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Coupled bunch instabilities and implications for RHIC RF cavity impedances

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RHIC/RF-27

RHIC Project

BROOKHAVEN NATIONAL LABORATORY

RHIC/RF Technical Note No. 27

COUPLED BUNCH INSTABILITIES AND IMPLICATIONS FOR RHIC RF CAVITY IMPEDANCES

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Coupled Bunch Instabilities and Implications for RHIC rf Cavity Impedances

Jim Rose

The impedances of higher order modes must be kept below certain limits to prevent emmittance blowup and possible beam loss due to coupled bunch instabilities. A mathCAD program was set up to perform growth rate calculations. Results were compared to ZAP[1] calculations for the non-degenerate case and then extended to the degenerate case where the radial modes are summed. The coupled bunch instability growth rate is highest at injection, and impedance limits of higher order modes are set to keep coupled bunch growth rates below $2s^{-1}$, a factor of five below the damping rate of the injection error damper.

The growth rates were initially calculated with ZAP using parabolic line densities. Because ZAP does not take into account the degeneracy of the higher order radial modes (and gives no physical insight) Baartman's[2] formulism was used to calculate growth rates based on bunch distributions of the form

$$\Psi(r)=K\left[1-\left(\frac{r}{\hat{r}_\phi}\right)^2\right]^\mu$$

where K is a normalization factor. A parabolic distribution in phase space is given by $\mu=1$ and is used in the following. The growth rates are calculated as

$$\frac{1}{\tau_{mk}}=\frac{\omega_\phi}{\hat{r}_\phi}\frac{I_0 R}{V_T\cos\phi_T}F_{mk}$$

where ω_ϕ is the angular synchrotron frequency, r_ϕ is the bunch half-length in radians of rf, I_0 is the DC beam current, R is the shunt impedance of the HOM, and V_T is the total rf voltage. F_{mk} is a form factor found analytically by Satoh[3] and is given as

$$F_{mk}=m\mu(\mu+1)\left(\frac{2}{\chi}\right)^{2\mu+1}\frac{(m+2k+\mu)\Gamma(k+\mu)\Gamma(m+k+\mu)}{k!(m+k)!}J^2_{(m+2k+\mu)}(\chi)$$

The factor F_{mk} depends only upon χ

$$\chi=\frac{\omega_{res}}{\omega_{rf}}\hat{r}_\phi$$

and the bunch distribution and mode indices. It is plotted for the dipole case (m=1, k=0,1,2,3) in figure 1, and for the quadrupole case (m=2, k=0,1,2,3) in figure 2. In this nomenclature the rigid dipole is the m=1, k=0 mode.

For narrow band resonators (i.e., rf cavities) using only the lowest (k=0) modes is incorrect since the higher order radial modes of a particular azimuthal mode are degenerate. Hence the growth rate is given as

$$\frac{1}{\tau_m}=\sum_{k=0}^{\infty}\frac{1}{\tau_{mk}}$$

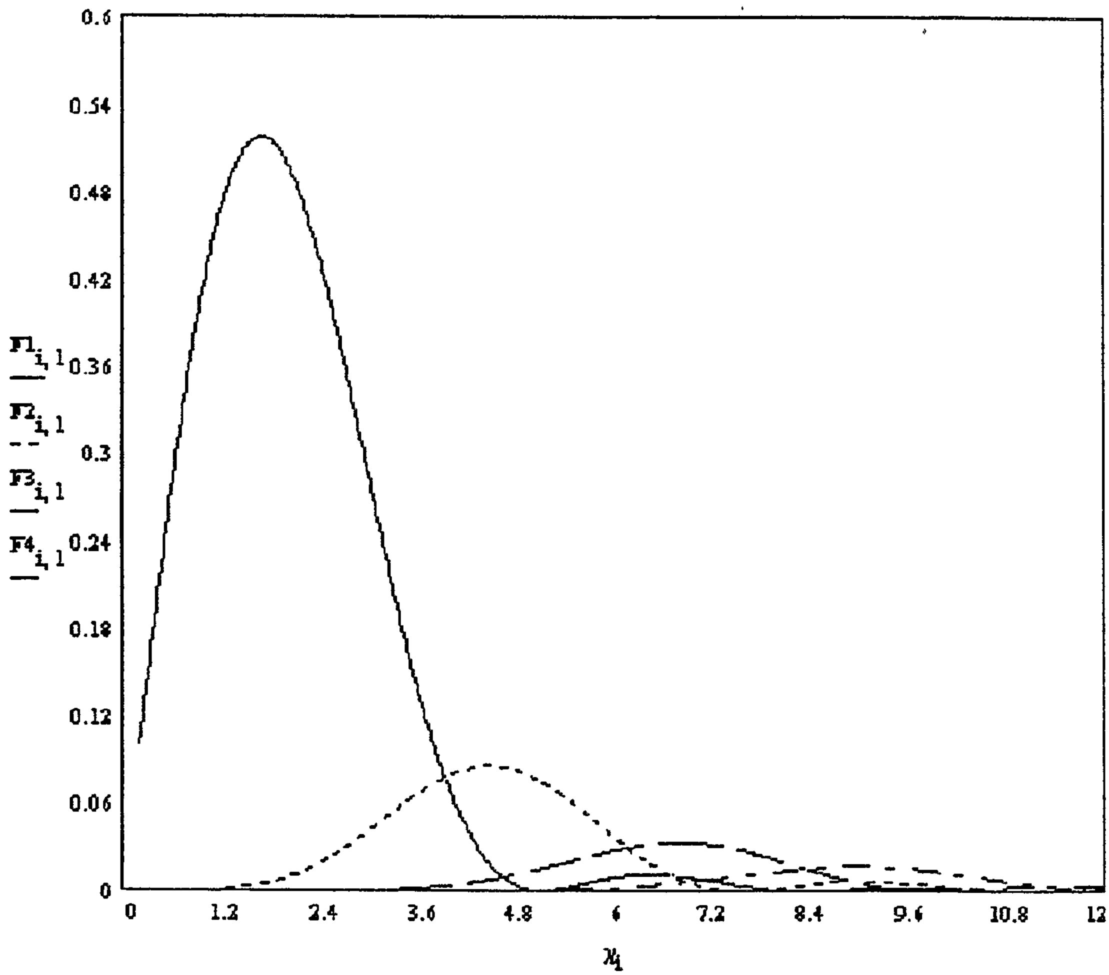

Figure 1 Dipole form factors for k=0,1,2,3

Figure 3 shows the form factors for the degenerate case for the dipole, quadrupole, sextupole and octupole for k=0 to 3.
The growth rates have been calculated for the dipole and quadrupole coupled bunch instabilities for both the non-degenerate and degenerate case and are shown in table I.
Since there is always some broadband impedance the radial modes are not fully degenerate, however the worst case of summing the radial modes was used for the calculation of impedance limits for the RHIC rf cavities.
Calculations with ZAP show the dipole and quadrupole coupled bunch modes to be unstable for any narrowband impedance due to the broadband impedance. This has been confirmed by E. Raka that proton bunches of 0.3 eV-sec have a broadband impedance threshold of nominally 0.25 ohms. Since RHIC has a broadband impedance about 1 ohm any real impedance causes the bunches to be unstable. (E. Raka has pointed out that raising the emmittance to 0.5 eV-sec would raise the threshold to over 1 ohm and could eliminate the problem for protons, and D.P. Deng confirmed that 0.5 ev-sec bunches can be rebucketed.) For the purposes of the design of the accelerating cavities it was decided to require the growth rates to be a factor of five below

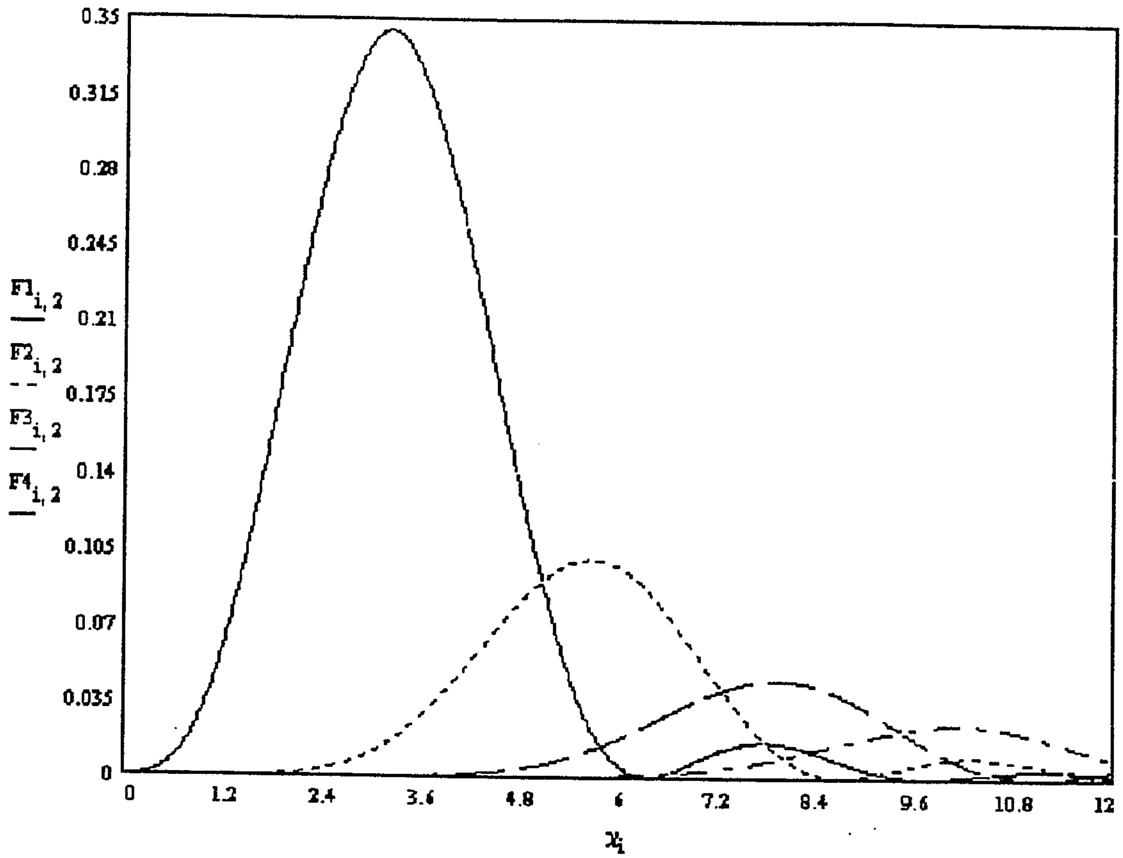

Figure 2 Quadrupole form factors for k=0,1,2,3

the damping rate of the injection error damper. Thus given the damping rate of $10s^{-1}$ of the wideband damper a growth rate limit of $2s^{-1}$ was set. One can then define an impedance limit as a function of χ (which is simply related to frequency) as

$$R_{\lim} = \frac{\frac{1}{\tau}\hat{r}_\phi V_T \cos\phi_T}{\omega_\phi I_0 F_m}$$

where $(1/\tau) = 2s^{-1}$. I have used the greater of either the dipole or quadrupole form factor in the above equation. This limit is plotted in figure 4 with the undamped HOM's superimposed. The peculiar peak at approximately 150 MHz is due to the changeover from the dipole to quadrupole form factors. Dampers for this cavity have been designed which bring these impedances below this limit and are described in rf technote # 25.

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I wish to thank Gene Raka for many helpful discussions on coupled bunch instabilities. Also helpful were discussions with M. Blaskiewicz and D.P. Deng.

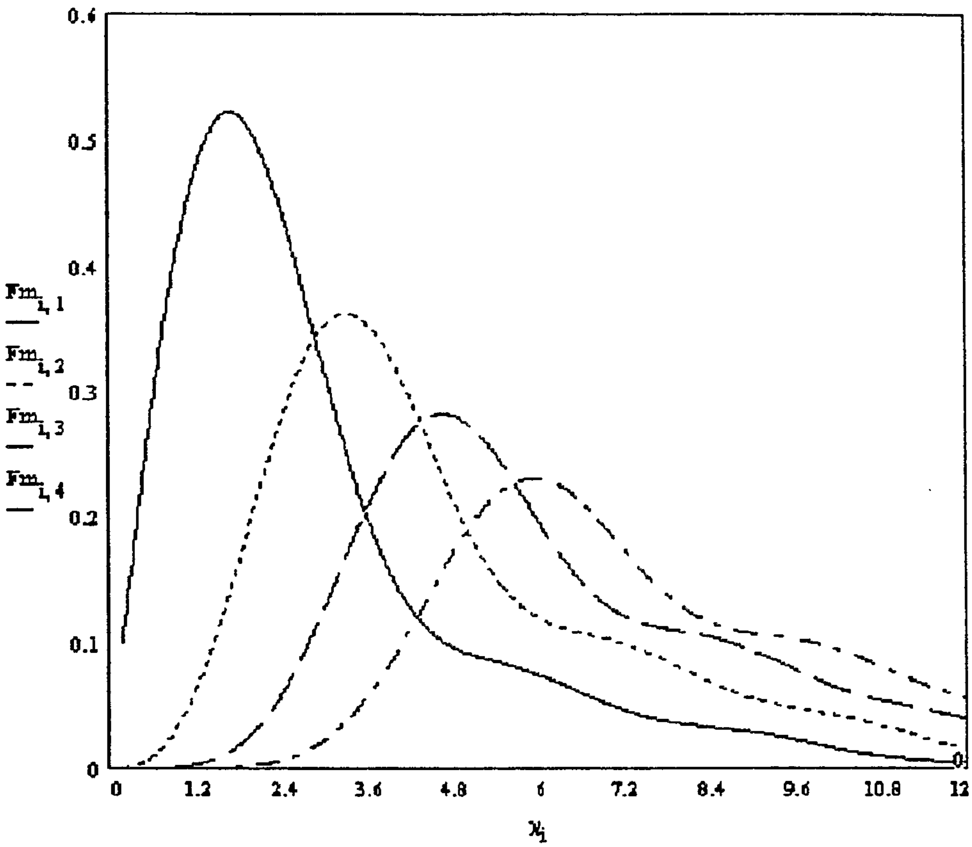

Figure 3 Form factors for dipole, quadrupole, sextupole and octupole with the first four radial modes summed (k=0,1,2,3)

References:

1) M.S. Zisman, S. Chattopadhhay, J.J. Bisognano "ZAP Users Manual" LBL-21270 UC-28

2) R. Baartman, USPAS accelerator school notes, January 1993

3) K. Satoh, "Stability of Higher -Order Longitudinal Modes in a Bunched Beam Without Mode Coupling" PEP note 357

HOM Frequency (MHz)	Shunt Impedance (MΩ)	Dipole Growth Rate (sec^{-1}) non-degen (degen)	Quad. Growth Rate (sec^{-1}) non-degen (degen)
103	0.129	14.2 (14.4)	4.7 (4.7)
192	0.077	2.9 (3.9)	5.6 (6.0)
276	0.174	.051 (3.5)	4.3 (7.6)
329	0.307	.48 (5.2)	0.77 (8.4)
394	0.156	.28 (1.7)	.28 (3.4)
479	0.219	.003 (1.5)	.49 (3.1)
507	0.234	.051 (1.4)	.2 (2.8)
520	0.031	.01 (.18)	.012 (.35)
538	0.089	.04 (.44)	.004 (.92)

Table I Growth rates for dipole and quadrupole coupled bunch instabilities

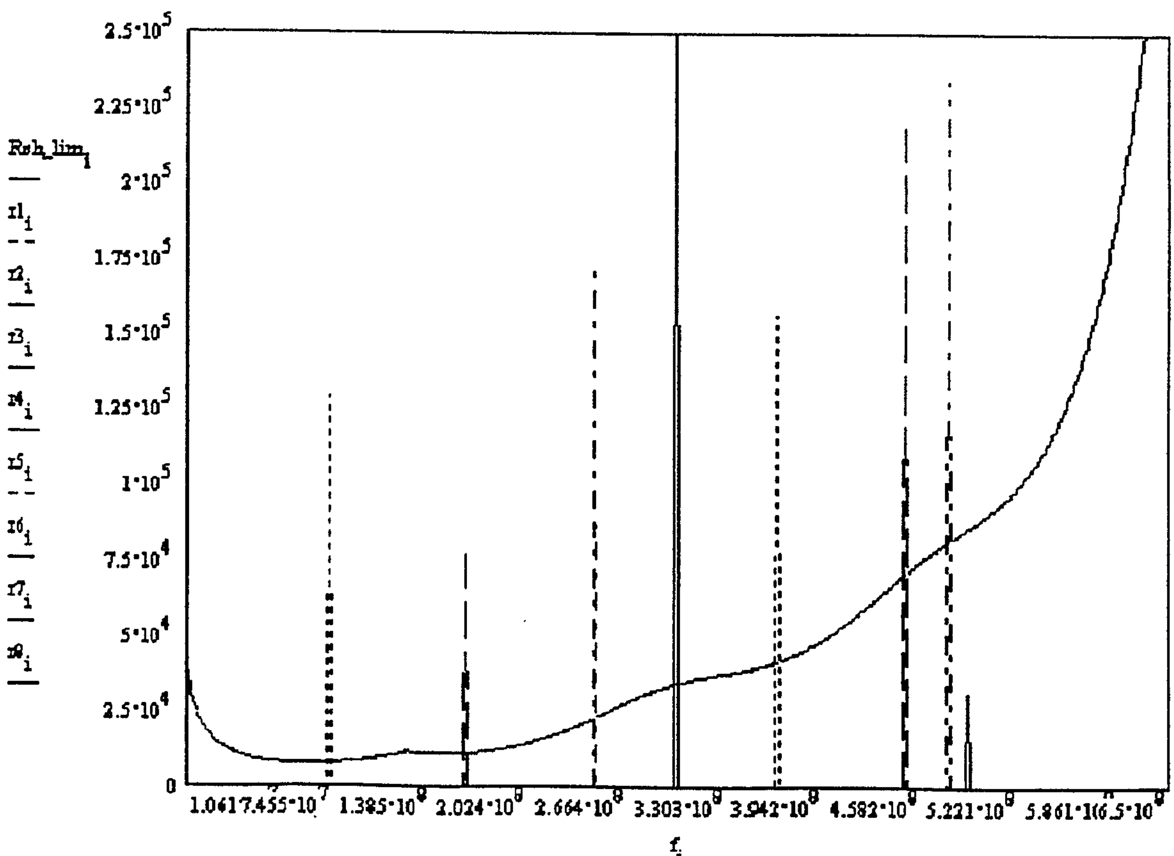

Figure 4 Impedance limit for growth rate of $2s^{-1}$ with HOM impedances superimposed. The y-axis is in ohms and the x-axis is in Hz.