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# Magnetic Field Error Coefficients ofr Helical Dipoles


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## 1 Introduction

The aim of this paper is to give a notation for the magnetic field error coefficients of helical dipoles. These coefficients shall be the magnetic multipole coefficients of ordinary dipoles when the helical wave length tends to infinity. Such a notation is different from Ref. [1].

For comparison, the magnetic field error notation for ordinary dipoles will be presented first. The notation for helical dipoles is given thereafter.

## 2 Magnetic Field Errors of Ordinary Dipoles

In a current free region in vacuum where the electrical field $\vec{E}$ is constant, the magnetic field $\vec{B}$ can be derived from a scalar potential $\psi$ as

$$\vec{B} = -\nabla\psi. \tag{1}$$

We will use a Cartesian coordinate system $(x, y, z)$ and a cylindrical coordinate system $(r, \theta, z)$. Here, $x$ denotes the horizontal, $y$ the vertical and $z$ the longitudinal direction. Furthermore we have

$$\begin{aligned} x &= r\ \cos\theta, \\ y &= r\ \sin\theta. \end{aligned} \tag{2}$$

We consider a dipole of infinite length, thus neglecting fringe fields. The symmetry condition of such an element reads

$$\psi(r, \theta, z) = \psi(r, \theta, z + \Delta z) \tag{3}$$

where $\Delta z$ is arbitrary. Therefore, the potential $\psi$ is independent of $z$:

$$\psi(r, \theta, z) = \psi(r, \theta). \tag{4}$$

Having a main field $B_0$ in $y$-direction, the solution of the Laplace equation $\Delta\psi = 0$ can be written in cylindrical coordinates as

$$\begin{aligned}\psi(r,\theta) = -B_0\Bigg\{ & r\sin\theta + \\ & + \sum_{n=0}^{\infty} \frac{1}{n+1}\frac{r^{n+1}}{r_0^n}\left[a_n\cos\left((n+1)\theta\right) + b_n\sin\left((n+1)\theta\right)\right]\Bigg\}.\end{aligned} \tag{5}$$

The term $-B_0 r\sin\theta$ gives the main field and the coefficients $a_n$ and $b_n$ denote deviations from the main field. The $b_n$ are called "normal" and the $a_n$ "skew" multipole coefficients. Here, the subscript "0" denotes a dipole, "1" a quadrupole etc. $r_0$ is a reference radius. For the RHIC dipoles $r_0 = \frac{5}{8}r_{coil}$ is used with $r_{coil} = 40$ mm.

From equations (1) and (5) the magnetic field can be obtained in cylindrical coordinates. We have

$$\begin{aligned} B_r &= B_0\left\{\sin\theta + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[a_n\cos\left((n+1)\theta\right) + b_n\sin\left((n+1)\theta\right)\right]\right\}, \\ B_\theta &= B_0\left\{\cos\theta + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[b_n\cos\left((n+1)\theta\right) - a_n\sin\left((n+1)\theta\right)\right]\right\}, \\ B_z &= 0.\end{aligned} \tag{6}$$

The Cartesian components of $\vec{B}$ can be written as

$$\begin{aligned} B_x &= B_0\left\{\sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[a_n\cos(n\theta) + b_n\sin(n\theta)\right]\right\}, \\ B_y &= B_0\left\{1 + \sum_{n=0}^{\infty}\left(\frac{r}{r_0}\right)^n\left[b_n\cos(n\theta) - a_n\sin(n\theta)\right]\right\}, \\ B_z &= 0,\end{aligned} \tag{7}$$

which can also be expressed as

$$B_y + iB_x = B_0\left[1 + \sum_{n=0}^{\infty}(b_n + ia_n)\left(\frac{x+iy}{r_0}\right)^n\right]. \tag{8}$$

Note that the European notation (see for example Ref. [2]) differs from the American one presented here. The transformation is

$$b_n(American) = b_{n+1}(European), \tag{9}$$

$$a_n(American) = -a_{n+1}(European). \tag{10}$$

# 3 Magnetic Field Errors of Helical Dipoles

We consider again a magnet of infinite length, thus neglecting fringe fields. The symmetry condition for a helical dipole is

$$\psi(r,\theta,z) = \psi(r,\theta - k\Delta z, z + \Delta z), \tag{11}$$

where $\Delta z$ is arbitrary. In other words, $\theta - kz = const.$ $k = 2\pi/\lambda$ is the wave number and $\lambda$ the wave length of the helix. $k$ shall have the positive sign for right-handed and the negative sign for left-handed helices. Introducing the new variable

$$\tilde{\theta} = \theta - kz, \tag{12}$$

the symmetry condition (11) leads to a potential $\psi$ which is only dependent on $r$ and $\tilde{\theta}$:

$$\psi(r,\theta,z) = \psi(r,\tilde{\theta}). \tag{13}$$

The tilde shall remind the reader of the fact that $\tilde{\theta}$ in a helix is similar to $\theta$ in a ordinary dipole. Using $(r,\tilde{\theta})$ as coordinates and having a transverse helical main Field $B_0$ a solution of the Laplace equation $\Delta\psi = 0$ is (cf. Eq. (5) and Ref. [1])

$$\begin{aligned}\psi(r,\tilde{\theta}) = -B_0 \Bigg\{ & \frac{2}{k} I_1(kr)\sin\tilde{\theta} + \\ & + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+2}} \frac{1}{r_0^n k^{n+1}} I_{n+1}((n+1)kr) \times \\ & \times \left[\tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta})\right]\Bigg\}\end{aligned} \tag{14}$$

where $I_n$ are modified Bessel functions. Similar to the ordinary dipole case, the term $-B_0\frac{2}{k}I_1(kr)\sin\tilde{\theta}$ yields the main field and the coefficients $\tilde{b}_n$, $\tilde{a}_n$ the deviations thereof. Here, the $\tilde{b}_n$ are called "normal" and the $\tilde{a}_n$ "skew" helical multipole coefficients (with respect to the direction of the main field $B_0$). The subscript "0" denotes a helical dipole, the subscript "1" a helical quadrupole etc. $r_0$ is again a reference radius.

The factors in (14) are chosen in such a way as to obtain the potential (5) when the helical wave length tends to infinity. In this case $k \to 0$, $\tilde{\theta} \to \theta$ and the Bessel function can be approximated by (cf. Ref. [3])

$$I_n(z) \approx \frac{1}{2^n}\frac{z^n}{n!}. \tag{15}$$

Now, the magnetic field can be computed as (cf. Ref. [1])

$$\begin{aligned} B_r &= B_0 \Bigg\{ 2 I_1'(kr) \sin\tilde{\theta} + \\ &\quad + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} I_{n+1}'((n+1)kr) \times \\ &\quad \times \left[ \tilde{a}_n \cos((n+1)\tilde{\theta}) + \tilde{b}_n \sin((n+1)\tilde{\theta}) \right] \Bigg\}, \\ B_\theta &= -\frac{1}{kr} B_z, \\ B_z &= -B_0 \Bigg\{ 2 I_1(kr) \cos\tilde{\theta} + \\ &\quad + \sum_{n=0}^{\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \frac{1}{r_0^n k^n} I_{n+1}((n+1)kr) \times \\ &\quad \times \left[ \tilde{b}_n \cos((n+1)\tilde{\theta}) - \tilde{a}_n \sin((n+1)\tilde{\theta}) \right] \Bigg\}, \end{aligned} \tag{16}$$

where $I_n'$ denotes the derivative with respect to the argument of the Bessel function.

Since the Bessel function is nonlinear, the magnetic field of a helical dipole is nonlinear too, even the main field given by $B_0$. Close to the magnet axis we have $r \to 0$ and the field can be approximated by

$$\begin{aligned} B_x &= -B_0 \sin(kz), \\ B_y &= B_0 \cos(kz), \\ B_z &= -B_0 k \left[ x \cos(kz) + y \sin(kz) \right], \end{aligned} \tag{17}$$

i.e. even close to the magnet axis there is a longitudinal field component that will lead to coupling.

# References


[1] V. Ptitsin, "Notes on the helical field", RHIC/AP/41 (1994).

[2] J. Rossbach and P. Schmüser, "Basic course on accelerator optics", Fifth General Accelerator Course, University of Jyväskylä, Finland, CERN 94-01 (1994).

[3] M. Abramowitz and I. Stegun, "Handbook of Mathematical Functions", Dover, New York (1972).