

# Measurements of Helical Magnetic Fields Using Flat Rotating Coils

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# Measurements of Helical Magnetic Fields Using Flat Rotating Coils

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## 1 Introduction

Rotating coils are commonly used to measure the magnetic field coefficients  $(a_n, b_n)$  inside straight magnets [1]. They can also be employed to determine the multipole coefficients  $(\tilde{a}_n, \tilde{b}_n)$  (cf. Ref. [2]) in helical magnets.

We use a cylindrical coordinate system  $(r, \theta, s)$  where  $s$  designates the coordinate along the longitudinal magnet axis. The area of a flat rotating coil ranges from  $r_1$  to  $r_2$  and from  $s_1$  to  $s_2$ . The magnetic flux through the coils is

$$\Phi(\theta) = N \int_{s_1}^{s_2} \int_{r_1}^{r_2} B_\theta(r, \theta) dr ds \quad (1)$$

where  $N$  is the number of coils windings. For rotating coils one has  $\theta = \omega t$  and the induced voltage

$$U = -\frac{d\Phi}{dt} \quad (2)$$

is proportional to the angular velocity  $\omega$ .

We will present formulae for the magnetic flux through a rotating coil for straight and helical magnets. Assuming the induced voltage is parameterized in terms of ordinary multipole coefficients  $(a_n, b_n)$  conversion formulas will be given to obtain the helical multipole coefficients  $(\tilde{a}_n, \tilde{b}_n)$ .

## 2 Straight Magnetic Fields

The azimuthal field in straight magnets can be expressed in multipole coefficients  $(a_n, b_n)$  as [2]

$$B_\theta = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[ b_n \cos((n+1)\theta) - a_n \sin((n+1)\theta) \right]. \quad (3)$$

$B_0$  is the main magnetic field and  $r_0$  a reference radius. The magnetic flux (1) becomes

$$\Phi(\theta) = NB_0(s_2 - s_1) \sum_{n=0}^{\infty} K_n \left[ b_n \cos((n+1)\theta) - a_n \sin((n+1)\theta) \right] \quad (4)$$

where the coefficients  $K_n$  are defined by

$$K_n = \frac{r_0}{n+1} \left[ \left( \frac{r_2}{r_0} \right)^{(n+1)} - \left( \frac{r_1}{r_0} \right)^{(n+1)} \right] \quad (5)$$

and result from the integration over  $r$  in (1).

### 3 Helical Magnetic Fields

The azimuthal helical field can be written in terms of helical multipole coefficients ( $\tilde{a}_n, \tilde{b}_n$ ) (cf. Ref. [2]) as

$$B_\theta = \frac{B_0}{kr} \sum_{n=0}^{\infty} f_n I_{n+1}((n+1)kr) \left[ \tilde{b}_n \cos((n+1)(\theta - ks)) - \tilde{a}_n \sin((n+1)(\theta - ks)) \right], \quad (6)$$

with

$$f_n = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n}. \quad (7)$$

Here  $B_0$  denotes the transverse component of the main field close to the magnet axis. This field is vertical at the location  $s = 0$ . The magnetic flux (1) can be expressed as

$$\Phi(\theta) = NB_0 \sum_{n=0}^{\infty} R_n \left[ \hat{b}_n \cos((n+1)\theta) - \hat{a}_n \sin((n+1)\theta) \right] \quad (8)$$

with new coefficients

$$R_n = \frac{f_n}{k} \int_{r_1}^{r_2} \frac{1}{r} I_{n+1}((n+1)kr) dr. \quad (9)$$

The integral in (9) can be computed numerically. In (8) new magnetic multipole coefficients

$$\begin{aligned} \hat{a}_n &= +\tilde{a}_n T_n + \tilde{b}_n S_n, \\ \hat{b}_n &= -\tilde{a}_n S_n + \tilde{b}_n T_n. \end{aligned} \quad (10)$$

are used for which

$$\begin{aligned} S_n &= \frac{1}{(n+1)k} \left[ \cos((n+1)ks_2) - \cos((n+1)ks_1) \right] \\ &= -\frac{2}{(n+1)k} \sin \frac{(n+1)k(s_2 - s_1)}{2} \sin \frac{(n+1)k(s_2 + s_1)}{2} \end{aligned} \quad (11)$$

and

$$\begin{aligned}
T_n &= \frac{1}{(n+1)k} \left[ \sin((n+1)ks_2) - \sin((n+1)ks_1) \right] \\
&= +\frac{2}{(n+1)k} \sin \frac{(n+1)k(s_2 - s_1)}{2} \cos \frac{(n+1)k(s_2 + s_1)}{2}
\end{aligned} \tag{12}$$

have been defined.

## 4 Conversion

We assume now a device that parameterizes the voltage (2) in terms of multipole coefficients  $(a_n, b_n)$  for straight magnets. If the measured magnetic field has helical symmetry, the coefficients  $(\tilde{a}_n, \tilde{b}_n)$  in Eq. (8) can be derived as

$$\begin{aligned}
\tilde{a}_n &= \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n T_n - b_n S_n}{S_n^2 + T_n^2}, \\
\tilde{b}_n &= \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n S_n + b_n T_n}{S_n^2 + T_n^2}.
\end{aligned} \tag{13}$$

We consider three special cases.

(a) Measuring coil of one helical wavelength with  $s_1 = s$ ,  $s_2 = s + \lambda$ .

From equations (11) and (12) we obtain

$$S_n = T_n = 0 \tag{14}$$

and with (10)

$$\hat{a} = \hat{b} = 0. \tag{15}$$

The magnetic flux (8) is therefore zero and the coefficients can not be obtained.

(b) Measuring coil of half helical wave length with  $s_1 = 0$ ,  $s_2 = \lambda/2$ .

In this case one has

$$S_n = \begin{cases} -\frac{2}{(n+1)k} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases} \quad \text{and} \quad T_n = 0.$$

Only coefficients with  $n$  even (i.e. helical dipole, sextupole etc. coefficients) can be measured. For those we have

$$\begin{aligned}
\tilde{a}_n &= +\frac{K_n}{R_n} \frac{(n+1)\pi}{2} b_n, \\
\tilde{b}_n &= -\frac{K_n}{R_n} \frac{(n+1)\pi}{2} a_n.
\end{aligned} \tag{16}$$

(c) Infinitely short measuring coil with  $s_1 = s$ ,  $s_2 = s + ds$ .  
Expanding (11) and (12) to first order in  $ds$  we obtain

$$\begin{aligned} S_n &= -\sin\left((n+1)ks\right)ds, \\ T_n &= +\cos\left((n+1)ks\right)ds \end{aligned} \tag{17}$$

and

$$\begin{aligned} \tilde{a} &= \frac{K_n}{R_n} \left[ +a_n \cos\left((n+1)ks\right) + b_n \sin\left((n+1)ks\right) \right], \\ \tilde{b} &= \frac{K_n}{R_n} \left[ -a_n \sin\left((n+1)ks\right) + b_n \cos\left((n+1)ks\right) \right]. \end{aligned} \tag{18}$$

If in addition  $s = 0$ , the  $(\tilde{a}_n, \tilde{b}_n)$  can be obtained from the  $(a_n, b_n)$  by multiplication with  $K_n/R_n$ .

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## References

- [1] P. Schmüser, “Magnetic measurements of the superconducting HERA magnets and analysis of systematic errors”, DESY HERA-p 92-1 (1992).
- [2] W. Fischer, “Magnetic field error coefficients for helical dipoles”, RHIC/AP/83 and AGS/RHIC/SN/17 (1996).