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# On Measurements of Helical Magnetic Fields Using Devices for Straight Magnets


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# On Measurements of Helical Magnetic Fields Using Devices for Straight Magnets

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**Abstract**

For the measurement of magnetic multipole coefficients $(a_n, b_n)$ inside straight magnets various methods and devices exist. The same devices can be used to measure the magnetic multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ of helical magnets.

Assuming that a measurement device parameterizes the measured magnetic helical field in terms of coefficients $(a_n, b_n)$, this note gives conversion formulae to obtain the coefficients $(\tilde{a}_n, \tilde{b}_n)$. The cases of radial rotating coils, tangential rotating coils and rotating Hall probes are treated.

## 1 Introduction

The magnetic field inside straight magnets can be parameterized in terms of multipole coefficients $(a_n, b_n)$ while the field inside helical magnets can be described by means of helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Sec. 2 and Ref. [1]). If a magnetic field measurement device always parameterizes its measurements in terms of $(a_n, b_n)$, conversion formulae are needed to obtain the coefficients $(\tilde{a}_n, \tilde{b}_n)$ when a helical magnetic field is measured.

Three types of measurement devices are treated in this note: first, rotating "radial" coils (cf. Fig. 1 (a)), second, rotating "tangential" coils (cf. Fig. 1 (b)) and third, rotating Hall probes (cf. Fig. 1 (c)). While the multipole measurements with rotating coils are obtained from the field within the coil area (shaded in Fig. 1), the multipole coefficients of Hall probe measurements are only obtained from the field on the circumference of a circle with given radius $r$.

In the following a cylindrical coordinate system $(r, \theta, s)$ is used where $s$ designates the coordinate along the longitudinal magnet axis (cf. Fig. 1). The field parameterization of straight and helical magnets in terms of multipole coefficients is given. For rotating coils the magnetic flux is computed for both parameterizations $(a_n, b_n)$ and $(\tilde{a}_n, \tilde{b}_n)$. The results are stated in a form that allows a direct comparison. For Hall probe measurements the magnetic fields in both parameterizations are also stated in a form that allows a comparison. For all cases conversion formulae from $(a_n, b_n)$ to $(\tilde{a}_n, \tilde{b}_n)$ are given in the same form.

This note is an extension of the earlier note RHIC/AP/98 (AGS/RHIC/SN/28) which treated only the case of radial rotating coils. However, the notation has slightly changed.

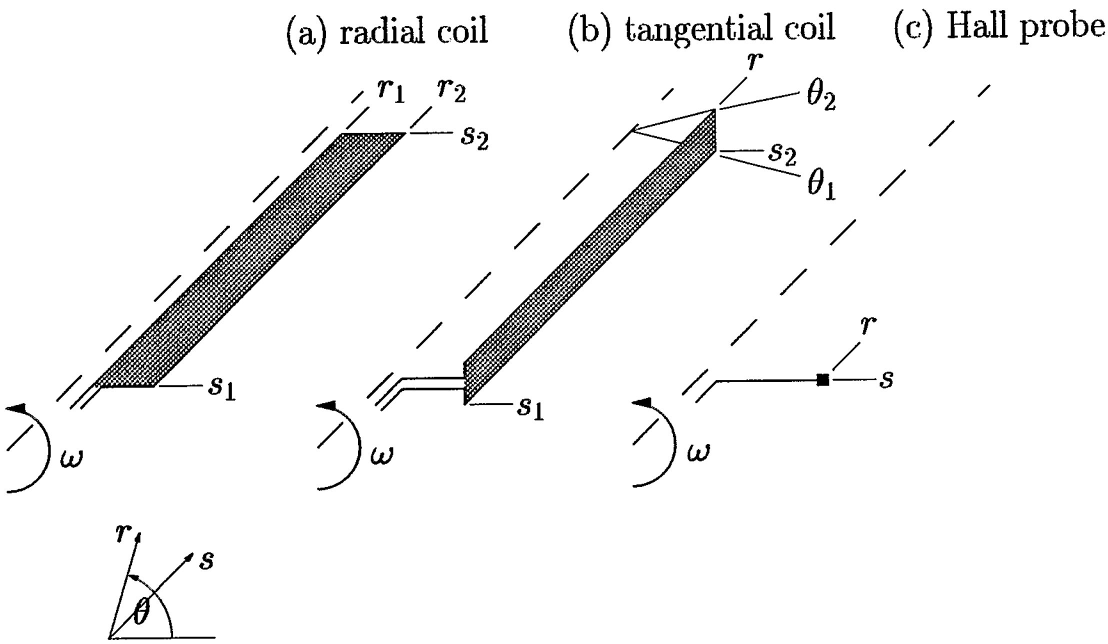


Figure 1: Three methods to measure magnetic multipole coefficients.

## 2 Magnetic Field Parameterization

The magnetic fields of straight magnets can be parameterized in terms of multipole coefficients $(a_n, b_n)$ as

$$B_r = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[a_n \cos\Big((n+1)\theta\Big) + b_n \sin\Big((n+1)\theta\Big)\right], \tag{1}$$

$$B_\theta = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[b_n \cos\Big((n+1)\theta\Big) - a_n \sin\Big((n+1)\theta\Big)\right], \tag{2}$$

$$B_s = 0, \tag{3}$$

where $B_0$ is a reference field strength and $r_0$ a reference radius. The helical magnetic field can be expressed in terms of $(\tilde{a}_n, \tilde{b}_n)$ as

$$B_r = B_0 \sum_{n=0}^{\infty} f_n I'_{n+1}\Big((n+1)kr\Big) \cdot \left[\tilde{a}_n \cos\Big((n+1)\tilde{\theta}\Big) + \tilde{b}_n \sin\Big((n+1)\tilde{\theta}\Big)\right], \tag{4}$$

$$B_\theta = -\frac{1}{kr} B_s, \tag{5}$$

$$B_s = -B_0 \sum_{n=0}^{\infty} f_n I_{n+1}\Big((n+1)kr\Big) \cdot \left[\tilde{b}_n \cos\Big((n+1)\tilde{\theta}\Big) - \tilde{a}_n \sin\Big((n+1)\tilde{\theta}\Big)\right]. \tag{6}$$

Here, $\tilde{\theta} = \theta - ks$ and $k = 2\pi/\lambda$ where $\lambda$ is the helical wave length. $k$ shall have the positive sign for right-handed helices. $I_n$ denotes the modified Bessel function of order $n$ and $I'_n$ its derivative with respect to the argument of the Bessel function. The coefficients $f_n$ are defined as

$$f_n = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n}. \tag{7}$$

Equations (4-6) give the same result as equations (1-3) if the helical wave length $\lambda$ tends to infinity. The transverse magnetic field component is vertical at $s = 0$ in this notation.

## 3 Rotating Coils

The magnetic flux through a coil is

$$\Phi(\theta) = N \int \vec{B}(r, \theta) \cdot d\vec{a} \tag{8}$$

where N is the number of coils windings. For rotating coils one has $\theta = \omega t$ and the induced voltage

$$U = -\frac{d\Phi}{dt} \tag{9}$$

is proportional to the angular velocity $\omega$.

### 3.1 Radial Coils

The area of a flat rotating radial coil ranges from $r_1$ to $r_2$ and from $s_1$ to $s_2$ (cf. Fig. 1). The magnetic flux (8) through the coils is

$$\Phi(\theta) = N \int_{s_1}^{s_2} \int_{r_1}^{r_2} B_\theta(r, \theta)\, dr\, ds. \tag{10}$$

Using (2) the magnetic flux (10) for straight magnets becomes

$$\left.\begin{aligned} \Phi(\theta) &= N B_0 (s_2 - s_1) \sum_{n=0}^{\infty} K_n \left[ b_n \cos\Big((n+1)\theta\Big) - a_n \sin\Big((n+1)\theta\Big) \right] \\ &\text{with} \\ K_n &= \frac{r_0}{n+1} \left[ \left(\frac{r_2}{r_0}\right)^{(n+1)} - \left(\frac{r_1}{r_0}\right)^{(n+1)} \right]. \end{aligned}\right\} \tag{11}$$

With (5) the flux for helical fields is

$$\left.\begin{aligned}\Phi(\theta) &= NB_0(s_2-s_1)\sum_{n=0}^{\infty} R_n\left[\hat{b}_n \cos\Big((n+1)\theta\Big) - \hat{a}_n \sin\Big((n+1)\theta\Big)\right] \\ &\text{with} \\ R_n &= f_n \int_{r_1}^{r_2} \frac{1}{kr} I_{n+1}\Big((n+1)kr\Big)\, dr.\end{aligned}\right\} \quad (12)$$

In (12) new magnetic multipole coefficients

$$\begin{aligned}\hat{a}_n &= +\tilde{a}_n T_n + \tilde{b}_n S_n, \\ \hat{b}_n &= -\tilde{a}_n S_n + \tilde{b}_n T_n.\end{aligned} \quad (13)$$

are used for which

$$\begin{aligned}S_n(s_1,s_2) &= +\frac{1}{(s_2-s_1)(n+1)k}\left[\cos\Big((n+1)ks_2\Big) - \cos\Big((n+1)ks_1\Big)\right] \\ &= -\frac{2}{(s_2-s_1)(n+1)k}\sin\frac{(n+1)k(s_2-s_1)}{2}\sin\frac{(n+1)k(s_2+s_1)}{2}\end{aligned} \quad (14)$$

and

$$\begin{aligned}T_n(s_1,s_2) &= +\frac{1}{(s_2-s_1)(n+1)k}\left[\sin\Big((n+1)ks_2\Big) - \sin\Big((n+1)ks_1\Big)\right] \\ &= +\frac{2}{(s_2-s_1)(n+1)k}\sin\frac{(n+1)k(s_2-s_1)}{2}\cos\frac{(n+1)k(s_2+s_1)}{2}\end{aligned} \quad (15)$$

have been defined. For later use the quantities $S_n(s,s)$ and $T_n(s,s)$ are defined as

$$\begin{aligned}S_n(s,s) &= \lim_{\Delta s\to 0} S_n(s,s+\Delta s) = -\sin\Big((n+1)ks\Big), \\ T_n(s,s) &= \lim_{\Delta s\to 0} T_n(s,s+\Delta s) = +\cos\Big((n+1)ks\Big).\end{aligned} \quad (16)$$

## 4 Tangential Coils

The magnetic flux through this type of coil is (cf. Fig. 1)

$$\Phi(\theta) = N\int_{s_1}^{s_2}\int_{\theta_1}^{\theta_2} B_r(r,\bar{\theta})\, r d\bar{\theta}\, ds. \quad (17)$$

The difference $\Delta\theta = \theta_2 - \theta_1$ is fixed and one can assume that

$$\theta_1 = \theta - \frac{\Delta\theta}{2} \quad \text{and} \quad \theta_2 = \theta + \frac{\Delta\theta}{2} \quad (18)$$

holds. With (1) the magnetic flux in straight magnets becomes

$$\left.\begin{aligned}\Phi(\theta) &= NB_0(s_2 - s_1)\cdot\frac{2}{n+1}\sin\frac{(n+1)\Delta\theta}{2}\times\\ &\quad\times\sum_{n=0}^{\infty}K_n\left[a_n\cos\Big((n+1)\theta\Big)+b_n\sin\Big((n+1)\theta\Big)\right]\\ &\text{with}\\ K_n &= \frac{r^{n+1}}{r_0^n}\end{aligned}\right\}\qquad(19)$$

and for helical fields one obtains with (4)

$$\left.\begin{aligned}\Phi(\theta) &= NB_0(s_2 - s_1)\cdot\frac{2}{n+1}\sin\frac{(n+1)\Delta\theta}{2}\times\\ &\quad\times\sum_{n=0}^{\infty}R_n\left[\hat{a}_n\cos\Big((n+1)\theta\Big)+\hat{b}_n\sin\Big((n+1)\theta\Big)\right]\\ &\text{with}\\ R_n &= f_n\ r\ I'_{n+1}\Big((n+1)kr\Big).\end{aligned}\right\}\qquad(20)$$

The $(\hat{a}_n, \hat{b}_n)$ are defined by (13) and the $(S_n, T_n)$ needed in this definition in (14,15).

# 5 Hall Probes

For Hall probe measurements the magnetic fields, either tangential or radial, can be compared directly.

## 5.1 Tangential Field Components

The tangential field in a straight magnet is (cf. Eq. (2))

$$\left.\begin{aligned}B_\theta(\theta) &= B_0\sum_{n=0}^{\infty}K_n\left[b_n\cos\Big((n+1)\theta\Big)-a_n\sin\Big((n+1)\theta\Big)\right]\\ &\text{with}\\ K_n &= \left(\frac{r}{r_0}\right)^n\end{aligned}\right\}\qquad(21)$$

and in a helical field (cf. Eq. (5))

$$\left.\begin{aligned} B_\theta(\theta) &= B_0 \sum_{n=0}^{\infty} R_n \left[\hat{b}_n \cos\Big((n+1)\theta\Big) - \hat{a}_n \sin\Big((n+1)\theta\Big)\right] \\ &\text{with} \\ R_n &= \frac{1}{kr} f_n I_{n+1}\Big((n+1)kr\Big). \end{aligned}\right\} \tag{22}$$

The $(\hat{a}_n, \hat{b}_n)$ are again defined by equations (13) and the $(S_n, T_n)$ needed in this definition by equations (16).

### 5.2 Radial Field Components

The radial field in a straight magnet is cf. Eq. (1))

$$\left.\begin{aligned} B_r(\theta) &= B_0 \sum_{n=0}^{\infty} K_n \left[a_n \cos\Big((n+1)\theta\Big) + b_n \sin\Big((n+1)\theta\Big)\right] \\ &\text{with} \\ K_n &= \left(\frac{r}{r_0}\right)^n \end{aligned}\right\} \tag{23}$$

and in a helical field cf. Eq. (4))

$$\left.\begin{aligned} B_r(\theta) &= B_0 \sum_{n=0}^{\infty} R_n \left[\hat{a}_n \cos\Big((n+1)\theta\Big) + \hat{b}_n \sin\Big((n+1)\theta\Big)\right] \\ &\text{with} \\ R_n &= f_n I'_{n+1}\Big((n+1)kr\Big). \end{aligned}\right\} \tag{24}$$

Also in this case the $(\hat{a}_n, \hat{b}_n)$ are defined by equations (13) and the $(S_n, T_n)$ by equations (16).

## 6 Conversion

It is now assumed that a device parameterizes the measured magnetic field in terms of multipole coefficients $(a_n, b_n)$ for straight magnets. If the measured magnetic field has helical symmetry, the coefficients $(\tilde{a}_n, \tilde{b}_n)$ can be derived by comparing (11) with (12), (19) with (20), (21) with (22) or (23) with (24). One obtains for all cases

$$\begin{aligned} K_n b_n &= R_n \hat{b}_n, \\ K_n a_n &= R_n \hat{a}_n. \end{aligned} \tag{25}$$

With (13) it follows that

$$\boxed{\begin{aligned}\tilde{a}_n &= \frac{K_n}{R_n}\cdot\frac{a_nT_n - b_nS_n}{S_n^2+T_n^2},\\ \tilde{b}_n &= \frac{K_n}{R_n}\cdot\frac{a_nS_n + b_nT_n}{S_n^2+T_n^2}.\end{aligned}} \tag{26}$$

Three special cases are considered here.
**(a)** Measuring coil of one helical wavelength with $s_1 = s$, $s_2 = s + \lambda$.
From equations (14) and (15) one obtains

$$S_n = T_n = 0 \tag{27}$$

and with (13)

$$\hat{a} = \hat{b} = 0. \tag{28}$$

The magnetic flux (10) or (17) is therefore zero and the coefficients can not be obtained.

**(b)** Measuring coil of half helical wave length with $s_1 = 0$, $s_2 = \lambda/2$.
In this case one has

$$S_n = \begin{cases} -\frac{2}{(n+1)\pi} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases} \qquad \text{and} \qquad T_n = 0. \tag{29}$$

Only coefficients with $n$ even (i.e. helical dipole, sextupole etc. coefficients) can be measured. For those one has

$$\begin{aligned}\tilde{a}_n &= +\frac{K_n}{R_n}\frac{(n+1)\pi}{2}b_n,\\ \tilde{b}_n &= -\frac{K_n}{R_n}\frac{(n+1)\pi}{2}a_n.\end{aligned} \tag{30}$$

**(c)** Infinitely short measuring coil with $s_1 = s$, $s_2 = s + ds$ or Hall probes.
For both cases the $S_n$ and $T_n$ are given by (16) and (26) becomes

$$\begin{aligned}\tilde{a} &= \frac{K_n}{R_n}\Big[+a_n\cos\Big((n+1)ks\Big) + b_n\sin\Big((n+1)ks\Big)\Big],\\ \tilde{b} &= \frac{K_n}{R_n}\Big[-a_n\sin\Big((n+1)ks\Big) + b_n\cos\Big((n+1)ks\Big)\Big].\end{aligned} \tag{31}$$

If in addition $s = 0$, the $(\tilde{a}_n, \tilde{b}_n)$ can by obtained from the $(a_n, b_n)$ by multiplication with $K_n/R_n$.

Table 1: Coefficients for BNL tangential measurement coils.

| Order n | $K_n/R_n$ | $U_n$ |
|---|---|---|
| 0 | 0.998730 | 0.984961 |
| 1 | 0.996578 | 0.940658 |
| 2 | 0.995191 | 0.869475 |
| 3 | 0.993849 | 0.775221 |
| 4 | 0.992530 | 0.662877 |
| 5 | 0.991225 | 0.538287 |
| 6 | 0.989929 | 0.407788 |
| 7 | 0.988639 | 0.277814 |
| 8 | 0.987355 | 0.154508 |
| 9 | 0.986075 | 0.0433542 |

# 7 Example

At BNL tangential coils are used with $r = 2.74$ cm and $s_2 - s_1 = 23$ cm. The helical wave length of magnets to be measured is $\lambda = 2.4$ m. For this set-up one has

$$\frac{K_n}{R_n} = \frac{(n+1)^{n+1}}{2^{n+1}(n+1)!} \cdot \frac{k^n r^n}{I'_{n+1}\Big((n+1)kr\Big)}. \tag{32}$$

New coefficients $U_n$ are used to denote

$$\begin{aligned} U_n &= -S_n(s_1, s_2) / \sin \frac{(n+1)k(s_2+s_1)}{2} \\ &= +T_n(s_1, s_2) / \cos \frac{(n+1)k(s_2+s_1)}{2}. \end{aligned} \tag{33}$$

The term $(s_2 + s_1)/2$ in the argument of the sin and cos function in (33) is the $s$-position of the middle of the coil. In Tab. 1 the coefficients $K_n/R_n$ are listed for orders 0 through 9 along with the $U_n$. Using these numbers and equations (26) and (33) the coefficients $(\tilde{a}_n, \tilde{b}_n)$ can be obtained from the coefficients $(a_n, b_n)$.

All coefficients $K_n/R_n$ are close to 1. This is due to the fact that $kr \approx 0.072$ is small compared to 1. For small arguments the Bessel function in Eq. (32) can be expanded in a power series as

$$I_n(z) = \frac{1}{2^n n!} z^n. \tag{34}$$

With this expansion $K_n/R_n = 1$ holds.

I would like to thank R. Gupta and E. Willen for discussions and reading the manuscript.

## References

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