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A Multipactoring Analysis of the Accelerating Cavity for RHIC

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AD/RHIC/RD-47

RHIC PROJECT

Brookhaven National Laboratory

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1.0 Introduction

Multipactoring has always been a concern in evacuated rf cavities. The problem can be addressed in several fashions, either by using computer codes to understand the effect [1] and to optimize the resonator shapes [2] or by coating the parts that showed particular problems with the appropriate materials, usually Titanium nitride or Carbon. Often both techniques proved to be necessary for the good success of a short conditioning period. This note describes the former method, based upon the use of simulations done with the code TWtraj [3], applied to the present configuration of the prototype cavity under development for the RHIC accelerating rf system. Results show that the first resonating discharge occurs in the high current end of the cavity at a gap voltage of 23.4 kV.

2.0 Description

Multipactoring is an electric discharge resulting from the emission, synchronous motion and multiplication of electrons in a resonant structure. The rf fields play a fundamental role in providing the synchronism necessary to have resonant discharges; if this occurs at energies where the emission of secondaries is optimal (typically in the 100 to 1500 eV range for copper [4,5]), the discharge can fully inhibit the operation of the cavity. Resonances can occur for electrons bouncing back to the point they were emitted (one point MP) or between two points on the surface (two point MP). For the intrinsic nature of the phenomenon, it typically occurs in regions of small radial electric field where the kinetic energy of the electrons is in the few hundred of eV range, which is optimal for the generation of secondaries.

The two most commonly used techniques to eliminate multiplication are either to increase the work function of the surface by evaporating in a Nitrogen atmosphere a Titanium film onto the critical areas, or using computer codes both to predict the locations of potential problems and to optimize the geometry of the resonator to provide the problem areas with longitudinal fields. Axial rf fields are important since they force electrons to drift out of resonant trajectories moving them into regions where the gradients are either too weak or too strong for any resonance to occur.

3.0 The 26.7 MHz cavity for RHIC

The 26.7 MHz cavity to be used in RHIC is a capacitively loaded coaxial quarterwave resonator. This cavity has been extensively studied using the electromagnetic analysis codes SUPERFISH and URMEL. Due to the nature of TWtraj, it was necessary to run an Oscar2D [6] simulation of this cavity in order to have the compatible output file containing the information about the field strength inside the geometry.

FIGURE 1. Oscar2D input file for the RHIC accelerating cavity

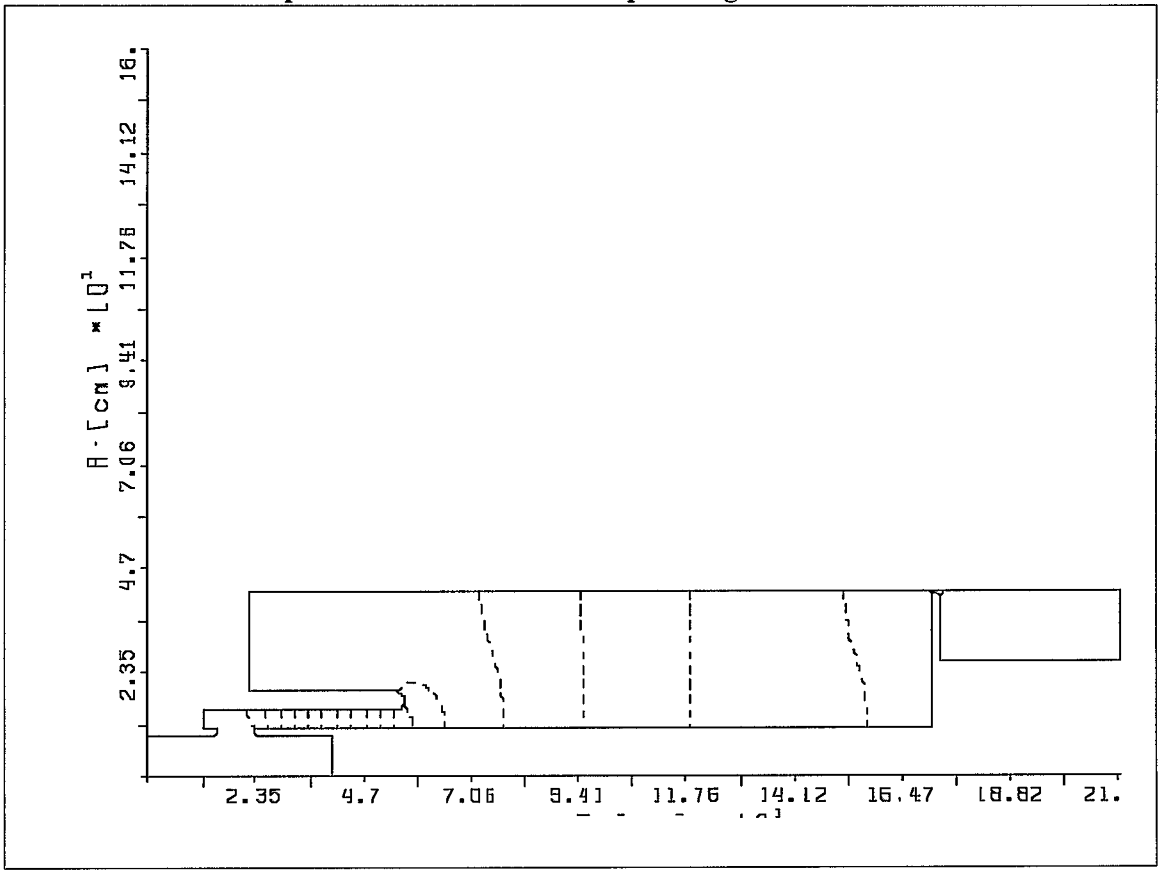
```
TEST MUSHROOM 26 MHZ RF CAVITY FOR RHIC
 $NINPUT
 NOL=21, FREQ=26.0, H=0.50,
 RB=  0.0, 2*9.0, 2*11.0, 2*15.0, 2*19.0, 2*42.0, 2*26.0, 2*40.0,
      2*11.0, 2*9.0, 2*0.0,
 ZB=  2*0.0, 2*14.0, 2*12.0, 2*54.0, 2*22.0, 2*212.0, 2*173.0, 2*171.0,
      2*24.5, 2*40.0, 0.0,
 RC=  2*0.0, 10.0, 3*0.0, 17.0, 6*0.0, 40.0, 2*0.0, 10.0, 3*0.0,
 ZC=  2*0.0, 14.0, 3*0.0, 54.0, 6*0.0, 172.0, 2*0.0, 24.5, 3*0.0,
 R =  2*0.0, 1.0, 3*0.0, 2.0, 6*0.0, 1.0, 2*0.0, 1.0, 3*0.0,
 TYPE= 21*0,
 METAL= 21*1,
 SIG = 3.45E7,
 FILNAM='finalpop', LOAD=2, RUNTIM=1500.,
 PC6= .F., scale=1., loadbg=0,
 NACC=1, autdril=1,
 W=0.17, graph=7,
 BCG=1,
 $END
```

Fig. 1 shows the Oscar2D input file for the RHIC Proof of Principle (PoP) accelerating cavity; the corresponding Oscar2D output is shown in Fig. 2 and 3 and is in good agreement with the SUPERFISH and URMEL calculations used to study the resonator [7].

FIGURE 2. Oscar2D output file for the input in Fig. 1

```
 MESH POINTS ALONG Z =  425
 MESH POINTS ALONG R =   85
 TOTAL MESH POINTS =    36125
 BOUNDARY POINTS =  1306
1
 LAMBDA =  3.2428915E-05  RESIDL =  5.335E-07  DL/L =  9.211E-02  IN 851 BCG I*
0ITERATION TIME =   1117
0FINAL LAMBDA =     0.3165394E-04         (FREQUENCY =        26.84449MHZ)
1 TEST MUSHROOM 26 MHZ RF CAVITY FOR RHIC                                        *
0NORMALIZATION FACTOR =  1.37451E+03
0STRUCTURE LENGTH L =  2.12000E+02 CENTIMETERS
0FREQUENCY =       26.84449 MEGA-HERTZ
0W (=V*T/L) =  1.70000E-01 MEGA-VOLTS/METER
0AXIAL VOLTAGE V =  6.05727E+05 VOLTS
0BETA (RELATIVE VELOCITY) = 1.000000
 PERIODIC LENGTH =     0.5583879E+03 CM,  DRIFT LENGTH = -0.1731939E+03 CM
0TRANSIT TIME FACTOR T =  5.94988E-01
0ENERGY STORED IN CAVITY U=5.93725E+00 JOULES POWER LOSS P = 9.59525E+04
0MAXIMUM SURFACE ELECTRIC FIELD = 2.16814E+07 V/M AT R = 1.09330E+01 *
0MAXIMUM AXIAL ELECTRIC FIELD = -2.46653E+06 V/M AT R =  0.00000E+00  Z*
0MAXIMUM SURFACE MAGNETIC FIELD = -1.06759E+02 GAUSS AT R = 1.10000E+01
0MAXIMUM AXIAL MAGNETIC FIELD =  0.00000E+00 H GAUSS     AT R =  0.00000E+00  Z*
0SHUNT IMPEDANCE Z (=V**2/P) =  3.82383E+06 OHMS
0Z/L =    1.80369E+06  Z/(L*Q) =   1.72822E+02  Z*T**2 =   1.35367E+06  Z*T**2/L*
0Q FACTOR =  1.04367E+04
 MAGNETIC GEOMETRIC FACTOR =  1.82920E+01 OHMS
 ELECTRIC GEOMETRIC FACTOR =  4.86744E+00 OHMS
```

FIGURE 3. Oscar2D plot of the field lines for the input in Fig.1.



4.0 The TWtraj code

TWtraj is a program that tracks the motion of an electron inside an rf cavity. Emitted at the initial energy of 2 eV from a user specified location on the cavity surface, the electron is made to travel according to the time varying electromagnetic fields given in the output of the eigenvalue solver Oscar2D; its path is tracked until it hits the cavity walls, recording the location and the impact energies. A new electron is then generated at the location and rf phase of the impact just happened and followed in its next journey.

As part of the input file, it is possible to specify the location, direction and rf phase at which the electron is emitted as well as the energy level of the fields in the cavity. The code computes the trajectory and energy of the particle and, in case of impact, a well tested empirical formula [3] is used to evaluate the distribution of secondaries emitted. One of these secondaries is then followed to the next impact.

FIGURE 4. TWtraj input file for the first level of MP found

```
1,1,  input data for trajectories-RHIC cav
 $files
  TW=.F.
  nnfile='finalpop'
  ddfile='biendd'
 $end
 $traject
  phimax=10000.,nim=10,
  iran=0,                        !random emiss. of sec.
  iampe=1,                       !# of field levels
   ampar=4.23e-2,                !scaling factor vs. Oscar
  iae=1,                         !# sources
   arr=26.5,                     !Source coord.  (radial)
   azz= 180.,29.,39.,59.,69.,    !          (transv.)
  iphe=5                     !#. of source rf phases and values
   phigrd=5.0, 10.0, 15.0, 20.0, 25.0,
  ife=1,                     !# source directions
   psigrd=180.,              !and values
  ncav=1,                    !max. # times electron penetrates.
 $end
```

FIGURE 5. Twtraj output file showing the first level of resonance

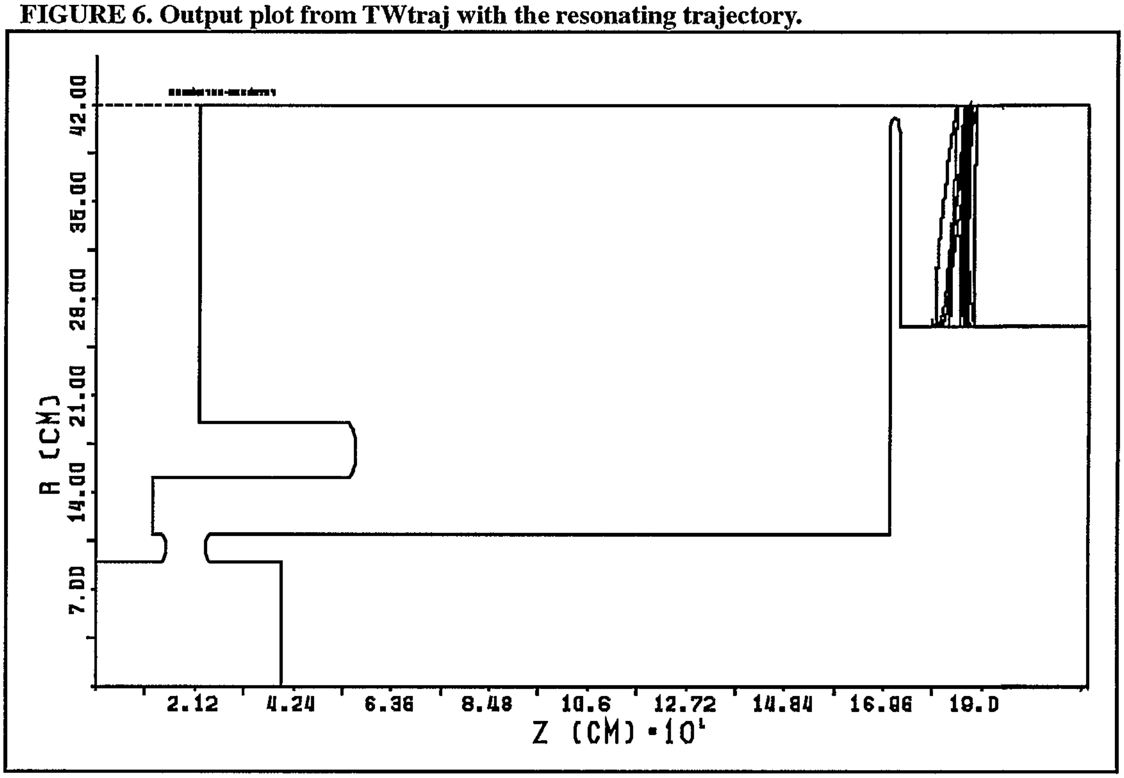
```
 FREQUENCY =   26.844MHZ,     EMAX(Z=24.250R=10.933CM) =      0.92MV/M, BMAX(Z=170.250R= 11.000CM) =     -4.52GAUSS,  EGAIN=VOLT
0----------------------------------------------------------------------------------------------------------------------------------
-
 @TRAJECT.@    PHASE  (GRAD)     @      ENERGY  (EV)      @       R   (CM)    @      Z   (CM)     @ PROBABILITY @    ANGLE  (GRAD)   @ R-VALUE @
 @ NUMBER @    START      IMPACT @     START      IMPACT  @    START  IMPACT  @    START  IMPACT  @             @    START  IMPACT   @   (CM)  @
-           ----------------------------------------------------------------------------------------------------------------------------------
-----------
@     1   @      5.00     45.36 @ 0.200E+01 0.263E+02  @ 26.5000 26.0000 @180.0000179.9268 @    1.0000     @ 180.00    -3.74   @  0.0000 @
@     2   @     45.36     77.56 @ 0.200E+01 0.298E+02  @ 26.0000 26.0000 @179.9268180.9956 @    0.6186     @ 180.00    80.38   @  0.0000 @
@     3   @     77.56    283.79 @ 0.200E+01 0.683E+03  @ 26.0000 42.0000 @180.9956185.2859 @    1.3029     @ 180.00   153.79   @  0.0000 @
@     4   @    283.79    441.99 @ 0.200E+01 0.927E+03  @ 42.0000 26.0000 @185.2859183.6028 @    2.7717     @   0.00    -1.99   @  0.0000 @
@     5   @    441.99    631.96 @ 0.200E+01 0.781E+03  @ 26.0000 42.0000 @183.6028187.1458 @    4.2748     @ 180.00   160.92   @  0.0000 @
@     6   @    631.96    809.70 @ 0.200E+01 0.775E+03  @ 42.0000 26.0000 @187.1458185.9791 @    8.1078     @   0.00     1.52   @  0.0000 @
@     7   @    809.70    992.20 @ 0.200E+01 0.746E+03  @ 26.0000 42.0000 @185.9791187.7843 @   13.4355     @ 180.00   165.21   @  0.0000 @
@     8   @    992.20   1173.19 @ 0.200E+01 0.720E+03  @ 42.0000 26.0000 @187.7843186.8290 @   24.9515     @   0.00     2.88   @  0.0000 @
@     9   @   1173.19   1353.30 @ 0.200E+01 0.725E+03  @ 26.0000 42.0000 @186.8290187.9756 @   42.9141     @ 180.00   166.70   @  0.0000 @
@    10   @   1353.30   1533.76 @ 0.200E+01 0.710E+03  @ 42.0000 26.0000 @187.9756187.0739 @   79.5013     @   0.00     3.22   @  0.0000 @
run terminated because:
MAXIMUM NUMBER OF IMPACTS IM EXCEEDED
@         @                     @                      @                  @                 @               @                    @         @
@     1   @     10.00     51.28 @ 0.200E+01 0.244E+02  @ 26.5000 26.0000 @180.0000179.9274 @    1.0000     @ 180.00    -3.48   @  0.0000 @
@     2   @     51.28    282.93 @ 0.200E+01 0.726E+03  @ 26.0000 42.0000 @179.9274189.4997 @    0.6025     @ 180.00   151.08   @  0.0000 @
@     3   @    282.93    465.86 @ 0.200E+01 0.557E+03  @ 42.0000 26.0000 @189.4997189.2111 @    1.2916     @   0.00     7.19   @  0.0000 @
@     4   @    465.86    640.37 @ 0.200E+01 0.639E+03  @ 26.0000 42.0000 @189.2111188.4428 @    2.4817     @ 180.00   170.74   @  0.0000 @
@     5   @    640.37    818.43 @ 0.200E+01 0.653E+03  @ 42.0000 26.0000 @188.4428187.7984 @    4.6494     @   0.00     4.75   @  0.0000 @
@     6   @    818.43    995.60 @ 0.200E+01 0.695E+03  @ 26.0000 42.0000 @187.7984188.1098 @    8.3781     @ 180.00   168.46   @  0.0000 @
@     7   @    995.60   1175.88 @ 0.200E+01 0.686E+03  @ 42.0000 26.0000 @188.1098187.3048 @   15.5346     @   0.00     3.80   @  0.0000 @
@     8   @   1175.88   1354.36 @ 0.200E+01 0.711E+03  @ 26.0000 42.0000 @187.3048188.0147 @   27.3597     @ 180.00   167.62   @  0.0000 @
@     9   @   1354.36   1534.46 @ 0.200E+01 0.703E+03  @ 42.0000 26.0000 @188.0147187.1470 @   50.6613     @   0.00     3.42   @  0.0000 @
@    10   @   1534.46   1713.88 @ 0.200E+01 0.716E+03  @ 26.0000 42.0000 @187.1470188.0528 @   88.2190     @ 180.00   167.27   @  0.0000 @
run terminated because:
MAXIMUM NUMBER OF IMPACTS IM EXCEEDED               High probability
@         @                     @                      @                  @                 @               @                    @         @
@     1   @     15.00     57.55 @ 0.200E+01 0.223E+02  @ 26.5000 26.0000 @180.0000179.9277 @    1.0000     @ 180.00    -3.21   @  0.0000 @
@     2   @     57.55    280.59 @ 0.200E+01 0.747E+03  @ 26.0000 42.0000 @179.9277188.5497 @    0.5844     @ 180.00   151.64   @  0.0000 @
@     3   @    280.59    459.81 @ 0.200E+01 0.637E+03  @ 42.0000 26.0000 @188.5497187.9592 @    1.2337     @   0.00     5.13   @  0.0000 @
@     4   @    459.81    636.29 @ 0.200E+01 0.686E+03  @ 26.0000 42.0000 @187.9592188.0548 @    2.2469     @ 180.00   168.95   @  0.0000 @
@     5   @    636.29    815.52 @ 0.200E+01 0.692E+03  @ 42.0000 26.0000 @188.0548187.2377 @    4.1683     @   0.00     3.71   @  0.0000 @
@     6   @    815.52    994.21 @ 0.200E+01 0.713E+03  @ 26.0000 42.0000 @187.2377187.9989 @    7.3138     @ 180.00   167.52   @  0.0000 @
@     7   @    994.21   1173.94 @ 0.200E+01 0.708E+03  @ 42.0000 26.0000 @187.9989187.1113 @   13.5413     @   0.00     3.29   @  0.0000 @
@     8   @   1173.94   1353.74 @ 0.200E+01 0.717E+03  @ 26.0000 42.0000 @187.1113188.0875 @   23.4962     @ 180.00   167.15   @  0.0000 @
@     9   @   1353.74   1535.32 @ 0.200E+01 0.691E+03  @ 42.0000 26.0000 @188.0875187.2489 @   43.5421     @   0.00     3.63   @  0.0000 @
@    10   @   1535.32   1714.18 @ 0.200E+01 0.713E+03  @ 26.0000 42.0000 @187.2489188.0348 @   76.3804     @ 180.00   167.48   @  0.0000 @
run terminated because:
MAXIMUM NUMBER OF IMPACTS IM EXCEEDED            Δϕ ~ 180
@         @                     @                      @                  @                 @               @                    @         @
@     1   @     20.00     64.25 @ 0.200E+01 0.198E+02  @ 26.5000 26.0000 @180.0000179.9276 @    1.0000     @ 180.00    -2.94   @  0.0000 @
@     2   @     64.25    279.59 @ 0.200E+01 0.755E+03  @ 26.0000 42.0000 @179.9276187.6597 @    0.5641     @ 180.00   151.85   @  0.0000 @
@     3   @    279.59    454.31 @ 0.200E+01 0.716E+03  @ 42.0000 26.0000 @187.6597186.7768 @    1.1833     @   0.00     3.17   @  0.0000 @
@     4   @    454.31    633.44 @ 0.200E+01 0.727E+03  @ 26.0000 42.0000 @186.7768187.8073 @    2.0430     @ 180.00   166.81   @  0.0000 @
@     5   @    633.44    813.44 @ 0.200E+01 0.717E+03  @ 42.0000 26.0000 @187.8073186.8722 @    3.7782     @   0.00     2.98   @  0.0000
@     6   @    813.44    993.39 @ 0.200E+01 0.724E+03  @ 26.0000 42.0000 @186.8722187.9723 @    6.5112     @ 180.00   166.81   @
@     7   @    993.39   1173.77 @ 0.200E+01 0.710E+03  @ 42.0000 26.0000 @187.9723187.0712 @   12.0604     @   0.00     3.22   @  0.0000 @
@     8   @   1173.77   1353.67 @ 0.200E+01 0.718E+03  @ 26.0000 42.0000 @187.0712188.0713 @   20.8993     @ 180.00   167.11   @  0.0000 @
@     9   @   1353.67   1535.15 @ 0.200E+01 0.693E+03  @ 42.0000 26.0000 @188.0713187.2255 @   38.7248     @   0.00     3.58   @  0.0000 @
@    10   @   1535.15   1714.12 @ 0.200E+01 0.713E+03  @ 26.0000 42.0000 @187.2255188.0349 @   67.8304     @ 180.00   167.44   @  0.0000 @
run terminated because:
MAXIMUM NUMBER OF IMPACTS IM EXCEEDED                        Optimal impact energy
@         @                     @                      @                  @                 @               @                    @         @
@     1   @     25.00     71.54 @ 0.200E+01 0.168E+02  @ 26.5000 26.0000 @180.0000179.9271 @    1.0000     @ 180.00    -2.65   @  0.0000 @
@     2   @     71.54    280.10 @ 0.200E+01 0.749E+03  @ 26.0000 42.0000 @179.9271186.8075 @    0.5413     @ 180.00   151.71   @  0.0000 @
@     3   @    280.10    449.21 @ 0.200E+01 0.795E+03  @ 42.0000 26.0000 @186.8075185.6358 @    1.1406     @   0.00     1.28   @  0.0000 @
@     4   @    449.21    631.97 @ 0.200E+01 0.753E+03  @ 26.0000 42.0000 @185.6358187.5412 @    1.8679     @ 180.00   164.92   @  0.0000 @
@     5   @    631.97    811.64 @ 0.200E+01 0.744E+03  @ 42.0000 26.0000 @187.5412186.5005 @    3.4650     @   0.00     2.33   @  0.0000 @
@     6   @    811.64    992.74 @ 0.200E+01 0.734E+03  @ 26.0000 42.0000 @186.5005187.9140 @    5.8651     @ 180.00   166.14   @  0.0000 @
@     7   @    992.74   1173.58 @ 0.200E+01 0.713E+03  @ 42.0000 26.0000 @187.9140186.9944 @   10.8686     @   0.00     3.10   @  0.0000 @
@     8   @   1173.58   1353.57 @ 0.200E+01 0.719E+03  @ 26.0000 42.0000 @186.9944188.0222 @   18.7896     @ 180.00   167.06   @  0.0000 @
@     9   @   1353.57   1534.47 @ 0.200E+01 0.702E+03  @ 42.0000 26.0000 @188.0222187.1497 @   34.8062     @   0.00     3.41   @  0.0000 @
@    10   @   1534.47   1713.88 @ 0.200E+01 0.716E+03  @ 26.0000 42.0000 @187.1497188.0540 @   60.6288     @ 180.00   167.28   @  0.0000 @
run terminated because:
```

As part of the output, the electron's path is plotted on the screen and an output file is written. This file lists all the interactions with the cavity surfaces, the rf phase and the kinetic energy of the electron at the time of the impact. Multipactoring occurs when a resonance is found, that is when the initial and final phase between each impact is the proper multiple of 180 degrees. A yield obtained from the number of secondaries generated each time is also given and is a good indication of the intensity of the discharge.

Fig. 4 shows the input file to TWtraj for the first resonance found. The parameter AMPAR is the coefficient used to change effectively the gap voltage level, ARR and AZZ are the radial and longitudinal coordinates of the emission point, and PHIGRD the starting rf phase.

The output trajectory (Fig. 6) clearly indicates a resonance, which is confirmed in the output file of Fig. 5: electrons are bouncing between the two locations r=42, z=185 and r=26, z=185; the impact energy is about 700 eV, the yields are high and the phase increment between two impacts is always around 180 degrees, indicating that this is the first level of two point MP.

FIGURE 6. Output plot from TWtraj with the resonating trajectory.



Unfortunately the program does not have any algorithm to "find" multipactoring: the resonances are found by the user, carefully scanning the gap voltage for different positions of the emission locations and many rf phases.

5.0 A simple model of MP

In order to verify the results just found, it is possible to derive a simple expression in the case of two conductors at a distance L, excited with a sinusoid at the frequency ω.

From the force balance

$$m\frac{d^2x}{dt^2} = eE\sin(\omega t)$$

where E is the field seen by the electron and e its charge; integrating twice

$$\frac{dx}{dt} = V_0 - \frac{eE\cos(\omega t)}{m\omega} = \frac{eE}{m\omega} \times (1 - \cos(\omega t))$$

and

$$x = \frac{eE}{m\omega} \times \left(t - \frac{\sin(\omega t)}{\omega}\right)$$

The resonance condition is met when $x = L$ and ωt is an odd multiple of π. Therefore

$$L = \frac{\pi eE}{m\omega^2} \times (2n - 1)$$

solving for E:

$$E = \frac{m}{e} \times \frac{\omega^2}{\pi} \times \frac{L}{(2n-1)}$$

and by substituting $\omega=2\pi c/\lambda$

$$V = L \times E = \frac{m \times c^2}{e} \times 4\pi \times \left(\frac{L}{\lambda}\right)^2 \times \frac{1}{(2n-1)}$$

This expression gives the discrete voltage levels V at which the resonance condition is met. These can be then scaled to a corresponding gap voltage by computing a step-up ratio using the electromagnetic codes mentioned before. It is now also possible to calculate the kinetic energy U of the electron when it impacts the opposite wall.

$$U = \frac{1}{2} \times m \times V^2 = \frac{8m \times c^2}{e}\left(\frac{L}{\lambda}\right)^2 \times \frac{1}{(2n-1)^2}$$

Using this two formulas it is possible to verify the resonance found for the PoP cavity.

n	1	2	3	4	
V	1301	433	260	186	V
U	828	92	33	16.9	eV

TABLE 1. The first four resonating modes as calculated in a region with L = 16 cm

For the case found, it can be seen that the rf phase between each impact increases by about 180 degrees every time, thus indicating that the resonance corresponds to the first level of MP; the voltage across the conductors is about 1.3 kV and the impact level 828 eV. Looking at the output file of Fig. 5, it is possible to notice impacts energies ranging from 645 to 950 eV. In order to check the voltage level, a run of the SUPERFISH [7] post processor SHY is required to get the voltage level at the position x=185 cm. inside the cavity. This peak voltage was calculated to be 1.217 kV for the estimated 23.4 kV at the gap. The impact energy corresponding to this value can then be calculated to be 726 eV. It can also be noted that the energies associated with the higher modes are too low for the generation of secondaries.

6.0 Conclusions

The Twtraj code has been installed on the BNL cluster computers and it has been first applied to study the multipactoring behavior of the RHIC PoP cavity. The first level of resonant discharge has been found in the tuner end of the cavity. A simple model of the electron dynamics was found in very good agreement, and provisions have been made for the coating using a Titanuim ball in that region, leaving several vacuum ports available for the insertion of the devices.

7.0 Acknowledgments

I am particularly grateful to dr. Renzo Parodi, one of the authors of the TWtraj program, who not only provided the source code but also shared a lot of his expertise and knowledge. Also, I would like to thank M. Brennan, W. Pirkl and A. J. McNerney for their continuous support and J. Rose for many valuable comments.

References

[1] - R. Boni et al. - Design and operation of a multipacting-free 51.4 MHz RF accelerating cavity - Nucl. Inst. and Meth. A274(1989), 49-55

[2] - R. J. Burton, et al. - Vacuum and multipactor performance of the HERA 52 MHz cavities - Proceedings of the 2nd European Particle Accelerator Conference, Nice, June 1990, 1017-19

[3] - P. Fernandes, R. Parodi - Newtraj, a computer code for the simulation of the electron discharge in accelerating structures, Proceedings of the IEEE Particle Accelerator conference, Washington DC (1987), 1857-9

[4] - C. Planner - High frequency electrical discharge by secondary electron resonance - M.S. thesis, Univ. of London, Feb.1968

[5] - CRC Handbook of Chemistry and Physics

[6] - P. Fernandes, R. Parodi - Oscar2d user's guide - INFN/TC-90/04, 27 Feb. 1990

[7] - J. Rose - private communications

[8] - Poisson/SUPERFISH reference manual - LANL - LA-UR-87-126