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Coherent Signals of the Bunched Beam at High Frequency

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Abstract

The longitudinal and transverse frequency spectra of the coherent and the Schottky signals of a multi-bunched particle beam have been investigated. Concerned with the performance of the stochastic cooling systems of the proposed RHIC collider, a comparison is provided for the amount of coherent and Schottky powers at GHz frequency range.

I. Introduction

It is observed in the bunched-beam stochastic cooling experiments that Schottky signals are often accompanied by sharply-peaked coherent signals up to high frequencies (e.g. 4–8 GHz). This phenomenon contradicts some people's intuition that the coherent contribution should become negligible at GHz frequency range.

In section II and II of this report, we study respectively the longitudinal and transverse frequency spectra of a beam of equally spaced particle bunches in synchrotrons or storage rings. The amount of coherent and Schottky powers is compared at GHz frequency range. A discussion is given in section IV.

II. Longitudinal Coherent and Schottky Signals

The observed longitudinal voltage signal is proportional to the current of the circulating beam.[1-2] For N_B bunches that are equally spaced along the circumference of the accelerator, each containing N_0 particles of electric charge qe, the current can be written as

$$I(t) = qe \sum_{m=-\infty}^{\infty} \sum_{l=1}^{N_B} \sum_{j=1}^{N_0} \delta\left(t - \frac{2\pi m}{\omega_0} - \frac{2\pi l}{N_B \omega_0} - \frac{\phi_{lj}}{h\omega_0}\right), \tag{1}$$

where $\delta(t)$ is the Dirac delta function, ω_0 is the revolution frequency of the synchronous particle, h is the harmonic number of the rf accelerating system, and ϕ_{lj} is the rf phase deviation of the jth particle in the lth bunch. The first summation being taken over an infinite number of revolutions implies that the observation time is much longer than the revolution period.

Substitute into eq. 1 the expression of synchrotron oscillation

$$\phi_{lj} = h\omega_0 \tau_{lj} \cos\left(\frac{2\pi n \Omega_{s,lj}}{\omega_0} + \varphi_{lj}^0\right), \tag{2}$$

where φ_{lj}^0, τ_{lj}, and $\Omega_{s,lj}$ are the initial phase, time amplitude, and frequency of the synchrotron oscillation of the jth particle in the lth bunch. The Fourier transform to $I(t)$ then becomes

$$I(\omega) = qe\omega_0 \sum_{m=-\infty}^{\infty} \sum_{l=1}^{N_B} \sum_{j=1}^{N_0} \sum_{k=-\infty}^{\infty} i^k \mathrm{J}_k(-\omega\tau_{lj}) \exp\left(ik\varphi_{lj}^0 - \frac{2\pi iml}{N_B}\right) \delta(\omega - m\omega_0 - k\Omega_{s,lj}), \quad (3)$$

where J_k is the Bessel function of kth order.

The coherent signals, which correspond to the $k = 0$ terms in Eq. 3, are sharply peaked at the multiple of the frequency $N_B\omega_0$. On the other hand, the Schottky signals, which correspond to all the $k \neq 0$ terms in Eq. 3, consist of sets of side-bands centered at the multiple of the revolution frequency ω_0. It is noted from Eq. 3 that the frequency spread of the kth ($k \neq 0$) side-band is $|k|$ times the synchrotron-frequency spread of the bunch. Furthermore, the number of those "significant" side-bands at the nth revolution harmonic is $2n\omega_0\hat{\tau}$, where $\hat{\tau}$ is the half time spread of the oscillation amplitude, i.e., the frequency spread of the side-bands is n times the revolution-frequency spread in the bunch.

Assume that initially particles are uniformly distributed in phase with $\langle\varphi_{lj}^0\rangle = 0$, and that side-bands of different revolution harmonic are not overlapping in the frequency range of interest. The power of the beam current at the nth revolution harmonic can then be derived from Eq. 3,

$$\langle|I(t)|^2\rangle = (qef_0)^2 \sum_{l=1}^{N_B} \left\{ \left[\sum_{j=1}^{N_0} J_0(n\omega_0\tau_{lj})\right]^2 + \sum_{j=1}^{N_0} \left[1 - J_0^2(n\omega_0\tau_{lj})\right] \right\}, \quad (4)$$

where $f_0 = \omega_0/2\pi$, and $\langle\ \rangle$ denotes the average over both the initial phase and the time that is long compared with the revolution period. Here, the relation[3]

$$J_0^2(x) + 2\sum_{k=1}^{\infty} J_k^2(x) = 1 \quad (5)$$

has been employed.

The first term in Eq. 4 represents the coherent power. At high frequencies of interest ($n\omega_0\hat{\tau} \gg 1$), this term is contributed only from the particles of small oscillation amplitude, i.e.

$$n\omega_0\tau_{lj} \leq 2.4, \quad l = 1, \cdots, N_B,\ j = 1, \cdots, N_0. \quad (6)$$

However, because this term is proportional to N_0^2, it will be shown that the coherent power is significant comparable with the Schottky power at these frequencies.

The second term in eq. 4 represents the so-called Schottky power. Compared with the coherent power, the Schottky power is linearly proportional to the number of particles N_0 in each bunch. In contrast to that of a coasting beam, the Schottky power at different revolution harmonic n is not the same for a bunched beam. Nevertheless, at high frequencies ($n\omega_0\hat{\tau} \gg 1$) it is very nearly a constant.

To obtain an order-of-magnitude estimate of the ratio of the coherent power to the Schottky power, consider the signals from the stored bunches each containing 10^9 ions (e.g. $^{197}Au^{79+}$) in the proposed RHIC[4], at the frequency range of 4–8GHz which is of primary interest for the stochastic cooling systems.[5] Suppose that the rms bunch length σ_l is 31 cm. The number of the particles in a bunch that significantly contribute to the coherent signals at frequency ω is

$$N_c \approx \frac{N_0 c}{2.5\sigma_l\omega},$$

where c is the speed of light. At the frequency of 8 GHz, the ratio of the coherent power to the Schottky power is

$$\frac{\langle |I_c(t)|^2\rangle}{\langle |I_{SH}(t)|^2\rangle} \approx \frac{N_c^2}{N_0} \approx \frac{N_0 c^2}{(2.5\omega\sigma_l)^2} \approx 5\times 10^4. \tag{7}$$

In reality, however, this ratio may be significantly less because machine imperfections and bunch-distribution fluctuation dilute the coherence.

III. Transverse Coherent and Schottky Signals

The observed transverse voltage signal is proportional to the product of the beam current and the transverse displacement, i.e. the dipole moment. Express the betatron motion as a superposition of a coherent betatron modulation and an incoherent betatron oscillation of a zero mean,

$$x_{lj} = \hat{x}_c e^{i\Psi_c} + \hat{x}_{lj} e^{i\Psi_{lj}}, \qquad l = 1,\cdots,N_B,\ j = 1,\cdots,N_0, \tag{8}$$

where the amplitude of the coherent modulation $\hat{x}_c$ is much smaller than the mean amplitude $\langle \hat{x} \rangle$ of the incoherent oscillation, and

$$\Psi_c(t) = \omega_\beta t, \;\; \Psi_{lj}(t) = \Psi^0_{lj} + \omega_\beta t + \xi \omega_\beta \phi_{lj}/\eta. \tag{9}$$

Here, Ψ^0_{lj} is the initial betatron phases, ω_β is the betatron-oscillation frequency, ξ is the machine chromaticity, and η is the frequency slip factor.

The dipole moment can be written as

$$d(t) = qe \sum_{m=-\infty}^{\infty} \sum_{l=1}^{N_B} \sum_{j=1}^{N_0} \left(\hat{x}_c e^{i\Psi_c} + \hat{x}_{lj} e^{i\Psi_{lj}} \right) \times \delta \left(t - \frac{2\pi m}{\omega_0} - \frac{2\pi l}{N_B \omega_0} - \frac{\phi_{lj}}{h\omega_0} \right). \tag{10}$$

The Fourier transform to $d_c(t)$, which is defined to be the first term of Eq.10 corresponding to the coherent modulation, is

$$\begin{aligned} d_c(\omega) &= qe\omega_0 \hat{x}_c \sum_{m=-\infty}^{\infty} \sum_{l=1}^{N_B} \sum_{j=1}^{N_0} \sum_{k=-\infty}^{\infty} i^k \mathrm{J}_k \left[-(\omega - \omega_\beta) \tau_{lj} \right] \\ &\quad \times \exp \left[ik\varphi^0_{lj} + \frac{2\pi i l (\omega_\beta - \omega + k\Omega_{s,lj})}{N_B} \right] \delta(\omega - m\omega_0 - \omega_\beta - k\Omega_{s,lj}). \end{aligned} \tag{11}$$

The coherent signals which are contributed from $k = 0$ terms are now sharply peaked at frequencies $mN_B\omega_0 \pm \omega_\beta$, with m integers. In the case that different revolution bands are not overlapping, the corresponding coherent power at the nth harmonic becomes

$$\langle |d_c(t)|^2 \rangle = (qef_0\hat{x}_c)^2 \sum_{l=1}^{N_B} \left[\sum_{j=1}^{N_0} J_0(n\omega_0\tau_{lj}) \right]^2. \tag{12}$$

Using the same argument as that of the previous section, this coherent power is found at high frequencies ($n\omega_0\hat{\tau} \gg 1$) to be contributed only from the particles of small synchrotron-oscillation amplitude (Eq. 6).

The second term $d_{SH}(t)$ of Eq.10 contributes to the Schottky signals . The Fourier transform to $d_{SH}(t)$ is

$$\begin{aligned} d_{SH}(\omega) &= qe\omega_0 \sum_{m=-\infty}^{\infty} \sum_{l=1}^{N_B} \sum_{j=1}^{N_0} \sum_{k=-\infty}^{\infty} i^k \hat{x}_{lj} \exp\left(i\Psi^0_{lj}\right) \mathrm{J}_k \left[-(\omega - \omega_\beta - \xi\omega_\beta/\eta) \tau_{lj} \right] \\ &\quad \times \exp \left[ik\varphi^0_{lj} + \frac{2\pi i l (\omega_\beta - \omega + k\Omega_{s,lj})}{N_B} \right] \delta(\omega - m\omega_0 - \omega_\beta - k\Omega_{s,lj}). \end{aligned} \tag{13}$$

The frequency spectrum of the transverse signals is similar to that of the longitudinal one except that corresponding to each revolution harmonic n, there exist two sets of side-bands located at $(n \pm \nu)\omega_0$, where ν is the fractional part of the betatron tune.

The machine chromaticity may change the profile of the signals. In the case that the chromaticity is corrected, the Schottky power at the nth harmonic can be evaluated,

$$\langle |d_{SH}(t)|^2 \rangle = (qef_0)^2 N_B N_0 \langle \epsilon_x \rangle \beta_x, \tag{14}$$

where $\langle \epsilon_x \rangle = \langle \hat{x}^2 \rangle / \beta_x$ is the mean transverse emittance, and β_x is the Courant-Snyder parameter at the location of the Schottky detector. The transverse Schottky power is a constant for different revolution harmonics.

Similar to the previous section, it can be obtained an order-of-magnitude estimate of the ratio of the coherent power to the Schottky power. Suppose that the amplitude of the coherent modulation can be controlled to be 0.1 mm, and that the mean un-normalized emittance is 0.1 πmm-mr. At the frequency of 8 GHz, the ratio of the coherent power to the Schottky power is

$$\frac{\langle |d_c(t)|^2 \rangle}{\langle |d_{SH}(t)|^2 \rangle} \approx \frac{N_c^2 \hat{x}_c^2}{6 N_0 \beta_x \langle \epsilon_x \rangle} \approx \frac{N_0 c^2 \hat{x}_c^2}{(15 \omega \sigma_l \beta_x \langle \epsilon_x \rangle)^2} \approx 10^1, \tag{15}$$

which may again be reduced by de-coherence due to machine imperfections.

IV. Discussion

Theoretical studies indicate that the contribution from the coherent signals is by no means negligible at GHz frequency range. During the stochastic cooling of the bunched beam, the required amplifier power is proportional to the measured power of the Schottky signals. In order to avoid the interference from the coherent signals, proper measures have to be taken to suppress the coherent signals at the specific frequencies.

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