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Experimental Cooling of Bunched Beams in FNAL's Accumulator Ring

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

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Accelerator Physics Technical Note No. 19

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Introduction

This report describes some experiments on stochastic cooling of bunched beams which were performed in FNAL's Accumulator ring in the beginning of September 1989. I precede that description with a rough overview of that ring in order to make the interpretation of the experimental results easier. The next section provides a very abridged parameter list. It is followed by a section on available diagnostic equipment and operating procedure. Then the experiments themselves are described. Some calculations are presented. These attempt to estimate the relevance of these experiments for RHIC. A conclusion follows.

FNAL's Accumulator Ring

Part of the information in this section is taken from chapter 5 of the "Design Report Tevatron 1 Project" issued by FNAL in September 1984. This report seems to describe the present (Sept. 89) ring rather accurately, as far as the basic machine parameters are concerned. The layout of the ring is given in Fig. 1, the behavior of the principal lattice functions is shown in Fig. 2. Note that there are three regions per superperiod in which the dispersion (α_p) is practically zero. The lattice contains sextupoles (labeled S7, S9, S10 and S12 in Fig. 2) and octupoles for chromaticity control and corrections. There are three rf systems: ARF1, ARF2 and ARF3. ARF1 provides for acceleration at h=2, ARF2 for acceleration at h=84, while ARF3, operating also at h=84, drives a heavily damped gymnastics cavity. Only ARF1 was available during my presence. A circuit diagram of these rf systems is given in Fig. 3. There are a number of cooling systems, of which we used only the so-called core cooling systems during the experiments. The information about the latter is somewhat sketchy, and measurement of their complex loop gains was one of the

unrealized goals of the experiments. I understood that there are no publications of much relevance on these systems. We used the 'original' system for cooling in longitudinal phase space because the 'new' one was not available; it works in 2–4 GHz band, while the new one should work in the 4–8 GHz band. We used the 'new' systems, which nominally operate in the 4–8 GHz band, for cooling in the two betatron phase spaces. However, I was told that their loop gains drop by a large factor (30 dB?) over this band because the transmission characteristics of the signal cables had not yet been properly compensated. The actually available band is thus much narrower, perhaps only 1 GHz, centered at 4.5 GHz. The cooling circuitry is shown in Figs. 4 and 5. Each cooling system contains a number of adjustable time delays and attenuators, its cooling rate is maximized by experimental manipulation of these parameters. In Figs. 4 and 5 the blocks labeled 'DELAY' represent delays which can be changed in discrete steps (switching lengths of transmission line) while the continuously variable 'trombones' are indicated by the symbol \nearrow , the numbers below these symbols indicate the time delay in psec inserted at that location. The attenuators are represented by the symbol \nearrow , the attached numbers reflect their settings.

FNAL is strongly interested in the cooling of bunched beams in the Tevatron in the horizontal and in vertical betatron phase spaces for protons as well as for \bar{p} 's; they are in an advanced stage of design and told me that they would know a lot more about it next year. By contrast, the first priority for RHIC should be cooling in synchrotron phase space, because this may offer a way to reduce or stop the continuous increase of bunch area with time due to intrabeam scattering. The resulting increased time average density in synchrotron phase space tends to increase the rate of growth in betatron phase space due to intrabeam scattering. This effect could be small, but cooling in betatron phase space could be used to counteract it. Cooling in betatron space is also of interest because it offers a mechanism to fight the development of transverse halos around the beam and a way to reduce the betatron emittance. Cooling while filling may allow longer filling periods than are possible without it, which is important for operation at high intensity, and cooling

during operation at low energies ($\gamma \ge 8$) may extend the luminosity life times considerably beyond the very small numbers to be expected without cooling.

Accumulator Parameters

1. General

Kinetic Energy (central orbit)	$7.94779~{ m GeV}$			
Bend field	$16.84~\mathrm{kG}$			
Magnetic bend radius (ρ)	17.46 m			
Circumference	474.07 m			
Revolution period	$1.59~\mu\mathrm{sec}$			
Superperiodicity	3			
Focusing structure	Separated function			
	FODO normal cell			
Nominal working point				
$ u_x$	6.61			
$ u_y$	8.61			
Nominal chromaticity				
$oldsymbol{\xi_{x}}$	-8.52			
ξ_y	-12.93			

${\bf Chromaticity-Corrected~Parameters}$

	$\begin{array}{c} \text{Injection} \\ \text{Orbit} \end{array}$	Stacking Orbit	$egin{array}{c} ext{Core} \ ext{Orbit} \end{array}$	$egin{array}{c} ext{Central} \ ext{Orbit} \end{array}$
Kinetic Energy (GeV)	8.02951	7.96229	7.89068	7.94779
$\Delta p/p~(\%)$	0.93	0.165	-0.690	
$ u_x$	6.616	6.611	6.614	,
$oldsymbol{ u}_{oldsymbol{y}}$	8.611	8.611	8.611	
ξ_x	2.05	1.13	-0.22	
ξ_y	0.21	0.32	0.33	
γ_t	5.37	5.42	5.50	
$1/\gamma_t^2 - 1/\gamma^2$	0.023	0.023	0.022	

Diagnostic Equipment

The behavior of the beam in the Accumulator was observed via a number of HP frequency analyzers, a Tectronix oscilloscope and a beam intensity monitor. There is also a tuneable narrow band radio receiver; tuned to one of the betatron side bands in the frequency spectrum of the signal coming from a beam position monitor its output for a given beam intensity is approximately proportional to the betatron emittance. This measure of the emittance became unreliable whenever the beam became to tightly bunched and there was some speculation on possible causes for this effect. The final amplifier in each cooling loop is a TWT (traveling wave tube), its output power is monitored and is proportional to the beam emittance for the coordinate in which the loop works. This was used as an alternative measure for the emittance after the difficulties with the receiver had made their appearance. If the output power of a TWT exceeds about 15 W its power supply trips. The locations of the spectrum analyzers in the system are indicated in Figs. 4 and 5 by the rectangular boxes enclosing an 'S'. Their outputs can be observed on screens that are part of the instruments; they are also digitized and sent to a central memory. From there they can be sent, singly or in combination, to any of the several computer display screens which are part of the accelerator control system. Different curves are distinguished by different colors. These screens can be used to show anything of interest and available to the control computer, e.g., the outputs of the radio receiver and the output powers of the TWTs. They can be hard copied, Figs. 3, 4 and 5 are examples of such hard copied screen images, as are most of the experimental results given in this note. The system is operated via conventional computer keyboard, touch panel and track-ball sets, spectrum analyzers, scope and receiver are controlled via their own (local) controls.

Experimental Procedure

The experiments were done with proton beams at $\gamma \approx 10.4$ (8.8 GeV/c), obtained directly from FNAL's booster. Production of a suitable circulating beam tended to require

considerable time, perhaps because this kind of experimentation falls outside the normal operating procedure. The ring was filled only once at the beginning of each 8 hour long session. Experimentation began with circulating currents of 6–7 mA, which dribbled down to ~ 4.5 mA at the end of the day. The loss rate was generally so low that it could not be judged by watching the multi-digit intensity monitor, except when the transverse beam dimensions became too large in the course of the experiments. This happened several times per session, during which the beam was repeatedly bunched, debunched, cooled and heated.

Experiments

The principal experimenters were J.D. Jackson and D. McGinnis, with an occasional help from J. Marriner while I acted mostly as an observer. Marriner had provided a list of experimental tasks, of which perhaps half was completed. Dave McGinnis kindly provided me with a write-up and evaluation, a copy of his report is attached. Hard copies of the raw experimental data are also attached. The first exercise after filling was to cool the coasting beam in all coordinates. This is a process that is well understood in theory and in practice. Doing this verified that everything was behaving properly and provided a reference against which the cooling of the bunched beam could be judged. Then the beam was bunched adiabatically, occasionally after reheating it in one or more coordinates, by increasing the rf voltage amplitude by hand to a predetermined value, and cooling along a particular coordinate, but primarily the vertical one, was attempted. The attached data records are grouped according to the rf voltage amplitude used while they were taken, these voltages were: 0 V (coasting beam), 100 V, 571 V, 1250 V and 3000 V.

Coasting Beam Cooling

The coasting beam run was used to make the travel times from pick up to kicker of the particles and the electrical signal equal. This was done by adjusting the appropriate trombone until the Schottky scans with the cooling loop open and closed were both sym-

metric relative to the same center line. Fig. 6a shows the initial condition, Fig. 6b the final one. In each, the curve with the large amplitude represents a scan with the cooling loop open, the small amplitude one has it closed. The difference between the two is the signal suppression in the cooling loop. Notice that these particular scans are centered on $4.499848 \text{ GHz} \approx 7156 \ f_{rev}$ and that they are 200 kHz wide (20 kHz/dev). Each took 6 sec to make. Then the beam was cooled in vertical betatron space, the development of the process is shown in Fig. 6c. Vertical emittance and beam current are plotted along the vertical axis, time along the horizontal one. The falling curve represents the changing vertical emittance, the flat one presumably the circulating beam current. The fact that the latter has an incorrect value and seems to increase with time was not noticed at the time the experiment was done and suggests a measuring error. Notice that the emittance decreases with a factor 3.3/1. in $(900-84)/60 \approx 13$ minutes. After this it seems to be close to but, not quite at its asymptotic limit. The notes on this figure are Marriner's and give an early estimate of various parameters, based on an estimated effective bandwidth of 0.5 GHz and an estimated mixing factor of 5 (1 for perfect mixing, >>1 for bad mixing). The experiment was repeated after a longitudinal cooling run. This reduces the momentum spread in the beam so that the mixing deteriorates and the mixing factor increases. Before this experiment was started the beam was heated in vertical betatron phase space by changing the time delay in the cooling link to produce positive rather than negative feedback. With the emittance at an agreed-upon value the heating was stopped by opening the loop, the beam was longitudinally cooled for 10 minutes, and recooled in betatron space. Before and after longitudinal Schottky scans are given in Figs. 6e and 6f. Note that they are centered at h=126 ($f_c=79.23598001~\mathrm{MHz}$) and that they took 20 sec to make. The reduction in momentum spread is associated with a reduction in the frequency spread and shows up as a relative reduction of the side band amplitudes, thus as a more triangular scan. The progression of the cooling in betatron space is shown in Fig. 6g, which also shows an essentially constant beam current. It may be seen that the

cooling rate is reduced relative to the first run, since now the relative change in emittance is about 2.8 in 13 minutes, rather than 3.3.

Bunched Beam Cooling

Figures 7 through 10 give the results for bunched beam cooling, each of them for a particular value of the rf voltage amplitude: Figs. 7 for $V_{rf} = 100 \text{ V}$, Figs. 8 for $V_{rf} = 571 \text{ V}$, Figs. 9 for $V_{rf} = 1200 \text{ V}$ and Figs. 10 for $V_{rf} = 3000 \text{ V}$. The sequence of sub-figures for each amplitude corresponds to the sequence of operations, which was:

- a. longitudinal Schottky scan,
- b. oscilloscope screen photo of rf voltage and beam current,
- c. cool in vertical betatron phase space,
- d. oscilloscope screen photo of rf voltage and beam current;
- e. vertical scans, one with cooling loop open, the other with cooling loop closed, for signal suppression measurement;
- f. longitudinal scan of cold beam,
- g. heat to restore initial betatron emittance,
- h. cool longitudinally during 10 minutes,
- i. oscilloscope screen photo on rf voltage and beam current,
- j. longitudinal scan of cooled beam, with longitudinally cold beam,
- k. cool in vertical betatron phase space,
- l. oscilloscope screen photo of rf voltage and beam current,
- m. longitudinal scan of cold beam,
- n. vertical scans, one with cooling loop open, the other with cooling loop closed, for signal suppression measurement.

Figures Xc and Xk, which describe the cooling process in vertical betatron phase space may show the emittance, as obtained from the radio receiver, for some cases the output power of the TWT tube, which is another measure for the emittance, and the beam current, all as functions of time. Note that the change in beam current during the 15 minute long cooling periods is negligible in all cases. The heating process was never recorded, thus there are no Figs. Xg. The set of records is incomplete for some of the rf voltages.

Some Calculations

As an assistance for judging the relevance of these results for RHIC, I compare the operating conditions in the two machines. For the accumulator I take the case of operation with 3 kV rf voltage, because it has the smallest phase spread in the bunches, for RHIC I assume operation at 30 GeV/u and at 100 GeV/u.

One reads from Fig. 10c a beam current of 7.3 mA and from the first oscilloscope photo in Fig. 10i a full width bunch length of $9/22.75 \times 1/2 \times 474.07 = 93.77 \text{ m} \Rightarrow 312.8$ nsec. The approximate revolution frequency is $c/474.07 \approx 632380$ Hz, the central frequency in the longitudinal scan of Fig. 10f, 79.235825 MHz is a harmonic h of the revolution frequency: h = int(79.235825/0.632380) = 125, thus the true revolution frequency is 79.235825/125 = 633886.6 Hz. The revolution period is then 1/633886.6 =1.5775 μ sec, instead of the 1.59 μ sec given in the parameter list. The charge per bunch is $0.5 \times 1.5775 \times 10^{-6} \times 7.3 \times 10^{-3} = 5.76 \times 10^{-9} \text{ C} \Rightarrow 5.76 \times 10^{-9} / (1.609 \times 10^{-19}) = 3.58 \times 10^{10} \text{ proposition}$ tons. The lineal charge density, averaged over the bunch length is $3.58 \times 10^{10}/93.77 =$ 3.81×10^8 protons/m, the peak lineal charge density is twice as high, or 7.6×10^8 protons/m, if I assume a triangular distribution, as suggested by the scope trace. The peak lineal density in RHIC is 1.32×10⁹, resp. 2.72×10⁹ particles/m for 10⁹ particles in gaussian, resp. triangular bunches with rms widths of 0.3 m, thus a factor 2-4 higher than that in the accumulator. This would reduce the cooling rate in RHIC by the same factor as compared with that in the accumulator, if one assumes every thing else to be equal. The absolute difference in revolution periods of particles with different momenta is $\Delta T =$ $T\eta\Delta p/p$, where $\eta = \gamma_t^{-2} - \gamma^{-2}$. This difference is inversely proportional to the required

frequency response of the longitudinal cooling loop. In the accumulator $\eta \approx 0.023,\,\mathrm{T} =$ 1.578 μ sec, while I calculate for the peak relative momentum error $\Delta p/p \approx 1.35 \times 10^{-3}$, thus $\Delta T \approx 49$ psec. A similar calculation for RHIC yields for a bunch area of 0.3 eVsec and a rms bunch length of 0.3 m $\Delta T \approx 10$ psec at $\gamma = 31.4$ and $\Delta T \approx 8$ psec at $\gamma = 101.$ (I assume that the 0.3 eVsec contains 0.95 of the particles, that the half length of the bunch in time corresponding to the rms bunch length is $2.5\times0.3/(\beta c)=2.5$ nsec and therefore the peak energy error $0.3/(\pi\times2.5\times10^{-9})=38.2$ MeV. This implies $\Delta p/p=1.3\times10^{-3}$ at $\gamma = 31.4 \text{ and } \Delta p/p = 4.1 \times 10^{-4} \text{ at } \gamma = 101, \text{ while } \gamma_t = 24.8, T = 3833.852/c = 12.78$ μ sec.) The accumulator's absolute slippage rate is larger than RHIC's by a factor 49/8 = 6. Taking the number of particles per unit slippage as a measure for the required peak frequency in the cooling loop one concludes that RHIC's peak frequency would have to be 20-40 times the accumulator's for equal performance. Such high frequencies, up to and beyond 100 GHz, do not seem feasible, thus one cannot expect a longitudinal cooling system to maintain the bunch area in RHIC at its initial value of 0.3 eVsec. The situation changes if one allows an increase of that area, something which will occur due to intrabeam scattering and which can be forced by intentional heating. Increasing the rf voltage amplitude simultaneously to preserve the bunch length, the increase in area will show up as an increased momentum spread. The slippage rate increases proportionally and the peak frequency requirement decreases. Accepting a final bunch area that is 10× the initial one reduces the peak frequency requirement by the same factor, i.e. to 8-16 GHz, which does seem feasible.

RHIC has a circumference of 3833.852 m, i.e. 3833.852/474.07 = 8.087 accumulator circumferences. If the cooling rate per turn were the same in the two rings, the rate per unit time in RHIC would be 0.123 of that in the accumulator: 10 minutes in the accumulator is equivalent to 80 minutes in RHIC.

Under the operating conditions assumed the accumulator has only 0.016 times the

longitudinal focusing strength of RHIC, so it approximates coasting beam conditions rather better than RHIC does. How this difference is reflected in the cooling rates is uncertain. I calculated the focusing strengths from the expression for the synchrotron phase advance per turn for small amplitude motion: $\Delta\psi_x\approx\{2\pi\hbar\eta eV/(\beta^2\gamma E_o)\cdot(Z/A)\}^{1/2}$ rad/turn: a synchrotron period in the accumulator takes 20119 turns for h=2, V=3000 V, while in RHIC it takes 334 turns at $\gamma=31.4$ and 343 turns at $\gamma=101$, assuming that h=2052, V=4.5 MV, $\gamma_t=24.8$, A/Z = 2.5. It would be interesting to repeat the experiments in the accumulator with the h=84, $V_{84}=126$ kV. This would increase its longitudinal focusing strength with a factor 42, which makes it 0.6 times that in RHIC. If the 126 kV is not available one could at least see how the cooling scales with harmonic number by making rungs at h=2 and h=84, keeping V/h constant in order to preserve the total bucket area hA_o and the peak energy error.

Conclusions

Stochastic cooling of bunched beams is a fascinating but difficult subject. Longitudinal cooling of bunched beams with reasonable cooling rates is obviously possible under the conditions that existed in the accumulator during this run, the rates were considerably better than I expected on the basis of an extremely preliminary and superficial exposure to the theory. It seems therefore desirable to develop a better understanding of existing theory, so that a more reliable comparison with these results may be made. It seems also desirable to take advantage of each opportunity for experimentation at FNAL's Accumulator, particularly, but certainly not exclusively, in the longitudinal coordinate.

Acknowledgment

I should like to thanks Drs. J. Marriner, J.D. Jackson and D. McGinnis for allowing me to be present at this exercise, and D. McGinnis for providing me with the hardcopies of the data and his write up.

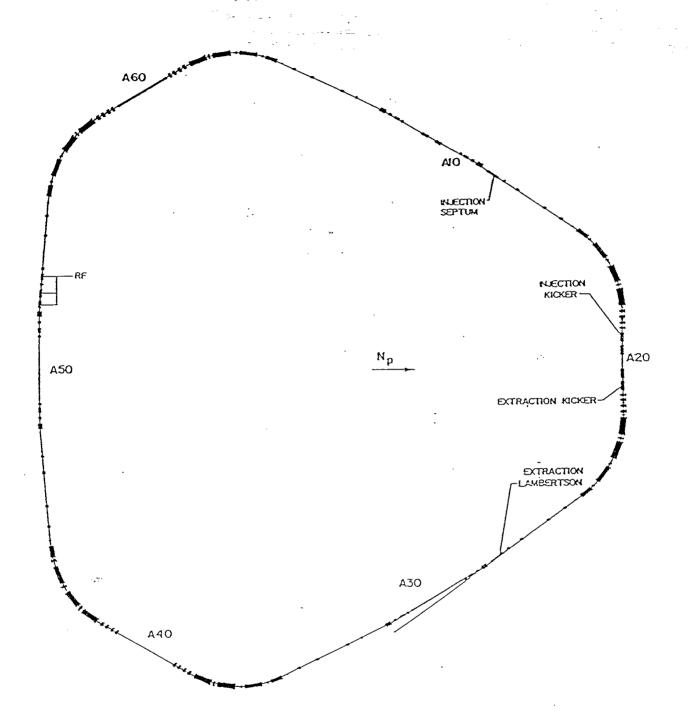


Figure 1.

ACCUMULATOR LAYOUT

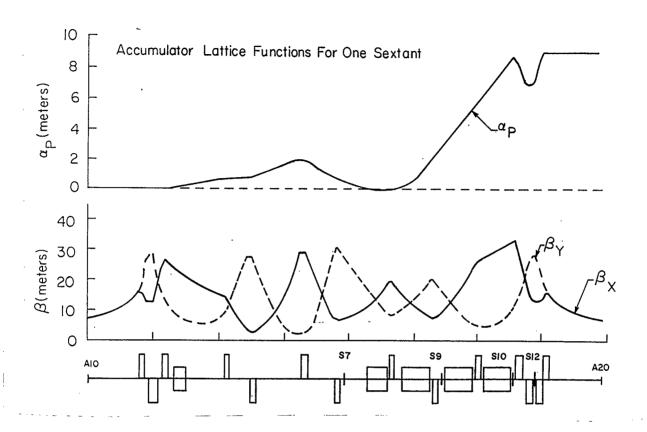


Figure 2.

SYSTEMS ACCUMULATOR RF

88-SEP-89 88:52:32

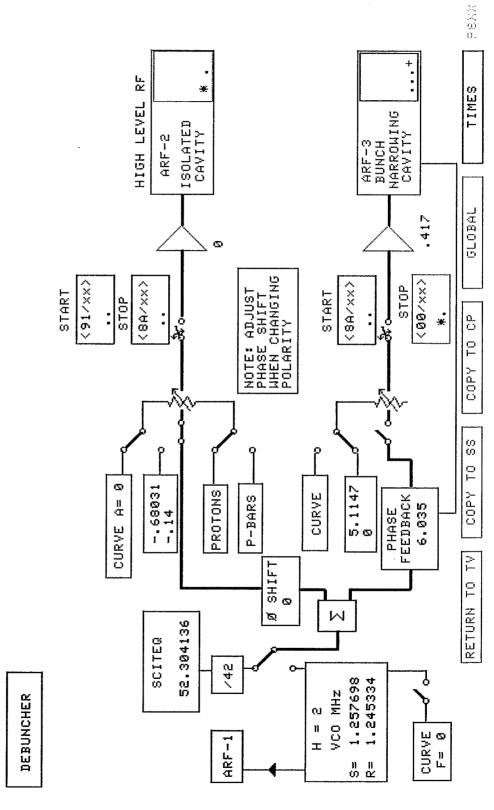
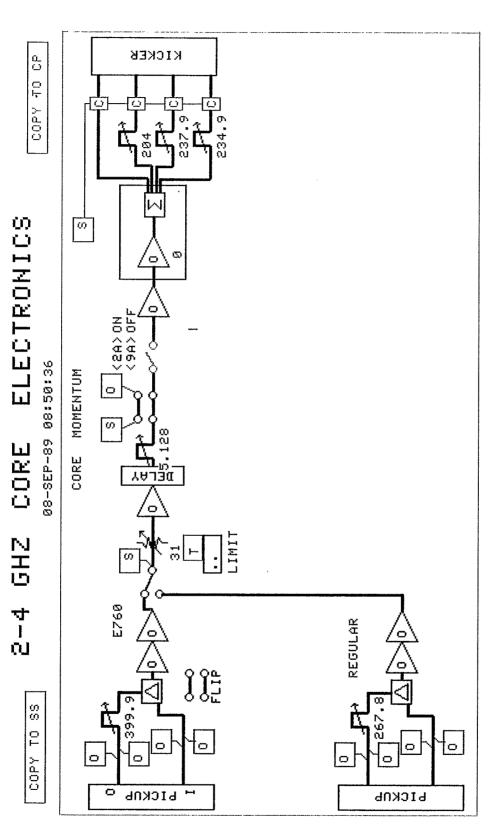


Figure 3.

COOLING STOCHASTIC



IF NO POWER CHECK TIMES

RETURN

RETURN TO TY

TIMES

4-8 GHZ

Figure 4.

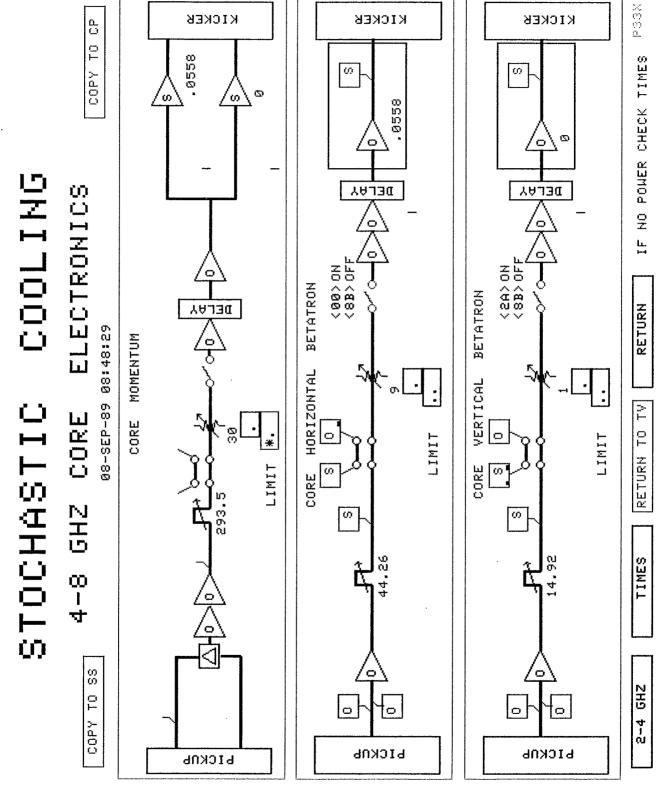


Figure 5.

89/86/89 1881	50	Vid BW 10 Hz Res BW 10 KHz	Asta = 0 dB	I'M BONE = SX, ST				8 GHz
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Figure 6a.

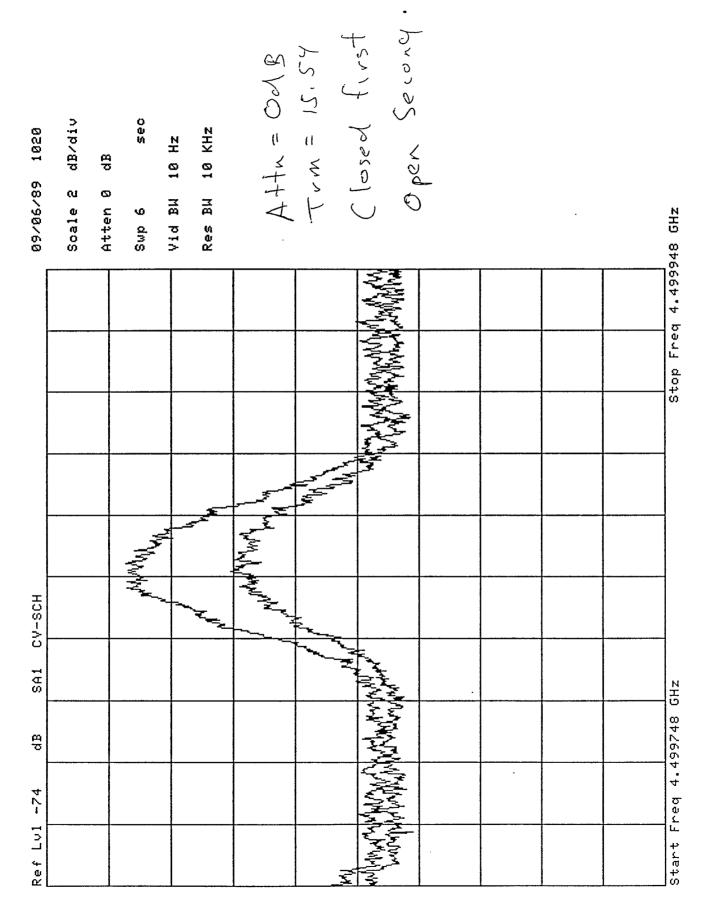


Figure 6b.

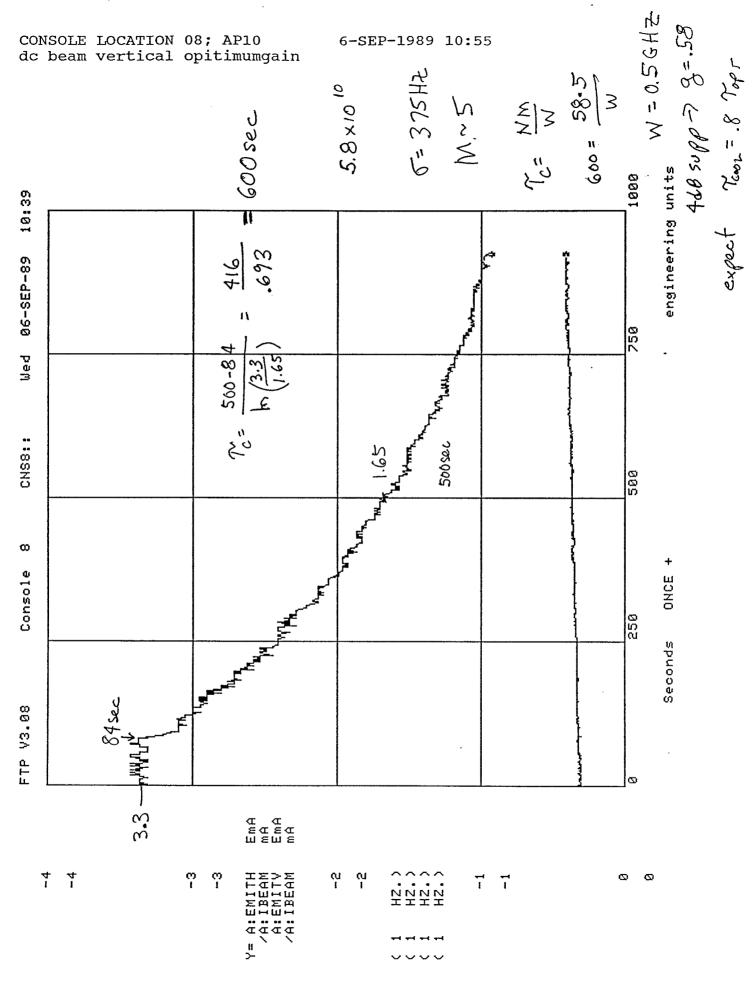


Figure 6c.

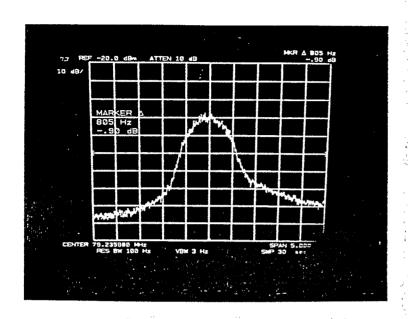


Figure 6e.

Longitudinal Shot Oby band. Boam Current = 5,8094 mA

No RF

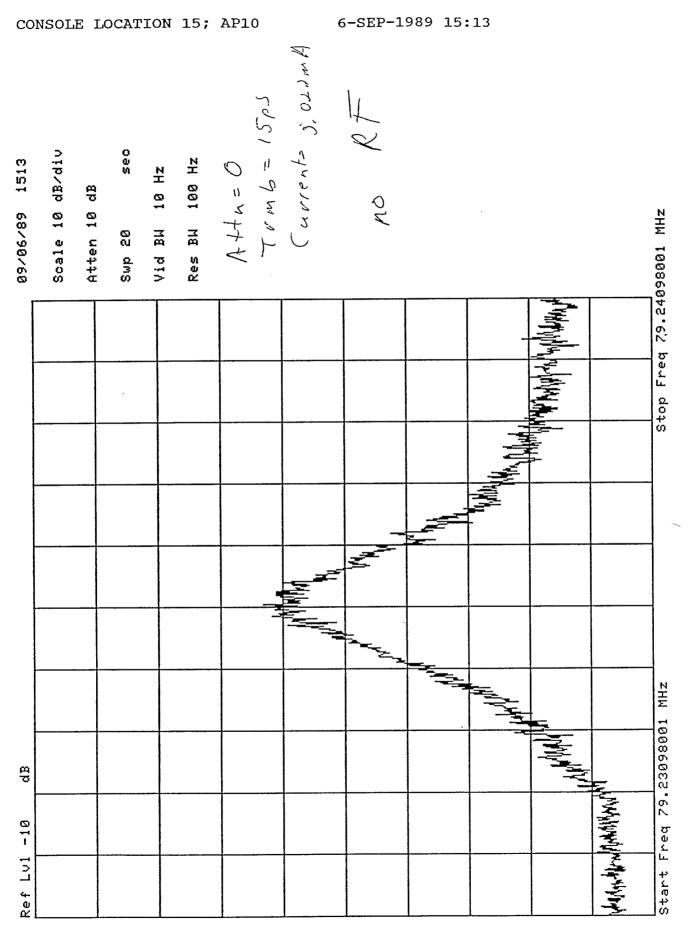
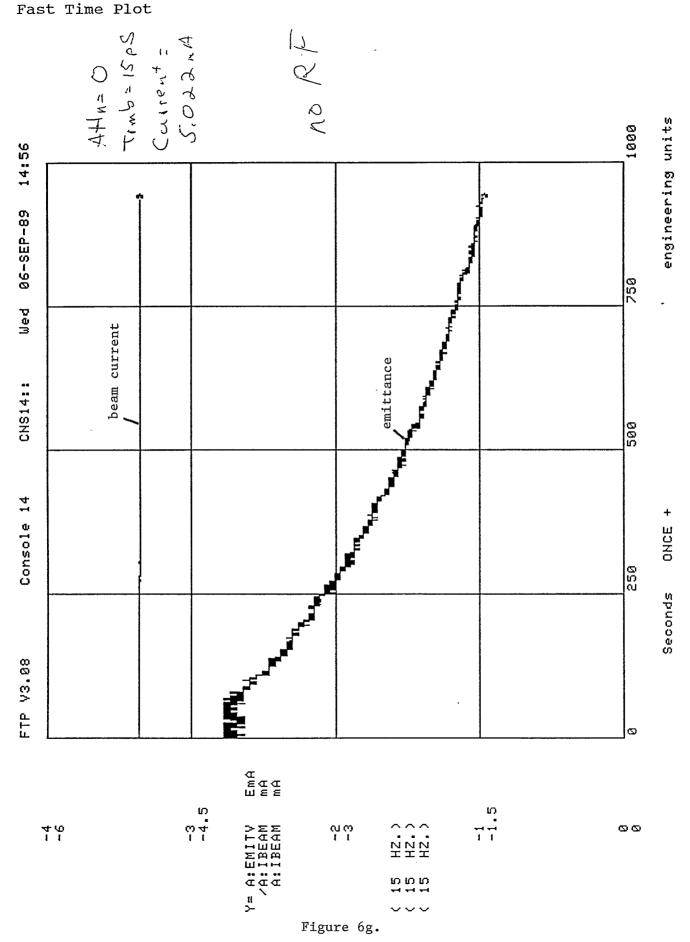


Figure 6f.



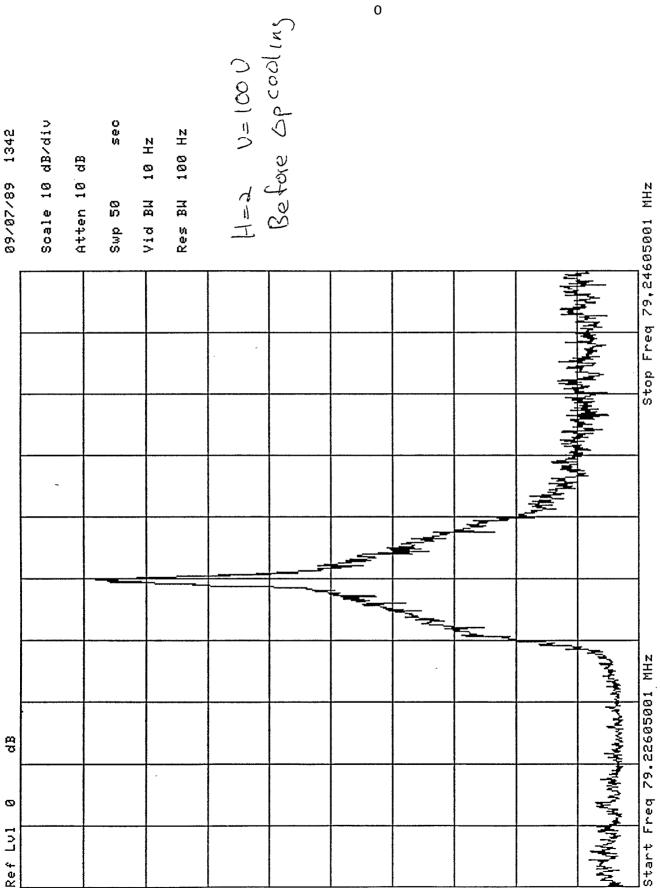
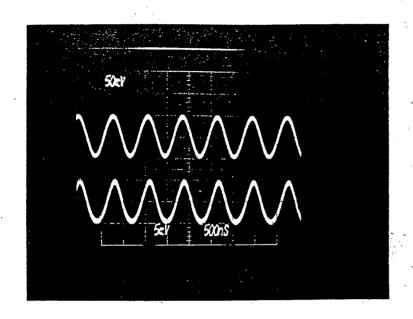


Figure 7a.



Before Δp Cooling.

Before Δp Cooling.

Figure 7b.

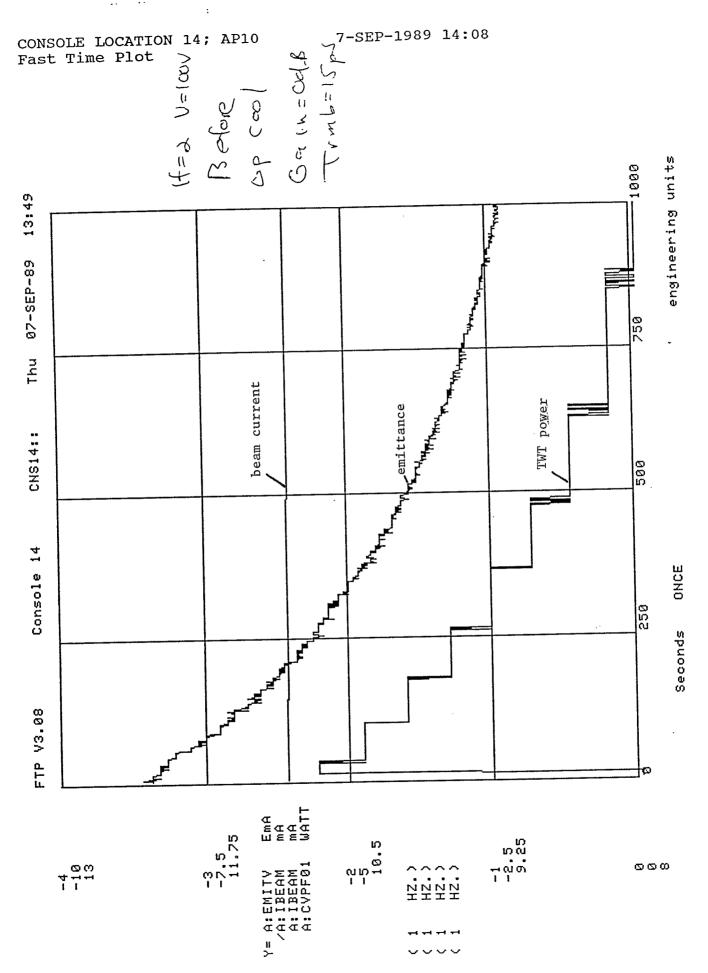


Figure 7c.

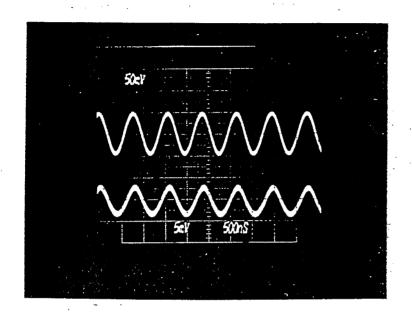


Figure 7d.

Before of cool

After verticool

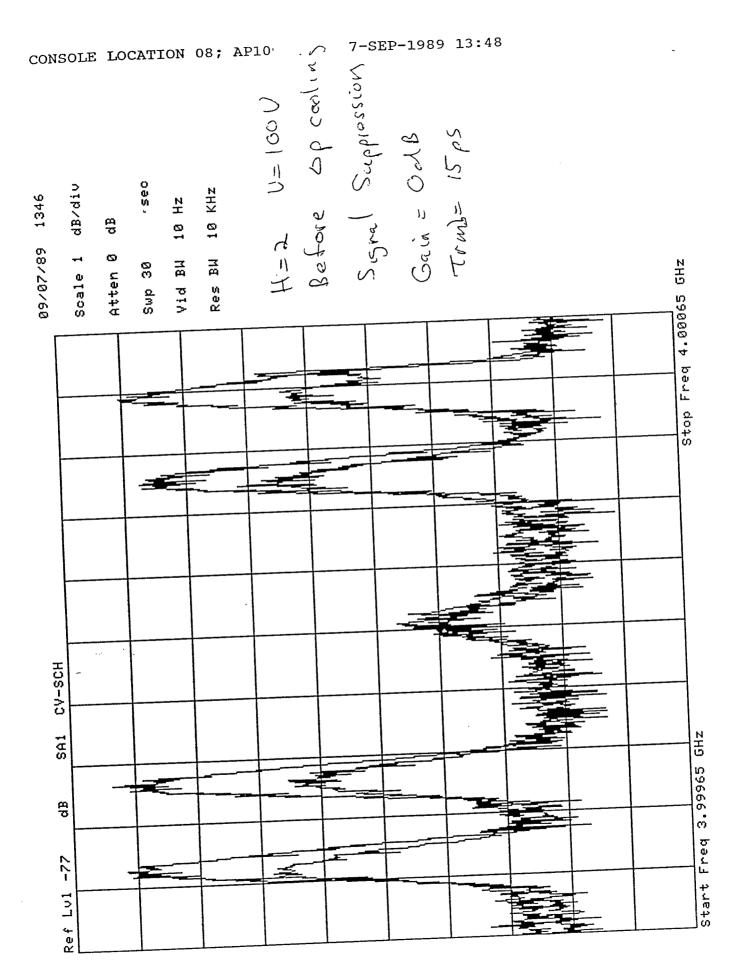


Figure 7e.

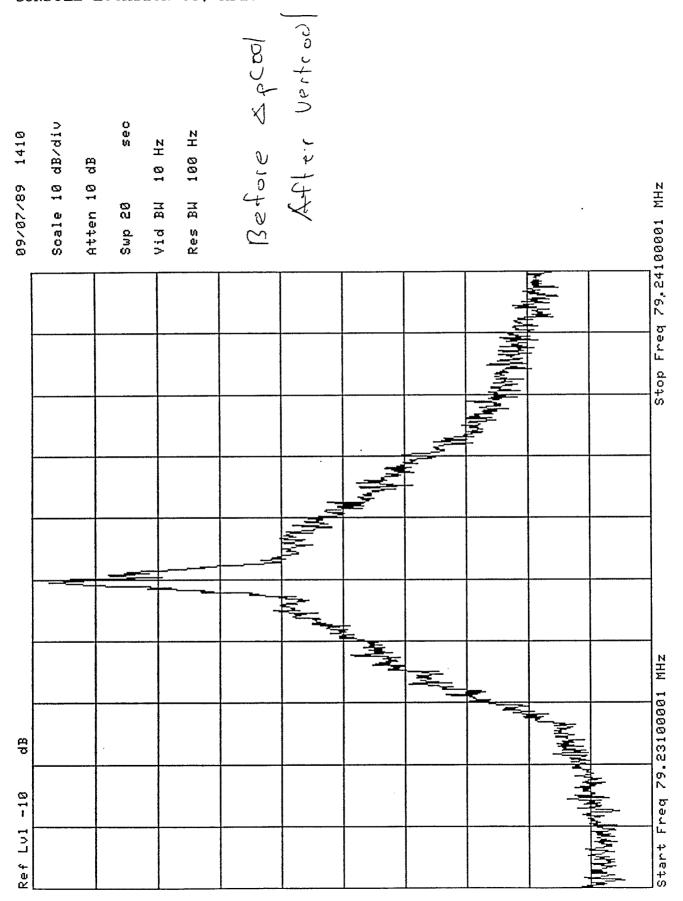


Figure 7f.

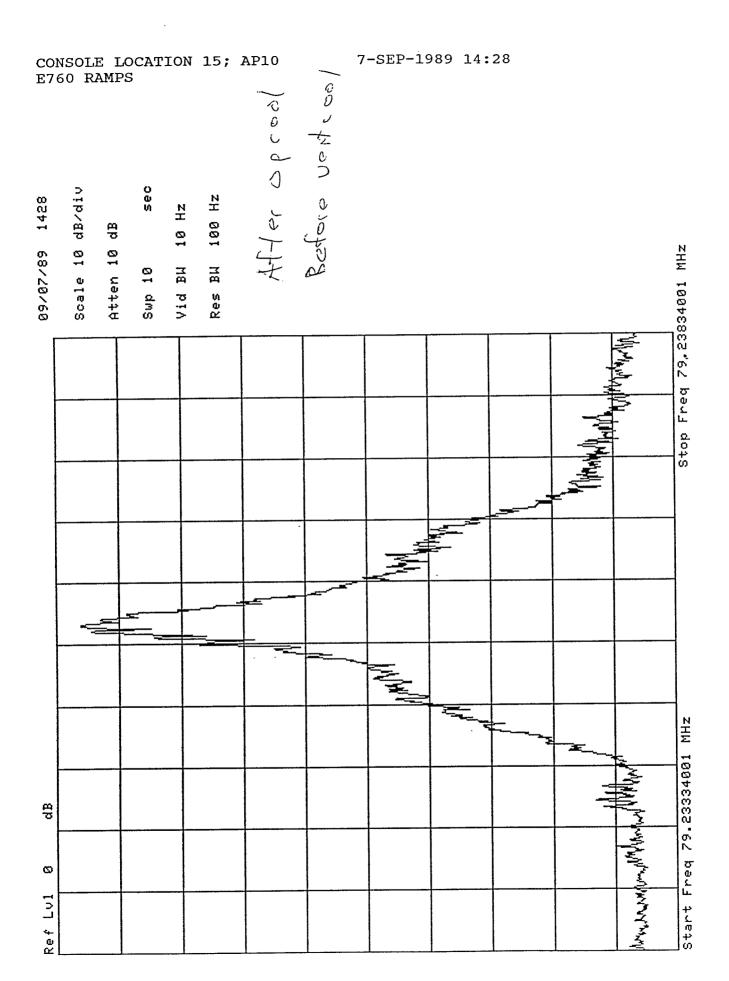


Figure 7j.

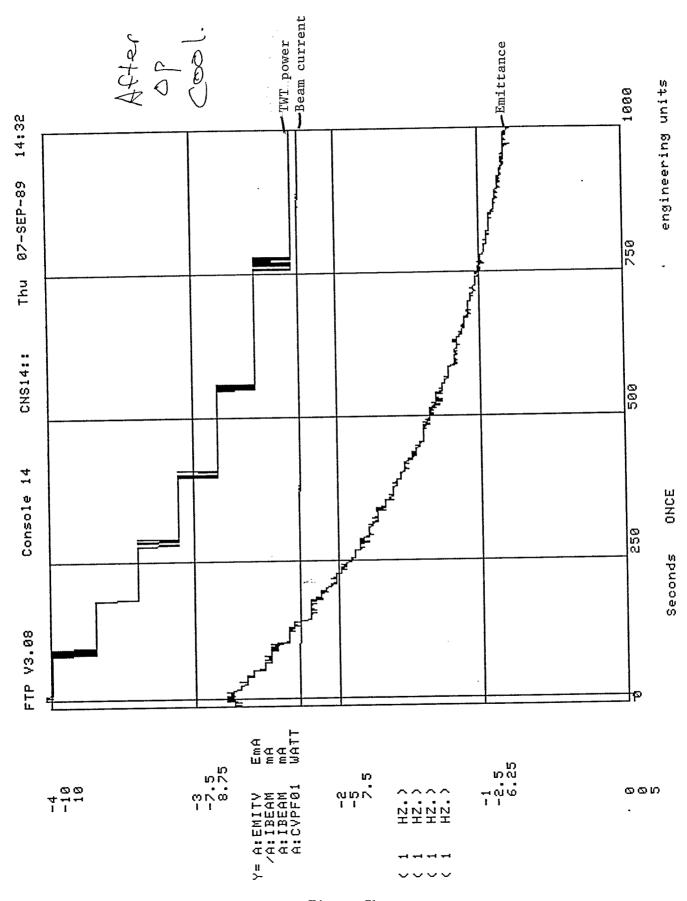


Figure 7k.

Figure 7m.

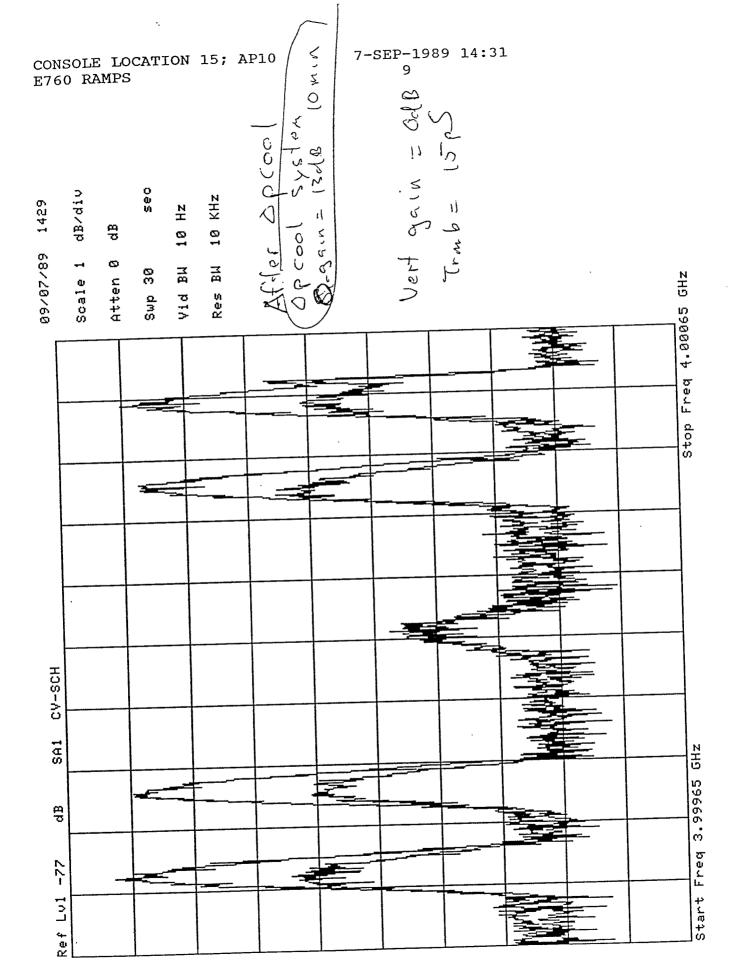
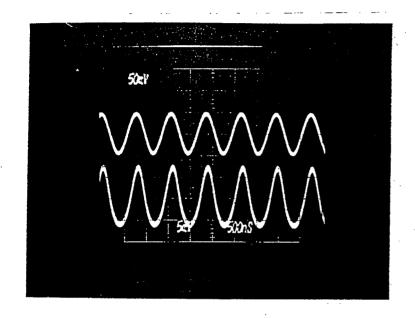


Figure 7n.

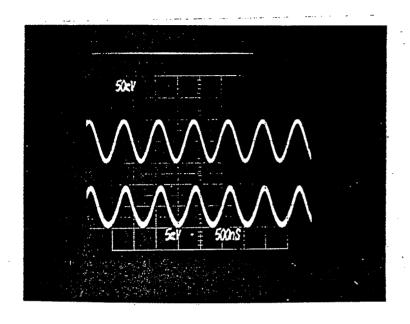


Affer ap Cool

Befor vort cool

Sp Goin = 13dB for 10min

Figure 7i.



After op copl

Batter

After Vert cool

Figure 71.

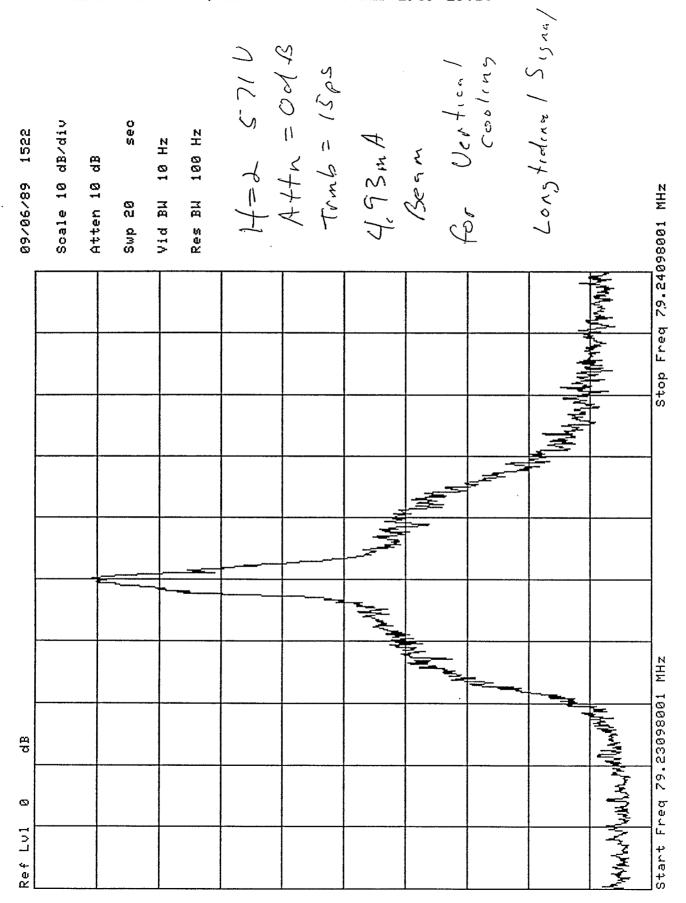
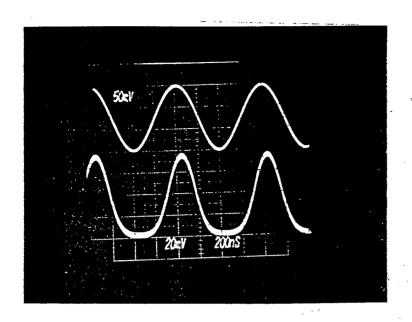


Figure 8a.



H=2 571.V for Vertical Coolin 4.93 mA Beam

Figure 8b.

Figure 8c.

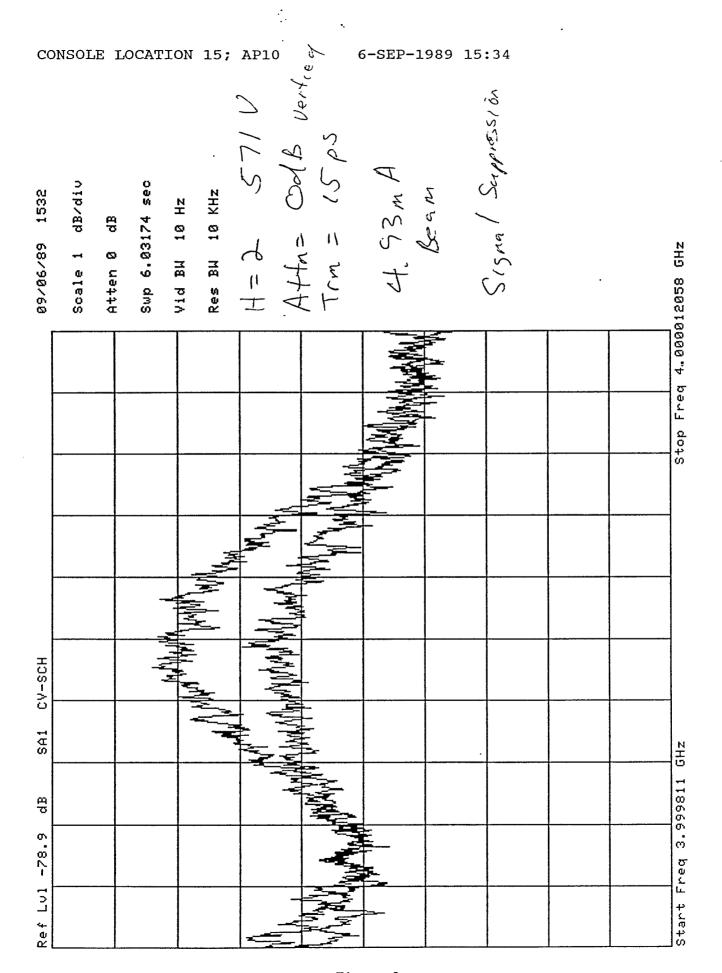
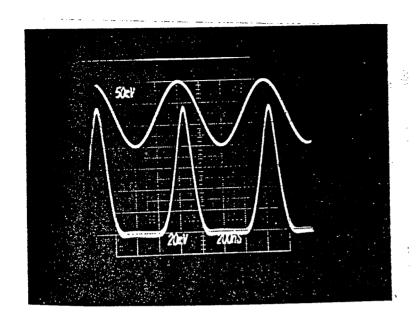


Figure 8e.



H=2 57/V

for Vertligal Cool

4.84 m A

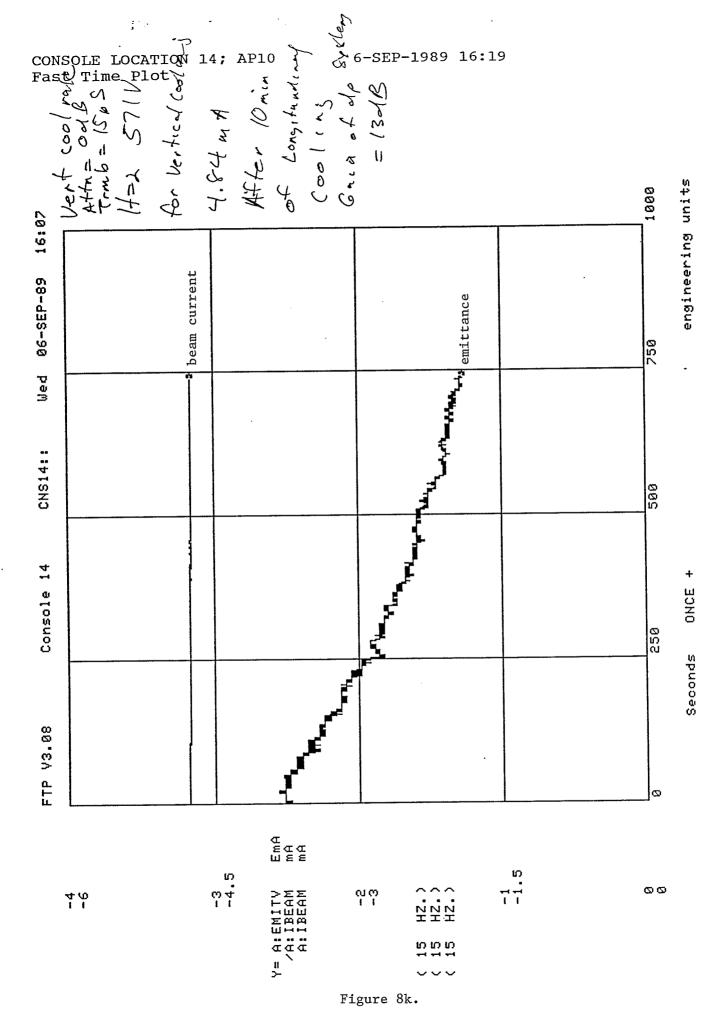
10 min of Long

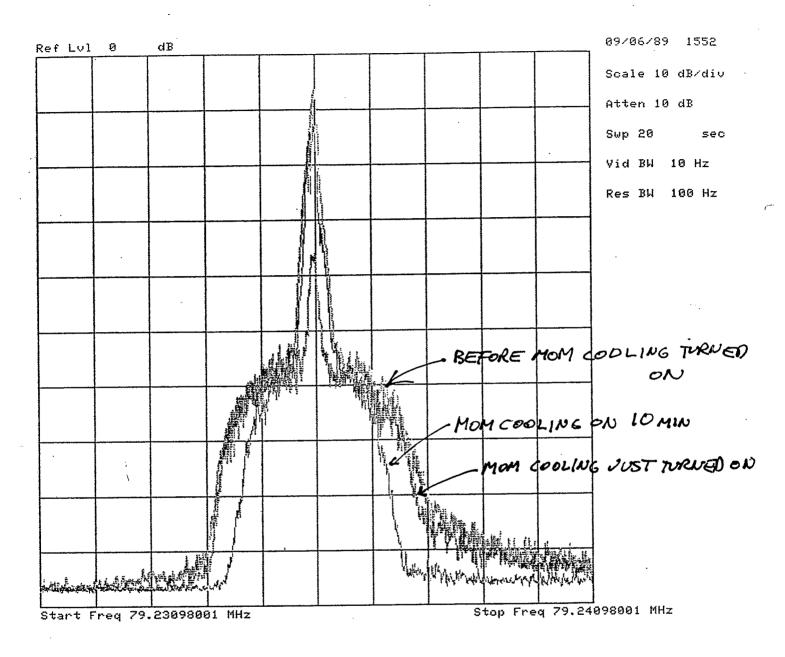
Cooling D

Gain of Long

System = 13 dB

Figure 8i.





Figures 8f,j.

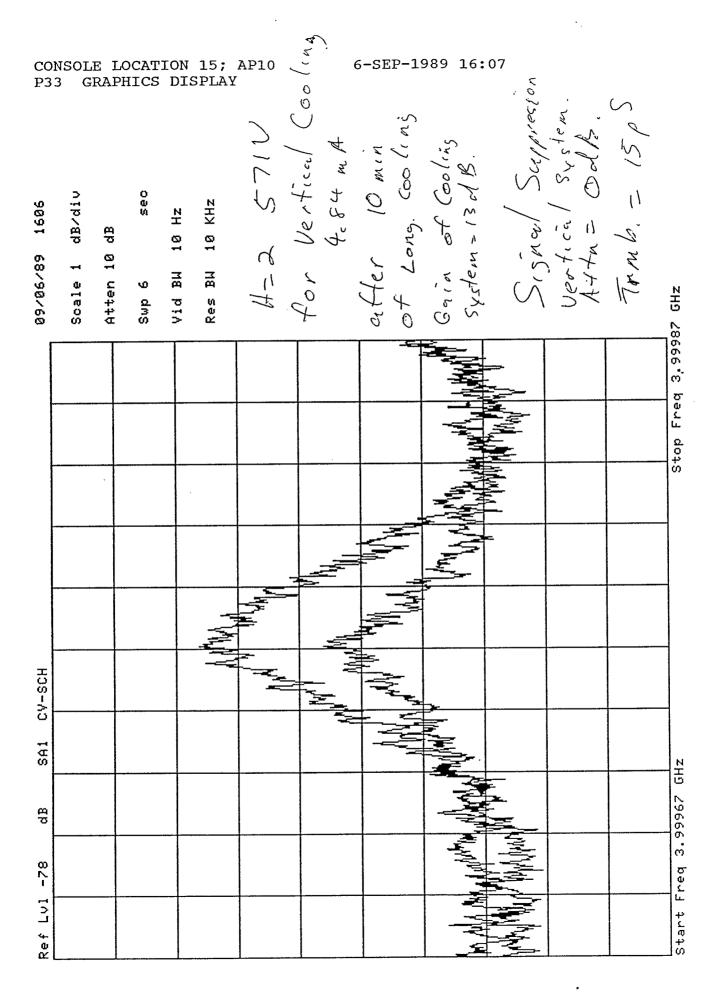


Figure 8n.

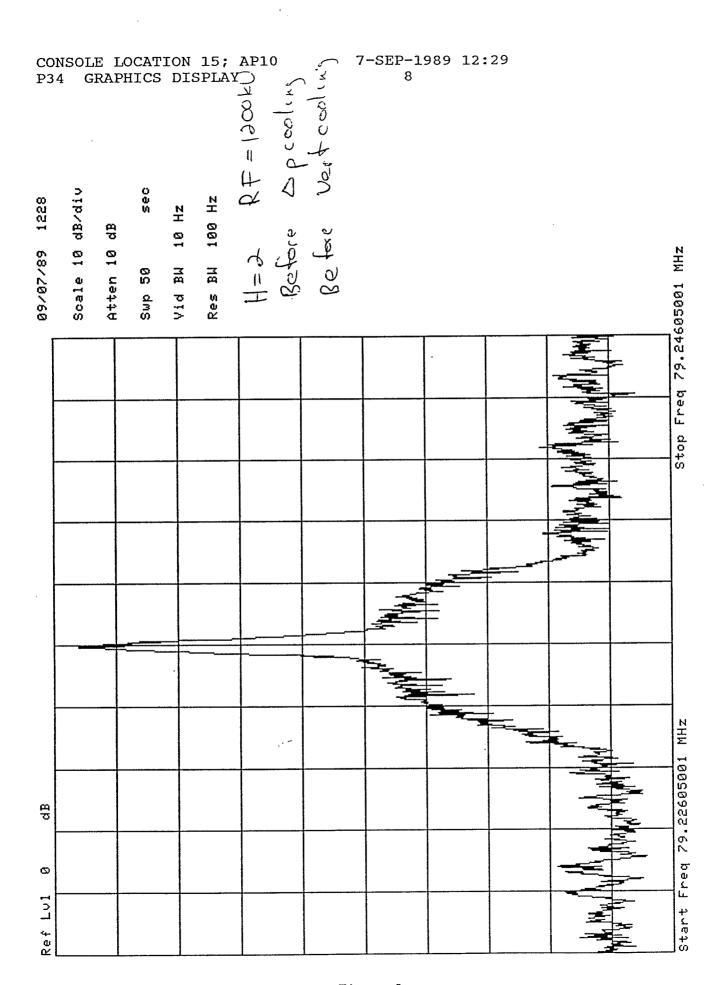
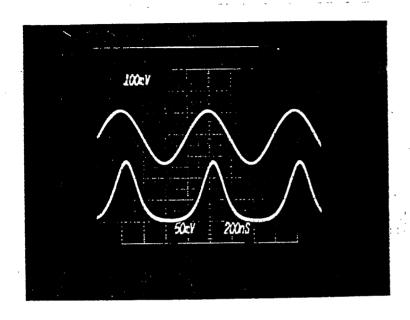


Figure 9a.



H=2 RF=1200KU

Before Opcoaling

Before Vertepoling

Figure 9b.

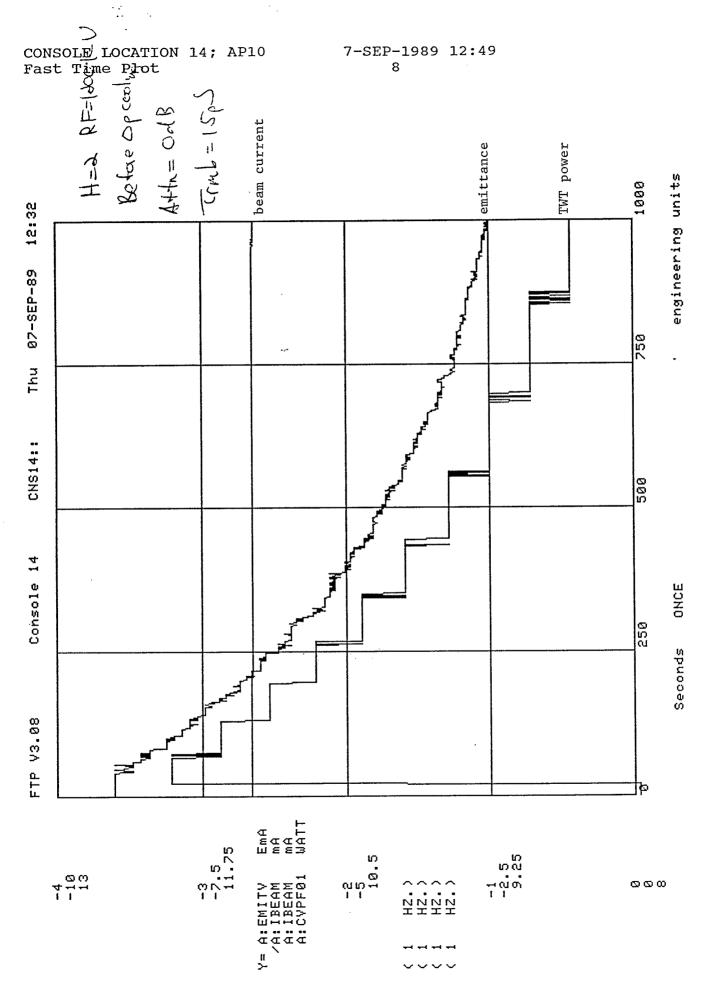
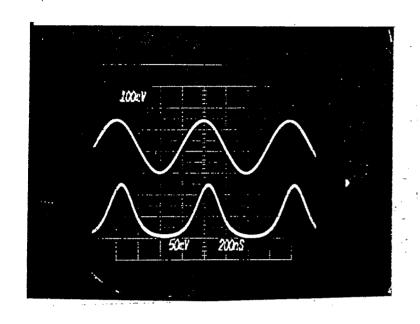


Figure 9c.



Before opcool. After Vert cooling

Figure 9d.

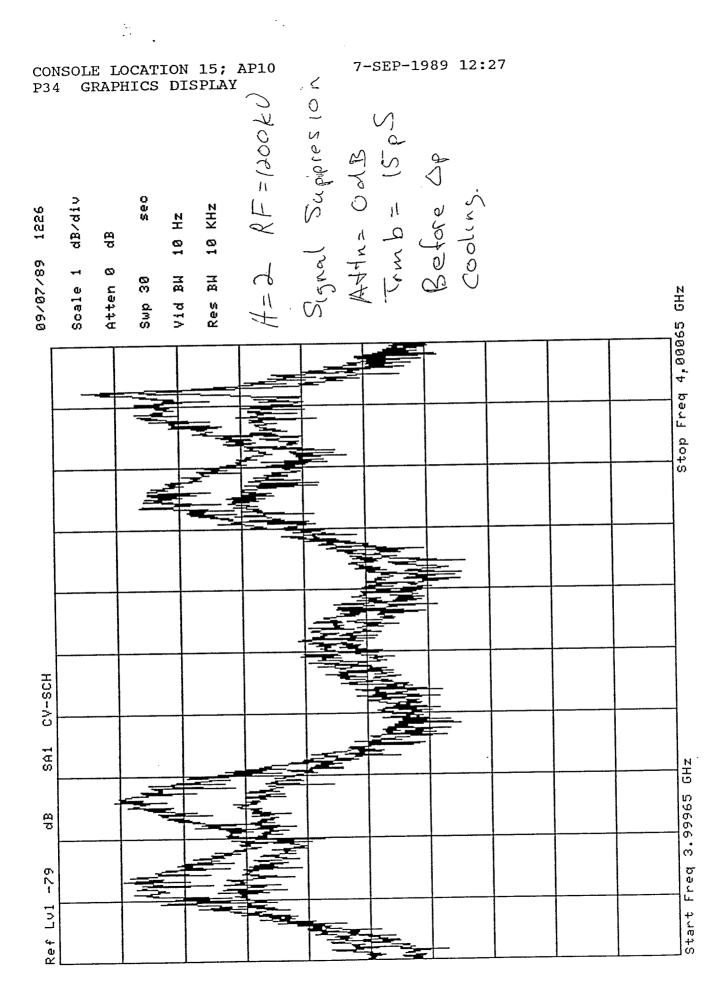


Figure 9e.

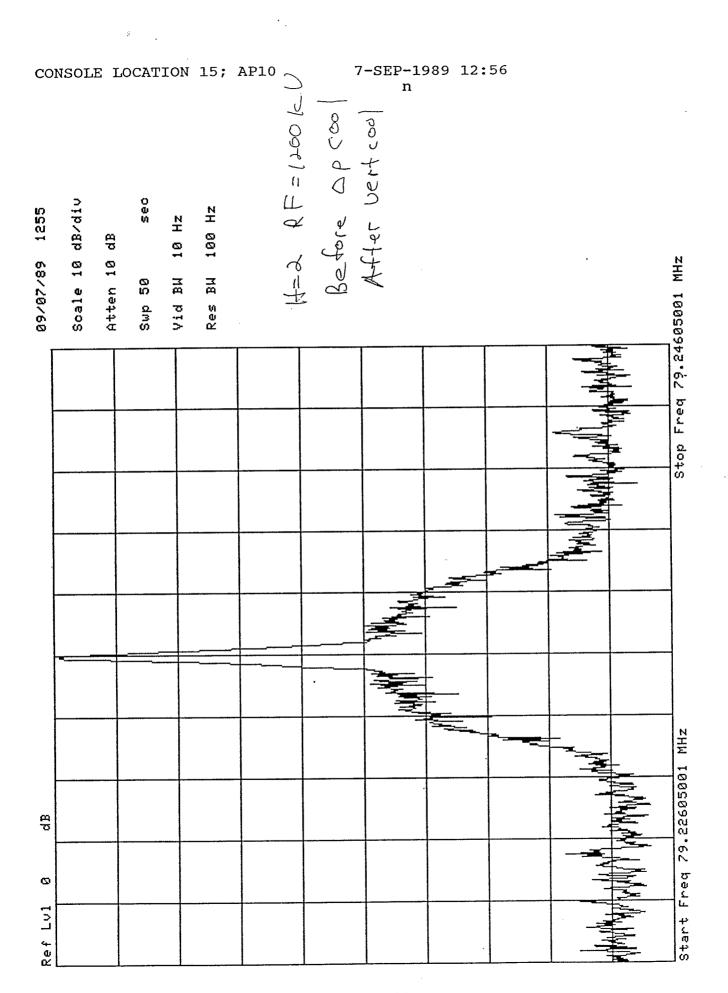


Figure 9f.

100cV
50cV 200nS

H=2 RF=1200 EU

After AB COOLING

CIP EXSTENT
10 min Gain 12dB

Before vert cool

Figure 9i.

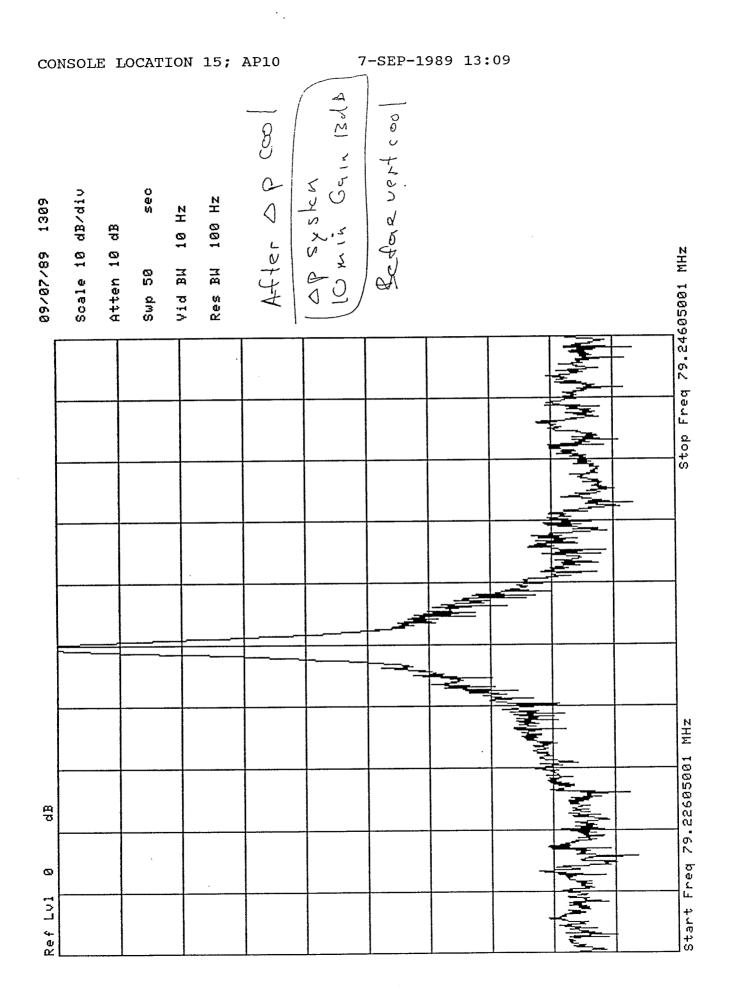
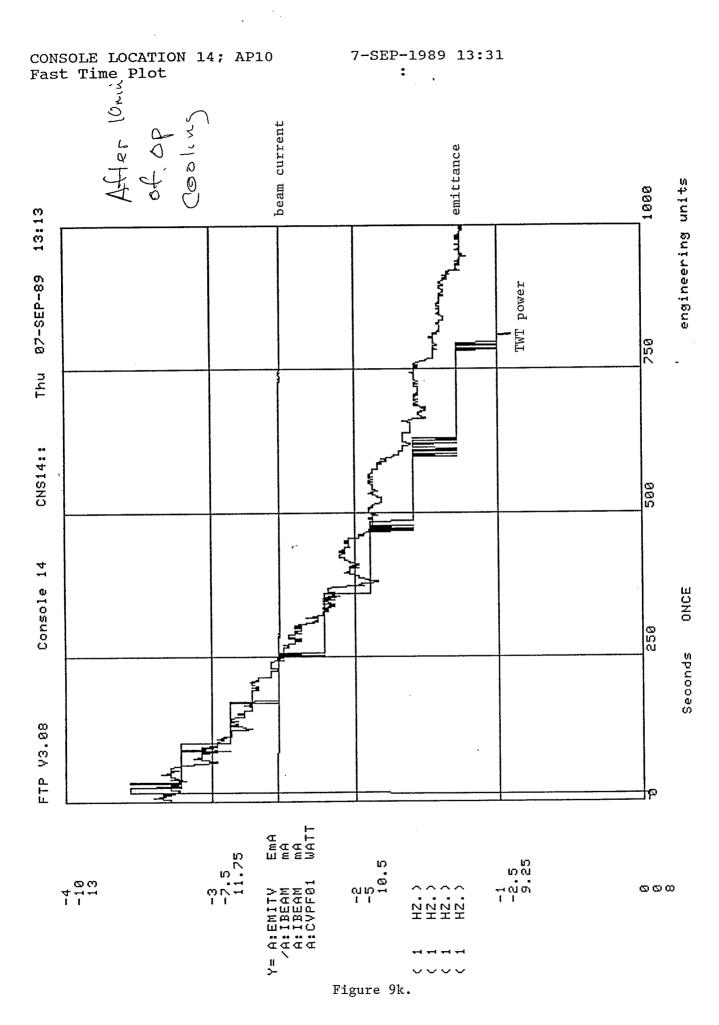
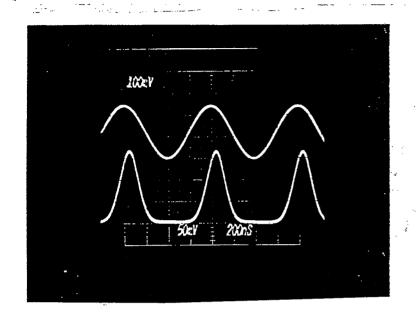


Figure 9j.





After verticed

Figure 91.

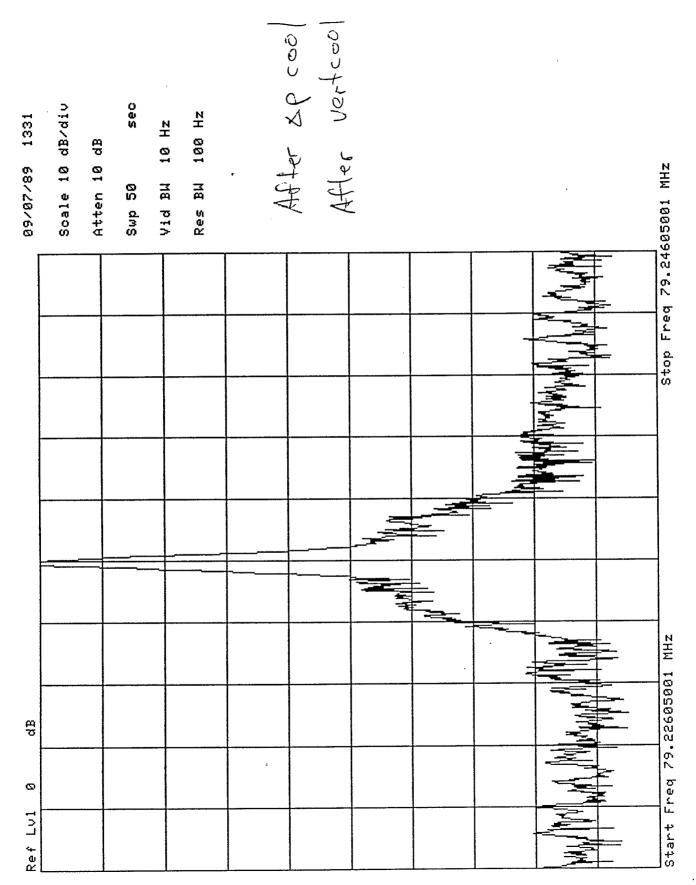


Figure 9m.

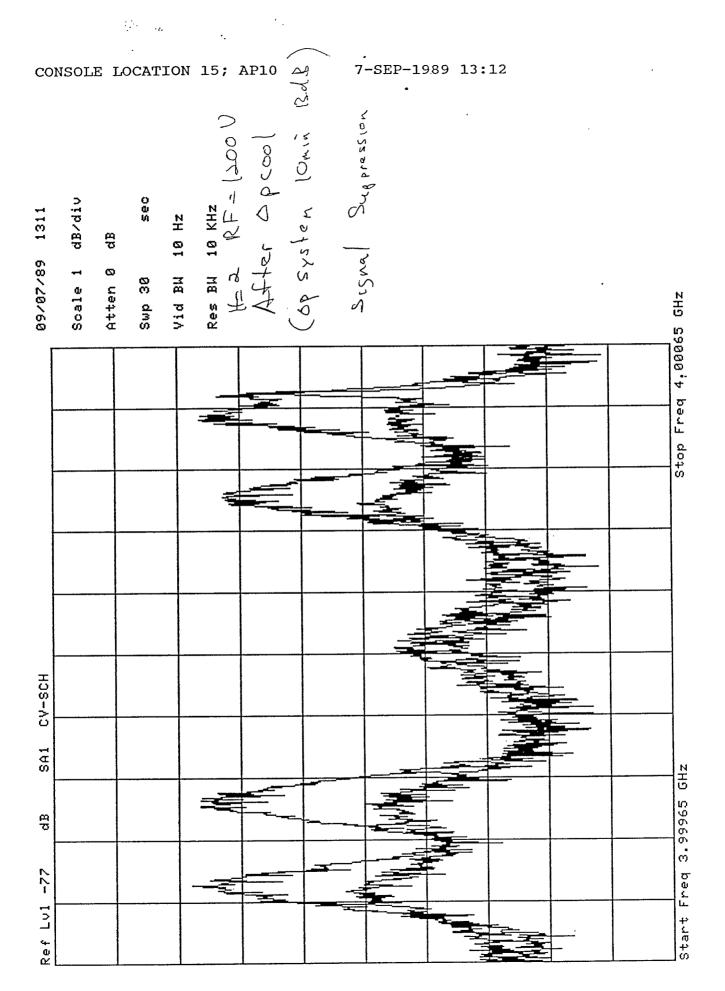
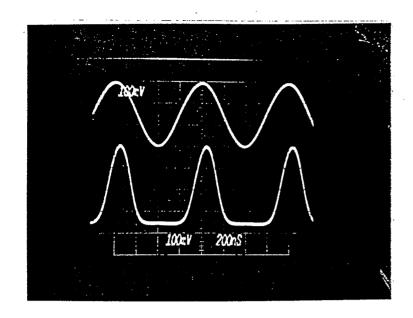


Figure 9n.

Figure 10a.



Longitudinal Signal

RF= 3kU H+2

Before Op cooling

Figure 10b.

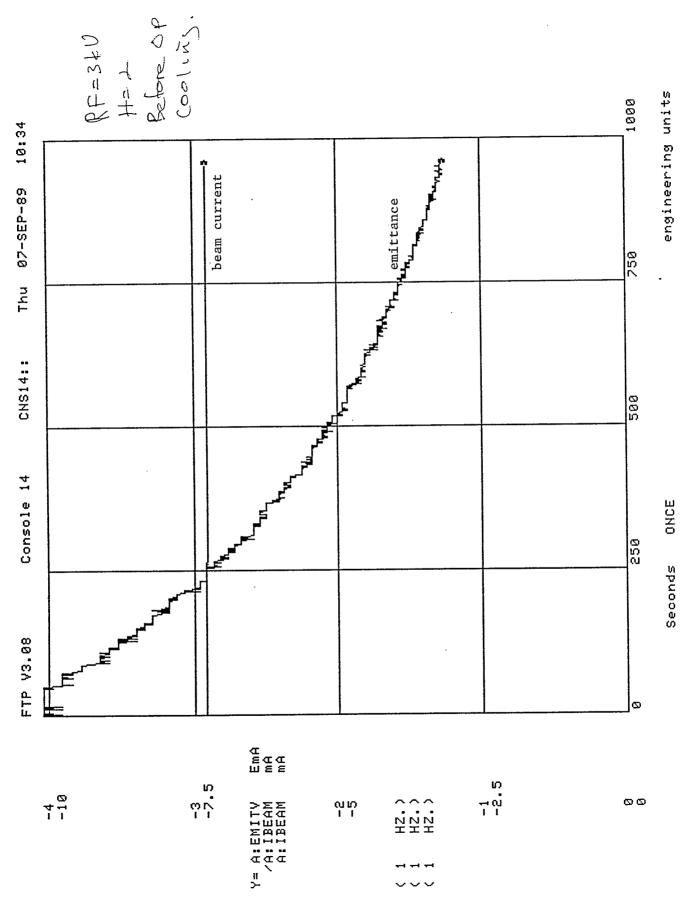


Figure 10c.

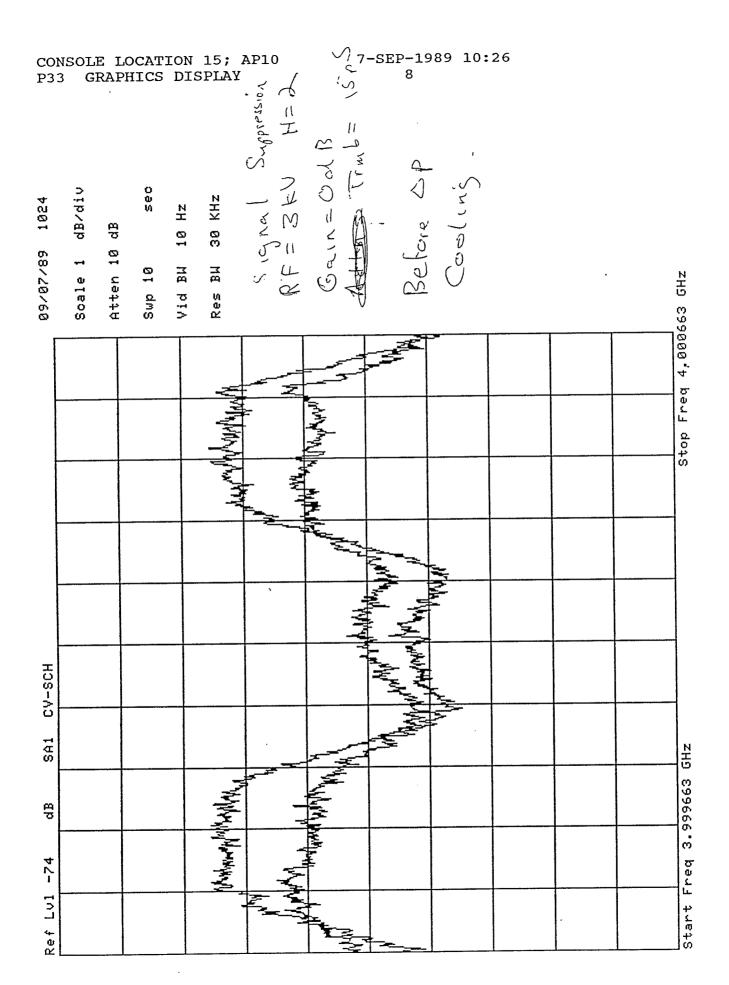
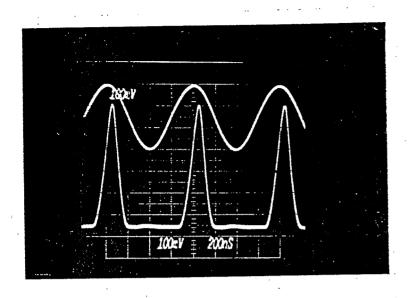
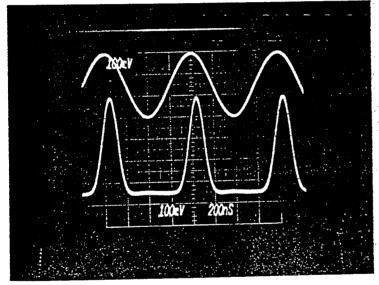


Figure 10e.

Figure 10f.





Long itudinal Signal

RF = 3EV 14=2

After 10 min of

Sp cooling

gain = Attl

of Dp system

was 13 dB

Ist time

2nd time
no op cooling
between 1st &
2nd time
(Refore Vert cooling
and time)

Figure 10i.

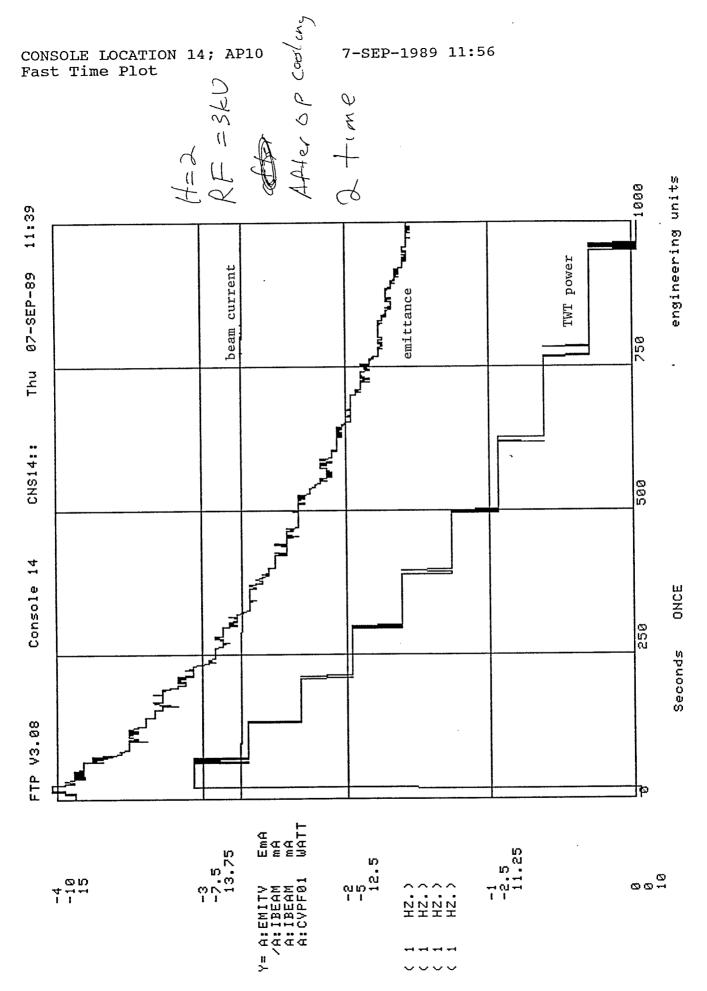
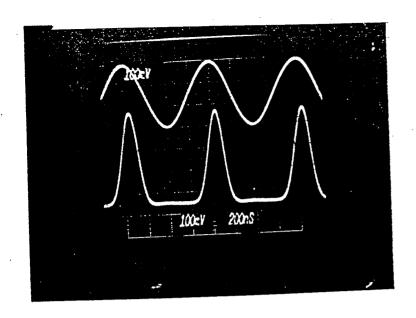


Figure 10k.



After op cool

2 time

After Vert cool.

Figure 101.

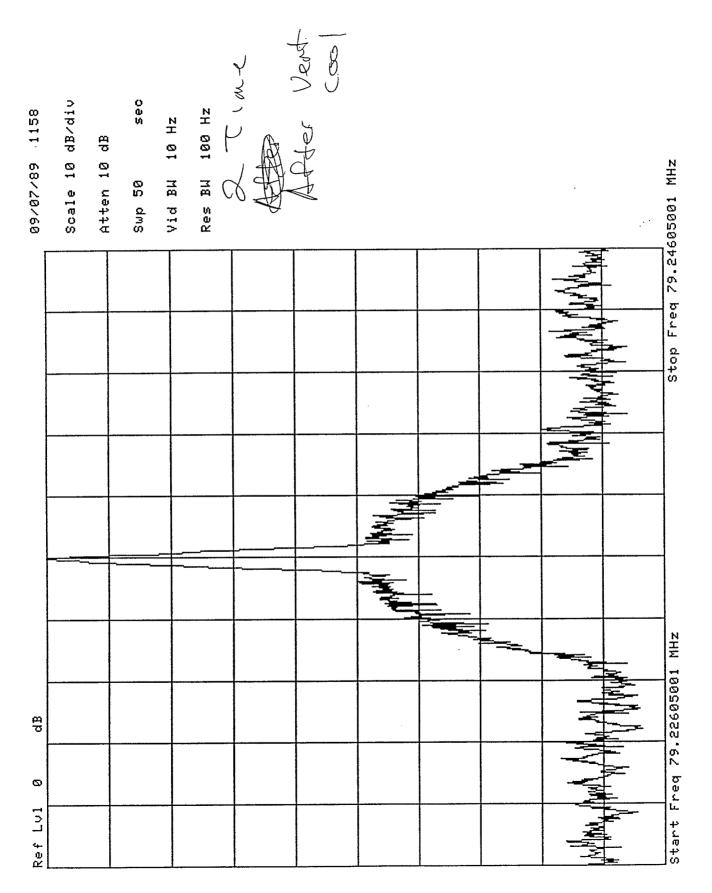


Figure 10m.

BUNCHED BEAM COOLING STUDIES IN THE ACCUMULATOR Sept. 4-8, 1989 David McGinnis

INTRODUCTION

Bunched beam cooling studies were performed on the Accumulator during Sept. 4-8, 1989. A proton beam from the BOOSTER was injected into the core and was bunched with the h=2 or h=84 RF systems. Problems with the h=84 RF system prevented detailed studies of bunch beam cooling at this harmonic. The rest of this report will deal only with a beam bunched at the h=2 harmonic. Because the vertical plane has a lower chromaticity than the horizontal plane, only vertical betatron cooling was studied in detail. However, results on cooling in the horizontal plane are qualitatively similar.

BEAM PREPARATION

As stated earlier, a proton beam of approximately 10 mA was injected from the BOOSTER into the Accumulator injection orbit. The beam was then moved to the central orbit with the h=2 RF systems and subsequently scraped to a momentum spread that could be accommodated by the 2-4 GHz momentum cooling system. The final beam intensity was about 5 mA. The beam was unbunched and the momentum spread was reduced further to approximately 0.06% by the 2-4 GHz central orbit momentum cooling system. Also the horizontal emittance was reduced by the 4-8 GHz core horizontal betatron system.

DC BEAM COOLING MEASUREMENTS

With the beam in a DC or coasting state, the following data was taken:

- 1. The frequency spectrum of the 126th longitudinal Schottky band.
- 2. The DC beam current.
- 3. The open loop and closed loop amplitudes of the vertical betatron Schottky bands at 4.0 GHz.

4. The vertical emittance as a function of time with the 4-8 GHz vertical cooling system on.

From these measurements, the mixing factor, cooling gain, signal/noise, and cooling bandwidth could be determined.

DC BEAM CALCULATIONS

Mixing Factor

The mixing factor is calculated from the longitudinal Schottky spectrum by assuming that the Schottky line is a guassian function of frequency.

$$\psi(f) = \frac{N}{\sqrt{2\pi} \sigma_f} e^{-\frac{f^2}{2\sigma_f^2}}$$

The mixing factor is defined as:

$$M(f) = \frac{\psi(f)_{peak}}{\psi_{average}}$$

Because the 10 dB full width of the guassian is:

$$\Delta f_{10 \text{ dB}} = 4.292 \, \sigma_{f}$$

and using the fact that:

$$\sigma_{f}(f) = \frac{f}{f_{o}} \sigma_{f}(f_{o})$$

the average mixing factor for a cooling system that operates over a frequency range from 4 GHz to 4+W GHz is given as:

$$M_{\text{average}} = \frac{85.31 \times 10^{3}}{\Delta f(hz)_{10 \text{ dB}_{h=126}} \left(4 + \frac{W(GHz)}{2}\right)}$$

Noise to Signal ratio

The average noise/signal ratio of a betatron Schottky band is found from measuring the peak Signal+Noise of a betatron Schottky line and the average noise floor. The peak Signal/Noise is:

$$\frac{S_{\text{peak}}}{N_{\text{avg}}} = \frac{(S+N)_{\text{peak}}}{N_{\text{avg}}} - 1$$

The average noise to signal in a Schottky band is:

$$U = \frac{1}{2} \frac{N_{avg}}{S_{avg}} = \frac{1}{2} \frac{N_{avg}}{S_{peak}} \frac{S_{peak}}{S_{avg}}$$

The factor of 1/2 appears because there are two betatron Schottky lines per band. The ratio between the peak signal and the average signal is the mixing factor. Thus the average noise to signal in a Schottky band is:

$$U = \frac{1}{2} \frac{N_{avg}}{(S + N)_{peak} - N_{avg}} M(f)$$

Since all the signal to noise measurements were done at 4 GHz, the average noise to signal is:

$$U = \frac{1}{2} \frac{N_{avg}}{((S + N)_{peak} - N_{avg})} \frac{10.66 \times 10^{3}}{\Delta f(Hz)_{1.0dB_{h=126}}}$$

Cooling Gain

The signal suppression is the ratio between the open loop gain and the closed loop gain. This ratio can be written as:

$$SS(dB) = 20\log_{10}\left(1 + \frac{g}{g_{opt}}\right)$$

The cooling gain is found by inverting the above equation:

$$\frac{g}{g_{opt}} = 10 \frac{SS(dB)}{20} - 1$$

Cooling Bandwidth

The cooling time, τ , is given as:

$$\frac{1}{\tau} = \frac{W}{N_p} \left(2g - g^2 (M + U) \right)$$

where N_p is the number of protons in the machine. The number of particles in the Accumulator is equal to:

$$N_{p} = 1x10^{10} I_{beam}(mA)$$

The cooling time is a minimum for a cooling gain given by:

$$g_{opt} = \frac{1}{M + U}$$

The equation for the cooling time can then be rewritten as:

$$\frac{1}{\tau} = \frac{W}{N_{p}(M+U)} \left(\frac{g}{g_{opt}}\right) \left(2 - \frac{g}{g_{opt}}\right)$$

Every quantity in this equation has been measured except for the cooling bandwidth. This equation can be inverted to determine W. Since the mixing factor is a function of cooling bandwidth, the resulting equation will be quadratic in W.

$$aW^2 + bW + c = 0$$

where W has units of GHz and:

$$a = \frac{\tau(Sec)}{10I_{heam}(mA)} \Delta f(Hz)_{10db} \left(\frac{g}{g_{opt}}\right) \left(2 - \frac{g}{g_{opt}}\right)$$

$$b = 8a - U \Delta f (Hz)_{1 \text{ 0dB}}_{h=126}$$

$$c = -170.62 \times 10^{3} - 8U \Delta f (Hz)_{1 \text{ 0dB}}_{h=126}$$

Coasting Beam Calculations

The results of the measurements and the above calculations are summarized in the following table:

10 dB full width of Longitudinal Schottky band at h=126	(Hz)	805	
Revolution Frequency (Hz)	6288	57	
Signal Suppression (dB)	3.59		
g/gopt	0.51		
Schottky Line Noise to Signal at 4 GHz	0.16		
Beam Current (mA)	5.81		
Cooling time (Sec)	627		
Cooling BandWidth (GHz)	2.46		
Mixing Factor at Center Frequency	20.25	;	

The cooling bandwidth is not 4 GHz because the gain equalizer for the coaxial trunk line had not been installed as of September 1989. Thus, the Schottky signal at the higher frequencies of 4-8 GHz band was severely attenuated.

Bunched beam measurements

The h=2 RF system (ARF3) was "adiabatically" raised to several different RF voltages. At each of these RF voltages, the vertical system was placed 180° from its optimum phasing and the beam was heated to a fairly large vertical emittance. The heated vertical emittance was approximately the same for each of the RF voltages. After the beam was heated, the vertical cooling system was returned to its proper phasing.

At each RF voltage, the vertical emittance as a function of time was measured with the 4-8 GHz vertical cooling system on. Along with each vertical emittance measurement the following data was also taken:

- 1. The ARF3 fanback voltage.
- 2. The bunch time domain structure was recorded using the resistive wall monitor. (This measurement was taken immediately before and after the vertical emittance measurement.)
- 3. The spectrum of the longitudinal Schottky band at h=126. (This measurement was taken immediately before and after the vertical emittance measurement.)
- 4. The open loop and closed loop amplitudes of the vertical betatron Schottky bands at 4.0 GHz.

These measurements were repeated for two different longitudinal emittances; a "hot" and a "cool" longitudinal emittance. The "cool" longitudinal emittance was obtained by engaging the 2-4 GHz momentum cooling system with an attenuation setting of 13dB for 10 minutes after the "hot" longitudinal emittance data was taken.

Bunched Beam Analysis

Because the Schottky line at h=126 has a large coherent signal with a bunched beam, the mixing factor derived from this information might be inaccurate. However, the mixing factor can be determined from analyzing the time domain response of the bunched beam.

The RMS half width, σ_T , can be determined by measuring the 1/2 maximum half width of a bunch:

$$\sigma_{T} = \Delta t_{1/2} \sqrt{\frac{-1}{2 \ln \left(\frac{1}{2}\right)}}$$

The RMS half width in RF phase is equal to:

$$\sigma_{\phi} = 2\pi\sigma_{T} h f_{o}$$

where h is the RF harmonic number (h=2 for our case) and f_0 is the beam revolution frequency. The RMS half width of the momentum spread can be found by considering the orbit of a single particle in phase space with a maximum phase error of σ_0 . The orbit is governed by:

$$\Omega^{2}\left(\cos(\phi) - \cos(\sigma_{\phi})\right) = \frac{\left(\omega_{RF}\eta\right)^{2}}{2} \left(\frac{\Delta p}{p}\right)^{2}$$

where Ω is the radial synchrotron frequency:

$$\Omega^2 = \frac{\omega_{RF}^2 \eta e V_{RF} \cos(\Psi_s)}{\beta c p 2\pi h}$$

 ψ_S is equal to -180° for a stationary bucket. The momentum for the central orbit is 8.837 GeV/c; β = 0.994; and η = -0.022. The maximum value of the momentum error occurs when ϕ = 0:

$$\left(\frac{\Delta p}{p}\right)_{\text{RMS}} = \frac{2\Omega}{\omega_{\text{RF}}\eta} \sin\left(\frac{\sigma_{\phi}}{2}\right)$$

The RMS frequency spread at the first Schottky band (h=1) is equal to:

$$\left(\frac{\Delta f_{o}}{f_{o}}\right)_{RMS} = \frac{2\Omega}{\omega_{RF}} \sin\left(\frac{\sigma_{\phi}}{2}\right)$$

Since the RMS longitudinal emittance (68% of the beam) is equal to:

$$\varepsilon_{\text{PRMS}} = \frac{2\pi \ \Omega \ \sigma_{\text{T}} p}{\omega_{\text{RF}} \eta} \sin \left(\frac{\omega_{\text{RF}} \sigma_{\text{T}}}{2} \right)$$

the mixing factor can be written as:

$$M(f) = \sqrt{\frac{\pi}{2}} \left(\frac{f_o}{f} \right) \left(\frac{p}{\eta \epsilon_{PRMS}} \right) \sigma_T$$

RF Duty Factor

Bunching the beam with RF increases the particle density that the cooling system samples. The distribution of particles in the RF bucket is approximately Gaussian:

$$\frac{\delta N}{\delta t} = \frac{1}{h} \frac{N_p}{\sqrt{2\pi} \sigma_T} e^{-\frac{1}{2} \left(\frac{t}{\sigma_T}\right)^2}$$

where N_p is the total number of particles in the machine. The peak density is:

$$\left(\frac{\delta N}{\delta t}\right)_{peak} = \frac{1}{h} \frac{N_p}{\sqrt{2\pi} \sigma_T}$$

The particle density for a DC or coasting beam is:

$$\left(\frac{\delta N}{\delta t}\right)_{DC} = \frac{N_{p}}{T_{rev}} = N_{p} f_{o}$$

The duty factor is defined as the ratio of the DC beam density to the peak beam density:

$$D = \frac{\left(\frac{\delta N}{\delta t}\right)_{DC}}{\left(\frac{\delta N}{\delta t}\right)_{peak}}$$

which can be rewritten as:

$$D = \sqrt{2\pi} h f_0 \sigma_T$$

The Duty factor for the RF harmonic and revolution frequency used in this study is:

$$D(\%) = (0.315) \sigma_{T}(nS)$$

Because the cooling system sees an increase density of particles with a

bunched beam, the cooling time becomes:

$$\frac{1}{\tau} = \frac{WD}{N_p} \frac{1}{M+U} \left(\frac{g}{g_{opt}} \right) \left(2 - \frac{g}{g_{opt}} \right)$$

Data Summary

A summary of the bunched beam study data is shown in the following table. The Duty and the Mixing factors were calculated assuming that the bandwidth of the cooling system for a bunched beam is the same as the DC or coasting beam bandwidth (2.46 GHz). Because the Signal/Noise measurements for this study were not trustworthy and considering the fact that the Signal/Noise is a function of the vertical emittance which is changing during the measurement, the U factor was not included in the calculations for the Duty factor. Following the data table is a graph of the duty factor as a function of σ_T . The slope of the graph is 0.362 % per nS change of σ_T which can be compared to the theoretical estimate of 0.315 % per nS change of σ_T derived earlier.

Conclusions

The data presented in this study indicates that the cooling duty factor is linearly proportion to the bunch length. Since the mixing factor is also linearly proportional to the bunch length, the cooling time for a given number of particles, cooling gain, and longitudinal emittance should be independent of bunch length. Also there was no evidence of any coherent signals in the 4-8 GHz stochastic cooling system. The following is list of suggestions for future studies on bunched beam cooling.

- 1. More accurate data on bunch shape in the time domain coupled with more bunch profiles taken during the betatron emittance measurement.
- 2. More accurate data of the Signal/Noise coupled with more Signal/Noise measurements during the betatron emittance measurement. Also, the Signal/Noise should be measured at other frequencies throughout the cooling band.

- 3. Cooling with various beam intensities. As an alternative to No. 2, the studies could be made with large beam intensities so that the signal to noise factor is not important.
- 4. Bunch beam cooling at h=84.
- 5. Horizontal bunch beam cooling.
- 6. Longitudinal bunch beam cooling both at 2-4 GHz and 4-8 GHz.

Bunched Beam Cooling Studies 9-1-89

, ,

Before delta P cooling	n	Using Time domain data	-	1.88	2.97	1.40	1.30	•	After delta P cooling	Cooling time	(Sec)	206	1012	873	866	ooling	Mixing Factor	Using Time domain data
	Shottky line	(S+N) at 4 GHz	(dgm)	-78.37	-80.58	-79.71	-80.49		Before delta P cooling	Cooling time	(Sec)	969	663	688	732	After delta P cooling	Duty Factor	Using Time domain data
	n	Using Time domain data		1.99	2.21	1.39	1.48	•		Beam Current	(mA)	5.65	4.77	6.61	6.82	ooling	Mixing Factor	Using Time domain data
	Shottky line	(S+N) at 4 GHz	(dBm)	-78.56	-80.95	-80.46	-81.51	-85.00	Sychrotron	Frequency	(Hz)	5.2	13.4	19.5	30.8	Before delta P cooling	Duty Factor	Using Time domain
		RF voltage	(Volts)	87	570	1200	3000	Noise Floor		RF voltage	(Volts)	8.7	570	1200	3000			RF voltage

1200 36 3000 31 2.79 NOTE: Signal to noise measurement is not used in calculation.

Average Duty Factor =

0.362 % per ns of RMS half width bunch length

(%) 52

10.37

RF voltage (Volts) 87

(%)

5.21 3.93 2.79

570

10.37

8.01 5.11 3.64

31 25 19

Cooling Duty Factor vs RMS Bunced Beam Length

Calculated Without Using the Noise to Signal Factor

