BNL-101506-2014-TECH

AD/AP 16;BNL-101506-2013-IR

Particle Losses in RHIC Due to Intrabeam Scattering Out of the Momentum Aperture

M. J. Rhodes-Brown

January 1990

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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AD/AP–16

Accelerator Development Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York 11973

Accelerator Physics Technical Note No. 16

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I. Introduction

The consequences of intrabeam scattering,[1–3] or the Coulomb–scattering of charged particles within a bunch, are of overriding importance to the design of RHIC.[4]

Above transition, the Coulomb–scattering within a bunch results in a growth[5] of the longitudinal and transverse emittance of the beam. In particular, the increasing longitudinal emittance dictates the magnitude of the rf system for RHIC.[4] At the present time, the rf system which is used to bunch the beam during the colliding stage is operated so as to keep the rms bunch length constant. This is achieved by increasing the rf voltage so that the rf bucket remains just large enough to contain the growth in beam energy spread σ_p. Within RHIC, at the present time, the maximum voltage is 4.5 MV. For ^{197}Au ions at top energy ($\gamma = 100$) we may assume $\Delta_B = 2\sigma_p$, where Δ_B is the half height of the bucket in $\Delta p/p$. For ^{197}Au ions in a storage mode at $\gamma = 30$, $\Delta_B = 2.5\sigma_p$ is allowed for a 4.5 MV rf system.

An important question for RHIC, "what are the particle losses from a single bunch, due to intrabeam scattering within the bunch?" The equalities $\Delta_B = 2\sigma_p$ or $\Delta_B = 2.5\sigma_p$ may be considered as boundary conditions or constraints on the longitudinal particle motion. This boundary represents the so–called momentum aperture for the bunched particle dynamics.

To date, intrabeam scattering theory for the average transverse and longitudinal emittance growth rates, agrees well with the available data measured for protons at CERN.[6] The theory as it stands, however, is not suitable for estimating particle losses across a momentum aperture, because it assumes the six dimensional form of the distribution function to be Gaussian in nature, without any boundary or constraint. The overall success of this parameterization for calculating emittance growth rates is readily understood by recognizing that the average emittance of the beam is the second moment of the distribution function. Such averaging is not particularly sensitive to the tail of the distribution function.

In this manuscript we introduce a model for particle losses across a momentum aperture or boundary due to intrabeam scattering. To achieve this result we derive a form for the longitudinal distribution function in the presence of a boundary. This distribution function is a solution of the Boltzmann equation[7] with a simplified collision integral that is renamed the Fokker–Planck equation.[8] Although the equation for the evolution of the distribution function is highly non–linear, we have found a closed final form for the time evolution of this distribution function.

In section II of this note the general dynamical equation for the distribution function is derived in the rest frame of the bunch. In section III, the complicated non–linear Fokker–Planck equation is solved exactly in longitudinal phase space, and the form of the particle losses extracted. In section IV this form is evolved to include accelerator, parameters, and results for RHIC are presented in section V. Finally in section VI, the results of the report are summarized, and future work utilizing this formalism is outlined.

II. General Formalism for Intrabeam Scattering

We initially work in the rest frame of the bunch, where all scattering dynamics may be considered non–relativistic in nature. Apart from the constraint in energy–phase space, that is imposed by the rf system, a particle in an accelerator is subject to external guiding and focusing forces (assumed to be dipoles and quadrupoles), and can scatter via the Coulomb force from neighboring particles in the bunch. We describe such a situation via the Boltzmann equation,[7] i.e.,

$$\frac{\partial f}{\partial t} + v^{\mu}\frac{\partial f}{\partial x^{\mu}} + \frac{F^{\mu}}{m}\frac{\partial f}{\partial v^{\mu}} = \left(\frac{\partial f}{\partial t}\right)_c \qquad (II.1)$$

where $\mu = 1,2,3$ for 3 dimensions, $f(\vec{x},\vec{v},t)$ is the projected distribution function of a single particle in the bunch with position ($\vec{x}$) and velocity ($\vec{v}$), F^{μ} is the applied external field (assumed to be dipoles and quadrupoles), and $(\partial f/\partial t)_c$ is the collision integral that describes scattering from neighboring particles in the bunch. The rf constraint is to be applied via a boundary– condition.

The Boltzmann form of the collision integral is given by[7]

$$\left(\frac{\partial f}{\partial t}\right)_c = \int d\vec{v}' d\vec{v}'' d\vec{v}''' \sigma(u,\Omega)\left[f''f''' - ff'\right] \times \delta(\vec{p}+\vec{p}' - \vec{p}'' - \vec{p}''')\delta(E+E' - E'' - E''') \qquad (II.2)$$

where $f \equiv f(\vec{x},\vec{v},t), f' \equiv f'(\vec{x},\vec{v}',t), f'' = f''(\vec{x},\vec{v}'',t), f''' = f'''(\vec{x},\vec{v}''',t)$. Equation (II.2) represents the difference between the number of particles scattered into and out of a region in six dimensional phase space. In eq. (II.2),

$$u = |v'^{\mu} - v^{\mu}| \quad \equiv \quad \text{Relative Velocity} \qquad (II.3)$$

$$\sigma(u,\Omega) = \frac{Z^4 e^4}{A^2 m^2 u^4}[Sin\theta/2]^{-4} \quad \equiv \quad \begin{array}{c}\text{Differential Cross Section}\\ \theta - \text{C.O.M. angle}\end{array} \qquad (II.4)$$

$$v'^{\mu}, v^{\mu} \quad \equiv \quad \text{Velocity of ions before collision} \qquad (II.5)$$

Equation II.4 is the usual Coulomb differential cross section, where mA is the mass of the ions. It is critical to simplify eq. (II.2). We follow the method introduced by Landau,[9] that is now called the Fokker–Planck equation.[8] The assumption is that only two–body scattering occurs and that the momentum change from a scattering event is *small on average*. This reflects the long range nature of the Coulomb interaction. Expanding the distribution functions up to second order in the velocity change, the collision integral is given by[8]

$$\left(\frac{\partial f}{\partial t}\right)_c = -\frac{\partial}{\partial v^{\mu}}(f < \Delta v^{\mu} >) + \frac{1}{2}\frac{\partial^2}{\partial v^{\mu} v^{\nu}}(f < \Delta v^{\mu} \Delta v^{\nu} >) \qquad (II.6)$$

where

$$< \Delta v^{\mu} >= \int d\vec{r}' f(v'^{\mu}) \int d\Omega \sigma(u,\Omega) \Delta v^{\mu} u \qquad (II.7)$$

$$< \Delta v^{\mu} \Delta v^{\nu} >= \int d\vec{v}' f(v'^{\mu}) \int d\Omega \sigma(u,\Omega) \Delta v^{\mu} \Delta v^{\nu} u \qquad (II.8)$$

$\Delta v^{\mu} \equiv$ Change in μth component of velocity of ion from collision.
$< \Delta v^{\mu} > \equiv$ Average change, per unit time, of μth component of velocity.
The first term on the right hand side of equation II.6 is called the *coefficient of dynamical friction.* The second term is called the *coefficient of diffusion.*

The derivation of eqs. (II.7) and (II.8), using the Rutherford form of eq. (II.4), is central to understanding the problem of intrabeam scattering. Indeed, much of Bjorken's paper on this subject[1] is concerned with the evaluation of these integrals, however, he assumes a Gaussian form for f and additionally integrates over the moment of f to extract the emittance.

Finally, combining the results of many previous authors,[1,9,10] the final collision of the Fokker–Planck equation gives us

$$\frac{m^2A^2}{4\pi Z^4 e^4 \log_{(\frac{2}{\theta_{min}})}}\left(\frac{\partial f}{\partial t}\right)_c = -\frac{\partial}{\partial v^\mu}\left(f\frac{\partial h}{\partial v^\mu}\right) + \frac{1}{2}\frac{\partial^2}{\partial v^\mu v^\nu}\left(f\frac{\partial^2 g}{\partial v^\mu v^\nu}\right) \qquad (II.9)$$

where

$$h(\vec{v}) = \int d\vec{v}' f(\vec{v}')|\vec{v} - \vec{v}'|^{-1} \qquad (II.10)$$

$$g(\vec{v}) = \int d\vec{v}' f(\vec{v}')|\vec{v} - \vec{v}'| \qquad (II.11)$$

This should be added to the right hand side of the Boltzmann equation. The log term, log $(2/\theta_{min})$, reflects the divergent nature of the point Coulomb interaction. θ_{min} is the minimum scattering angle (in the center of mass), and has to be determined separately. Normally log $(2/\theta_{min})$ takes the constant value 10.

In the following section we show the new analytic solution we have found for eq. (II.9) in longitudinal phase space. In this way, both the form of f and a closed expression for the particle losses will be derived. In section IV we extend the results of the next section to include those variables of interest in accelerators.

III. Evaluation of Distribution Function, and Particle Losses From Separatrix

We restrict our discussion to evolution of the distribution function in the longitudinal phase space. Equation (II.9) reduces to,

$$\frac{1}{\eta}\left(\frac{\partial f}{\partial t}\right)_c = -\frac{\partial}{\partial v}\left(f\frac{\partial h}{\partial v}\right) + \frac{1}{2}\frac{\partial^2}{\partial v^2}\left(f\frac{\partial^2 g}{\partial v^2}\right) \qquad (III.1)$$

where

$$\eta^{-1} = \frac{m^2A^2}{4\pi Z^4 e^4 \log_{(\frac{2}{\theta_{min}})}} \qquad (III.2)$$

$$h(v) = \int_{-\infty}^{\infty} dv' f(v')|v - v'|^{-1} \qquad (III.3)$$

$$g(v) = \int_{-\infty}^{-\infty} dv' f(v')|v - v'| \quad . \qquad (III.4)$$

In the absence of dissipation, we are only interested in the diffusion term of equation (III.1). From equation (III.4) we find

$$\frac{\partial g(v)}{\partial v} = \int_{-\infty}^{\infty} dv' f(v')\theta(v-v') \quad ,$$

where $\theta(v-v')$ is a step function defined by (see figure 1),

$$\theta(v-v') = -1 \quad v < v'$$
$$\theta(v-v') = +1 \quad v > v' \qquad (III.5)$$

Hence,

$$\frac{\partial^2 g(v)}{\partial v^2} = \int_{-\infty}^{\infty} dv' f(v')\delta(v-v') = f(v) \qquad (III.6)$$

From equations (III.1) and (III.6) we find

$$\frac{1}{\eta}\left(\frac{\partial f}{\partial t}\right)_c = \frac{\partial}{\partial v}\left(f\frac{\partial f}{\partial v}\right) \qquad (III.7)$$

or

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{\partial}{\partial v}\left(D(v,t)\frac{\partial f}{\partial v}\right) \qquad (III.8)$$

where the diffusion function is introduced as

$$D(v,t) = \frac{4\pi^2 Z^4 e^4}{m^2 A^2}\log\left(\frac{2}{\theta_{min}}\right) f(v,t) = \eta f(v,t) \qquad (III.9)$$

Equations III.8 and III.9 are the diffusion equations for intrabeam scattering in one–dimension. Unlike the well–known one–dimensional diffusion equation used to estimate beam–gas scattering, the diffusion coefficient is no longer a constant, but is proportional to the distribution function. *Such a form emphasizes the importance of the tail of the distribution*, for if $D(v,t) \propto f(v,t)$ the diffusion rate from the tail of the distribution will be significantly reduced from the standard Gaussian form for f associated with a constant diffusion rate.

The friction component of equation (III.1) represents the microscopic basis for possible future studies on stochastic cooling of bunched beams. Utilizing both the diffusion and friction term in equation (III.1) may allow the strong Coulombic forces between fully stripped heavy ions to be included in a stochastic cooling study.

Let us solve the non–linear partial differential equation (II.8) i.e.

$$\frac{1}{\eta}\left(\frac{\partial f}{\partial t}\right)_c = \left(\frac{\partial f}{\partial v}\right)^2 + f\frac{\partial^2 f}{\partial v^2} \qquad (III.10)$$

What is extremely fortunate (and indeed remarkable), is that this equation is separable, i.e. let

$$f(v,t) = \mathcal{G}(v)S(t)$$

$$\frac{1}{\eta}\mathcal{G}(v)\dot{S}(t) = [\mathcal{G}'(v)]^2 S^2(t) + S^2(t)\mathcal{G}(v)\mathcal{G}''(v) \qquad (III.11)$$

or

$$\frac{1}{\eta}\frac{\dot{S}(t)}{S^2(t)} = \frac{[\mathcal{G}'(v)]^2}{\mathcal{G}(v)} + \mathcal{G}''(v) \qquad (III.12)$$

where $\dot{S} \equiv ds/dt$ and $\mathcal{G}' \equiv d\mathcal{G}/dv$. Thus,

$$\dot{S}(t) = -2B\eta S^2(t) \qquad (III.13)$$

$$\mathcal{G}(v)\mathcal{G}''(v) + [\mathcal{G}'(v)]^2 = -2B\mathcal{G}(v) \qquad (III.14)$$

where B is a constant to be determined later.

III.1 Time Solution

Now from equation III.13 we have

$$\frac{ds}{dt} = -2B\eta S^2(t) \qquad (III.15)$$

or

$$S(t) = \frac{1}{C - 2B\eta t} \qquad (III.16)$$

where C is a constant of integration.

If we impose $S(t=0) = 1$ then $C = 1$ and the time solution is given by

$$S(t) = \frac{1}{1 - 2B\eta t} \qquad (III.17)$$

III.2 Longitudinal Solution

From equation III.14 we have

$$\frac{d}{dv}\left[\mathcal{G}(v)\mathcal{G}'(v)\right] = -2B\mathcal{G}(v) \qquad (III.18)$$

Let us define

$$Y(v) = \mathcal{G}^2(v) \qquad (III.19)$$

Thus

$$Y''(v) = -4BY^{1/2}(v) \qquad (III.20)$$

Let

$$W = \frac{dY}{dv} \qquad (III.21)$$

Thus

$$\frac{dW}{dv} = \frac{dW}{dY}\frac{dY}{dv} = \frac{dW}{dY}W \qquad (III.22)$$

Substituting in (III.20) and integrating over (Y) we find

$$\frac{W^2}{2} = -4BY^{3/2} \times \frac{2}{3} + C \qquad (III.23)$$

or

$$\left(\frac{dY}{dv}\right)^2 = 2C - \frac{16}{3}BY^{3/2} \qquad (III.24)$$

If we let $\frac{16}{3c}BY^{3/2} = X^3$ then the <u>general solution</u> of (III.14) can be reexpressed in the integral equation form,

$$\frac{3\sqrt{2C}}{4B}\left(\frac{B}{3C}\right)^{1/3}\int\frac{X\,dx}{(1-X^3)^{1/2}} = v - v_m \qquad (III.25)$$

where v_m is a constant which is to be determined later.

In order to extract $\mathcal{G}(v)$ from equation (III.25), the integral equation must be inverted and X, and hence Y, replaced by equation (III.19). Equation (III.25) represents the general velocity solution to the partial differential equation (III.10).

III.3 Longitudinal Solution with Boundary Conditions Relevant to RHIC

The planned rf mode of RHIC described in the Introduction makes the solution of the integral equation (III.25) easier. Within RHIC the particles are always expected to fill the bunch up to a maximum v_m, thus we can impose the boundary condition

$$\mathcal{G}(|v| \geq v_M) = 0$$
$$Y(|v| \leq v_M) = 0 \qquad (III.26)$$

Under these conditions $C \equiv 0$ in equation (III.23), and we can integrate (III.24),

$$\int dy \left(\frac{3}{16B}\right)^{1/2} Y^{-3/4} = i \int dv$$

or

$$Y(v)^{1/4} = i\sqrt{\frac{B}{3}}v + d \qquad (III.27)$$

where d is an integration constant. From (III.26) we have at

$$v = v_M \quad d = -i\sqrt{\frac{B}{4}}v_M$$
$$v = -v_M \quad d = i\sqrt{\frac{B}{3}}v_M \qquad (III.28)$$

The symmetric solution of the velocity component of (III.8), that satisfies the boundary conditions (III.26) is given by

$$\mathcal{G}(v) = -\frac{B}{3}(v - v_M)^2 \quad 0 \leq v \leq v_M$$
$$\mathcal{G}(v) = -\frac{B}{3}(v + v_M)^2 \quad 0 \geq v \geq -v_M \qquad (III.29)$$

This solution has a discontinuous first derivative at $v = 0$. This can be understood on physical grounds, for in the rf bucket the Coulomb forces between particles at a finite distance apart are always acting. Hence we may expect the velocity equals zero point of the distribution function not to be an absolute extremum of the distribution function. Any particle at this point will be subject to the long range Coulomb forces due to the other particles in the bunch.

Now we demand at $t = 0$

$$\int_{-v_M}^{v_M} f(v, t = 0) = N_B \qquad (III.30)$$

where N_b is the density of particles in the bunch. From this definition of N_B we find $B = 9N_B/2v_M^3$. Collecting together (III.27), (III.31) and (III.17) we can write the final form of the distribution function as

$$f(v,t) = \frac{3N_B(v - v_M)^2 m^2 A^2}{2v_M^3 m^2 A^2 + 18\pi \ell n N_B Z^4 e^4 t} \quad 0 \leq v \leq v_M$$

$$f(v,t) = \frac{3N_B(v + v_M)^2 m^2 A^2}{2v_M^3 m^2 A^2 + 18\pi \ell n N_B Z^4 E^4 t} \quad 0 \geq v \geq -v_M \qquad (III.31)$$

where we have defined $\ell n = \log(2/\theta_{min})$. In Appendix A we verify that equation (III.31) is a solution of equation (III.8).

III.4 Particle Losses Across Momentum Aperture

We define the time development of the particle density $N_B(t)$ within a bunch by

$$N_B(t) = \int_{-v_M}^{v_M} f(v,t)dv \quad . \qquad (III.32)$$

Using the solution for $f(v,t)$, given by equation (III.31) we find

$$N_B(t) = \frac{N_B(t=0)}{1 + \frac{9\pi \ell n N_B(t=0) Z^4 e^4 t}{m^2 A^2 v_M^3}} \quad . \qquad (III.33)$$

Hence if we write $N_B(t) = N(t)/V$, where N is the number of particles in the bunch at time t, and V the spatial volume of the bunch, equation (III.33) also gives the number of particles in the bunch as a function of time. Remember (III.33) is defined in the particle rest frame p.

From equation (III.33) the half life $t_{1/2}^p$ in the particle rest frame p is given by

$$t_{1/2}^p = \frac{m^2 A^2 v_m^3}{9 N_B \pi Z^4 e^4 \ell n} \quad . \qquad (III.34)$$

If we separate the heavy ion physical parameters from the dynamical variables of the bunch, equation (III.34) can be written in the lab frame L as

$$t_{1/2}^L = \frac{\gamma}{9\pi Z^4 e^4 m_N A \ell n} \times \frac{1}{\alpha} \qquad (III.35)$$

where γ is the Lorentz function, m_N is the mass of a nucleon, and A is the atomic mass of the ion. α is the all important critical dynamical variable, and is a measure of the phase space density occupied by the bunch, i.e.

$$\alpha = \frac{N}{V^L m_N^3 A^3 (v_m^L)^3} \quad , \tag{III.36}$$

where V^L is the spatial volume in the lab frame L.

From equations (III.33) – (III.36) we can see that the half life for particle losses from the bunch is critically determined by the beam density in phase space. The selection of the constraint v_m^L will severely effect the value of α, and hence $t_{1/2}^L$.

IV. Evolution to Accelerator Variables

During the ten hour beam lifetime, there are no expected constraints or particle losses in the transverse direction. For this reason we choose the transverse 4–dimensional phase space volume of the beam as that volume that contains 99.9% of the beam[11] at time $(t = 0)$. This volume is taken to be a constant over the ten hour period. With this model, equation II.1 becomes,

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial t}\right)_c \tag{IV.1}$$

and much of the preceding analysis can be directly utilized.

From equations (III.32) – (III.36) we can write for the fraction of particle retained,

$$\frac{N(t^L)}{N(t^L = 0)} = \frac{1}{1 + \frac{9\pi \ell_n Z^4 e^4 m_N A \alpha t^L}{\gamma}} \tag{IV.2}$$

where t^L is the storage time of the beam in the lab frame, and α is now given by

$$\alpha = \frac{N\gamma^3}{p^3 (\frac{\Delta p}{p})^3 \pi \epsilon \sqrt{\hat{\beta}\check{\beta}} \ell_B} \quad . \tag{IV.3}$$

In equation (IV.3), p is the beam momentum in the lab frame ($p = m_N A \gamma \beta c$), $\Delta p/p$ is the momentum spread of the bunch, and ℓ_B is the length of the bunch. The ratio $N/\pi\epsilon\sqrt{\hat{\beta}\check{\beta}}$ is simply the peak value of a Gaussian representation for the particle density[11] at time $t^L = 0$, where $\sqrt{\hat{\beta}\check{\beta}}$ is the geometric mean of the lattice function around the ring, and ϵ is the beam emittance. A value of ϵ is chosen such that the Gaussian parameterization of the beam contains 99.9% of the particles[11] in a 4–D phase space.

V. Estimation of Particle Losses from RHIC

In this section we apply formulas (IV.2) – (IV.3) to estimate particle beams from RHIC for $\gamma = 30$. For RHIC operation, ℓ_B is a constant for the ten hour beam lifetime, and the separatrix Δ_B is adjusted such that $\Delta_B = d\sigma_p (\equiv d\Delta p/p)$ for all t^L. The constant parameter d is limited by the available rf voltage[4,5] of 4.5 MV. For $\gamma = 30$ we consider both $d = 2$ and $d = 2.5$. Numerically, equation IV.2 becomes

$$\frac{N(t^L)}{N(t^L = 0)} = \frac{1}{1 + \frac{1.88 \times 10^{-21} N t^L}{(d\sigma_p)^3 \epsilon_H \gamma \ell_B}} \qquad (V.1)$$

for ^{197}Au beams in RHIC. For the RHIC lattice $\sqrt{\hat{\beta}\breve{\beta}} = 20.71$ m, and to include 99.9% of the beam in a 4–D phase space we take[11] $\epsilon = 6\epsilon_H$, where ϵ_H corresponds to 95% of the beam in a projected 1–D phase space. For RHIC operation we have $N = 10^9$ and the values of σ_p were taken from the intrabeam scattering calculations of Parzen.[5]

In Figure 1 the particles retained for $\gamma = 30$ ^{197}Au beams are plotted for the projected ten hour beam lifetime within RHIC. It can be seen that for $\Delta_B = 2\sigma_p$ this calculation predicts up to 30% beam loss from the longitudinal momentum aperture. The cubic dependence of the particle losses on Δ_B is evident, for when Δ_B is increased to $\Delta_B = 2.5\sigma_p$, the particle losses across the momentum aperture have decreased to 20% after a ten hour beam lifetime. The charge in the particle loss rate over the ten hour lifetime may be easily understood from equation V.1. Initially the available phase space area is at its smallest value and hence the loss rate is maximum. As t^L increases $\Delta_B (\equiv d\sigma_p)$ also increases as the rf voltage is increased. In this way the particle density of the bunch decreases and the diffusion rate across the momentum aperture falls as a consequence.

V.1 Discussion of Results

In this paper we have derived a closed form for the particle losses across the momentum aperture in RHIC. The boundary conditions relevant to RHIC lend themselves to a simplified solution of the diffusion equation. The formalism is essentially microscopic in nature, that is the non–linear diffusion equation is derived starting from the two body interaction within a bunch. Although some approximations have been utilized, i.e. the guiding and focusing fields were assumed to be dipoles and quadrupoles, the lattice functions were averaged around the ring, and the rf voltage was assumed to simply impose a constraint on longitudinal motion, the overall results would seem to reflect the essential features of particle losses. These features are:

1) The particle distribution in longitudinal space, for the boundary conditions relevant to RHIC, are given by Equation (III.31).

2) As a direct consequence of this distribution, if all other parameters remain constant, the particle losses depend on the third power of the momentum aperture value Δ_B.

3) Overall, the critical dynamical variable for particle losses is the particle density of the bunch. Hence we may expect the largest loss rate of the particles during the initial storage times.

4) For the rf conditions at RHIC (4.5 MV), we may expect some particle losses over a ten hour period. The theory derived here predicts 30% losses for $\Delta_B = 2\sigma_p$ and 20% for $\Delta_B = 2.5\sigma_p$. Although these losses remain to be experimentally verified, the theoretical predictions derived here indicate that the particle losses may be significantly reduced by a modest future upgrade of rf voltage. This modest upgrade reflects the cubic dependence of the losses on Δ_B.

Figure Caption

1. Definition of the function $\theta(v - v')$.
2. Figure showing the loss rate of ^{197}Au ions across a longitudinal separatrix based on equation V.1. The dashed line corresponds to $\Delta_B = 2\sigma_p$ and the solid curve $\Delta_B = 2.5\sigma_p$. Within equation V.1, ϵ_H takes the value 10π mm mrad and ℓ_B takes the value $\ell_B = 1.58m$ for $\Delta_B = 2\sigma_p$, and $\ell_B = 1.17$m for $\Delta_B = 2.5\sigma_p$

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APPENDIX A

In this appendix we verify that the solution given earlier for the distribution function is indeed a solution of the non–linear partial differential equation (III.8). We want to show,

$$f(v,t) = -\frac{B}{3}(v \pm v_M)^2 \times \frac{1}{1-2B\eta t} \qquad (A.1)$$

is a solution of

$$\frac{1}{\eta}\left(\frac{\partial f}{\partial t}\right)_c = \left(\frac{\partial f}{\partial v}\right)^2 + f\frac{\partial^2 f}{\partial v^2} \quad . \qquad (A.2)$$

Now

$$\begin{aligned} \frac{1}{\eta}\frac{\partial f}{\partial t} &= \frac{B}{3}(v \pm v_M)^2 \times \frac{\partial B}{(1-2B\eta t)^2} \\ \frac{\partial f}{\partial v} &= -\frac{2B}{3}(v \pm v_M) \times \frac{1}{1-2B\eta t} \qquad (A.3) \\ \frac{\partial^2 f}{\partial v^2} &= -\frac{2B}{3} \times \frac{1}{(1-2B\eta t)} \end{aligned}$$

Thus

$$\left(\frac{\partial f}{\partial v}\right)^2 = \frac{4}{9}B^2(v \pm v_M)^2 \times \frac{1}{(1-2B\eta t)^2} \qquad (A.4)$$

and

$$f\left(\frac{\partial^2 f}{\partial v^2}\right) = \frac{2B^2}{9}(v \pm v_M)^2 \times \frac{1}{(1-2B\eta t)^2} \qquad (A.5)$$

Hence on combining (A.3), (A.4) and (A.5) we see (A.1) is a solution of (A.2).

FIGURE 1

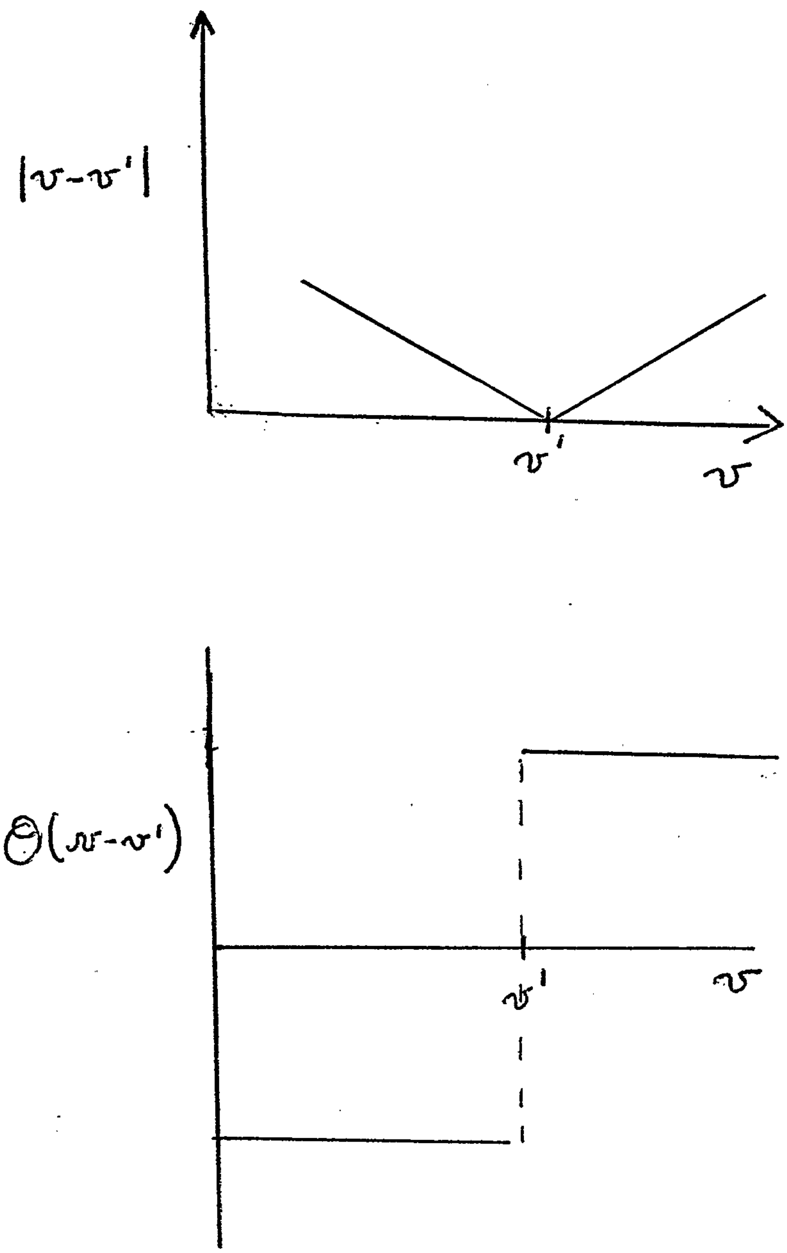

FIGURE 2: FRACTION OF PARTICLE LOSSES FOR ^{197}Au BEAMS, $\gamma = 30$