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Implementation of the TRL Algorithm for Improved Impedance Measurements

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Brookhaven National Laboratory

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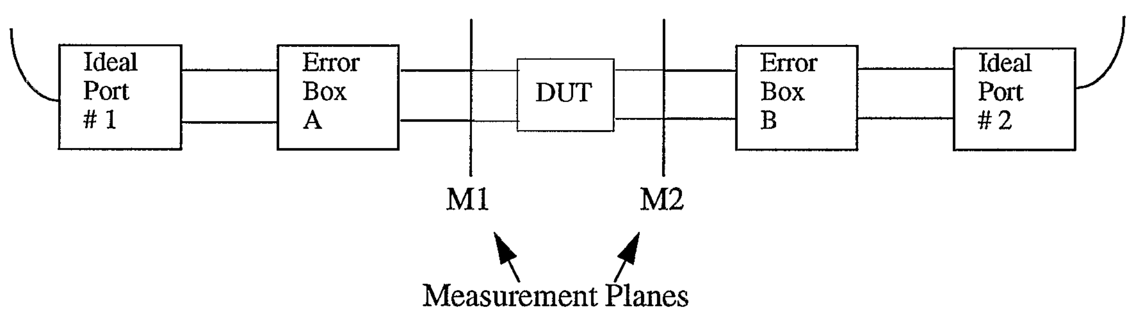
1 Introduction

The Thru-Reflect-Line (TRL) algorithm for deembedding the scattering parameters and hence the impedance of a Device Under Test (DUT) has been implemented in LabVIEW. This algorithm helps obtain the correct impedance of a device placed between mismatched ports.

The nonideal port at each end of the two-port DUT is modeled by an ideal port in cascade with an error box (Figure 1). The scattering parameters are measured for three known conditions between the measurement planes M1 and M2, using the Network Analyzer. The scattering parameters of the Error Boxes A and B are then determined, and the scattering parameter and the impedance of the DUT is calculated.

FIGURE 1.

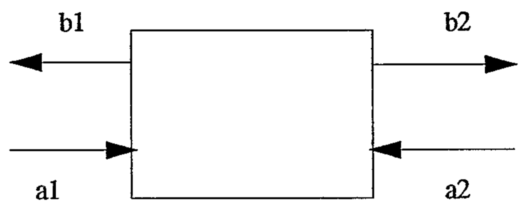


2 Scattering Parameters and Impedance

A common way to measure the longitudinal coupling impedance of accelerator devices is to model the beam with a wire and then measure this system with a vector network analyzer. Unfortunately, the high impedance of the wire pipe system is difficult to match to the 50 ohm ports of the network analyzer and the resulting reflections can severely degrade the accuracy of the measurement. The network analyzer typically used in these measurements provides accuracy enhancement through the use of Open, Short and Load standards. These standards are difficult to construct in the pipe and wire geometry. Therefore, the TRL calibration scheme that relies on simpler standards is used to regain the desired accuracy.

The scattering parameters of a microwave network give the relationship between the incident waves and the reflected waves at its terminals[1].

FIGURE 2.



$$\begin{bmatrix} b1 \\ b2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a1 \\ a2 \end{bmatrix}$$

a1,a2 are the incident waves; b1,b2 are the reflected waves;

$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ is the Scattering Matrix

$S11 = \left.\frac{b1}{a1}\right|_{a2=0}$ reflection coefficient at Port 1 when Port 2 is match terminated.

$S21 = \left.\frac{b2}{a1}\right|_{a2=0}$ forward gain.

$S12 = \left.\frac{b1}{a2}\right|_{a1=0}$ reverse gain.

$S22 = \left.\frac{b2}{a2}\right|_{a1=0}$ reflection coefficient at Port 2 when Port 1 is match terminated.

The HP 8753 Network Analyzer measures the four scattering parameters upto a frequency of 6 GHz.

Also defined is the transmission matrix which gives the relationship between the input quantities and the output quantities.This is useful when two ports are connected in cascade, because the transmission matrix of the cascaded ports is equal to the product of the transmission matrix of each port.

$$\begin{bmatrix} b2 \\ a2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} a1 \\ b1 \end{bmatrix}$$

$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$ is the transmission matrix.

The transmission matrix can be obtained from the scattering matrix as follows:

$$R = \begin{bmatrix} -|S| & S_{11} \\ -S_{22} & 1 \end{bmatrix} \frac{1}{S_{21}}$$

For the measurement of the longitudinal coupling impedance of accelerator devices, the beam is simulated by a current in a wire placed at the axis of the pipe. The longitudinal impedance is given by[2]

$$Z(\omega) = 2Z0 \frac{(S_{21}(ref) - S_{21}(DUT))}{S_{21}(DUT)}$$

where S21(ref) is the scattering parameter with DUT replaced by a reference pipe, Z0 is the characteristic impedance of the pipe wire system.

This report describes the scattering parameter correction.

3 Program Algorithm

The scattering parameters between the measurement planes M1 and M2 (Figure 1) are measured for the following four cases:

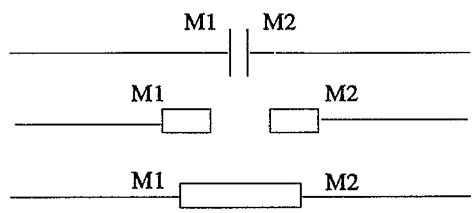
Thru Measurement: The measurement planes M1 and M2 are connected and all the S parameters are measured.

Reflect Measurement: A reflective load of unknown reflection coefficient Γ is connected at the measurement plane M1 and S_{11} is measured. The reflective load is connected at measurement plane M2 and S_{22} is measured.

Line (Delay) Measurement: A nonreflecting transmission line of arbitrary length is connected between M1 and M2 and the S parameters are measured. The length of the transmission line should not be a multiple of half wavelength. Therefore a second set of S parameter measurements are done with a second transmission line such that its length is not a multiple of the length of the first transmission line. At the frequencies where the length of the first transmission line is a multiple of half wavelength, the program uses the second set of measurements. See Sec 4.

Device Measurement: The device is placed between the measurement planes M1 and M2, and the S parameters are measured.

FIGURE 3.



The transmission matrices R_A and R_B of the error ports A and B are determined as follows[3],[4],[5]:

$R_T = R_A R_B$ (1)

R_T is the transmission matrix obtained from the thru measurement

$R_D = R_A R_l R_B$ (2)

R_D is the transmission matrix obtained from the line measurement

$R_l = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{\gamma l} \end{bmatrix}$ is the transmission matrix of the nonreflecting delay line.

From (1) and (2)

$$R_M R_A = R_A R_l$$

where $R_M = R_D R_T^{-1} = \begin{bmatrix} m11 & m12 \\ m21 & m22 \end{bmatrix}$

<u>Determining R_A and R_B</u>

For Port A, write

$$R_A = \begin{bmatrix} ka & pb \\ k & p \end{bmatrix}$$

$$R_M R_A = R_A R_l$$

i.e. $\begin{bmatrix} m11 & m12 \\ m21 & m22 \end{bmatrix} \begin{bmatrix} ka & pb \\ k & p \end{bmatrix} = \begin{bmatrix} ka & pb \\ k & p \end{bmatrix} \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{\gamma l} \end{bmatrix}$ (3)

Solving equation (3) for a and b,

$$a, b = -\frac{(m22 - m11)}{2m21} \pm \sqrt{\left(\frac{m22 - m11}{2m21}\right)^2 + \frac{m12}{m21}}$$

Select a and b such that |a| > |b|.

Similarly for Port B

$$R_B R_N = R_l R_B$$

where $R_N = R_T^{-1} R_D = \begin{bmatrix} n11 & n12 \\ n21 & n22 \end{bmatrix}$

$$R_N^T = \begin{bmatrix} nt11 & nt12 \\ nt21 & nt22 \end{bmatrix}$$

$$R_N^T R_B^T = R_B^T R_l^T$$

write

$$R_B^T = \begin{bmatrix} rc & sd \\ r & s \end{bmatrix}$$

$$R_N^T R_B^T = R_B^T R_l^T$$

i.e. $\begin{bmatrix} nt11 & nt12 \\ nt21 & nt22 \end{bmatrix} \begin{bmatrix} rc & sd \\ r & s \end{bmatrix} = \begin{bmatrix} rc & sd \\ r & s \end{bmatrix} \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{\gamma l} \end{bmatrix}$ (4)

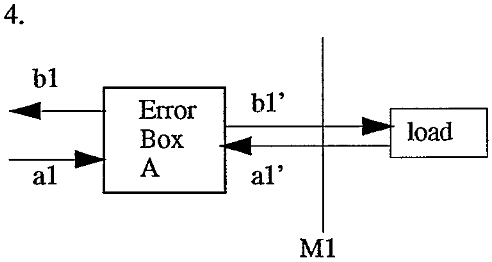
Solving equation (4) for c and d,

$$c, d = -\frac{(nt22 - nt11)}{2nt(21)} \pm \sqrt{\left(\frac{nt22 - nt11}{(2nt21)}\right)^2 + \frac{nt12}{nt21}}$$

Select c and d such that $|c| > |d|$.

The reflection coefficient Γ is determined as follows

FIGURE 4.



With Port A connected to the load,

$\begin{bmatrix} b1 \\ b1' \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a1 \\ a1' \end{bmatrix}$ gives

$$p = \pm\sqrt{\frac{\Gamma(a - w1)}{(a-b)(w1-b)}} \qquad (5)$$

$w1 = S_{11}$ when Port A is connected to the reflective load, Γ is the reflection coefficient of the load.

Also for a reciprocal network $|R_A| = 1$ gives

$$k = \frac{1}{p(a-b)} \qquad (6)$$

With Port B connected to the load,

$$s = \pm\sqrt{\frac{\Gamma(w2+c)}{(c-d)(w2+d)}} \qquad (7)$$

$w2 = S_{22}$ when Port B is connected to the reflective load.

For the reciprocal network B, $|R_B| = 1$ gives

$$r = \frac{1}{s(c-d)} \qquad (8)$$

From $\quad R_T = R_A R_B$

$$R_T = \begin{bmatrix} karc + pbsd & kar + pbs \\ krc + psd & kr + ps \end{bmatrix}$$

Therefore

$$S_{11T} = \frac{p^2\left(\frac{s}{r}\right)(a-b)b + a}{p^2\left(\frac{s}{r}\right)(a-b) + 1} \qquad (9)$$

From equations (5), (7) and (9)

$$\Gamma = \pm\sqrt{\frac{(S_{11T}-a)(w1-b)(w2+d)}{(S_{11T}-b)(w1-a)(w2+c)}}$$

After determining Γ, p and s are determined from equations (5) and (7); k and r are determined from (6) and (8). In the Thru-Short-Delay (TSD) algorithm, Γ is set to -1.

The sign of p, k and r, s is selected by comparing the sign of $R_A R_B$ with that of R_T. The sign of Γ is determined by knowing whether the load is closer to a short or an open.

Determining the Scattering matrix of the device

$$R_{MDUT} = R_A R_{DUT} R_B$$

R_{MDUT} is the transmission matrix obtained from the device measurement. R_{DUT} is the transmission matrix of the device.

$$R_{DUT} = R_A^{-1} R_{MDUT} R_B^{-1}.$$

S_{DUT}, the scattering matrix of the calibrated device, is obtained from R_{DUT}.

4 Experimental Results

A LabVIEW program with user interface is written to implement the above algorithm. The program can collect data from a Network Analyzer via GPIB bus or read data from a file. Measurements were done with a 400 MHz filter as a device. The device is connected to the two ports of the Network Analyzer and a mismatch is placed at Port B. Fig A1 shows the S21 parameter of the uncalibrated device. Fig A2 shows the S21 parameter of the device obtained from the above calibration algorithm. This matches with S21 parameter in Fig A3, which shows the device measured with the Network Analyzer and no mismatches at either Port. As mentioned in Section 2, the Network Analyzer cannot in general deembed S Parameters, as it requires gating of the mismatches and also requires precision standards (short, open and matching load) which are not available for a 300 ohm system.

Length of delay

The frequencies at which the length of the delay line is an integer multiple of half wavelength, i.e. $l = \mp n\frac{\lambda}{2}$, both $e^{\gamma l} and e^{-\gamma l} = \mp 1$ and the transmission matrix is degenerate. Therefore, at these frequencies, measurements from the second delay are used. The length of the second transmission line should not be a multiple of the length of the first transmission line.

$R_M = \mp 1\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ at these values. The 'delayvariable' (dv), which gives the difference between the diagonal elements of R_M, is set to one at these values and gives a criteria for switching to the second delay. Figs A4 and A5 indicate where dv = 1, and the value of S21 at these frequencies. The length of the delay is 16 cm. Fig A6 and A7 give the value of dv and S21, with a 59 cm delay. Fig A8 gives the corrected S21 of the device, using the two delays. The value of dv =1 should not overlap for the two delays. Delays of lengths say 23 cm and 59 cm would be better. Delays of appropriate lengths have to be built for experiments.

5 References

[1] Fields and Waves in Modern Radio, S. Ramo, J. Whinnery, John Wiley & Sons, 1956.

[2] On Coaxial Wire Measurements of the Longitudinal Coupling Impedance, H. Hahn, F. Pedersen, BNL 50870, April 1978.

[3] A Unified Mathematical Approach to Two-Port Calibration Techniques and some Applications, R. A. Soares, P. Gouzien, P. Legaud and G. Follot, IEEE Trans. Microwave Theory Tech, vol. 37, pp. 1669-1673, Nov 1989

[4] "Thru-Reflect-Line": An Improved Technique for calibrating the Dual Six-Port Automatic Network Analyzer, G. F. Engen and C. A. Hoer, IEEE Trans. Microwave Theory Tech., vol. MTT-27. pp. 987-998, Dec 1979

[5] Transmission Line Impedance Measurements for an Advanced Hadron Facility, L.S. Walling, D.E. McMurray, D.V. Neuffer, H.A. Thiessen, Nucl. Instr. and Meth in Phys Res. A281 (1989) 433-447.

Fig A1

Front Panel

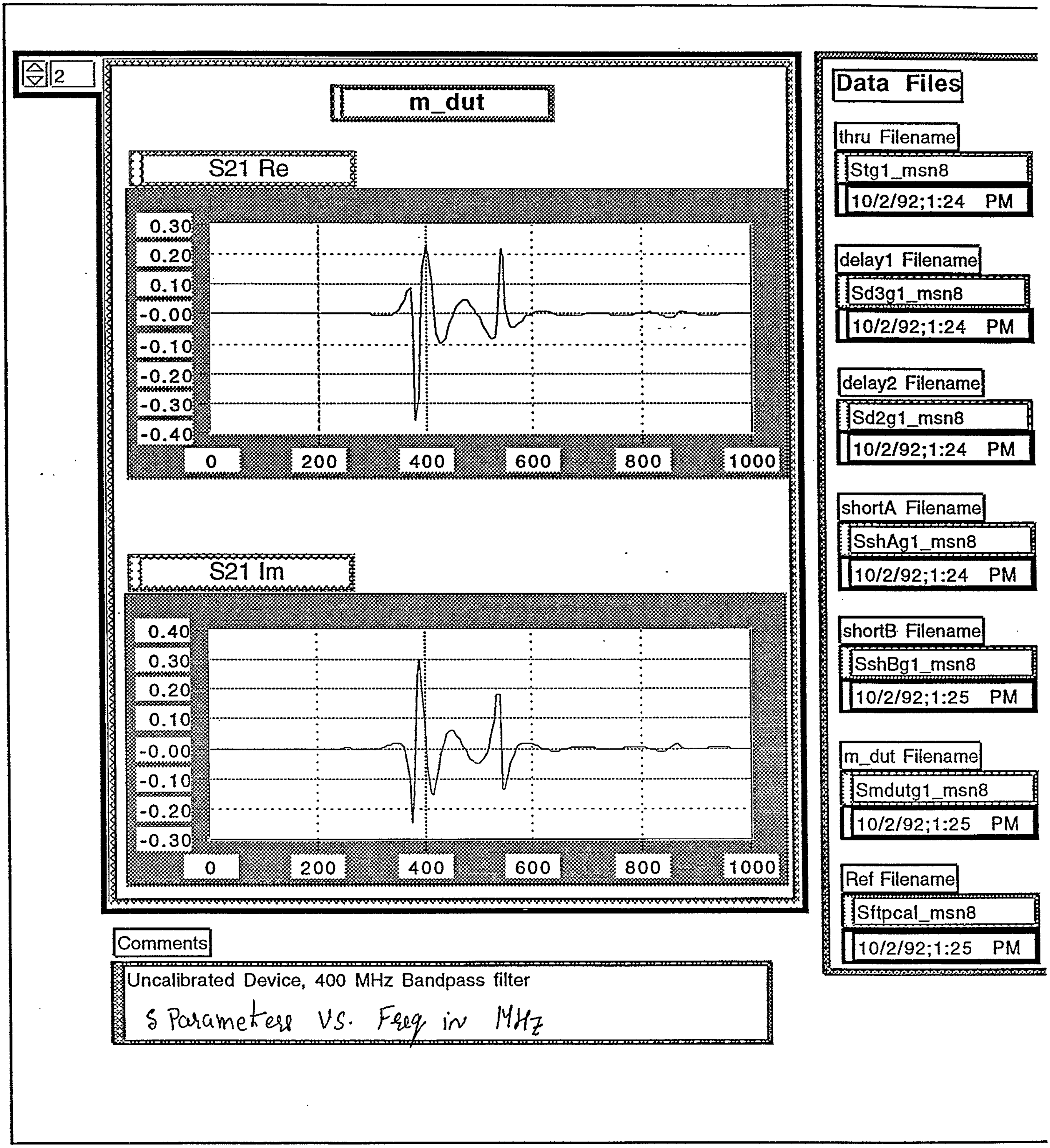


Fig A2

Front Panel

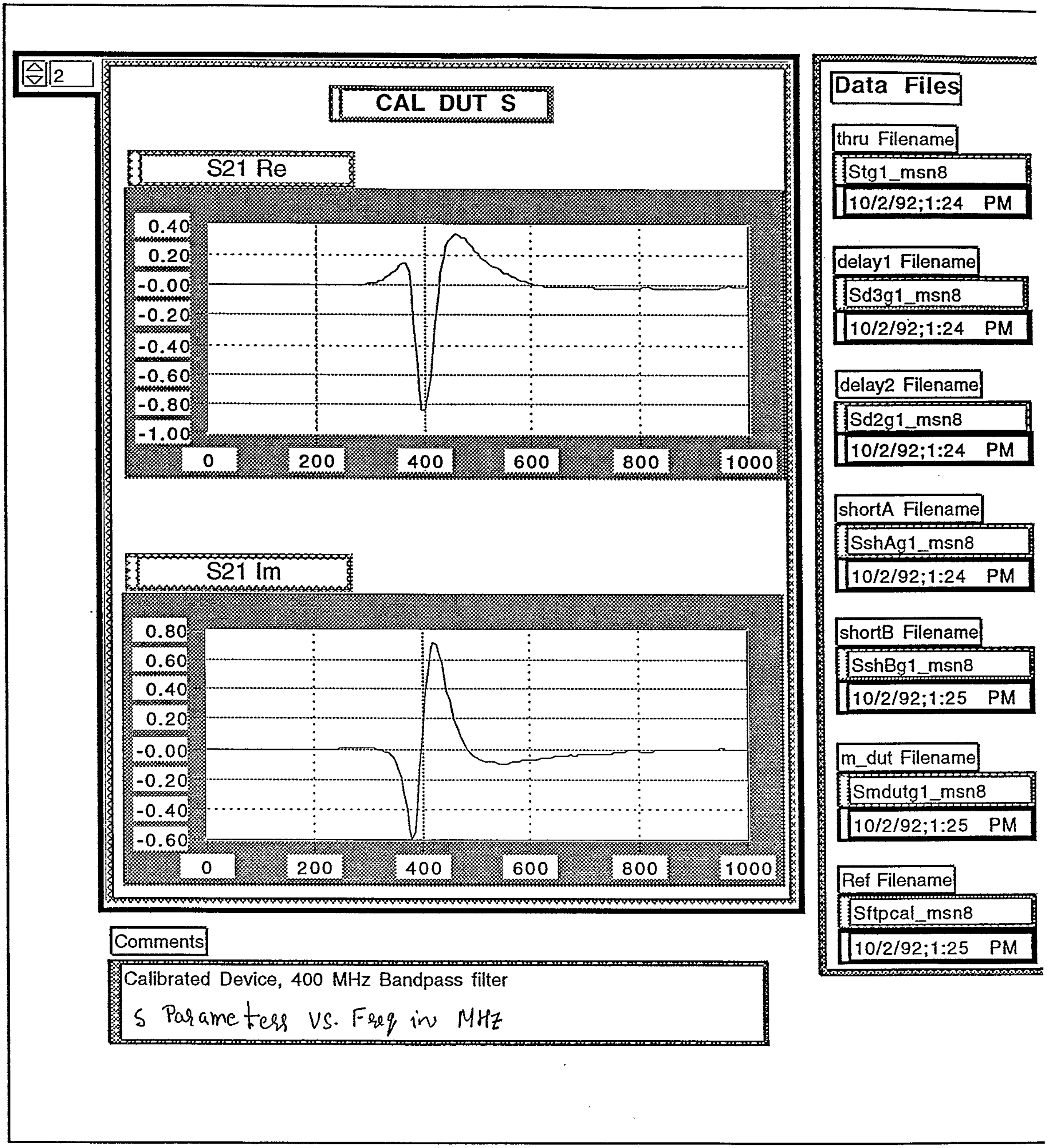


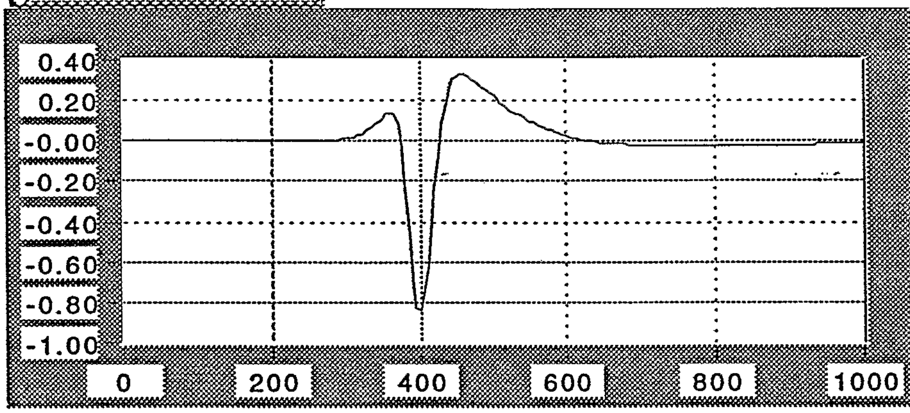
Fig A3

Front Panel

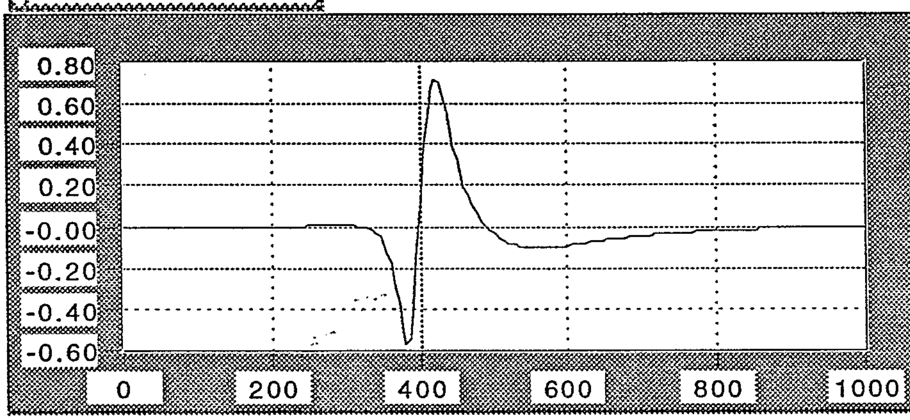
2

Ref

S21 Re



S21 Im



Data Files

thru Filename
Stg1_msn8
10/2/92;1:24 PM

delay1 Filename
Sd3g1_msn8
10/2/92;1:24 PM

delay2 Filename
Sd2g1_msn8
10/2/92;1:24 PM

shortA Filename
SshAg1_msn8
10/2/92;1:24 PM

shortB Filename
SshBg1_msn8
10/2/92;1:25 PM

m_dut Filename
Smdutg1_msn8
10/2/92;1:25 PM

Ref Filename
Sftpcal_msn8
10/2/92;1:25 PM

Comments

Device Measured with NA, no mismatches; 400 MHz Bandpass filter
S Parameters vs. Freq in MHz

Fig A4

Front Panel

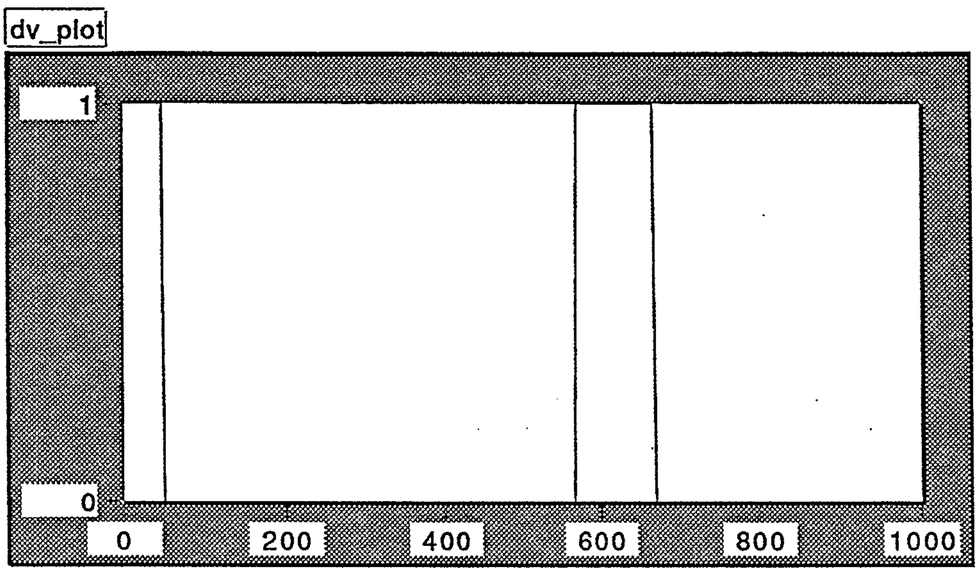


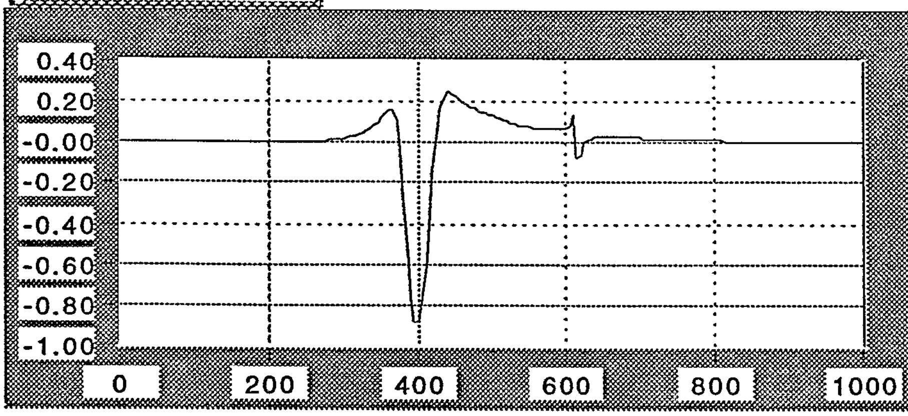
Fig A5

Front Panel

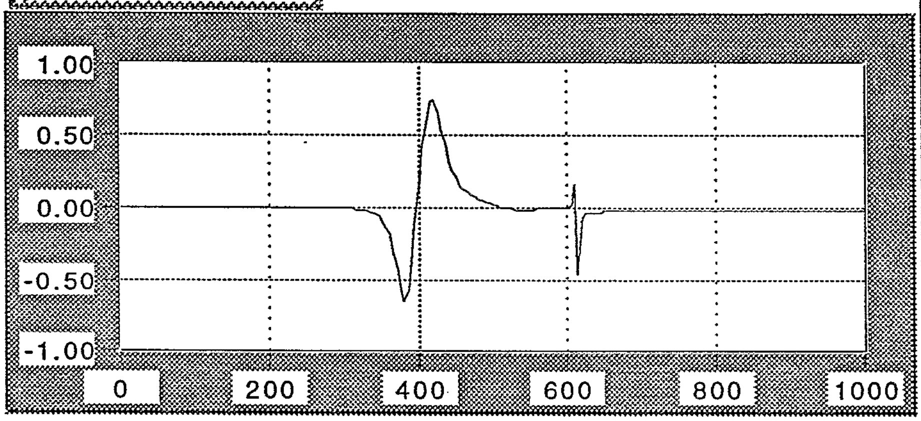
2

CAL DUT S

S21 Re



S21 Im



Data Files

thru Filename	St_nl3	10/2/92;1:35 PM
delay1 Filename	Sd1_nl3	10/2/92;1:33 PM
delay2 Filename	Sd1_nl3	10/2/92;1:34 PM
shortA Filename	SshA_nl3	10/2/92;1:34 PM
shortB Filename	SshB_nl3	10/2/92;1:34 PM
m_dut Filename	Smdut_nl3	10/2/92;1:34 PM
Ref Filename	Sdevice_nl1	10/2/92;1:34 PM

Comments

Device Measured with 1 delay of length 16 cm
S Parameters vs. Freq in MHz

Front Panel

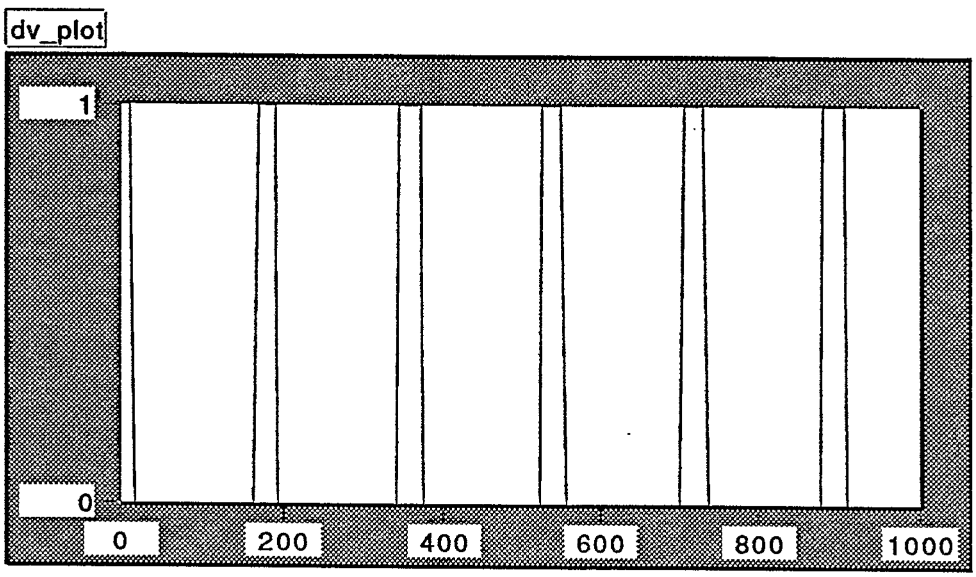


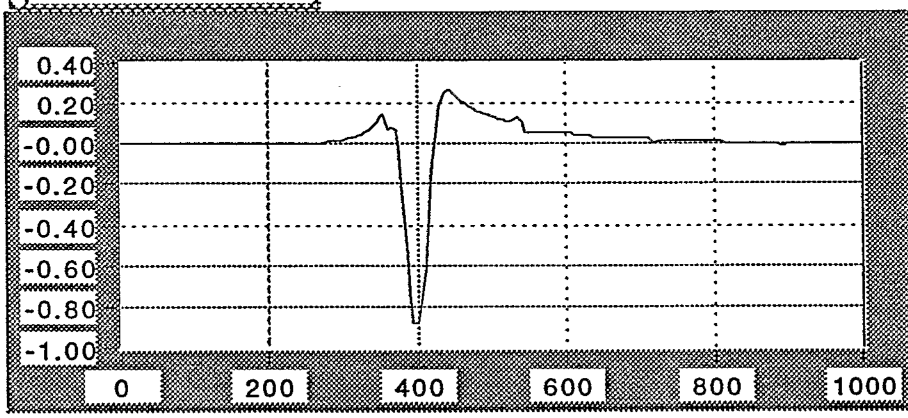
Fig A7

Front Panel

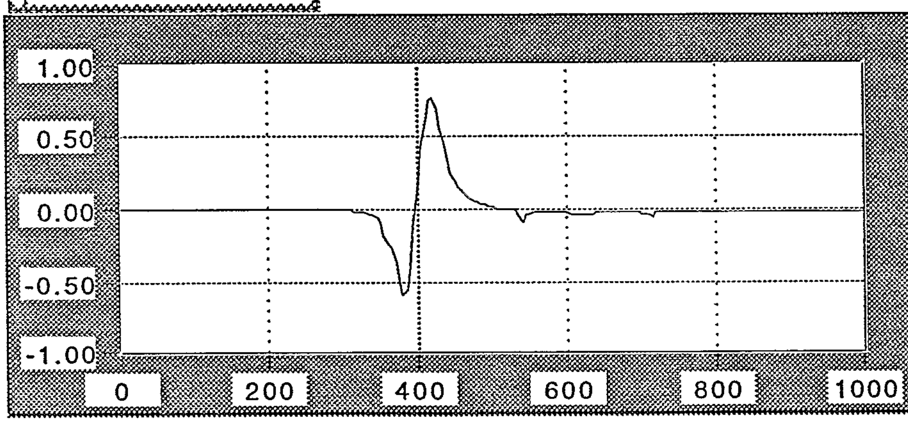
2

CAL DUT S

S21 Re



S21 Im



Data Files

thru Filename	St_nl3	10/2/92;1:35 PM
delay1 Filename	Sd4_nl3	10/2/92;1:38 PM
delay2 Filename	Sd4_nl3	10/2/92;1:38 PM
shortA Filename	SshA_nl3	10/2/92;1:34 PM
shortB Filename	SshB_nl3	10/2/92;1:34 PM
m_dut Filename	Smdut_nl3	10/2/92;1:34 PM
Ref Filename	Sdevice_nl1	10/2/92;1:34 PM

Comments

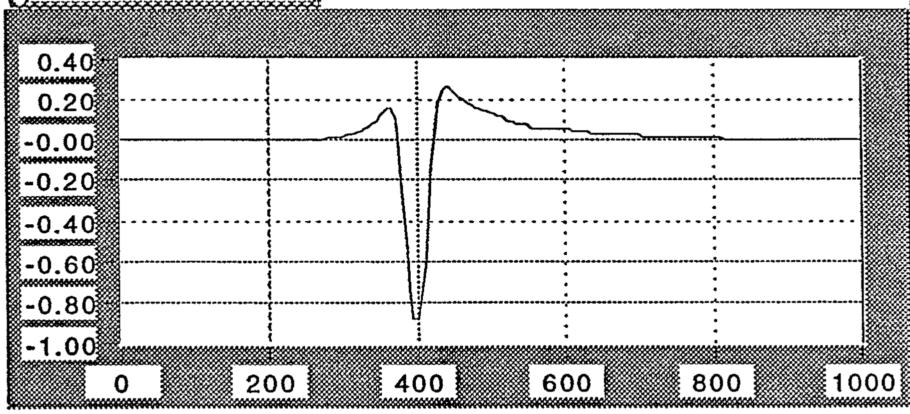
Device Measured with 1 delay of length 59 cm
S Parameters vs. freq in MHz

Front Panel

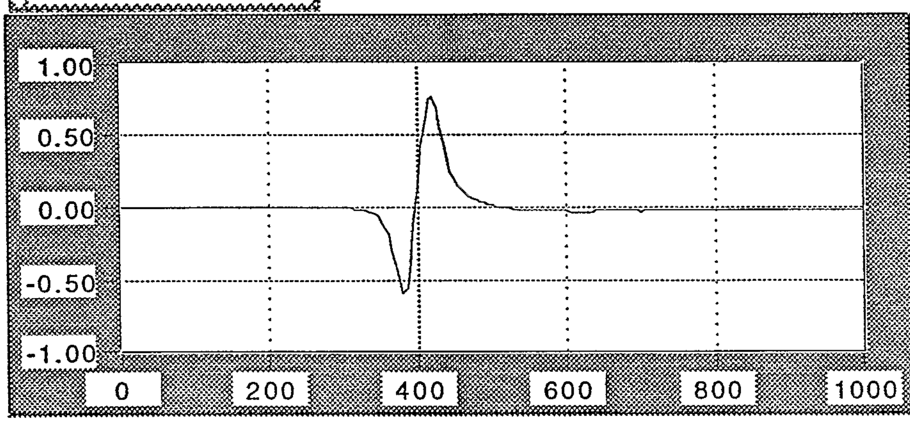
2

CAL DUT S

S21 Re



S21 Im



Comments

Device Measured with 2 delay of lengths 16 cm and 59 cm, Freq in MHz

Data Files

thru Filename	St_nl3	10/2/92;1:35 PM
delay1 Filename	Sd4_nl3	10/2/92;1:41 PM
delay2 Filename	Sd1_nl3	10/2/92;1:41 PM
shortA Filename	SshA_nl3	10/2/92;1:34 PM
shortB Filename	SshB_nl3	10/2/92;1:34 PM
m_dut Filename	Smdut_nl3	10/2/92;1:34 PM
Ref Filename	Sdevice_nl1	10/2/92;1:34 PM