 564.03 A for Al13+ ions. The maximum available current is 1100 A.

13 Aluminum in Booster

Parameter	Injection	Merge porch	Extraction	Unit
Q	5	5	5	
mc^2	25.1305883178	25.1305883178	25.1305883178	GeV
W/A	1.97523297380	49.2338483823	105.879558420	MeV
cp/A	60.6699862083	306.714874793	456.407638293	MeV
E/A	0.932737763264	0.979996378673	1.03664208871	GeV
$B\rho$	1.09281576898	5.52468976347	8.22102484908	Tm
β	0.0650450626079	0.312975518550	0.440275041178	
$\gamma - 1$	0.00212216640606	0.0528962509555	0.113755716387	
η	-0.953	-0.859	-0.763	
ϵ_H (95%)	12.1π	12.1π	12.1π	mm mrad
ϵ_V (95%)	5.68π	5.68π	5.68π	mm mrad
h	4	1	1	
hf	386.560	465.000	654.157	KHz
R	$201.780/(2\pi)$	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

14 Aluminum in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	13	13	13	
mc^2	25.1265065964	25.1265065964	25.1265065964	GeV
W/A	105.232448673	6.97958516567	10.8146436224	GeV
cp/A	0.454900969049	7.85526393621	11.7083293855	GeV
E/A	1.03584380410	7.91019652110	11.7452549778	GeV
$B\rho$	3.15149462610	54.4202447696	81.11378003	Tm
β	0.439159810823	0.993055471537	0.996856126802	
γ	1.11307883582	8.5000	12.6210097366	
η	-0.793	0.0	0.00756	
ϵ_H (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
ϵ_V (95%)	$\leq 12\pi$	$\leq 12\pi$	$\leq 12\pi$	mm mrad
h	16	12	12	
hf	2.610000	4.42642071890	4.44317772988	MHz
R	128.4526	128.4526	128.457917578	m

15 Aluminum in RHIC

Parameter	Injection	PP Injection	Store	Unit
Q	13	13	13	
mc^2	25.1265065964	25.1265065964	25.1265065964	GeV
W/A	10.8146436224	22.6093571067	97.6843642139	GeV
cp/A	11.7083293855	23.5215662255	98.6105844676	GeV
E/A	11.7452549778	23.5399684621	98.6149755693	GeV
$B\rho$	81.11378003	162.954345233	683.161277225	Tm
β	0.996856126802	0.999218255683	0.999955472263	
γ	12.6210097366	25.2951657262	105.967947838	
η	-0.00437	0.000346	0.001820	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
f	77.9504864891	78.1352629775	78.1929095058	KHz
h	360	360	360	
hf	28.0621751361	28.1286946719	28.1494474221	MHz
δC	-1.9045	-5.1767	-5.120372	mm

16 Center-of-Mass Energy for Proton-Ion Collisions in RHIC

Let E_1 and P_1 be the energy and momentum of an ion circulating in RHIC, and let E_2 and $-P_2$ be the energy and momentum of the counter-circulating ion. The counter-circulating ion may be identical to the circulating one or it may be some other kind of ion.

The center-of-mass (CM) mass-energy equivalent, Mc^2, is given by the Lorentz invariant

$$M^2c^4 = (E_1 + E_2)^2 - (cP_1 - cP_2)^2 \tag{73}$$

where

$$E_1 = m_1c^2\gamma_1, \quad cP_1 = m_1c^2\beta_1\gamma_1 \tag{74}$$

and

$$E_2 = m_2c^2\gamma_2, \quad cP_2 = m_2c^2\beta_2\gamma_2. \tag{75}$$

Thus we have

$$M^2 = (m_1\gamma_1 + m_2\gamma_2)^2 - (m_1\beta_1\gamma_1 - m_2\beta_2\gamma_2)^2 \tag{76}$$

$$M^2 = m_1^2\left(\gamma_1^2 - \beta_1^2\gamma_1^2\right) + m_2^2\left(\gamma_2^2 - \beta_2^2\gamma_2^2\right) + 2m_1m_2\gamma_1\gamma_2\left(1 + \beta_1\beta_2\right) \tag{77}$$

and

$$M^2 = m_1^2 + m_2^2 + 2m_1m_2\gamma_1\gamma_2\left(1 + \beta_1\beta_2\right). \tag{78}$$

Here we have used the identities

$$\gamma_1^2 - \beta_1^2\gamma_1^2 = 1, \quad \gamma_2^2 - \beta_2^2\gamma_2^2 = 1. \tag{79}$$

For Proton-Ion collisions we take

$$\beta_1 = \beta_p, \quad \gamma_1 = \gamma_p \tag{80}$$

$$\beta_2 = \beta_I, \quad \gamma_2 = \gamma_I \tag{81}$$

and

$$m_1 = m_p, \quad m_2 = m_I/A \tag{82}$$

where A is the atomic number (number of nucleons) of the ion, and the subscripts p and I refer to the proton and ion respectively. Thus (78) becomes

$$M^2 = m_p^2 + \left(\frac{m_I}{A}\right)^2 + 2m_p\left(\frac{m_I}{A}\right)\gamma_p\gamma_I\left(1 + \beta_p\beta_I\right) \tag{83}$$

where

$$\beta_p\gamma_p = \left\{\gamma_p^2 - 1\right\}^{1/2}, \quad \beta_I\gamma_I = \left\{\gamma_I^2 - 1\right\}^{1/2}. \tag{84}$$

The mass-energy equivalent of the proton is [2]

$$m_pc^2 = 0.938272046(21) \text{ GeV}. \tag{85}$$

For the Al13+ ion, the atomic number is

$$A = 27 \tag{86}$$

and the mass-energy equivalent is

$$m_Ic^2 = 25.1265065964 \text{ GeV}. \tag{87}$$

For polarized protons, the desired values of γ_p are quantized by the relation

$$G\gamma_p = k + \frac{1}{2} \tag{88}$$

where k is a non-negative integer and

$$G = (g_p - 2)/2. \tag{89}$$

Here the proton g factor is [2]

$$g_p = 5.585694713(46) \tag{90}$$

which gives

$$G = 1.79284735650. \tag{91}$$

For polarized protons with

$$k = 198 \tag{92}$$

and Al13+ ions with

$$\gamma_I = 105.967947838 \tag{93}$$

we then have CM mass-energy equivalent

$$Mc^2 = 202.429782994 \text{ GeV}. \tag{94}$$

17 Al13+ Energy Loss in the BTA Stripper Foils

The stripper used to strip aluminum ions consists of a 6.45 mg/cm^2 aluminum foil followed by a 8.39 mg/cm^2 "glassy" carbon foil [12, 13]. We can estimate the energy loss in the foils as follows:

The kinetic energy of a proton that has the same velocity as the Al13+ ion just upstream of the aluminum foil is

$$W_p = 106.7 \text{ MeV}. \tag{95}$$

The rate of energy loss of a proton passing through the foil with kinetic energy W_p is [14]

$$-\frac{dE_p}{dx} = 5.416 \text{ MeV cm}^2/\text{g}. \tag{96}$$

The rate of energy loss of the Al13+ ion is obtained by scaling the Bethe-Bloch result for protons [15]. Thus

$$-\frac{dE}{dx} = -Z^2 \frac{dE_p}{dx} \text{ cm}^2/\text{g} \tag{97}$$

where $Z = 13$. Multiplying this by the surface density of the aluminum foil (6.45 mg/cm^2) gives

$$\Delta E_a = 0.2187 \text{ MeV per nucleon}. \tag{98}$$

This is the energy lost by the Al13+ ion upon passing through the aluminium foil. The kinetic energy of a proton that has the same velocity as the Al13+ ion just downstream of the aluminum foil is then

$$W_p = 106.5 \text{ MeV}. \tag{99}$$

The rate of energy loss of a proton passing through the carbon foil with this kinetic energy is [14]

$$-\frac{dE_p}{dx} = 6.193 \text{ MeV cm}^2/\text{g}. \tag{100}$$

Using this result in (97) with $Z = 13$, and multiplying by the surface density of the carbon foil (8.39 mg/cm^2) gives

$$\Delta E_c = 0.3252 \text{ MeV per nucleon}. \tag{101}$$

The total energy lost upon passing through both foils is then

$$\Delta E = \Delta E_a + \Delta E_c = 0.5439 \text{ MeV per nucleon}. \tag{102}$$

This agrees reasonably well with the value 0.63 MeV per nucleon obtained by Peter Thieberger using the SRIM code.

18 Atomic Binding Energies

To obtain the energy required to remove a certain number of electrons from a given atom we follow Brown and Thieberger [16] and use the tables given in Ref. [17]. Here the table numbered N gives the energy required to remove all electrons from atoms consisting of N electrons and Z protons, with Z running from N to 118. Tables are given for $N = 3$ (Lithium-like atoms) through $N = 105$ (Dubnium-like atoms).

Let E_Q be the energy required to remove the outer Q electrons from a neutral atom containing Z protons, and let $\mathcal{E}_{Z-Q}$ be the energy required to remove the remaining $Z - Q$ electrons. Then we have

$$E_Q = E_Z - \mathcal{E}_{Z-Q} \tag{103}$$

where E_Z is the energy required to remove all Z electrons. Here E_Z is obtained from the first entry of Table Z and $\mathcal{E}_{Z-Q}$ is obtained from entry Z of Table $Z - Q$.

For the case of the fully stripped aluminum ion we have $Q = Z = 13$ and

$$E_Z = 6592 \text{ eV}. \tag{104}$$

The ion binding energy E_Q is qiven in **Table 1** for various charge states Q.

Table 1: Aluminum Ion Binding Energies E_Q

Q	$Z - Q$	$\mathcal{E}_{Z-Q}$ (eV)	E_Q (eV)
4	9	6423	169
5	8	6270	322
6	7	6082	510
10	3	4831	1761

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