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On the radius of convergence for multipole expansion

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Abstract

Analytic evaluation of multipole expansion coefficients gives us a tool to study the validity and the radius of convergence for the multipole expansion of the magnetic field. We found that the multipoles (up to 18^{th} terms) can accurately describe the magnetic field within 80% of the coil radius. Outside the radius of convergence, the multipole expansion deviates appreciably from the exact magnetic field.

I. Introduction

Multipole expansion is widely used in the calculation of the dynamical aperture for particle beam dynamics. Using the analytic expression for the multipole expansion derived[1] for the current filament in the parallel sheets of irons with infinite permeability, we can study the radius of convergence. Since multipole coefficients can be calculated accurately, our result will not be limited by the accuracy of the finite mesh in the numerical integration programs such as POISSON or PD2E.

The analytic expression in ref. 1 does assume that the infinite sheets of iron and infinite permeability of the iron. These assumptions are valid approximately in most of the magnet design before iron saturation. Without looking for mathematical vigorous, we shall use some simple configurations of four coils to compare the magnetic field of the exact solution with that obtained from the multipole expansion. The radius of convergence is obtained by observing the large deviation in multipole expansion. Since the magnetic field has a simple pole at the coil location, one may argue that the radius of convergence is the radius of the coil location. However, one may need infinite many multipoles to obtain a credible result. In realistic magnet design, one can not calculate reliably the higher multipoles due to finite mesh. We thus define the radius of convergence to be the region that the magnetic field can be accurately represented by the multipole expansion up to N^{th} term, where N is a reasonable multipole, say $N = 18$ (Note that the magnet designer refer to our N^{th} term as $2(N+1)^{th}$ multipole). In fact $N = 18$ is normally too difficult to be obtained reliably in the magnet design. Fortunately we shall see that within the radius of convergence in our prescription, multipoles up to $N = 10$ (the 22–pole) can represent fairly well the exact magnetic field.

II. Field due to a Current Filament in Two Sheets of Parallel Plates

The magnetic field due to a single current filament at (x_c, y_c) in two sheets of infinite permeability iron is given by[1]

$$H(z) = \frac{I}{4g}\left(\tanh\frac{\pi(z - Z_c^*)}{2g} + \coth\frac{\pi(z - Z_c)}{2g}\right) \tag{1}$$

where g is the gap distance between iron plates, I is the current on the filament, $z = x + iy$ is the location of measuring the field, $Z_c = x_c + iy_c$, $H = H_y + iH_x$ is the magnetic field. The magnetic field in Eq. (1) can be expanded in multipole as

$$H = \frac{I}{4g}\sum_n \frac{1}{n!}(\alpha_n + \beta_n)\left(\frac{\pi}{2g}\right)^n z^n \tag{2}$$

where α, β coefficients are tabulated in ref. 1.

There are some interesting features in Eq. (2), i.e. when the coil location x_c is much larger than the gap, i.e. $\pi x_c/2g >> 1$, the dipole component of the flux density becomes the familiar equation as,

$$B_0 = \frac{\mu_0 I}{g} \ , \tag{3}$$

which is independent of x_c. The higher multipoles depends on x_c exponentially as

$$B_n \cong \frac{\mu_0 I}{g}\left(\frac{\pi}{2g}\right)^n z^n e^{-\pi x_c/g} \tag{4}$$

Thus the multipoles would be smaller, when the coils are far away from the center of the magnet.

III. Radius of Convergence

Since Eq. (1) has a simple pole at $z = Z_c$, we might expect that the radius of convergence is within the coil radius, $|Z_c|$. The validity of multipole expansion depends also on the number of multipoles used in the expansion. In the present study, we shall use 0–18th terms. In reality, we shall see that 0–10th terms are sufficient to represent the magnetic field inside the radius of convergence. The 12th to the 18th terms in Eq. (2) hardly improve the result by increasing the radius of convergence.

To simplify the analysis, we shall study the 4 coils configuration as following:

$$\begin{array}{ll} I \ \text{at} \ (x_c, y_c) \ \text{and} \ (x_c, -y_c) \\ -I \ \text{at} \ (-x_c, y_c) \ \text{and} \ (-x_c, -y_c) \end{array}$$

These four coils are located inside two parallel sheets of irons with gap, $g = 0.08255$ m.

Case 1) $x_c = 0.06$ m, $Y_c = 0.02456$ m

Figure 1 shows the exact field measured along the x axis in comparison with the multipole expansion as

$$H_y(x) = \sum_{m=0}^{N} b_m x^m \tag{5}$$

Within the radius of convergence, the field can be represented by the multipole expansion. The radius of convergence is about 0.052 m or 80% of the coil radius.

Case 2) $x_c = 0.02$ m, $Y_c = 0.03147$ m

Figure 2 shows the field $H_y(x)$ for case 2. Similarly we found that the radius of convergence is also 80% of the coil radius.

Figure 3 shows the situation for $x_c = 0.01$ m and $Y_c = 0.02$ m and 0.04 m respectively. Here the radius of convergence is about 75% of the coil radius.

Figure 4 shows the $1 + \Delta B/B_0$ for the eddy current correction coils to obtain $b_2 = -0.785$ m^{-2} for the compensation of sextupoles due to the eddy current on the vacuum chamber [see ref. 1 for the exact geometry]. The multipole expansion is a good description of the field up to 36 mm, which is about 80% of the radius of the first coil. The multipole coefficients b_n, are tabulated in Table 1. Note that b_6 and b_{10} in ref. 1 are mistabulated.

Table 1. Multipole Coefficient for the Eddy Current Correction Coils

n	$b_n[m^{-n}]$
0	0.62727E–02
2	-0.78500E+00
4	-0.31479E+03
6	0.96962E+05
8	0.89013E+08
10	-0.20831E+11
12	-0.14487E+14
14	0.17398E+16
16	0.42971E+19
18	-0.15410E+21

IV. Conclusion

Some simple examples are used to demonstrate the radius of convergence for the multipole expansion for the magnetic field. Using 0^{th} to the 18^{th} terms in multipole expansion, the radius convergence is about 80% of the coil radius. Within 80% of the coil radius 0 to 10^{th} terms (2 to 22 poles in magnet notation) gives good representation of the magnetic field. Beyond 80% of the coil radius, the multipole expansion becomes slowly convergent. Using the multipole expansion in this region is infertile and unrealistic.

The result of our analysis may not be applicable to some specially designed magnets such as $\cos\theta$ or H and C magnets etc., where the coil and/or irons are shaped to obtain uniform field beyond the radius of convergence discussed in the paper.

References

1. S.Y. Lee, "Multipole Expansion for the Eddy Current Correction Coil" Acc. Phys. Technical Note AD/AP/TN–12.

Fig. 1 The exact magnetic field is compared with that of the multipole expansion of Eq. (5) summed up to n=N terms for Case 1.

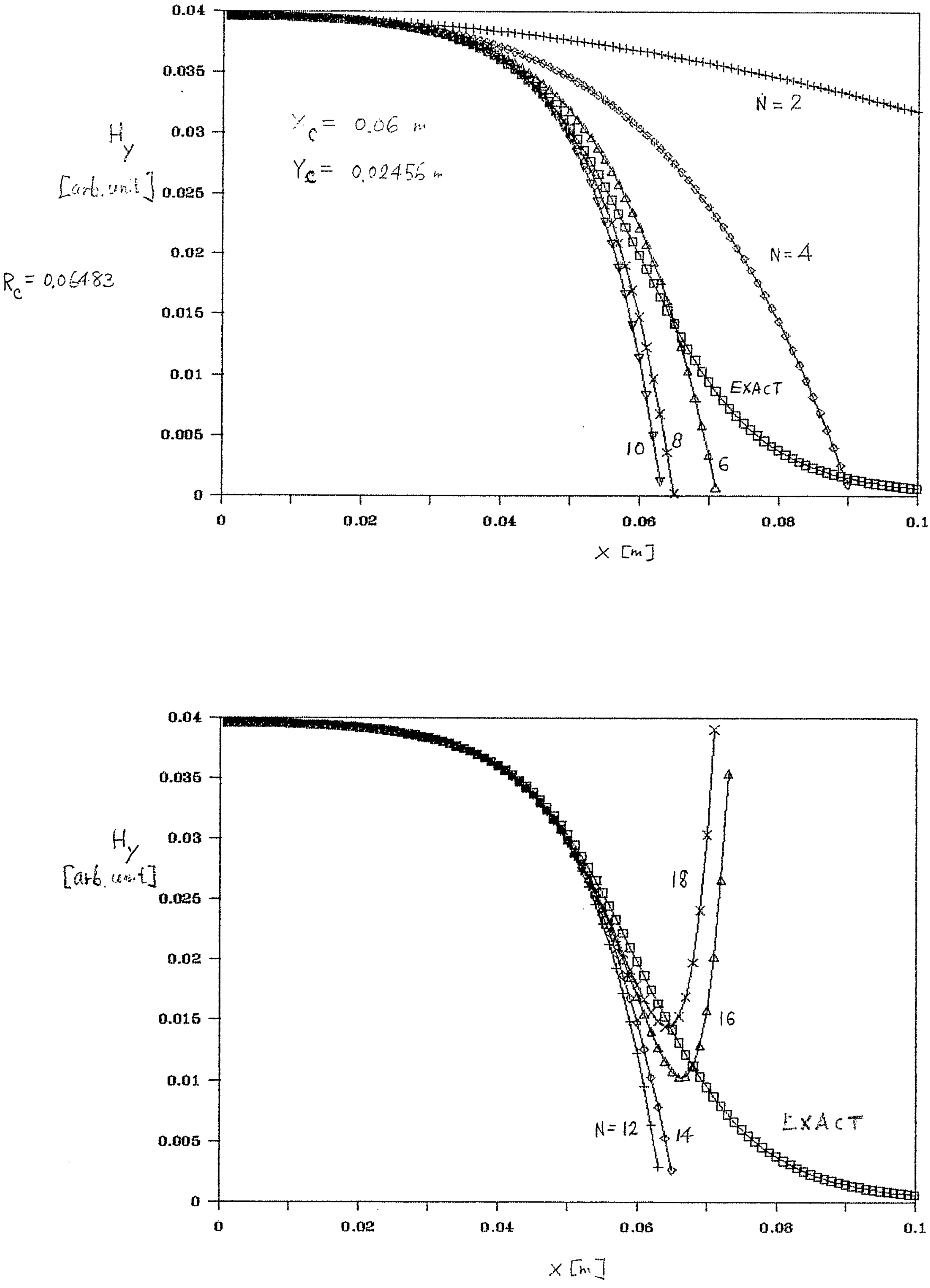


Fig 1

Fig. 2 Similar to the caption of Fig. 1 for Case 2 coil configuration.

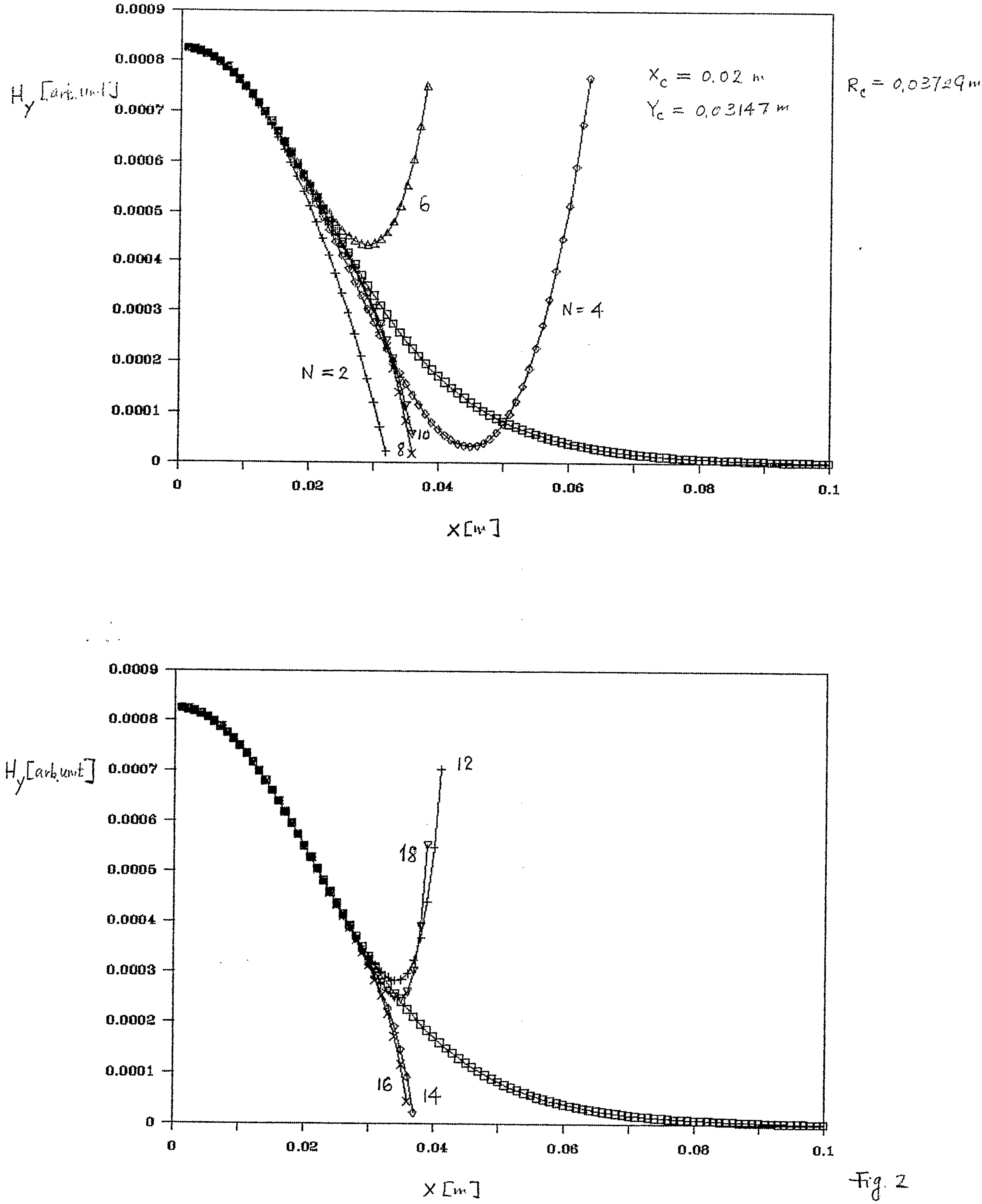
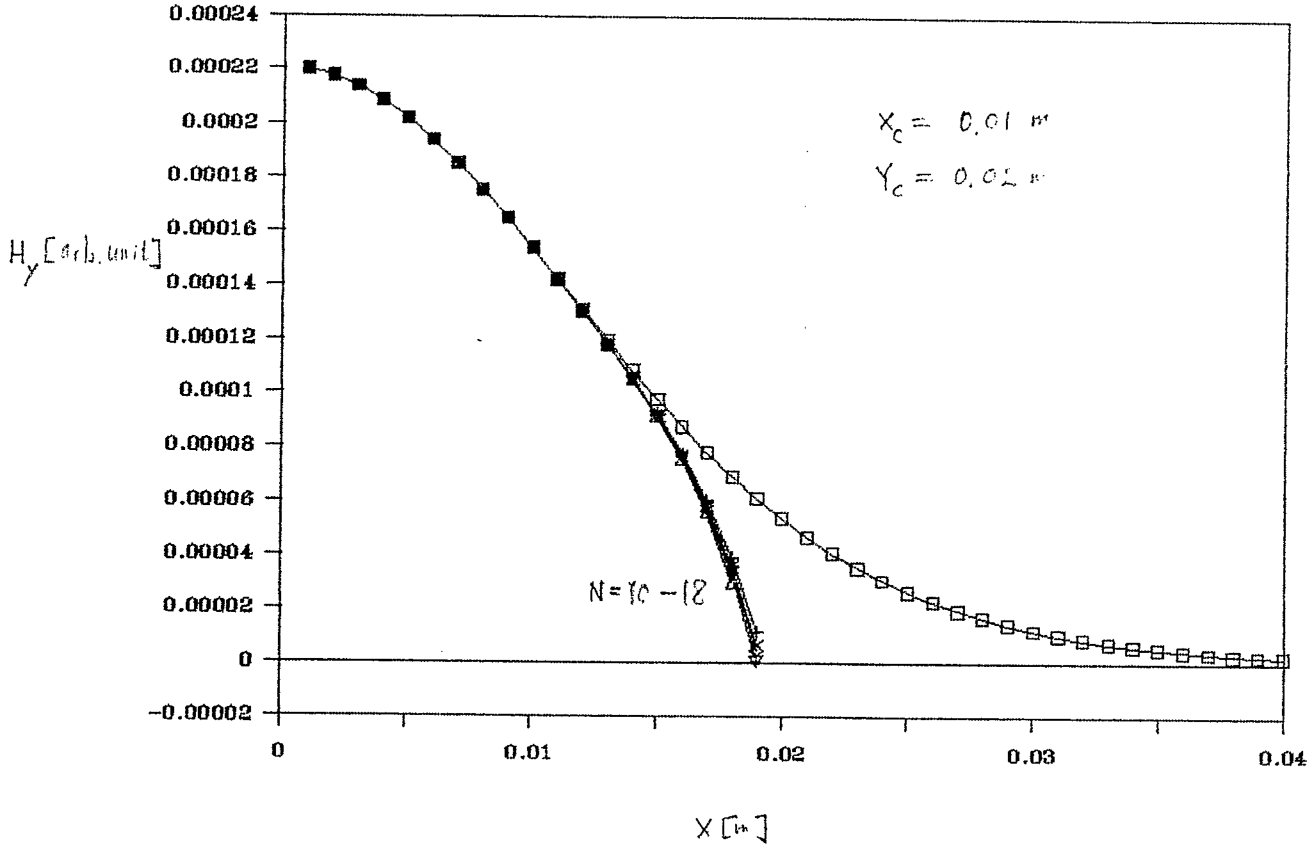


Fig. 2

Fig. 3 Similar to that of Fig. 1 for the coil configurations shown on the Figures.



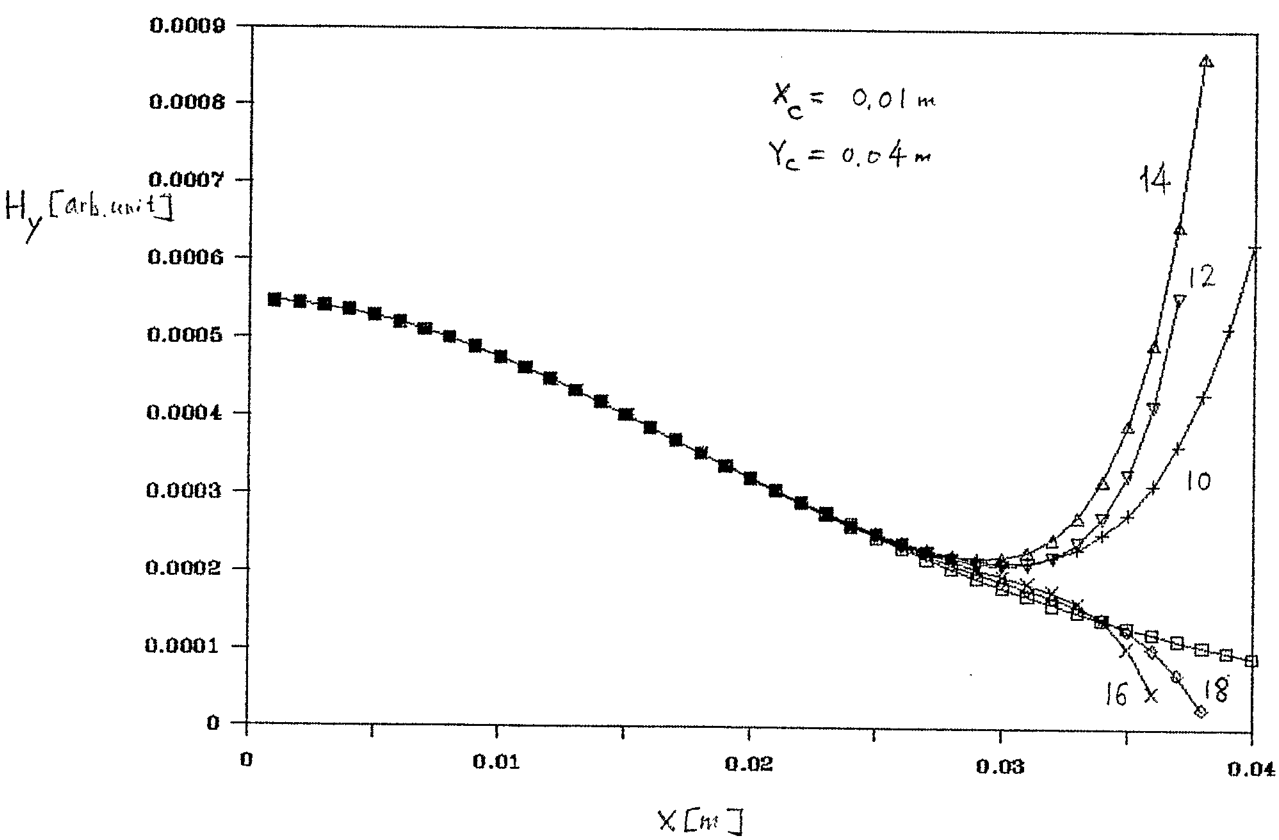


Fig. 4 Exact $1+\Delta B/B_0$ is compared with the multipole expansion for the booster correction coils. [The coil configuration is shown in Ref. 1.]