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Longitudinal stability in RHIC

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ABSTRACT

Introduction

Unless specific procedures are adapted the beam bunches in RHIC will be succeptable to longitudinal coupled bunch instabilities. These will be driven by the higher order parasitic modes present in the accelerating and storage cavities. At the design intensity per bunch the longitudinal density of the protons and in some cases the heavy ions will, in conjuction with the space charge impedance and/or the broadband impedance of the vacuum chamber, shift the coherent synchrotron frequency of the bunches outside of the incoherent band during some part of the injection or acceleration cycle. However once the bunches are transferred to the much smaller bucket of the storage rf system the resulting significant increase is synchrotron frequency spread will in general result in stability. One can insure stability by transferring the bunches into the "storage rf buckets" at the lowest possible energy. Another method of obtaining stability is to increase the synchrotron frequency spread of the bunches in the accelerating rf system buckets. This can be done by exciting a "Landau Cavity" at some multiple of the bunch frequency so as to increase the no-linearity of the rf voltage that the bunch sees. In a later section we will discuss both of these schemes and will in particular investigate using one of the storage cavities as a Landau Cavity.

Stability Limits

We shall assume that the real coherent frequency shifts produced by the space charge or broadband vacuum chamber impedance are much large than any imaginary shifts produced by the damped parasitie modes of the rf cavities. Hence a constant value of (Z/n) can be used in estimating these shifts. Next we shall assume a longitudinal phase space density of the form $(1-r^2)^{3/2}$ which leads to a line change density of the form $[1-(\phi/\hat{\phi})^2]^2$. The latter has both the first and second derivative zero at the bunch edge and resembles the proton bunches observed in the AGS at RHIC intensities. (In the early 90's).

The coherent frequency shift $\Delta\omega_m = (\omega_m - m\omega_s)$ of the bunch away from the incoherent synchrotron frequency at the center of the bunch ω_s is calculated using the Sacherer[1] integral equation. Here ω_m is the coherent synchrotron frequency of a bunch and m = 1, 2, -- the azimuthal mode number i.e. dipole, quadrupole etc. It is also assumed that there is no mode coupling i.e. $\Delta\omega_m << m\omega_s$. Using the method of expansion into orthogonal polynomials one must solve the equation[2]

$$det(M-\Delta\upsilon \bullet I)=0 \tag{1}$$

where M is called the interaction matrix and $\Delta\nu = (\omega_m - m\omega_s)/\omega_s$.

Numerical solutions of equation 1 yield eigenvalue $\Delta\upsilon_k$ with corresponding eigenfunctions $R_k(r)$ for a given value of m. For the case of a constant (Z/n) one has[2],

$$M_{kl}=\frac{m\xi}{2\pi}\left|\frac{Z}{n}\right|\int_{-\hat{\varphi}}^{\hat{\varphi}} g_k(\varphi)\; g_l^*(\varphi)\; d\varphi$$

where,

$$g_k(\varphi)=\int_{-\infty}^{\infty} w(r)\ f_k(r)\ e^{jm\theta}\ \frac{d\varphi}{\omega_s}$$

$\xi = \eta\ I_o/\beta^2 v_s^2 E/e$ with $v_s = \omega_s/\omega_o$ the rotation frequency, and I_o is the average beam current. Here $w(r) = (d\ \Psi\ /dr)/r$ is the weight function such that orthogonal polynomials $f(r)$ form a complete set of normalized functions i.e.:

$$\int_o^a w(r)\ f_k(r)\ f_l\ (r)\ rdr = \delta_{k\ell}$$

Now Ψ_o is the stationary distribution function of the bunch in longitudinal phase space (r,θ) or $(\phi,\dot{\phi}/\omega_s)$ with a the maximum value of r of the bunch and $\pm\hat{\phi}$ the bunch length. The functions f_k are given by Satoh [(3)] for distribution functions of the form $(1 - r^2)^{n/2}$, $n \geq -1$.

The M_{kl} with k,l = 0, 1,2 for the case m = 1 and n = 3 have been calculated (note $M_{20} = M_{02} = 0$). Then the frequency shifts and corresponding eigenfunctions were obtained for the three dipole modes in this approximation.

In order to obtain a stability diagram one must include the spread in synchrotron frequency $\omega_s = \omega_s(r)$ in the analysis. The source of this spread, which is a function of r, is the non-linear part of the rf focusing force and the space change force (the latter is linear only for the distribution with n = 1). Following Zotter[(4)] one has,

$$\omega_s = \omega_{so} - \Delta\omega_{sc} \cdot G(r) - \omega_{so} \cdot H(r) \qquad (2)$$

where ω_{so} is the zero current value and $\Delta\ \omega_{sc}$ the shift of the center frequency i.e. $\Delta\ \omega_{sc} = \omega_{so} - \omega_{sc}$ with ω_{sc} the value at the center of the bunch. The function G(r) can be evaluated for a given n and H(r) is the contribution due to the rf non-linearity.

Equation 2 can be written as $\omega_s(r) = \omega_{sc} + D(r)$. It can be shown [(4)] that, again expanding in orthogonal polynomials, the Sacherer integral equation reduces to the eigenvalue problem.

$$det(M+mK-\Delta\omega I)=0 \tag{3}$$

Where again M is the interaction matrix, m the azimuthal mode number and

$$\Delta\omega=\omega-m\omega_{sc}$$

with ω the coherent frequency and K the dispension matrix given by

$$K_{kl}=\int_o^a D(r)w(r)f_k(r)f_l(r)rdr \tag{4}$$

Now the matrix elements $(M_{k\ell}+K_{kl})=M'_{k\ell}\ ,k,l,=0,1,2$ have been evaluated (m=1) for n =3 with the following results:

$$M'_{00}=\Delta\omega_{sc}-(4/7)S, M'_{11}=.6818\Delta\omega_{sc}-.5175S,$$

$$M'_{22}=.5806\Delta\omega_{sc}-.509S, M'_{01}=.233S=M'_{10}$$

$$M'_{00}=\Delta\omega_{sc}-(4/7)S, M111=.6818\Delta\omega_{sc}-.5175S,$$

$$M'_{21}=M'_{12}=-.0685\Delta\omega_{sc}+.24325$$

where $S = \omega_{so} H(r)$ and

$$\Delta\omega_{sc}=\frac{15}{8}\frac{\xi}{\varphi_0^3}|Z/n|\omega_{so} \tag{5}$$

for a constant Z/n. Again the $M'_{20}=M'_{02}$ elements are zero and for the r=0 particle so are the $M'_{01}=M'_{10}$ elements. Hence if one included the effect of space charge and calculates the frequency shift $\Delta\omega$ for no rf frequency spread i.e. S=0 then the lowest order radial eigen mode is $\sim f_o(r)$ and the frequency shift is just $\Delta\omega_{sc}$. That is the coherent frequency shift in just equal to the incoherent single particle frequency shift at the center of the bunch. This corresponds to rigid dipole motion and is the same result obtained for the distribution n=1 i.e. the parabolic line charge bunch. We note here that for a given beam current this frequency shift is 1.52 times greater than that obtained for the lowest order radial mode using equation(1) and the $M_{k\ell}$ for $k,\ell = 0, 1, 2$ as mentioned above.

Since here the lowest order radial eigenmode contains only $f_o(r)$ the synthetic kernel approximation[4] can be used to obtain a dispersion relation and hence a stability diagram. This has been done and the result is shown in figure 1. Here

$$\Delta\omega_1 = \frac{\omega_{so} I_o h^2 Z/n}{2.633 B^3 V_T} \tag{6}$$

where h=rf harmonic number, B = bunching factor, V_T the total rf voltage. In a stationary bucket $S \cong (\hat{\varphi}^2/16)\omega_{so}$ where $\hat{\varphi}$ is the bunch half length in radians. In principal one could use the matrix equation (3) to obtain the stability limit but this would require a much greater number of elements $M_1^{\prime}$ than we have evaluated. The curve shown in figure 1 has also been obtained by other authors(5)(6) for the distribution n=3.

We see then that for a broad band impedance i.e. a constant Z/n the stability limits are either $\Delta\omega_1/S \leq 0.4$ or 0.20 depending upon the sign of Z and η. Let us consider protons at injection with V_T = 196 Kv, $\omega_{so} = 2\pi\ 45$, I_o = 1.25ma, B = (6.5/37.5) for a bunch area of 0.3 evsec, and h = 342. Then $\Delta\omega_1$ = .054 (Z/n) ω_{so} and S = [(π B)^2/16] ω_{so} = 1.85 x 10^{-2} ω_{so} so that (Z/n)$\leq$ j0.14 Ω for stability. If the bunch area were increased to 0.5 evsec this limit would increase to 0.5 Ω and to .78 Ω for a .6 evsec bunch. Now the broad band impedance due to the vacuum chamber, bellows, etc. is .75 - 1 Ω (7) up to about 300 MHZ or just above where the first zero of a 6.5 nsec n=3 bunch spectrum would occur. However one must also consider the space charge impedance given by $(Z/n)_{sc} = g_o Z_o /2\ \beta\gamma^2$ where Z_o = 377 Ω and g_o is a form factor that depends upon the beam size and vacuum chamber radius b. It also depends upon the longitudinal distribution(8) and we use the results of this reference for the n=3 case to obtain a g_o = 5.2 with b =3 cm and σ = 1.4 mm as the rms beam size (3 mmm rad emittance). Hence for γ = 30 one obtains a space charge impedance of ≈ - 1.09 jΩ. This implies close to a null in the total longitudinal impedance seen by the proton beam at injection. Since the results quoted above are only approximations the actual value can only be determined by measurements made with the beam during turn on. In any event in order to possibly obtain stability without additional efforts, to be described later, the proton bunch area should be increase to 0.5 evsec rather than the 0.3 evsec originally planned.

Next let us consider the Au beam at injection (γ =11.1) where V_T = 170Kv and B= (15.5/37.5) for a bunch area of .25 evsec/AMU. We again take I_o = 1.25 ma and obtain a $\Delta\omega_1/S$ = .044. Hence if we take the criteria $|Z/n| \leq .4/(\Delta\omega_1/S)$ we obtain $|Z/n| \leq 9.1\Omega$. Now if we again take the transverse emmittance as $10\pi\mu$m then the space charge impedance becomes ≈-8jΩ which dominates the total broad band impedance. Since injection is below transition η is negative and one again uses the upper half of the stability diagram which give the 9.1Ω limit. If the bunch area is reduced to .2 evsec /AMU so that B =(13.5/37.5) then the (Z/n) limit would be reduced to≈-4.6jΩ since $(\Delta\omega_1/S) \sim B^5$. This implies that the limit on the Au beam longitudinal emittance is fairly tight since it should not be larger than .3 evsec/AMU when transition is reached. Again the actual effective impedance can only be determined by appropriate measurement with a beam.

Finally let us consider both protons and gold at top energy. First protons at γ = 268 or 250 Gev and a bunch area of .5 evsec. At 600 kv the bunch length of 4.45 nsec would fit into a 197 MHZ bucket so that this area could be transferred with no rf gym-nastics. At 300kv the nominal accelerating voltage the final bunch length would be ≈5.3 nsec. Putting this into equation (6) gives a $\Delta\omega_1 = .0654\ \omega_{so}$ (Z/n) while S = .0123 ω_{so}. This results in a Z/n≤ .075Ω. The design limit for gold bunches is 0.35 evsec/AMU which at 300 Kv for the accelerating voltage gives a bunch length of ≈ 7nsec at 100 Gev. Again using the above expressions for $\Delta\omega_1$ and S we obtain a limit on the broad band impedance of $\leq 0.3\Omega$ at the nominal 1.25 ma beam current. Since the space change impedance decreases as $1/\gamma^2$ only the broadband vacuum chamber impedance is important at these energy. Hence we conclude that both species will become potentially unstable against coupled bunch longitudinal dipole oscillations driven by the high order parasitic modes of the acceleration and storage cavities at some point during the acceleration cycle. Since the bunch areas are limited to.5 evsec for protons and .35 evsec for Au ions stabilization without feedback can only be accomplished by increasing S. Methods for accomplishing this are discussed below.

Growth Rates

As pointed out above the proton bunches and the gold bunches at nominal design intensities will be potentially unstable at one or more times during injection or acceleration. We assume that any resulting coupled bunch instability will be due to a higher order resonant mode in one of the rf cavities. In order to calculate the growth rate due to a single mode we will use the formulations of Baartman [(6)] i.e.

$$1/\tau = \frac{\omega_s}{\hat{\varphi}} \frac{I_o R}{V_T \cos Q_S} F_m(\chi) \tag{7}$$

Where I_o is the total DC beam current R the shunt impedance of the resonance and $\hat{\varphi}$ is the bunch half length in radians. F_m is a form factor given by

$$F_{mk} = m\mu\ (\mu+1) \left(\frac{2}{\chi}\right)^{2n+1} \frac{(m+2k+\mu)\ \Gamma\ (k+\mu)}{k!(m+k)!}\ \Gamma\ (m+k+\mu)\ J^2_{(\mu+m+k)}\ (\chi) \qquad (8)$$

Here *m* is the azmithal mode number, $\mu=n/2$, *k* is the radial mode number, and $\chi=\omega_{res}\tau_\varphi$ with τ_φ the bunch half length in seconds. Since n=3 for our distribution we have μ=3/2 and consider first only the m=1 dipole mode. Also we shall assume k=o i.e. that only the lowest order radial mode is excited. This is reasonable since as noted above the lowest frequency eigen mode for $n=3$ has $k=0$ in the 3x3 matrix approximation. Then equation 7 is correct since only the growth rate for one radial mode need be considered. The Bessel functions are then $J_{5/2}(\chi)$ and the form factor $F_{1,0}(\chi)$ is plotted in figure 2.

First we consider proton bunches at injection with an area of 0.5 ev sec and V_T = 196KV. The first parasitic mode of the revised accelerating cavity is expected to have resonant frequency of 104.7 MHZ, a R= 166 KΩ and a Q = 29,000.[9] If we assume this to fall on top of a coupled bunch mode line then the growth rate for the m=1 mode would be given by

(χ= 2π 104.7 x 10^6 x 4.2 x 10^{-9})

$$1/\tau = \frac{2\pi 45}{.705 rad} \frac{57x1.25x10^{-3}166x10^3}{196x10^3} F_1(2.76)=10.6\sec^{-1}$$

Now at 250 Gev with $V_T = 300$ Kv the bunch length is ≈ 5.3 nsec so that $\chi = 1.74$ while $f_s = 174\ \text{sec}^{-1}$. Then $1/\tau = \frac{2\pi x174}{.445}\ \frac{57x1.25x10^{-3}\ 166x10^3}{300x10^3}\ F_1(1.74) = 8.55\ \text{sec}^{-1}$.

Next let us calculate the frequency shift due to a broad band impedance of Z/n = 1Ω. Using equation 6 and the injection parameters we obtain

$$\Delta\omega_1 = \frac{2\pi\ 45 \times 1.25 \times 10^{-3}\ 342^2 \times 1}{2.633 \times (8.4/37.5)^3\ 196 \times 10^3} = .025\omega_{so} = 7.1\ \text{sec}^{-1} \qquad \textbf{(17)}$$

Thus our assumption that the real frequency shift due to space charge and the vacuum chamber broad band impedance is much greater than the imaginary shift caused by a single cavity resonance would not be valid if the higher order cavity modes were not damped. At present computer simulations indicate that with two dampers installed the first parasitic mode in the new accelerating cavity will have shunt impedance of only 1.3 KΩ at 104.7 MHZ with an R/Q ≈ 5.7[9]. This is a reduction factor of 128 and would result in a growth rate of .083 sec^{-1} or an e-folding time of 12 sec. Since the nominal acceleration time to top energy is 60 sec or 5 e-folding times no significant growth should occur[6] even if the bunches were unstable at injection. If the number of bunches is doubled the growth rate will increase and the mode number change. However, since the resonance will spread across many rotation lines the driving impedance will be essentially the same and hence a doubling of the rate would probably occur. Increasing the intensity per bunch will also increase the growth rate and shorten the time the beam will be stable. It should also be noted that the growth rate for the same bunch area during acceleration for $V_T = 300$ Kv, change by less than 20% from the injection value.

In the case of Au ions at injection we have a bunch that is 15.5 nec long for an area of .25 evsec/ AMU this gives a $\chi = 2\pi \times 7.75 \times 10^{-9} \times 104.7 \times 10^{6} = 5$ for the first parasitic accelerating cavity mode. We see from figure 3 that $F_{1,0}(5) = .015$ so that the lowest order radial dipole mode if unstable would be driven very weakly i.e. .0035 sec^{-1} growth rate. Now the k=0 quadrupole mode if it excited and were unstable would have a growth rate 6 2/3 greater since $F_{2,0}(5) = .1$ as shown in figure 3. This would result in an e-folding time of 42 sec so that even in the highly unlikely event that such a mode were unstable it could not grow before acceleration takes place.

During acceleration the bunch length shrinks significantly so that the growth rate of any unstable mode driven by the 104.7 MHZ impedance will vary considerably. When $\gamma = 20.95$ the bunch length for a given area will be the same as for $\gamma \approx 26$ and $\gamma = 108$. If the area is .25 ev sec AMU the width would be 5.8 nsec which gives a χ(/04.7) = 1.91 which is at the peak of the $F_{1,0}$ form factor. The resulting growth rate for V= 300 Kv is .059 sec^{-1} for the current of 1.25 ma/bunch in 57 bunches. This is still a very low value considering that it would take 5.7 sec to change γ from 11.8 to 20.95 and another 3.17 sec to reach $\gamma = 26$ at a $\dot{B} = .5$T/sec. From γ = 21-26 the growth rate will of course vary and ω_s will pass through zero and the rotation of the bunches in phase space will change sign. Thus, even if a coupled bunch instability were to occur before transition it is not clear if it would persist in the non-adiabatic region around transition. Hence, unless there were an impedance considerably larger than those expected in the accelerating cavities or the storage cavities the occurance of a coupled bunch instability before the transition energy is reached that would have a measurable effect on the Au bunch area highly unlikely at the design intensity or several times this current.

At γ=26 the bunch length would be 6.4 nsec assuming that in passing transition there is a growth in area from .25 to .3 evsec/AMU. The growth rate for the 104.7 MHZ resonance driving the dipole mode would be essentially the same as at γ=20.95 i.e. $\approx$.059 $sec.^{-1}$ For $\gamma=\sqrt{3}\ \gamma_{tr} = 39.5$ the bunch length would increase to 7.5 nsec but the growth rate would change very little to .057 sec^{-1}. Finally at 100 GeV the bunch would again be 6.4 nsec wide and the growth rate would be .052 sec^{-1} for the same resonance. Since it would take about 52 sec to accelerate from γ=26 to γ=108 we see that even if the bunches were unstable during the period it would represent only 2.8 e-folding periods. Using the Baartman[(6)] criterion of 4-5 e-folding times as being safe one can conclude that at the design intensity and perhaps 50% greater the Au bunches should be exhibit no oscillations due to the lowest frequency, accelerating cavity resonance. Again if the current per bunch is increased stability will be lost sooner and the growth rate increased while increasing the number of bunch only changes the growth rate. In the case of the Au ions it is possible to transfer the bunches to the storage rf system buckets at $\gamma \approx 26$ and then use that system for acceleration[10]. This would then greatly increase the intensity threshold for instability due to the much larger synchrotron frequency spread within the bunch since it would then occupy a much larger fraction of the bucket area. We shall discuss this in more detail later.

So far we have considered only the m=1 or dipole mode driven by the lowest frequency parasitic resonance expected to be present in the accelerating cavities. Table 1 shows the predicted resonances and the effect of proposed damping loops. For protons at injection $\chi = 5.2$ for the 197.9 MHZ resonance and we find $F_{20} \approx 0.2$, $F_{30} \approx 0.18$ and a negligible value for F_{10}. Since the damped resonant impedance is essentially the same as the 104 MHZ line the growth rates would be smaller than for the dipole case. At what intensity the m=2 or 3 modes would be become unstable has not been estimated. One can only state that it will be progressively larger as m increases.

Next let us consider the highest frequency mode in table 1 and protons at 250 GeV. Then $\chi = 5.37$ and one finds that $F_{30} \approx .2$. The growth rate would be .264 sec^{-1} at the design intensity but it is highly unlikely that the sextuple mode would be unstable at this level.

Finally let us look at the storage cavity modes. In particular the first resonance above the 196 MHZ fundamental i.e. at 308.4 MHZ would produce a $\chi = 8.1$ at injection, 7.5 at $\gamma = 39.5$, and 5.1 at 250 GeV for protons. The mode with the largest growth rate would be m= 4 at top energy. Again, at design intensity it would be $\approx .82$ sec^{-1}. At lower energy the m= 5,6 modes would be most strongly driven as can be seen from figure 2. For this resonance Q ≅ 4400 corresponding to a 70Kc band width while the line spacing for the coupled bunch modes is $f_o \cong 78$ Kc and the harmonic number is ≈3950. Since f_o the rotation frequency changes by over 40Hz from injection to maximum energy the spectrum line corresponding to a given coupled bunch mode n near the cavity resonance at 308MHz will change by about 160Kc. Hence a given mode n would not be driven at the same rate during the acceleration cycle for this reason as well as due to the change in χ. As far as the stability of the m = 4, 5, 6 modes is concerned again the real frequency shifts at a given intensity will decrease with m so the thresholds will be greater. The maximum growth rate of .82 sec^{-1} for the 308 MHZ resonance (m=4) is still more than eight times smaller than the broadband dipole mode frequency shift of 7.1 sec^{-1} . For m = 4,5,6 of course the latter will be smaller so that one would be in the region where the real and imaginary frequency shifts are comparable and thresholds are difficult to determine.[6] At the design intensity it is safe to assume however, that the 308 MHz resonance will not present a problem. As the intensity is increased the threshold for higher m modes could be reached but by then other methods for suppressing instability could be employed. As for the other parasites modes in the storage cavities the resonant impedances are much lower and or the χ values much higher so that they should not be a problem at design intensity or several times larger.

We should point out that the growth rates calculated above are for a single cavity. In the case of the accelerating cavities one should multiply by a factor of two since the damped resonance will certainly overlap. For the storage cavities since the band width of the 308 MHz mode is less than the mode frequency spacing only a worst case scenario where the same parasitic mode of more than one cavity falls on top of another would alter the results.

Stabilizing Methods:

As mentioned in the introduction one can increase the intensity threshold for stability by increasing the synchrotron frequency spread within the bunched beam. We shall consider first the increase obtained when the Au beam is transferred into the storage buckets at $\gamma = 26$ instead of $\gamma = 108$.[10] At γ=26 and 0.4 eV sec/AMU bunch area with $V_{ACC} = 300$ kv the bunch half length will be .714 rad with B = .197. Using equation with h = 360 and Io = 1.25 ma we obtain a $\Delta\ \omega_1 = .0265\ \omega_{so}$ (z/n) while $S=(\hat{\varphi}^2/16)\ \omega_{so} = .239\ \omega_{so}$ so that $(\Delta\ \omega_1/s) = 1.1$ (z/n). Now if the bunches are transferred to the 196 MHZ buckets with h = 2,520 and V= 6MV then B=.44 and we obtain a $\Delta\ \omega_1 = 5.9 \times 10^{-3}\ \omega_{so}$ (z/n). For this case S= .134 ω_{so} so that $(\Delta\ \omega_1/s) = .044$ (z/n) hence for the same intensity the allowed (Z/n) can be 25 times greater.

If the bunch area is 0.35 eV sec/AMU then at γ= 26 or 24.4 GeV rebucketing with 2MV on the storage cavities should be possible with no loss [10][14]. For this area and $V_{acc} = 300$ Kv we find a $(\Delta\ \omega_1/s) = 1.5$ (z/n). With $V_{acc} = 600$ Kv which is required prior to rebucketing it is increased by $2^{1/4}$ i.e. $(\Delta\ \omega_1/s) = 1.78$ (z/n). In the 2MV, h= 2,520 bucket one has a $\Delta\ \omega_1 = .0086$ ω_{so} (Z/n) with S= .232 ω_{so} so that $(\Delta\ \omega_1/s) = .0373$ (Z/n) which would allow an impedance 40 times greater than in the nominal accelerating bucket. We note that at $\gamma = 26$ the space charge impedance for a 3mmm rad emittance beam would be ~ -1.45jΩ while again the nominal broad band vacuum chamber impedance is expected to be ~ 0.8jΩ. Hence in the region around $\gamma = 26$ the cancellation of these impedance can be expected to occur. Thus the reduction in the stability threshold due to the two fold increase in V_{acc} prior to rebucketing will be of less importance when performed at low energy.

Next let us consider the proton case. In principal one could also transfer the bunches from the h=360 to the h = 2,520 rf buckets at a low value of γ where the ratio of η/γ is the same as at γ =268. For protons this would occur for a $\gamma = 23.875$ well below the nominal injection value of ~ 30. This is quite close to $\gamma = 22.8$ so that the effects of the magnetic nonlinearity $\propto$ would have to be considered. Also the required matching voltage for the transfer from the AGS to the h=360 buckets would be considerably reduced at this lower γ. In addition the space charge impedance would increase and hence there would less likely to be a cancellation between it and the broad band impedance. Hence this option is not worth considering.

Now the primary reason for suggesting the $\gamma = 23.875$ injection was the fact that one could in principal then transfer to the h = 2520 system just by increasing the h = 360 rf voltage to

600 Kv. At larger values of γ one would have to perform some bunch rotation manipulations in order to effect he transfer with only 600 Kv available in the h = 360 system. These should be easier to perform without losses than for Au ions since the proton bunches will be relatively smaller. The choice of γ within the limits of the AGS extraction energy could be dictated by where the expected cancellation between the space charge and broadband impedance occurs. The stability threshold could be quite large if one could inject near the null. Then a transfer to the h = 2520 system would insure stability throughout the acceleration and storage cycles.

As an alternate strategy for obtaining stability of the proton bunches one could excite one of the storage cavities at some fraction of the h = 360 cavity voltage. This would constitute a "Landau" cavity by providing an additional spread in the Synchrotron frequency within a bunch. The use of the seventh harmonic is of course dictated by the ratio of the frequencies of the two rf systems. Stabilization using the sixth harmonic was carried out on the ISR at CERN. However in more extensive tests ratios of 3 and 4[11] were used. An important parameter is (1+ kn) where n is the ratio of the cavity frequencies and $k = V_1/V_2$. Here V_1 is the Landau cavity voltage and V_2 that of the accelerating rf system. In their tests the phase of V_1 was opposite to that of V_2 at the bunch center and k was chosen so that $1 + kn \approx 0$. This produces a large frequency spread since $\omega_s \approx 0$ at the bunch center and increase almost linearly for a bunch that occupies a fraction of the Landau cavity period. In figure 3 we have plotted the synchrotron frequencies for $V_1 = \pm V_2/7$,

$V_2 = 300$ Kv at $\gamma = \sqrt{3}\ \gamma_{tr} \cong 39.5$ (where the bunch length is a maximum) and a $\varphi_s = 180$.

A 0.5 evsec bunch would have a half width of ≅ 39° or 219° in the figure. The fact that slope of f_s vs the bunch length is falling rapidly at the edge of our nominal bunch size is a potential drawback of using such a large value of n. Observations[12] and calculations[13] for the case of n=2 indicate that if the slope becomes zero at the bunch edge local Landau damping is lost and the instability threshold is lowered. Now for k = ± 1/7 the bunch shape will of course be altered so that our stability analysis that assumes a particular distribution would no longer apply. For k = - 1/7 the bunch will have a flat top and steep edges while for k= 1/7 the density will be peaked at the center.

We note that for k= -1/7 the incoherent space charge tune shift would be zero at the center of the bunch and for an inductive wall impedance it would be come negative at the edge of the bunch i.e. larger amplitude particles would have a lower synchrotron frequency. This variation would of course be superimposed on that due to the Landau cavity. For k = + 1/7 the incoherent space charge tune shift would be maximum at the center of the bunch and much greater than without the Landau cavity. The shift would decrease toward the edge of the bunch and again is negative for the inductive wall. Hence the variation of frequency with amplitude would be the same as that due to the Landau cavity (and to the accelerating rf cavity). However as noted above the stability threshold depends upon the third power of the bunch length and inversely with the frequency spread. It can be shown[14] that for short bunches $S_-/S_+ = 2.34\ \Phi_{oo}^{3/4}$ where S_- is the spread for k = -1/7 and S_+ for +1/7 and Φ_{oo} is the bunch length for k=0. Hence for a given Landau cavity voltage the negative sign is most likely to give the larger stability limit. As noted above the choice of a negative k was used operationally at the ISR[11].

One additional option in operating with a Landau cavity for proton bunch stabilization would be to increase the accelerating voltage V_2 to 600 Kv and V_1 to 85.7 Kv. Then a 0.5 evsec bunch half width will be about 30° and hence occupy a smaller fraction of the effective rf bucket. The absolute frequency spread will actually be larger since Ω_o the synchrotron frequency with $V_1 = 0$ will be $\sqrt{2}$ larger while the slope of $\dot{\Omega}_s$ vs bunch length is much smaller at the edge of the bunch than at the center. The edge of the bunch will also be further away from the region where this slope is zero than for the $V_2 = 300$ Kv case. Thus any effects due to a near zero slope will be reduced. Note also that while the real frequency shift given by equation 6 will only increase by 2¼ (for $V_1 = 0$) the spread with $V_1 = 85.7$ Kv will be > 0.9 $\sqrt{2}$. Hence one would not expect the stability threshold to be lowered by this increase in V_2.

Conclusion

Although the proton bunches and the heavy ion bunches are expected to be potentially unstable at the design intensity per bunch it has been shown that for the anticipated impedances of the higher order resonant modes in the accelerating and storage cavities it should be possible to suppress the growth of any coupled bunch instabilities. Once the value of the effective broadband coupling impedance is known it should be possible to control any such instabilities at several times the design intensity using the methods outlined above. At some point it may become necessary to employ feedback to aid in suppressing any dipole or quadrupole mode that arises as the bunch intensity is increased[(15)].

Acknowledgement:

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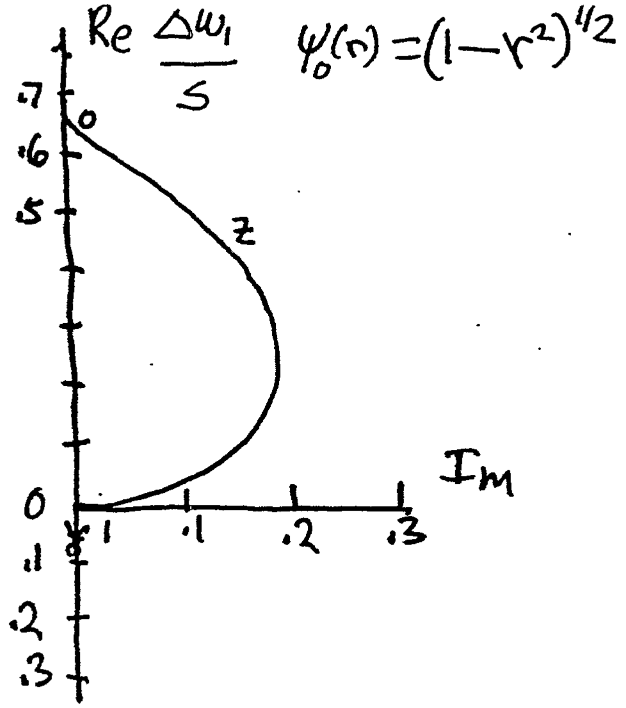
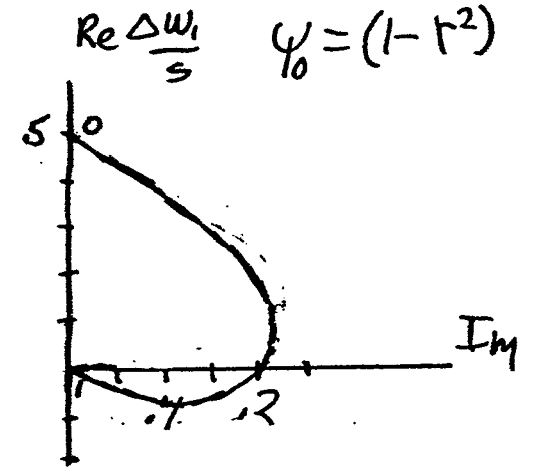
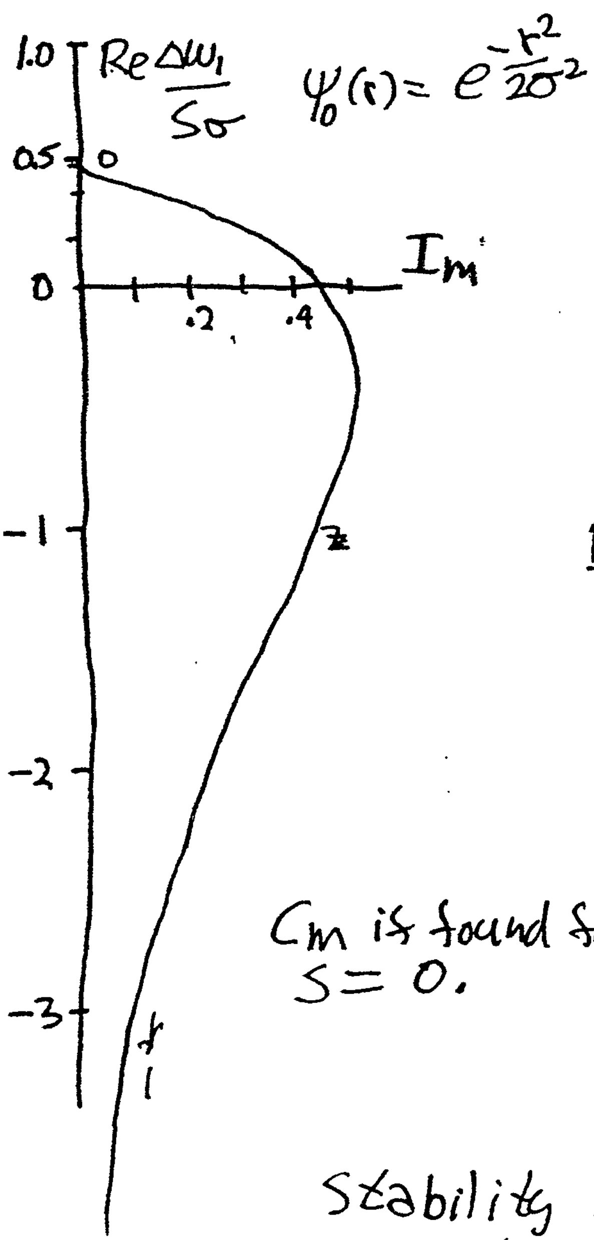
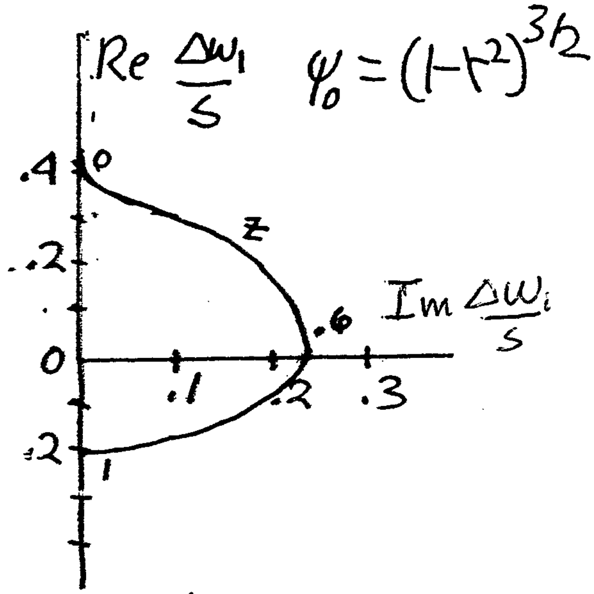
Figure 1









Dispersion Relation

$$1 = C_m \int_0^1 \frac{W(r)\, r^{2m+1}\, dr}{(\omega - \omega_s(r))}$$

$W(r) = \frac{1}{r}\frac{d\psi_0}{dr}$ $\qquad \omega_s(r) = \omega_{sc} - S r^2$

C_m is found for $S = 0$.

$$z = \frac{\omega_{sc} - \omega}{S}$$

ω_{sc} is frequency at bunch center

Stability plot is for $\mathrm{Im}\, z = 0$, $m = 1$, $C_1 = -\pi$ [illegible]

HOM dampers performances (two damping loop-longitudinal modes only). MAFIA results.

NO HOM INSTALLED					2 DAMPERS INSTALLED	DAMPING FACTOR
Nr.	F[MHz]	R_{sh}[kΩ]	Q[-]	R/Q[Ω]	R_{sh}'[kΩ]	R_{sh}/R_{sh}'
1	27.7	1120	17900	62.6	1120	1
2	104.7	166	29250	5.7	1.3	128
3	197.9	54	22800	2.4	1.4	38.6
4	269.2	86.3	21400	4.0	7.0	12.3
5	279.0	105.0	20400	5.2	10.1	10.4
6	324.3	342.5	31400	10.3	14.5	23.6

Table 1

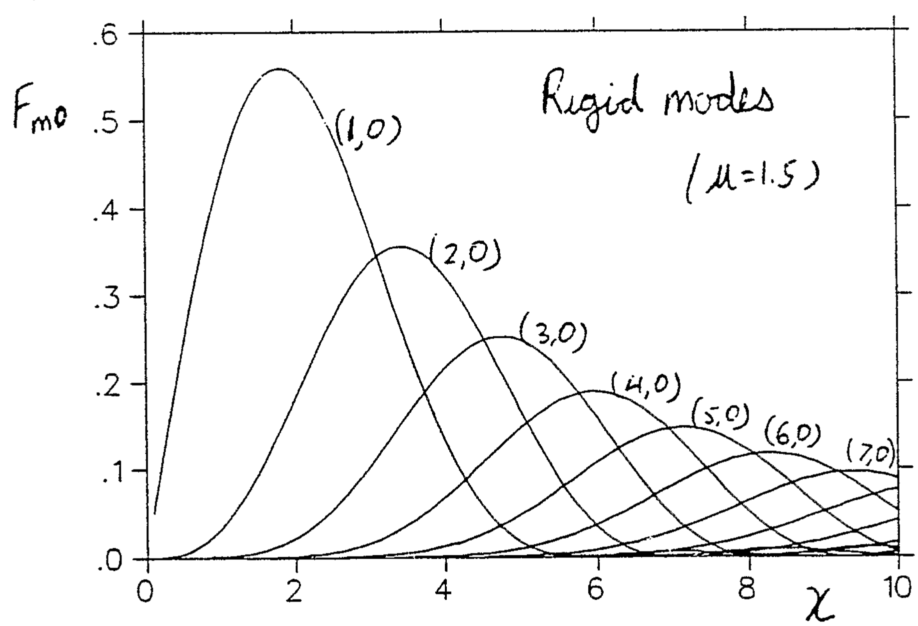
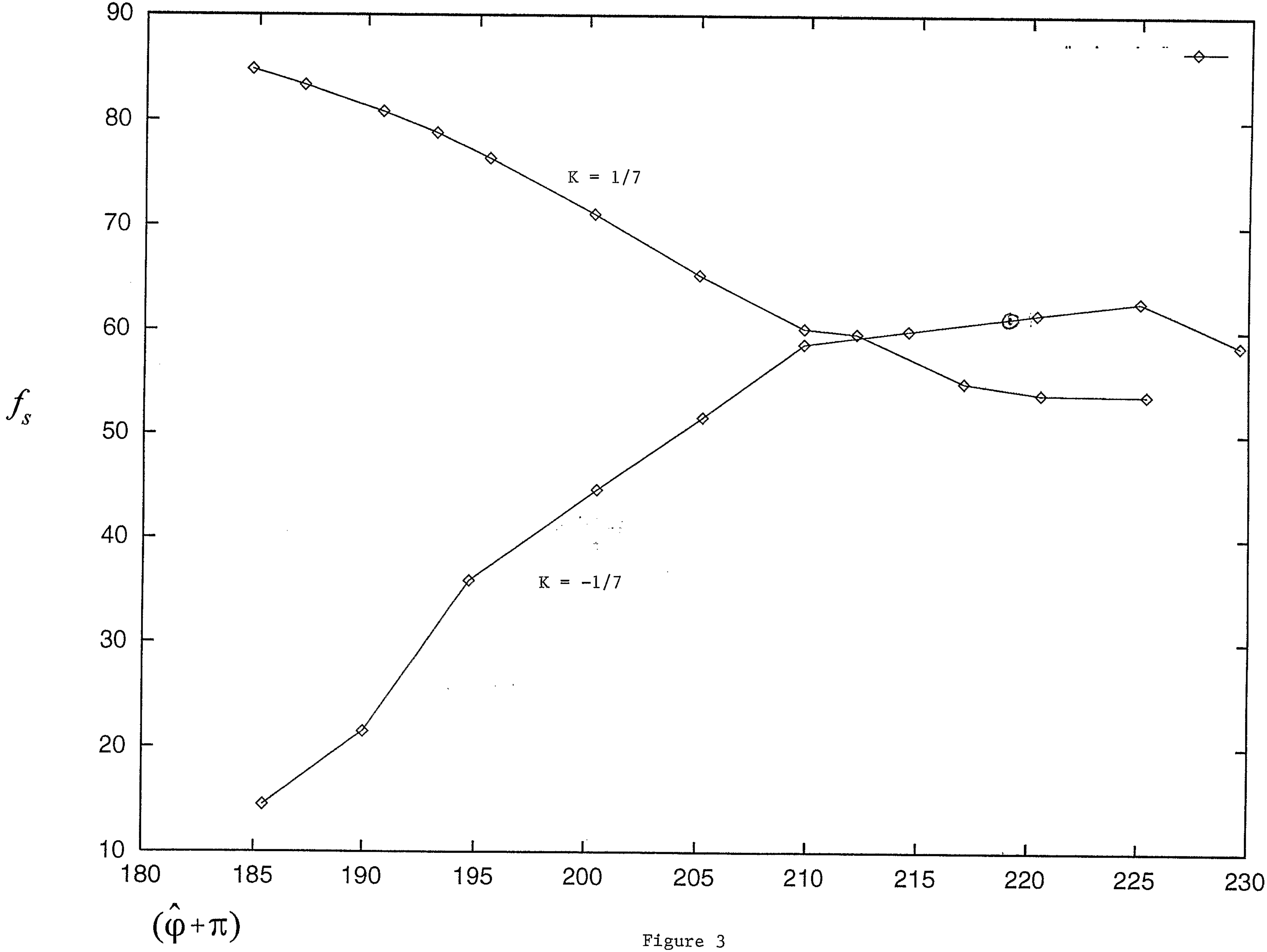


Figure 2



90
80
70
60
50
40
30
20
10
180
185
190
195
200
205
210
215
220
225
230
K = 1/7
K = -1/7
f_s
$(\hat{\varphi}+\pi)$

Figure 3