

BNL-101917-2014-TECH AD/RHIC/5;BNL-101917-2013-IR

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September 1984

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U.S. Department of Energy

USDOE Office of Science (SC)

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High Energy Facilities Advanced Projects

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RHIC Technical Note No. 5

BEAM LOSS DUE TO THE APERTURE LIMITATION RESULTING FROM INTRABEAM SCATTERING

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September 12, 1984

ABSTRACT

Diffusion equation is used to evaluate the beam loss in the presence of aperture limitation resulting from the intrabeam scattering. We discuss the effect of different boundary conditions. Satisfactory beam intensity can be maintained within the proposed RHIC operation time.

I. Introduction

Intrabeam scattering (IBS)¹ has become one of the important topics in the heavy ion collider design study. Detailed calculation of the emittance $\varepsilon_{\rm N}$, momentum spread $\sigma_{\rm E}$ and bunch length σ_{ℓ} blow up due to IBS has been performed by G. Parzen.² Fig. 1 summarizes these results, where we have plotted $\sigma_{\rm E}$, $\varepsilon_{\rm N}$ and σ_{ℓ} vs. $t^{\frac{1}{2}}$. It is tempting to argue that $\sigma_{\rm E}$, $\varepsilon_{\rm N}$ and σ_{ℓ} satisfies certain statistical random diffusion process. In fact, the situation is not very simple. First, the Einstein relation of Brownian motion is not satisfied at all. However Fig. 1 shows that within this limited region of time scale, the Einstein relation is not a very bad approximation after all.

With the diffusion equation in mind, we would like to ask the question of beam survival rate in these diffusion processes. Section 2 reviews briefly the Fokker-Planck equation and Section 3 discusses the transversed and longitudinal losses and the Conclusion is given in Section 4.

2. Fokker-Planck diffusion equation

For Markovian Processes, the distribution function $\rho(y,t)$ satisfies the Fokker-Planck equation.³

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial y} (A_1 \rho) + \frac{1}{2} \frac{\partial^2}{\partial y^2} (A_2 \rho)$$
(1)

where

$$A_{1} = \lim_{\Delta t \to 0} \left[\langle \Delta y \rangle / \Delta t \right]$$
(2)

$$A_{2} = \lim_{\Delta t \to 0} \left[\langle (\Delta y)^{2} \rangle / \Delta t \right]$$
(3)

Higher order contributions in eq. (1) are neglected. For dynamical system where a Hamiltonian governs the motion of dynamical variables,⁴ one has

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$$A_1 = \frac{1}{2} \frac{\partial}{\partial y} A_2 \quad . \tag{4}$$

With this relation, eq. (1) becomes

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial y} \left(\frac{A_2}{2} \frac{\partial \rho}{\partial y} \right) .$$
 (5)

A special interesting situation is $A_1 = \langle \Delta y \rangle / \Delta t = \text{constant} = D$, then

$$A_2 = 2D(t) y \tag{6}$$

Eq. (5) then becomes

$$\frac{\partial \rho}{\partial t} = D \frac{\partial}{\partial y} y \frac{\partial}{\partial y} \rho$$
 (7)

The fundamental solution of eq. (7) is

$$\rho(y,t) = \frac{1}{\langle \Delta y(t) \rangle} e^{-y/\langle \Delta y(t) \rangle}$$
(8)

On the other hand, another interesting situation is that $A_1 = 0$. We have then $A_2 = \langle (\Delta y)^2 \rangle / \Delta t = D(t)$. The fundamental solution in this case is

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$$\rho(y,t) = \frac{1}{\sqrt{4\pi \langle \Delta y^2(t) \rangle}} e^{-y^2/2 \langle \Delta y^2(t) \rangle}$$
(9)

3. Applications

3.1. Transverse phase space

The Fokker-Planck equation for the transverse phase space is given

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} (D\varepsilon \frac{\partial \rho}{\partial \varepsilon})$$

where ε is the emittance of the beam and

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$$D = \frac{d \langle \varepsilon \rangle}{dt} = D(t)$$

Let us define new variable τ and y as

$$\tau = \frac{1}{\varepsilon_a} \int D dt = \frac{1}{\varepsilon_o} (\langle \varepsilon_f \rangle - \langle \varepsilon_i \rangle)$$
$$y = \varepsilon / \varepsilon_o$$

The diffusion equation becomes then

$$\frac{\partial \rho}{\partial \tau} = \frac{\partial}{\partial y} \left(y \frac{\partial \rho}{\partial y} \right)$$
(10)

Besides the fundamental solution of equation (8), there are alternative solutions to equation (10). Teng,⁵ MacLachlan⁶ and Ruggiero⁷ used the sink boundary condition

$$\rho (y=1,\tau) = 0$$
 (11a)

and initial condition

$$\rho(y, \tau=0) = \rho(y)$$
, (11b)

where $\rho_o(y)$ is the initial distribution.

The solution is given by

$$\rho(y,t) = \sum_{n=1}^{\infty} \frac{J_{o}(\lambda_{n}\sqrt{y}) e^{-\frac{\lambda_{n}^{2}}{4}\tau}}{J_{1}^{2}(\lambda_{n})} \int_{0}^{1} \rho_{o}(y') J_{o}(\lambda_{n}\sqrt{y'}) dy'$$
(12)

where \boldsymbol{J}_{0} and \boldsymbol{J}_{1} are Bessel functions and $\boldsymbol{\lambda}_{n}\, 's$ are given by

 $J_{0}(\lambda_{n}) = 0$

Due to the aperture limitation, the number of particles confined is given by

$$N(\tau) = \int_{0}^{1} \rho(y,\tau) \, dy = 2 \Sigma \frac{e^{-\lambda_{n}^{2} \tau/4}}{\lambda_{n} J_{1}(\lambda_{n})} \int_{0}^{1} \rho_{0}(y) J_{0}(\lambda_{n} \sqrt{y}) \, dy$$
(13)

Table 1 summarizes the result calculated with boundary conditions (11a) and (11b). While the loss rate calculated from the fundamental solution in eq. (8) is also shown for comparison. With the Courant-Snyder invariant, the fundamental solution of eq. (8) is a Gaussian distribution of the phase space variables x and x'. The loss rate shown on the last row of Table 1 is therefore determined by the available phase space, $\sqrt{6\sigma}_a/\sigma_x$.

Since the aperture acts only as a beam scraper, which gives no dynamical action to the diffusion process, I believe that eq. (11a) is incorrect. The similarity between the diffusion process here and heat conduction is inappropriate, because the heat loss is proportional to the temperature gradient. In the present situation, the diffusion process comes from the interaction between beam particles alone.

Table 1									
	γ	5 (.65hr)	7 (2hr)	12*	30*	100*			
	δ (10 ⁻³)	0.93	1.15	1.56	1.98	1.10			
	ϵ_{i}^{N}	10π	10π	10π	10π	10π			
	$\epsilon_{\tt f}^{\tt N}$	37.2π	52 . 1π	44π	33.2π	27.8π			
(±26mm good field))ε a	6x9.42π	6x8.79π	6x7.68π	6x6.62π	6x8.93π			
Analysis with	h boundary	conditions in	n eqs. (lla)) and (11b))+				
τ		.096	.114	.061	.019	.0033			
loss rate(%)		<1	<1	<1	<1	0			
Analysis with the fundam $\epsilon_a / (\epsilon_f^N / \gamma)$ $\sqrt{6}\sigma_a / \sigma_x$ loss rate(%)		mental solut: 7.60 2.76 2.2	ion of eq. 7.09 2.66 2.9	(8). 12.6 3.5	36 6. 0	19.3 14. 0			
δ is ε _a is	δ is the rms momentum spread ε_{a} is the admittance space available for betatron oscillation								
$arepsilon_{i}^{N},arepsilon_{f}^{N}$ ar	e the norma	alized initia	al and final	l emittance	es respect	ively			
τ = (ε	$\varepsilon_{\rm f}^{\rm N} - \varepsilon_{\rm i}^{\rm N})/\gamma \varepsilon_{\rm i}$	3							
√6σ _a /σ _{si}	the ratio ze	of aperture	to the beam	n rms trans	verse phy	sical			
* 10	10 hr operation time is assumed								
+ se	e ref. 7 fo	or details.							

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3.2. Diffusion in the longitudinal phase space

Let us define the momentum spread as

$$\delta = \frac{\Delta p}{p} \tag{14}$$

The Fokker-Planck equation becomes

$$\frac{\partial \rho}{\partial t} = \frac{1}{2} \frac{\partial}{\partial \delta} \left(A_2 \frac{\partial \rho}{\partial \delta} \right)$$
(15)

where

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$$A_2 = \frac{d}{dt} < \delta^2 > ,$$

is assumed to be a function of time only. Let $\Delta_{\!\!\!a}$ be the aperture limit. Defining Z and τ as

$$z \equiv \frac{\delta}{\Delta_a}$$
(16)

$$\tau = \frac{1}{2\Delta_{a}^{2}} \int_{0}^{t} A_{2}(t')dt' = \frac{1}{2\Delta_{a}^{2}} \left(\delta^{2}(t) - \delta^{2}(0)\right) , \qquad (17)$$

we obtain the following Fokker-Planck equation

$$\frac{\partial \rho}{\partial \tau} = \left(\frac{\partial^2}{\partial z^2} \rho\right) \tag{18}$$

Using the similar condition as that of eqs. (11a) and (11b), we obtain the solution

$$N(\tau) = 2 \sum_{n=0}^{\infty} \frac{(-)^n}{\beta_n} e^{-\beta_n^2 \tau} \approx \frac{4}{\pi} e^{-\frac{\pi^2}{4}\tau}$$
(19)

with

$$\beta_n = \frac{\pi}{2} (2n+1)$$

for a delta function initial distribution.

Table 2 summarizes the loss rate calculated from eq. (19) and that calculated from the fundamental Gaussian distribution of eq. (9). We found that the loss rate obtained in these two analysis are about the same. However in reality they are very different. Fig. 2 shows the survival number as a function of parameter τ in equation (17) for these two cases. Curve (A) obtained from the boundary conditions (11a) and (11b), while (B) is equal to $\operatorname{erf}(1/2\sqrt{\tau})$ obtained from the fundamental solution of eq. (9). We feel that boundary conditions (11a) and (11b) are inappropriate in this case too.

Ŷ	5	7	12	30	100				
$\Delta_a = (\Delta p/p)$ bucket (10^{-3})	2.28	2.72	3.82	9.95	2.68				
δ(t=0)	.818	.751	.678	1.261	.359				
$\delta_{\mathbf{F}} = \delta(\mathbf{t}=\mathbf{t}_{\mathbf{F}})$.93	1.14	1.563	1.985	1.099				
Analysis with boundary conditions (11a) and (11b)									
τ _o	•064	.038	.016	.008	.009				
τ _t	.083	.088	.084	.024	.085				
loss(%)	4	5	4	0	4				
Analysis with the fundamental solution of eq. (9).									
Δ_a / δ_F	2.45	2.39	2.44	5.01	2.44				
loss(%)	3	3	3	0	3				

4. Conclusion

In conclusion, we have applied the Fokker-Planck diffusion equation to estimate the beam loss due to the intrabeam scattering. Due to many particle dynamics, diffusion equation should be appropriate in describing this random process. We discuss however the implication of boundary conditions on the beam loss. We argue to prefer one special model to the other. The effect of the intrabeam scattering process is found to contribute only about 5% of total loss of beam as a whole. Details are however summarized in Tables 1 and 2 respectively for transverse and longitudinal phases.

Table 2

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