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Study on the Tolerances of the Big Bend

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RHIC PROJECT

Brookhaven National Laboratory

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1. Introduction

The Big Bend of the RHIC injection line consists of 26 combined function dipole magnets. It has 6 regular OFoDOODoFO cells and an irregular half cell at the beginning for dispersion matching, where O is 0.715m long straight section and o is 0.45m short straight section. Its structure and parameters are described in reference 1. The tolerances of this section have been studied and the results are summarized in this note.

2. Systematic Errors

On the geometrical center line of the combined function dipoles the nominal value of $\frac{B}{\frac{\partial B}{\partial x}}$ is ± 0.340356m. If the actual magnet field distribution does not fulfill this requirement, on the central orbit there will be a field error ΔBL or gradient error $\Delta K_1 L$ or both when we locate the magnet geometrical center line on the ideal central orbit. The field error ΔBL, will cause both dispersion function change and orbit distortion in the x-direction. The gradient error $\Delta K_1 L$ will affect the beam Twiss parameters and the x dispersion functions. The effect of ΔBL are shown in Table 1, and those of $\Delta K_1 L$ are shown in Table 2.

Table 1: Effect of Systematic Field Error ΔBL

$\frac{\Delta BL}{BL}$	η_x	η'_x	F_η	x_{co}(mm)	x'_{co}(mrad)
0	2.182	0	1	0	0
$+10^{-3}$	2.165	-0.001	1.014	-3.776	0.048
-10^{-3}	2.200	0.001	1.014	3.776	-0.048

Table 2: Effect of Systematic Gradient Error $\Delta K_1 L$

$\frac{\Delta K_1 L}{K_1 L}$	β_x	α_x	F_x	η_x	η'_x	F_η	β_y	α_y	F_y
0	26.603	0.007	1	2.182	0	1	5.954	0.034	1
10^{-3}	26.591	0.010	1.003	2.175	0	1.003	5.925	0.031	1.006
-10^{-3}	26.612	0.004	1.003	2.190	0	1.003	5.993	0.036	1.005
10^{-2}	26.322	0.034	1.03	2.113	0.002	1.038	5.71	0.006	1.05
-10^{-2}	26.524	−0.022	1.03	2.268	−0.002	1.045	6.279	0.048	1.056

In the above and following tables, the beam parameters and central orbit distortions are datas at the end of the Big Bend. F is the phase space dilution factor. $F = \frac{A}{\epsilon_0}$, where ϵ_0 is the area of the nominal beam emittance, A is the area of the ellipse which is similar to the nominal admittance of the Big Bend and circumscribes the admittance distorted by these errors with an area equal ϵ_0. In F_η, A corresponding to a momentum error $\Delta P/P_0 = 1 \times 10^{-3}$. From the above datas, it is clear that the effects of a systematic field error of 10^{-3} are small, the induced dilution factor F is about or less than 1.01, but for a 10^{-2} $\frac{\Delta K_1 L}{K_1 L}$, F is about $1.03 \sim 1.05$.

The induced orbit distortion x_{co} along the big bend by a field error $\frac{\Delta BL}{BL} = 1 \times 10^{-3}$ is shown in Figure 1, the maximum distortion is about 4 mm.

These systematic field errors can be eliminated by intentionally offsetting the magnets by an amount $DX = -a$ (for $B' > 0$, F magnets) or $DX = a$ (for $B' < 0$, D magnets) if on the geometrical center line of the magnets $\frac{\int B d\ell}{\int B' d\ell} = \pm(0.340356 - a)$. The exciting current should be $I\left(1 - \frac{a}{0.3403778}\right)$, where I is the required current in order to produce the nominal $\int B d\ell$ on the geometrical center line of the magnets. The effects of these systematic field errors can also be eliminated by tuning the quadrupoles in the upstream or downstream sections if one knows these small changes in beam parameters. The induced orbit distortion can be corrected by correctors.

The ripple or instability of the exciting current of these magnets will cause both ΔBL and proportional $\Delta K_1 L$ at the same time. This effect can not be compensated by offsetting magnets, correctors or tuning upstream or downstream sections. The effects are shown in Table 3 and the induced central orbit distortion in x-plane is the same as shown in Figure 1.

Table 3: Effects of ΔBL and Proportional $\Delta K_1 L$

$\frac{\Delta BL}{BL} = \frac{\Delta K_1 L}{K_1 L}$	β_x	α_x	F_x	η_x	η'_x	F_η	β_y	α_y	F_y
10^{-3}	26.590	0.010	1.003	2.157	0	1.011	5.925	0.031	1.006
-10^{-3}	26.613	0.004	1.003	2.208	0	1.011	5.983	0.036	1.005

3. Effect of Errors of SWM, G1D and G2F

The effects of the field error ΔBL of SWM, ΔBL and $\Delta K_1 L$ of G1D and G2F and the longitudinal alignment error of G1D and G2F are shown in Tables 4 through 7.

Table 4: The Effect of 10^{-3} Field Error of SWM

$\frac{\Delta BL}{BL}$ of SWM	η_x	η'_x	F_η	x_{co}	x'_{co}
10^{-3}	2.195	0.001	1.01	1.596	−0.048
-10^{-3}	2.170	−0.001	1.01	−1.596	0.048

Table 5: The Effect of 10^{-3} Field Error of G1D and G2F

$\frac{\Delta BL}{BL}$ of G1D	$\frac{\Delta BL}{BL}$ of G2F	η_x	η'_x	F_η	x_{co}	x'_{co}
10^{-3}	10^{-3}	2.169	0	1.006	0.582	0.043
-10^{-3}	-10^{-3}	2.196	0	1.006	−0.582	−0.043
-10^{-3}	10^{-3}	2.174	0	1.003	−0.297	0.28
10^{-3}	-10^{-3}	2.191	0	1.004	0.297	−0.028

Table 6: The Effect of 10^{-3} Gradient Error of G1D and G2F

$\frac{\Delta K_1 L}{K_1 L}$ of G1D	$\frac{\Delta K_1 L}{K_1 L}$ of G2F	β_x	α_x	F_x	β_y	α_y	F_y
10^{-3}	10^{-3}	26.572	0.01	1.003	5.992	0.036	1.007
-10^{-3}	-10^{-3}	26.634	0.005	1.002	5.916	0.031	1.007
10^{-3}	-10^{-3}	26.614	0.004	1.003	5.982	0.036	1.005
-10^{-3}	10^{-3}	26.590	0.01	1.003	5.925	0.031	1.006

The effects on η_x and η'_x are negligibly small.

Table 7: The Effects of Transverse Misalignment of G1D and G2F

G1D ds(mm)	G2F ds(mm)	β_x	α_x	F_x	η_x	η'_x	F_η	β_y	α_y	F_y
3	3	26.628	0.008	1.001	2.177	0	1.002	5.351	0.34	1.001
−3	−3	26.578	0.007	1.001	2.187	0	1.002	5.950	0.033	1.001
3	−3	26.609	0.006	1.001	2.178	0	1.002	5.961	0.035	1.002
−3	3	26.597	0.009	1.002	2.187	0	1.002	5.946	0.032	1.002

All these effects are negligibly small.

4. Random Alignment Errors

The effect of random alignment errors with DX = 0.25 mm, DY = 0.25 mm and DPSI (rotation around the beam axis) = 0.25 mrad for 13 different random number with Gaussian distribution truncated at 2.5 σ have been calculated. The central orbit distortions are shown in Figure 2 and Figure 3 for x and y directions respectively. The effects on the dispersion functions are shown in Table 8 and Table 9 for x and y direction respectively. Their effects on β and α are negligibly small.

Table 8: Effect of Random Alignment Errors on η_x and η'_x

η_x	η'_x	$F_{\eta a}$
2.159	0	1.01
2.171	0	1.005
2.215	0.001	1.018
2.198	−0.001	1.014
2.168	0.001	1.013
2.174	0	1.003
2.173	0	1.004
2.199	−0.001	1.014
2.196	0.001	1.013
2.179	0	1.001
2.200	−0.001	1.014
2.178	0	1.002
2.202	0.001	1.014

The maximum change of η_x is $^{+0.033}_{-0.023}$ m, η'_x is ± 0.001 and $F_\eta \leq 1.018$.

Table 9: The Effect on η_y and η'_y by Random Alignment Errors

η_y	η'_y	F_y
0.014	0	1.013
0.005	−0.002	1.012
0.013	0.003	1.02
0.012	−0.004	1.024
0.019	0.003	1.024
−0.001	−0.002	1.01
−0.011	0	1.01
−0.006	0	1.006
0.009	−0.001	1.01
−0.001	0.004	1.02
−0.008	−0.004	1.02
−0.002	0.001	1.006
−0.005	−0.002	1.012

The maximum change of η_y +0.019m to −0.011 and $\eta'_y \pm 0.004$, the corresponding dilution factor is $F_{\eta y} \leq 1.024$.

5. Effect of Random Field Errors

The random field errors ΔBL and ΔK_1L will cause central orbit distortion in the x-direction and changes of beam parameters β_x, α_x, β_y, α_y and dispersion functions η_x and η_x'. The central orbit distortion x_{co} is shown in Figure 4, the changes of β_x, α_x, β_y, α_y, η_x and η_x' are summarized in Table 10.

Table 10: Effect of Random Field Errors $\left(\frac{\Delta BL}{BL} = 10^{-3}, \frac{\Delta K_1 L}{K_1 L} = 10^{-3}, \text{rms}\right)$
Truncated at 2.5 σ

β_x	α_x	F_x	η_x	η_x'	F_η	β_y	α_y	F_y
26.55	0.002	1.005	2.172	−0.003	1.004	6.004	0.032	1.009
26.547	0.022	1.015	2.205	−0.001	1.015	5.962	0.030	1.004
26.452	−0.001	1.01	2.208	−0.001	1.016	6.112	0.041	1.027
26.568	0.001	1.006	2.172	0	1.004	5.981	0.040	1.007
26.551	0.023	1.016	2.165	0.001	1.012	5.921	0.024	1.011
26.600	0.010	1.003	2.177	0.001	1.012	6.011	0.035	1.01
26.599	0.019	1.012	2.182	0.001	1.012	5.994	0.035	1.007
26.730	0.009	1.005	2.149	0.001	1.019	5.849	0.033	1.018
26.664	0.007	1.002	2.168	0.001	1.013	5.909	0.034	1.008
26.621	0.017	1.01	2.186	0.001	1.01	5.889	0.025	1.014
26.502	0.015	1.009	2.195	0	1.006	6.033	0.038	1.014
26.550	−0.009	1.016	2.114	0.001	1.03	6.100	0.046	1.027

6. Overall Effect of Both Random Field and Alignment Errors

The overall effects on β_x, α_x, β_y, α_y are the same as shown in Table 10 (due to field error), the overall effects on η_y and η'_y and central orbit distortion in y-direction are the same as shown in Table 9 and Figure 3 respectively. The overall effects on central orbit distortion in the x-direction are shown in Figure 5 and the changes of η_x and η'_x are shown in the following Table 11.

Table 11: Overall Effects of Random Field and Random Alignment Errors

η_x	η'_x	F_η
2.149	−0.002	1.027
2.158	0.001	1.016
2.229	0.002	1.03
2.198	0	1.007
2.205	0.001	1.015
2.202	0	1.009
2.185	−0.001	1.011
2.155	−0.002	1.026
2.177	0	1.002
2.184	0	1.001
2.217	−0.002	1.028
2.191	0	1.004
2.206	0.002	1.026

7. Correction of the Central Orbit Distortion

The maximum allowable orbit distortion in the y-direction is about $\pm$ 5 mm and is much longer in the x-direction. From Figure 5, one can find that, there are four cases out of 13 in which the maximum x_{co} is larger than 5 mm when DX = 0.25 mm $\frac{\Delta BL}{BL} = 1 \times 10^{-3}$ random error. As shown in figure 3, there is only one case out of 13, the maximum y_{co} is larger than 5 mm when random errors are DY = 0.25 mm DPSI = 0.25 mm truncated at 2.5 σ. But if the random errors are not truncated at 2.5 σ, there are 3 cases out of 13, the maximum y_{co} is larger than 5 mm. The results are shown in Figure 6.

Besides, in order to avoid the accumulation effect of the errors in different sections of the injection line, it would be better to keep the orbit distortions at the end of each section to be zero. For this purpose, in the Big Bend, there should be at least 2 dipole correctors and two beam position monitors in each direction. The beam position monitors should be installed at the end of Big Bend or the beginning of the downstream section. The phase advance between the two BPM's should be about 90^o and located at $\beta_{\max}$. From Figure 1, 3, 5, 6, it is clear that in most cases, at the upstream half section, orbit distortion is much smaller. In order to avoid large over compensation at the upstream half section, the two correctors would be installed near the center of the Big Bend and 90^o phase advance between them located at $\beta_{\max}$.

Figures 7 to 11 show the initial and corrected orbit distortions in y-direction for different random error distributions. The two correctors are located between G8D, G9D and G12D, G13D, respectively. The maximum required corrector strength is about 0.3 mrad bending angle, corresponding to BL = 300 Gauss·M. At the exit of the Big Bend, the corrected orbit $y_{co} = 0$, $y'_{co} = 0$ and the maximum residual orbit distortion in the whole section is less than 4 mm.

In Figures 12 to 16, the initial and corrected orbit distortion in x-direction for 5 different random error distributions are shown. The two correctors are located between G10F, G11F and G14F, G15F respectively. The required maximum corrector strength is about the same as shown in y-direction. At the end of the Big Bend, the corrected orbit $x_{co} = 0$, $x'_{co} = 0$ and the maximum residual distortion in the whole section is about 4 mm in most cases.

In order to decrease the over compensation effect or the large initial distortion in the upstream half section (as shown in Figure 15, at 40M, $x_{co} = 5$ mm). We can shift the proper magnet or by installing one or two corrector near the beginning of the section. For this purpose, two more BPM's should be installed for each direction near the center of this section. Again, the two BPM's should be located 90^o apart and at $\beta_{\max}$.

8. The Effect of Initial Beam Position and Direction

The effects of the initial beam position and direction at the beginning of SWM have been studied. The induced orbit distortion by $x_o = 0.25$ mm, $x'_o = 0.025$ mrad, $y_o = 0.25$ mm and $y'_o = 0.025$ mrad are shown in Figures 17 and 18 respectively. The maximum orbit distortion is about 0.6 mm, 1 mm in x-direction and about half mm in y-direction.

9. Summary

1. The $\frac{\Delta BL}{BL} = \frac{\Delta K_1 L}{K_1 L} = 10^{-3}$ systematic error induced by exciting current ripple will cause a maximum x_{co} of about 4 mm and change of η_x of ± 0.025 m, corresponding to $F_\eta \simeq 1.01$ for $\frac{\Delta P}{P} = 10^{-3}$.
2. The systematic field error $\frac{\Delta BL}{BL} = 10^{-3}$ will cause a maximum x_{co} of 4 mm, and change of η_x of ± 0.017 m, $\eta'_x \pm 0.001$. The corresponding dilution factor is about 1.01. The systematic gradient error $\frac{\Delta K_1 L}{K_1 L} = 10^{-3}$ will cause a change of η_x of ± 0.007 m, and small change in β_x, α_x, β_y, α_y. The corresponding dilution factor is less than 1.01. This systematic error can be eliminated by offsetting the magnets or compensated by tuning and correctors.
3. A field error $\frac{\Delta BL}{BL} = 10^{-3}$ of SWM, will cause a change of η_x of about ± 0.012 m, $\eta'_x \pm 0.001$ and maximum x_{co} of about 1.6 mm.
4. A field error of $\frac{\Delta BL}{BL} = 10^{-3}$ in G1D and G2F will cause maximum change of η_x of ± 0.013 m, and the maximum x_{co} is about 0.6 mm. The effect of gradient error of G1D, G2F $\frac{\Delta K_1 L}{K_1 L} = 10^{-3}$ is very small. The induced changes of β and α will cause a maximum dilution factor of 1.007. The effect of longitudinal misalignment of G1D and G2F is negligibly small.
5. The random alignment error DX = 0.25 mm, DY = 0.25 mm, DPSI = 0.25 mrad will cause maximum change of $\eta_x{}^{+0.033\text{m}}_{-0.023\text{m}}$, $\eta'_x \pm 0.001$, $\eta_y{}^{+0.019\text{m}}_{-0.011\text{m}}$ and $\eta'_y \pm 0.004$. The maximum orbit distortions are about $x_{co,\max} \simeq 5$ mm $y_{co,\max} \simeq 6$ mm.
6. The random field error $\frac{\Delta BL}{BL} = 10^{-3}$, gradient error $\frac{\Delta K_1 L}{K_1 L} = 10^{-3}$ will cause a maximum change of $\eta_x{}^{+0.026\text{m}}_{-0.068\text{m}}$, $\eta'_x{}^{+0.001}_{-0.003}$ and the maximum x_{co} is about 6 mm. The induced changes in β_x, α_x, β_y, α_y will cause a phase space dilution factor F less than 1.02.
7. The overall effect of these random errors will cause a maximum change of $\eta_x{}^{+0.047\text{m}}_{-0.033\text{m}}$, $\eta'_x \pm 0.002$, maximum x_{co} 8 mm. The effect on y-direction is the same as random alignment errors.
8. The maximum orbit distortion x_{co} induced by $x_o = 0.25$ mm is about 0.6 mm and 1 mm by $x'_o = 0.025$ mrad. The maximum y_{co} induced by $y_o = 0.25$ mm is about 0.5 mm and induced by $y'_o = 0.025$ mrad is about 0.5 mm also.

Acknowledgments

I am thankful to Drs. J. Claus and H. Foelsche for their valuable discussions.

References

1. J. Claus and H. Foelsche, Beam Transfer from AGS to RHIC, AD/RHIC-47, 1988.

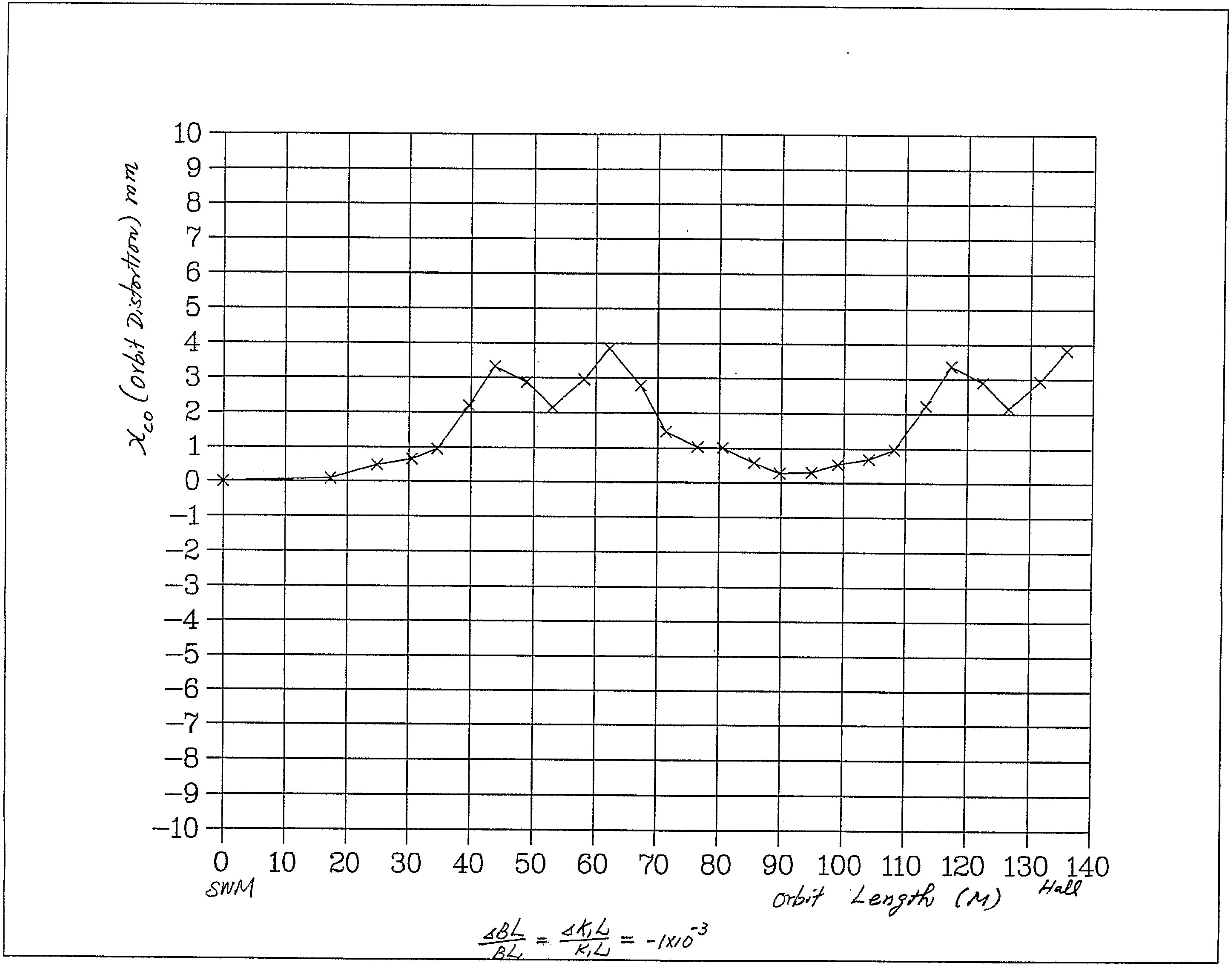


Fig. 1. Orbit Distortion Induced by systematic Field Error $\frac{\Delta BL}{BL} = 1 \times 10^{-3}$

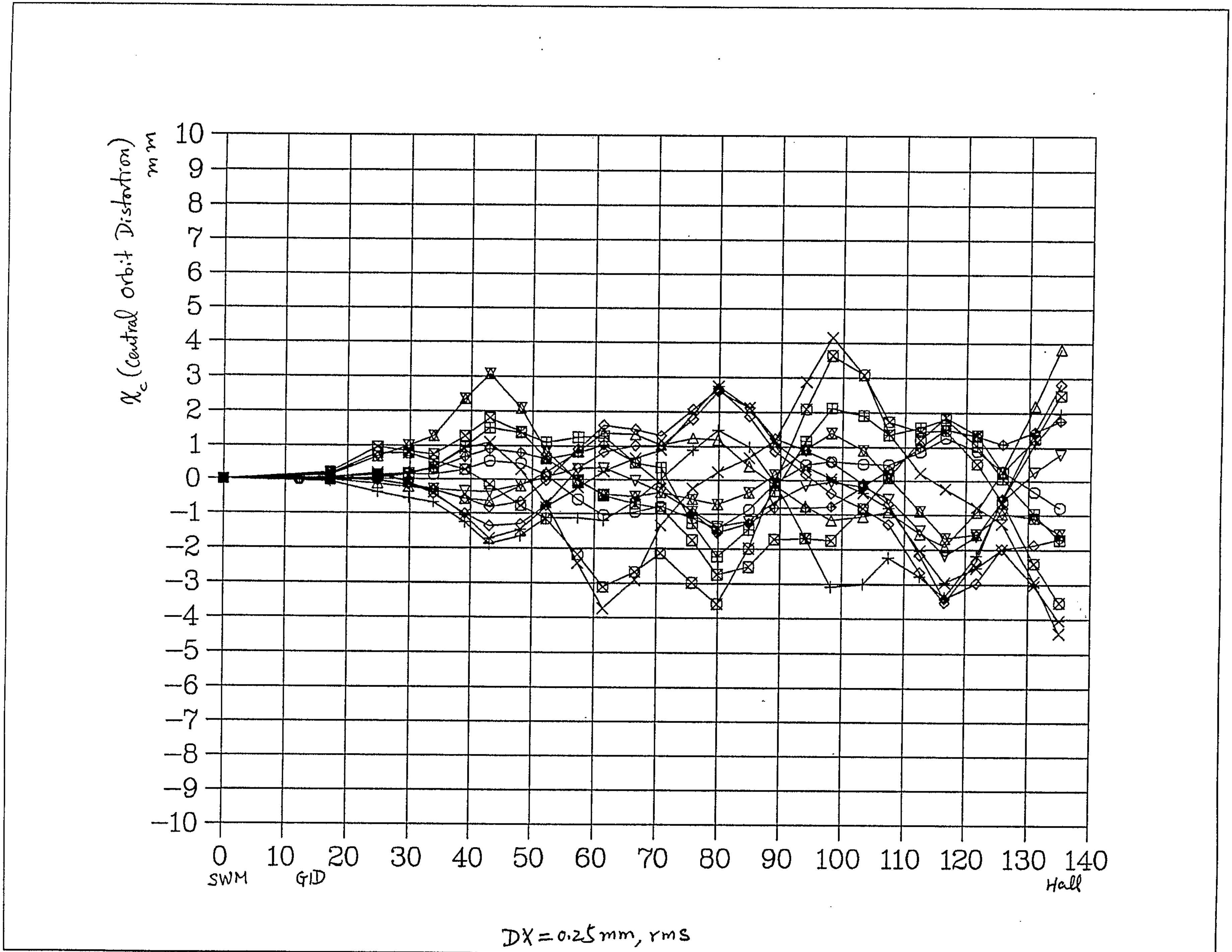


Figure 2. Central orbit Distortion Induced by Misalignment

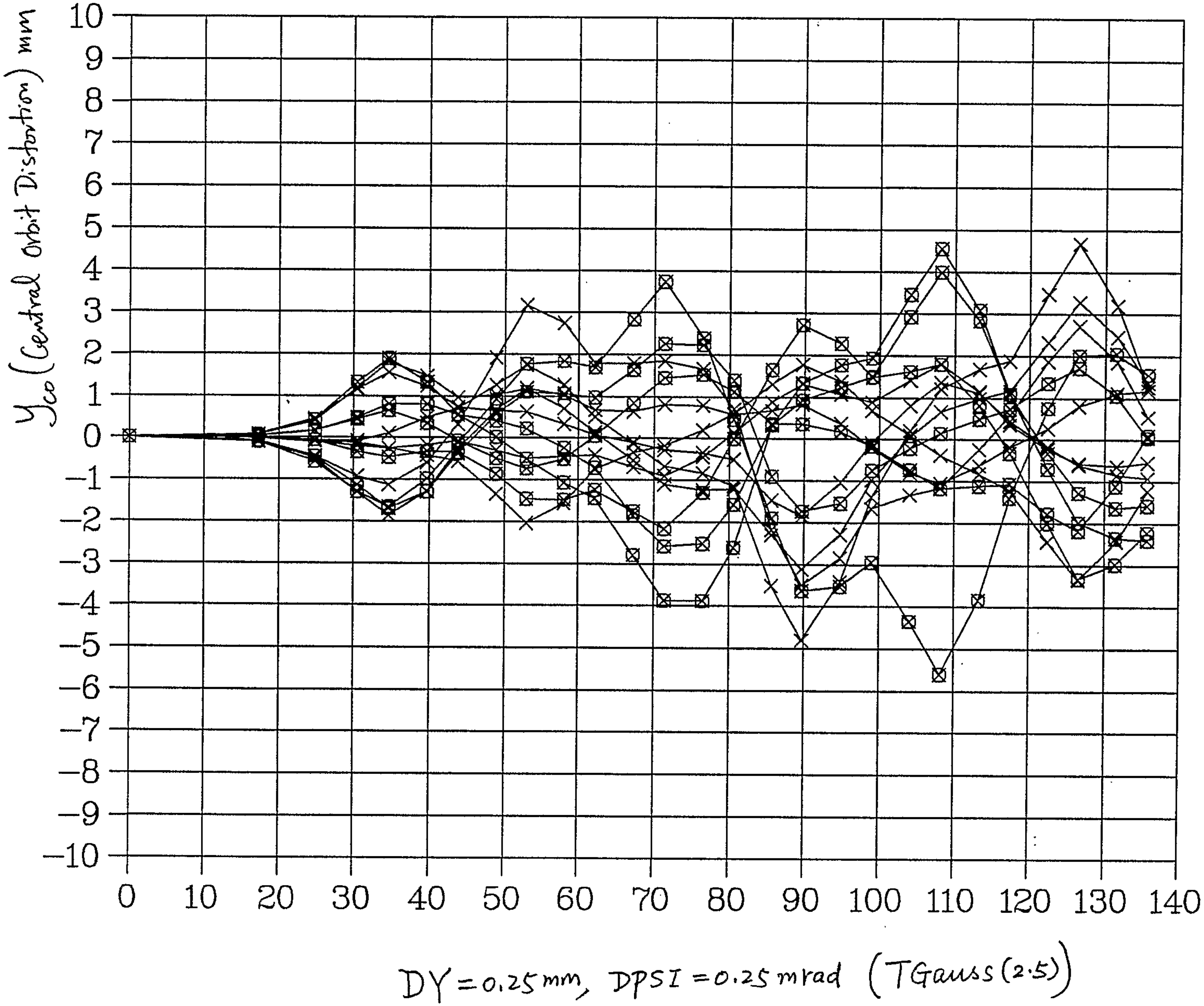


Figure 3. Central orbit distortion induced by misalignment

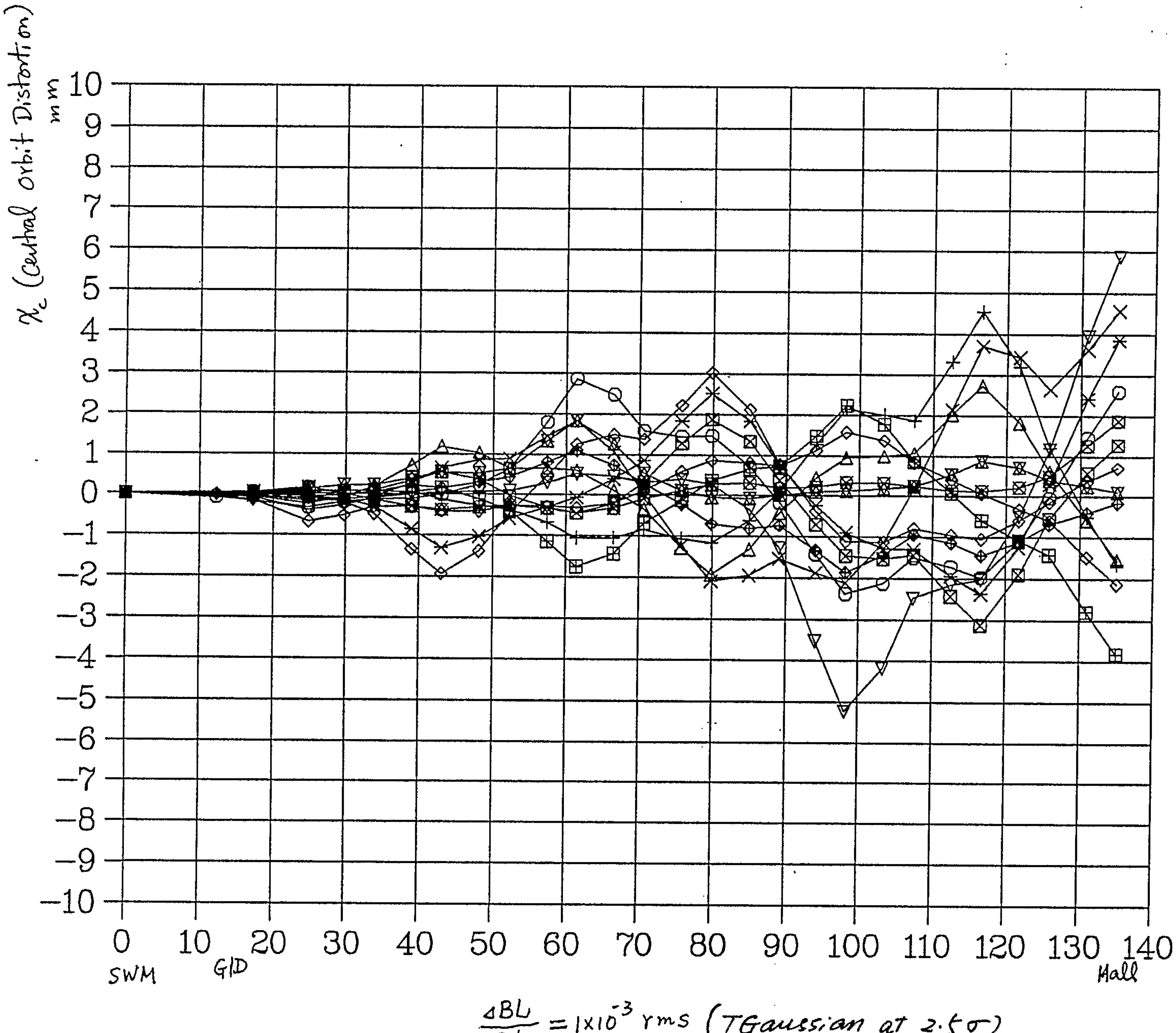
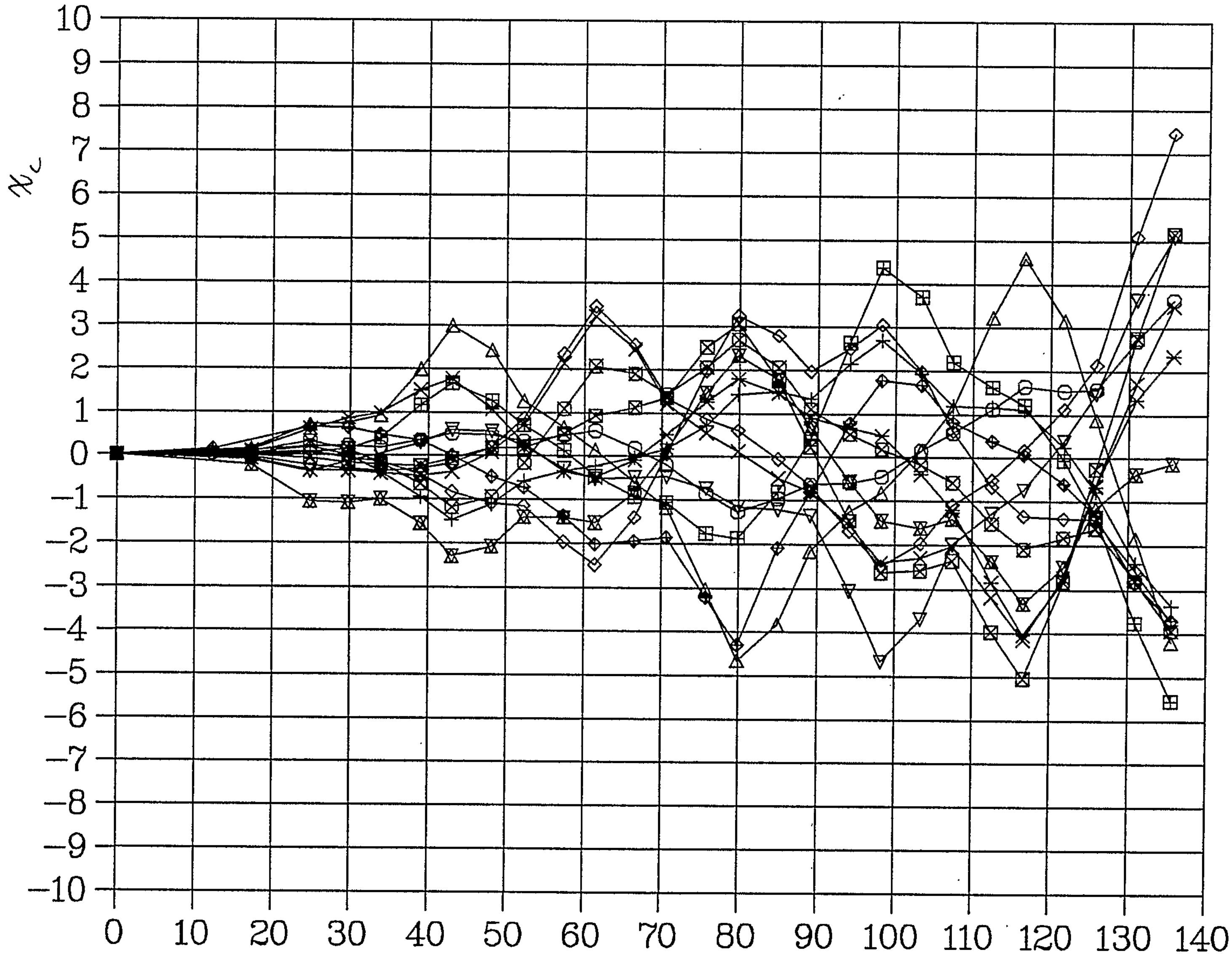


Figure 4. Central Orbit Distortion Induced by Random Field Errors



$DX = 0.25\,mm$, rms; $\frac{\Delta BL}{BL} = 10^{-3}$ rms (TGaussian at 2.5σ)

Figure 5. Central orbit Distortion Induced by Misalignment and Field Error

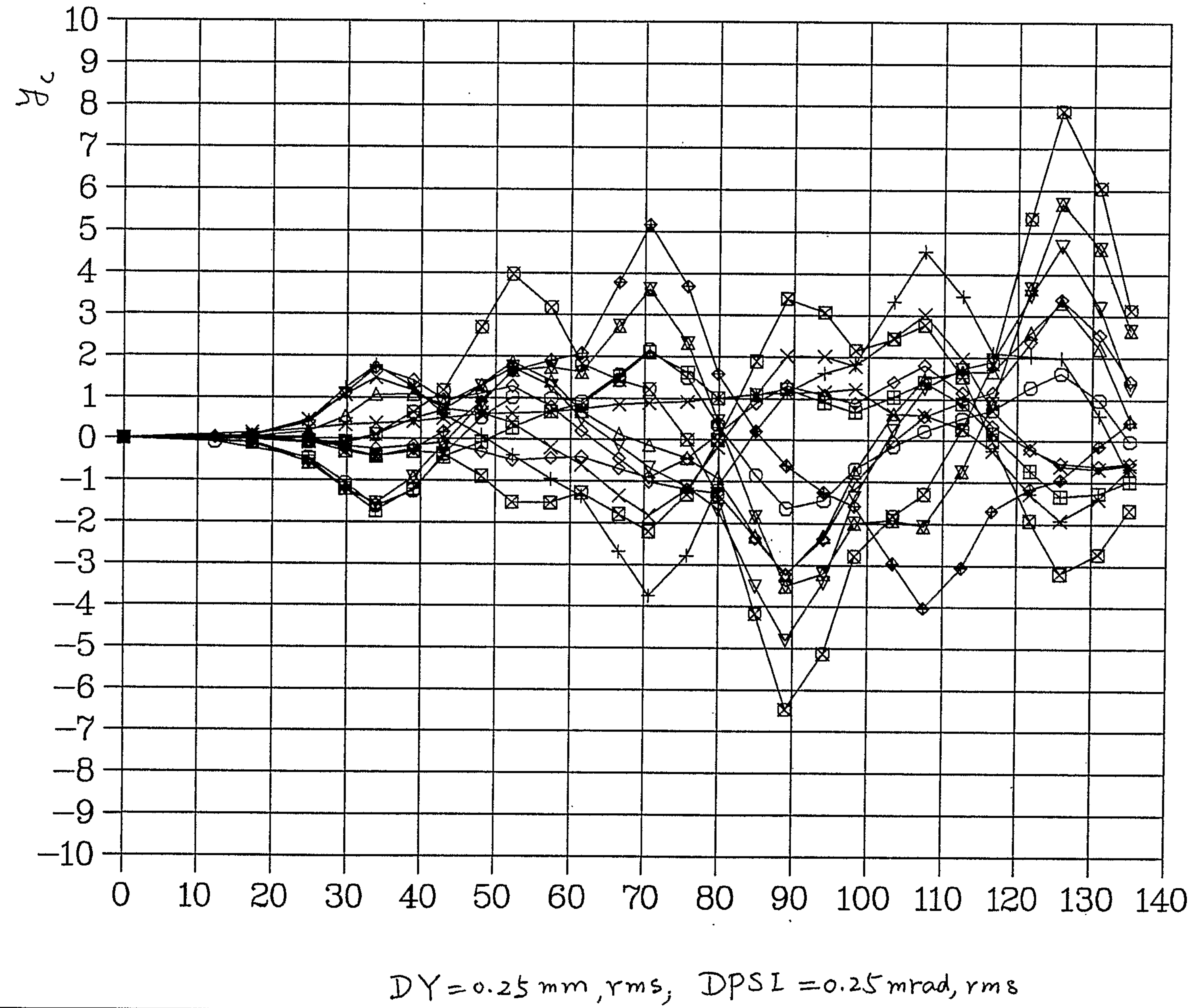


Figure 6. Central Orbit Distortion Induced in Y-Direction

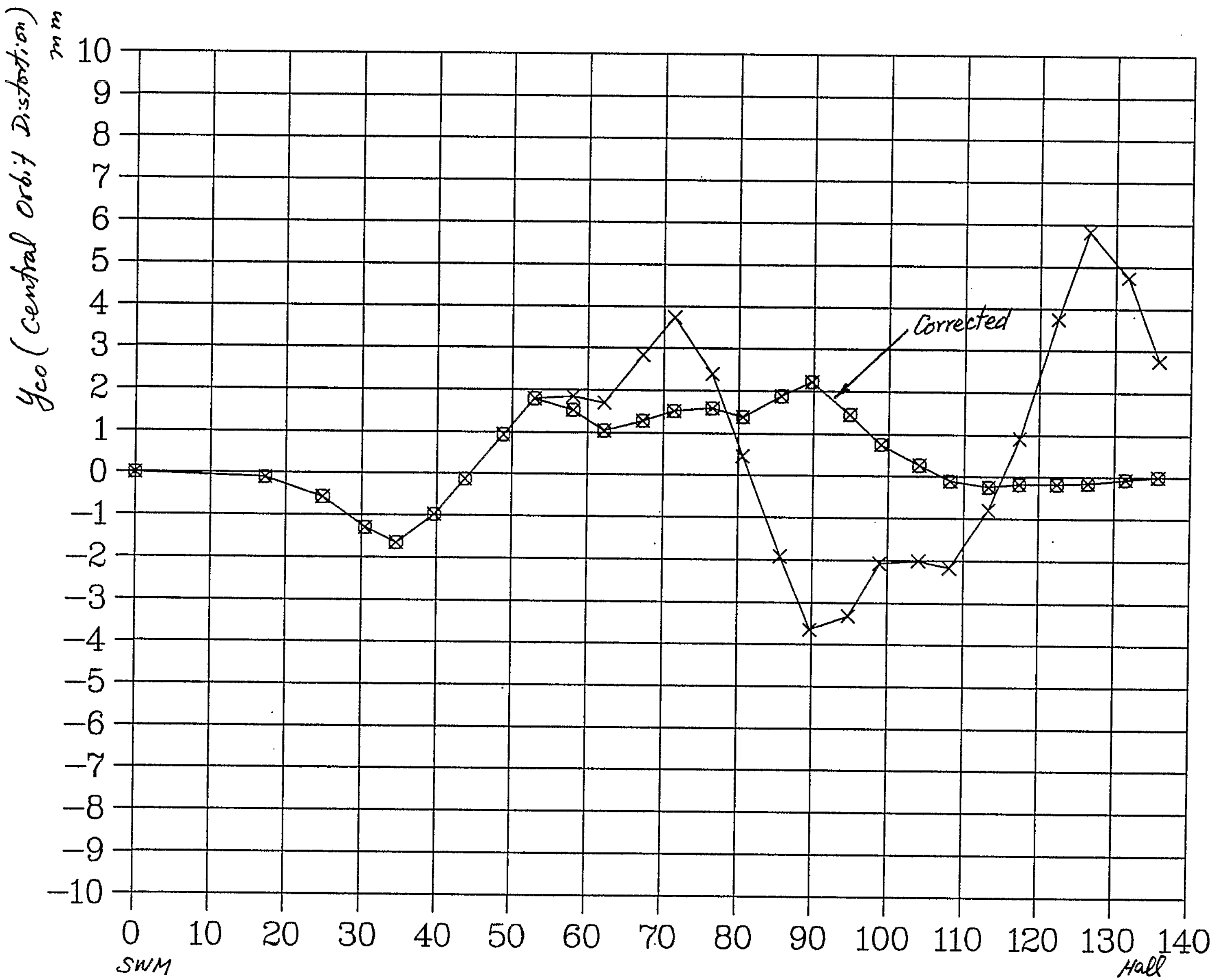


Figure 7. Initial and Corrected Orbit Distortion

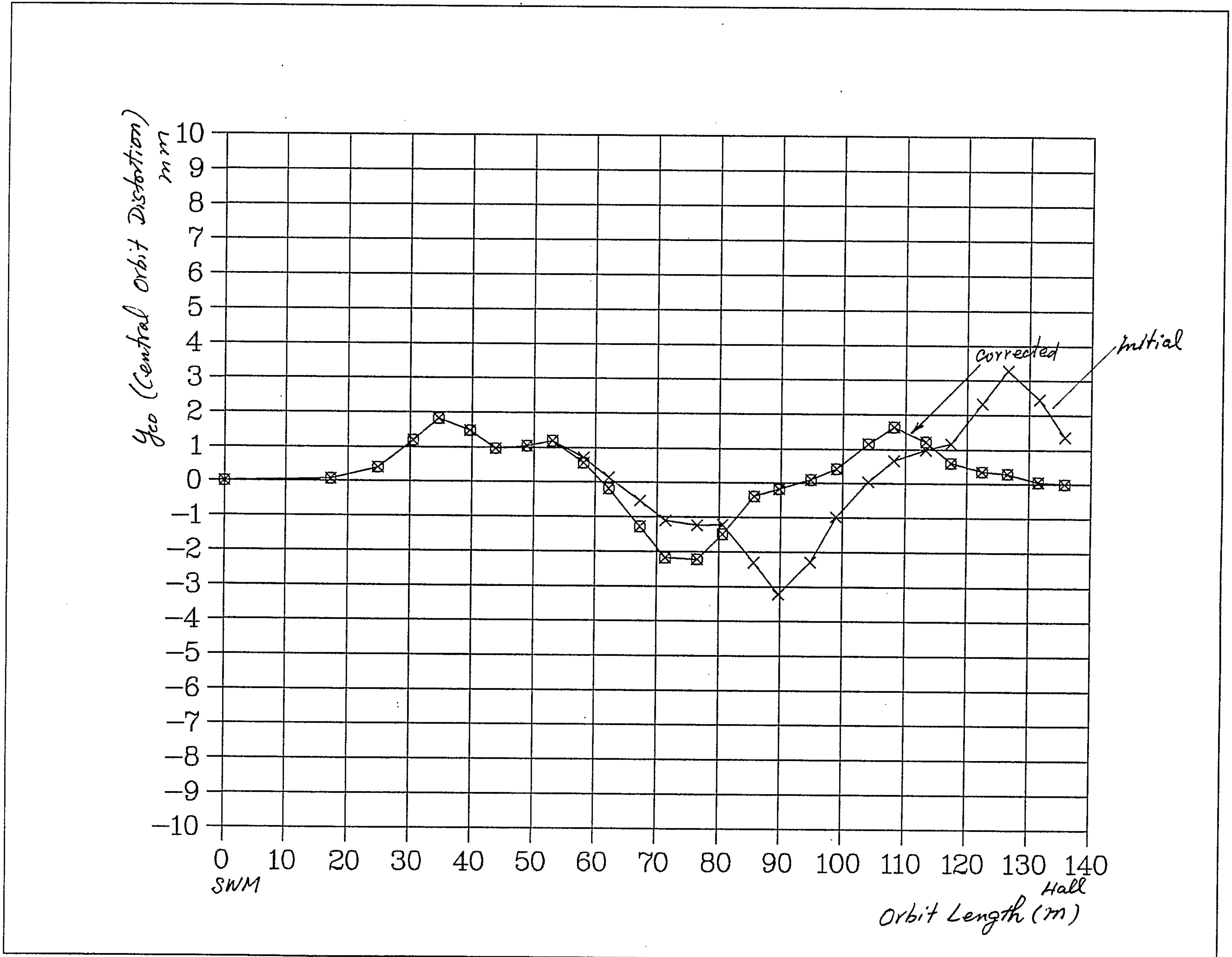


Figure 8. Initial and Corrected Orbit Distortion

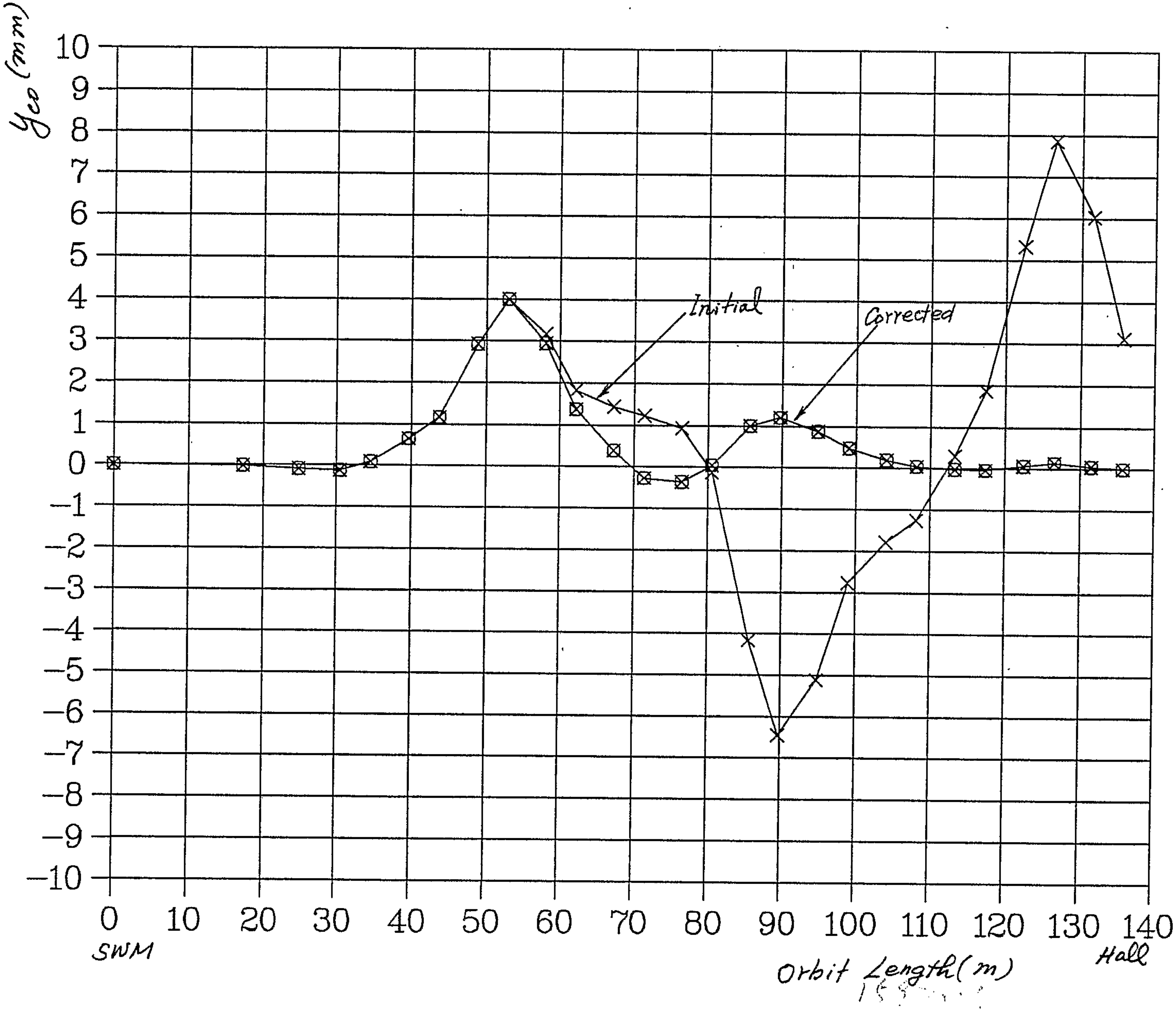


Figure 9. Initial and Corrected Orbit Distortion

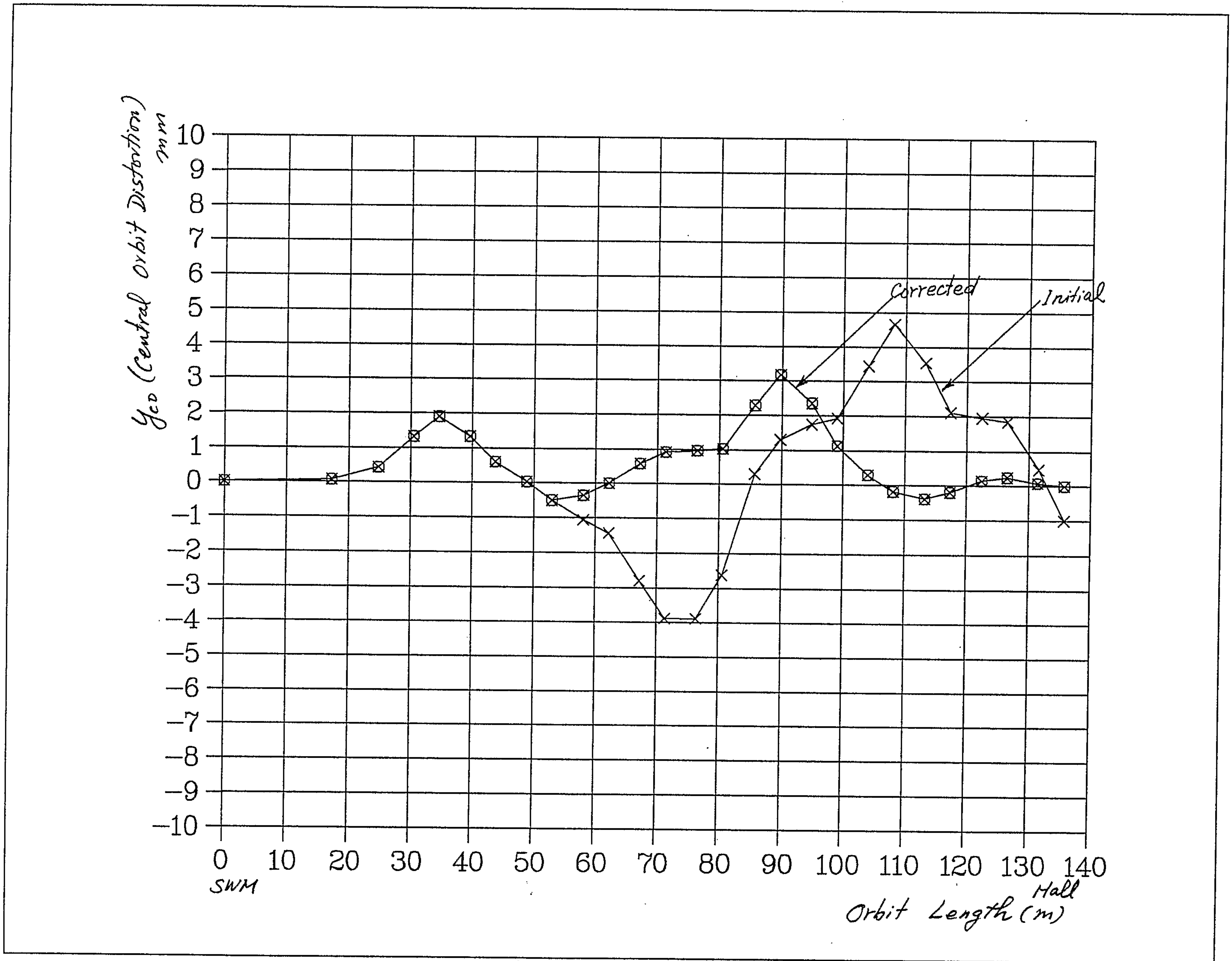


Figure 10. Initial and Corrected Orbit Distortion

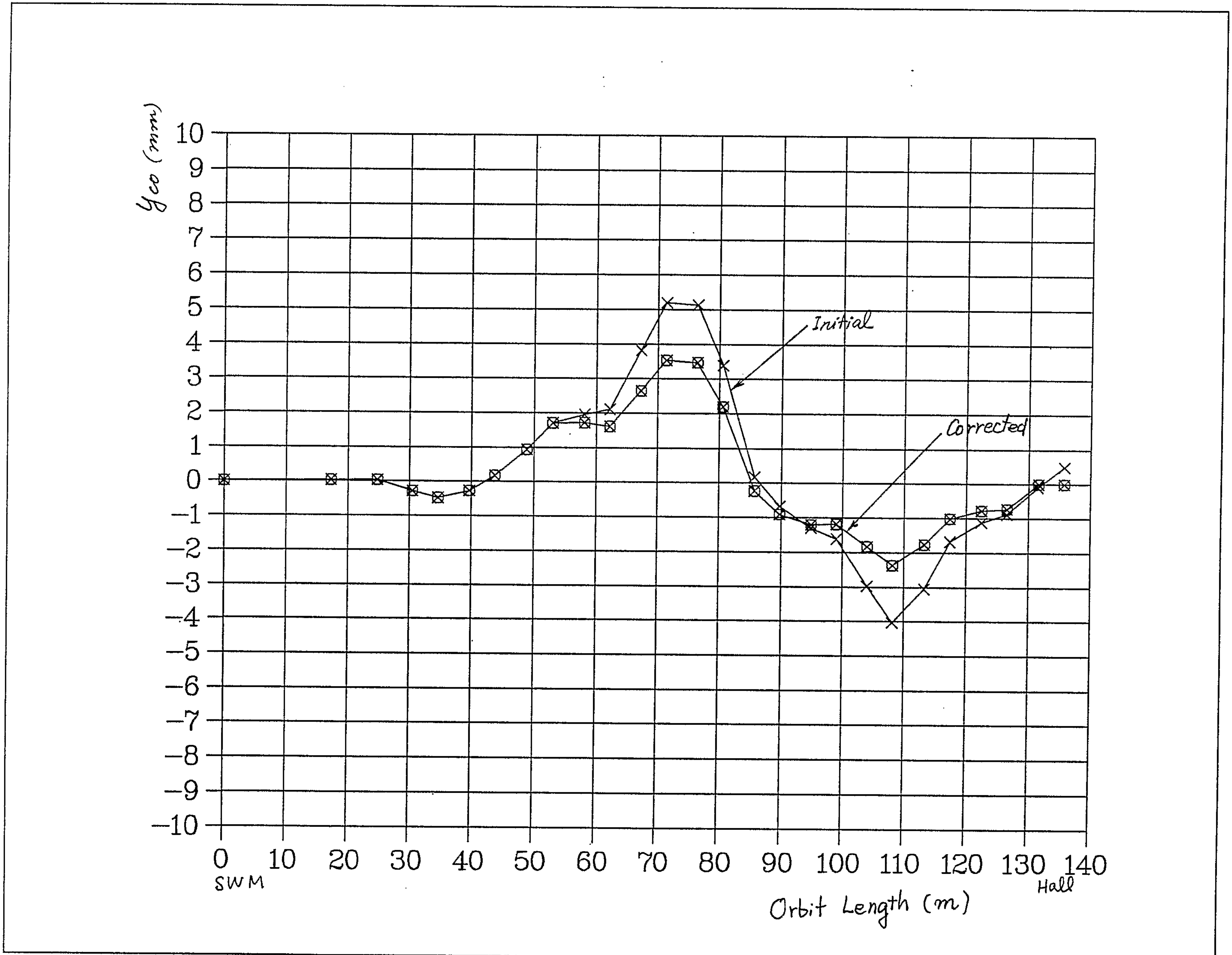


Figure 11. Initial and Corrected Orbit Distortion

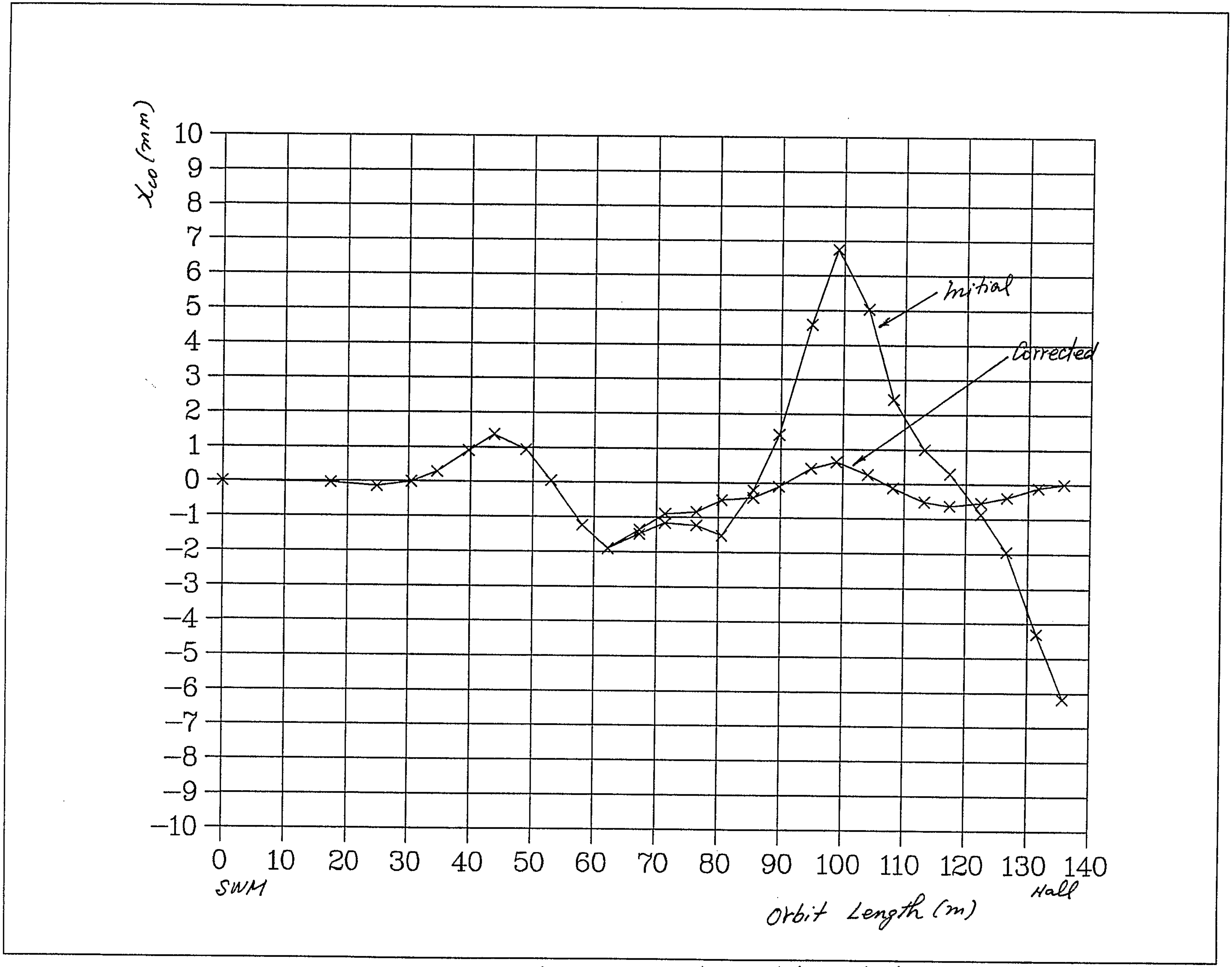


Figure 12. Initial and Corrected Orbit Distortion

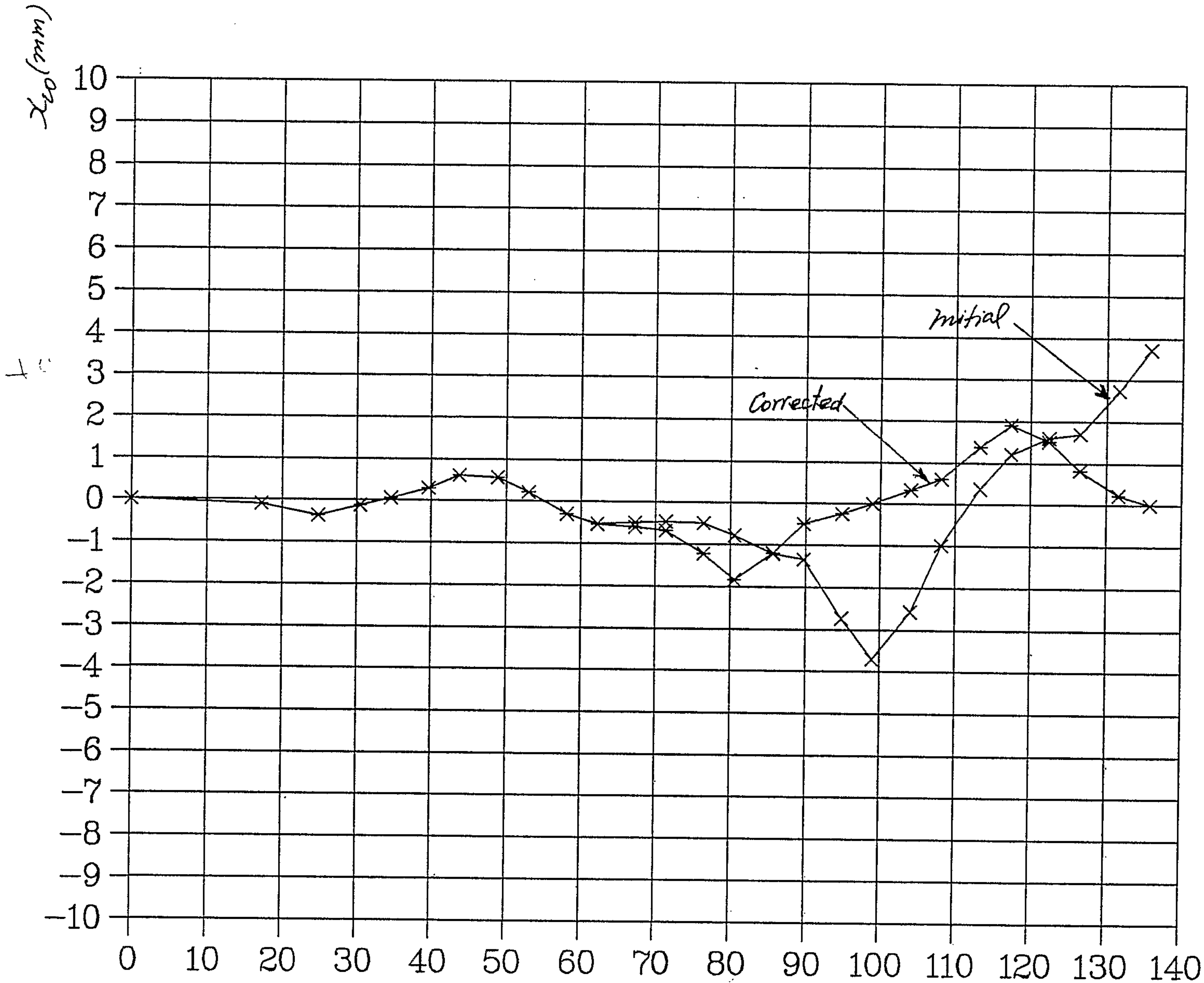


Figure 13. The Initial and Corrected Orbit Distortion

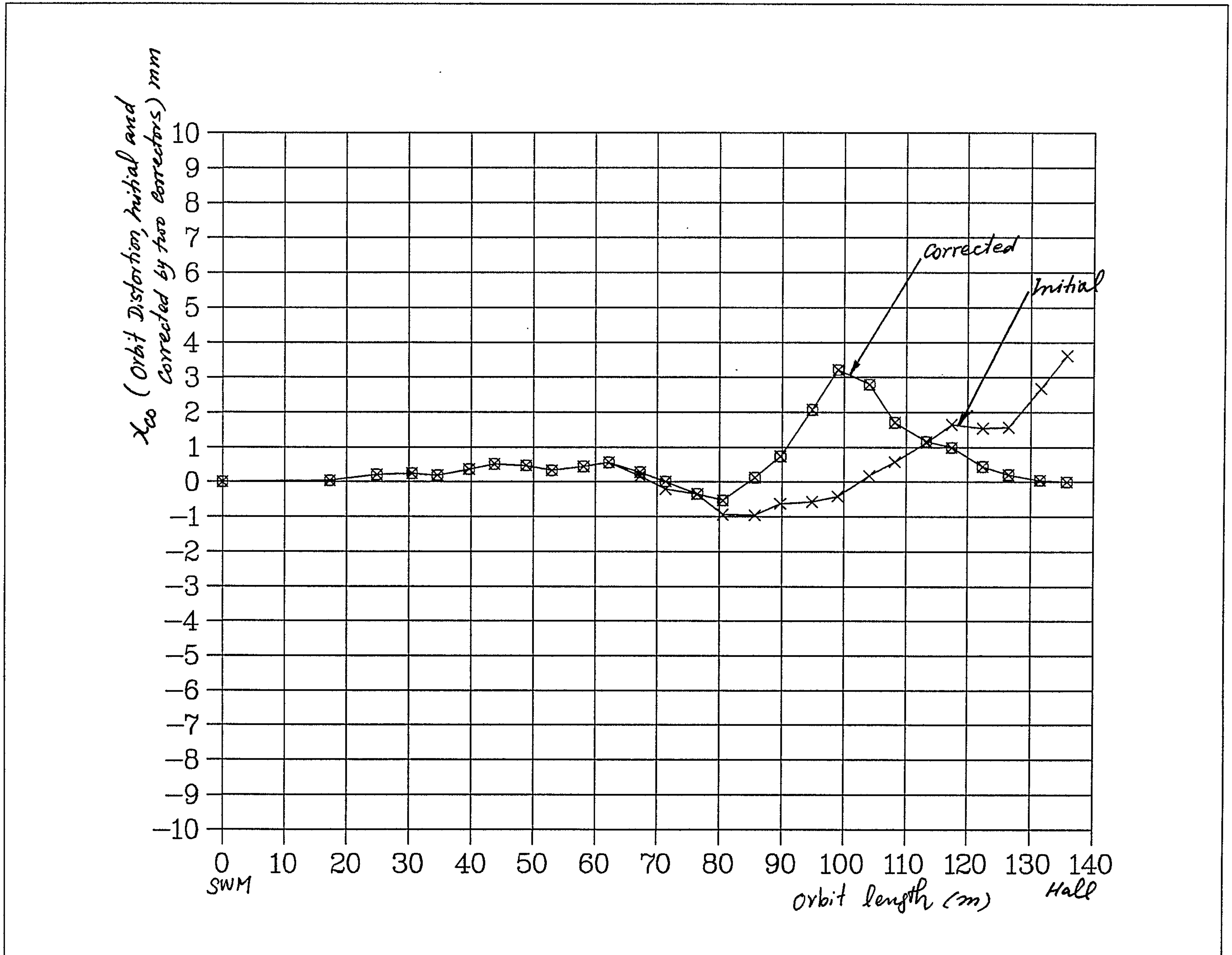


Figure 14 The Initial and Corrected Orbit Distortion

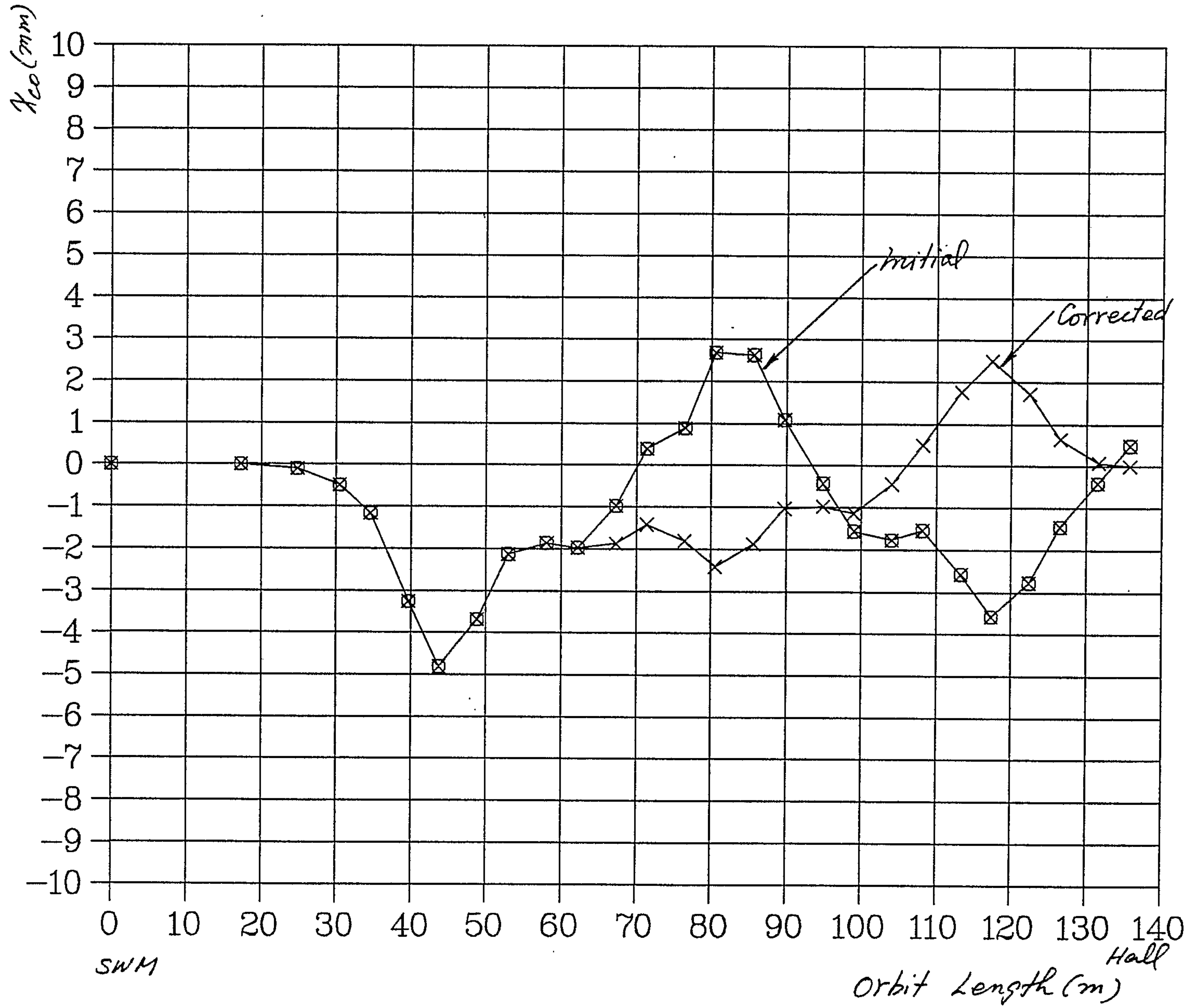


Figure 15. The Initial and Corrected Orbit Distortion

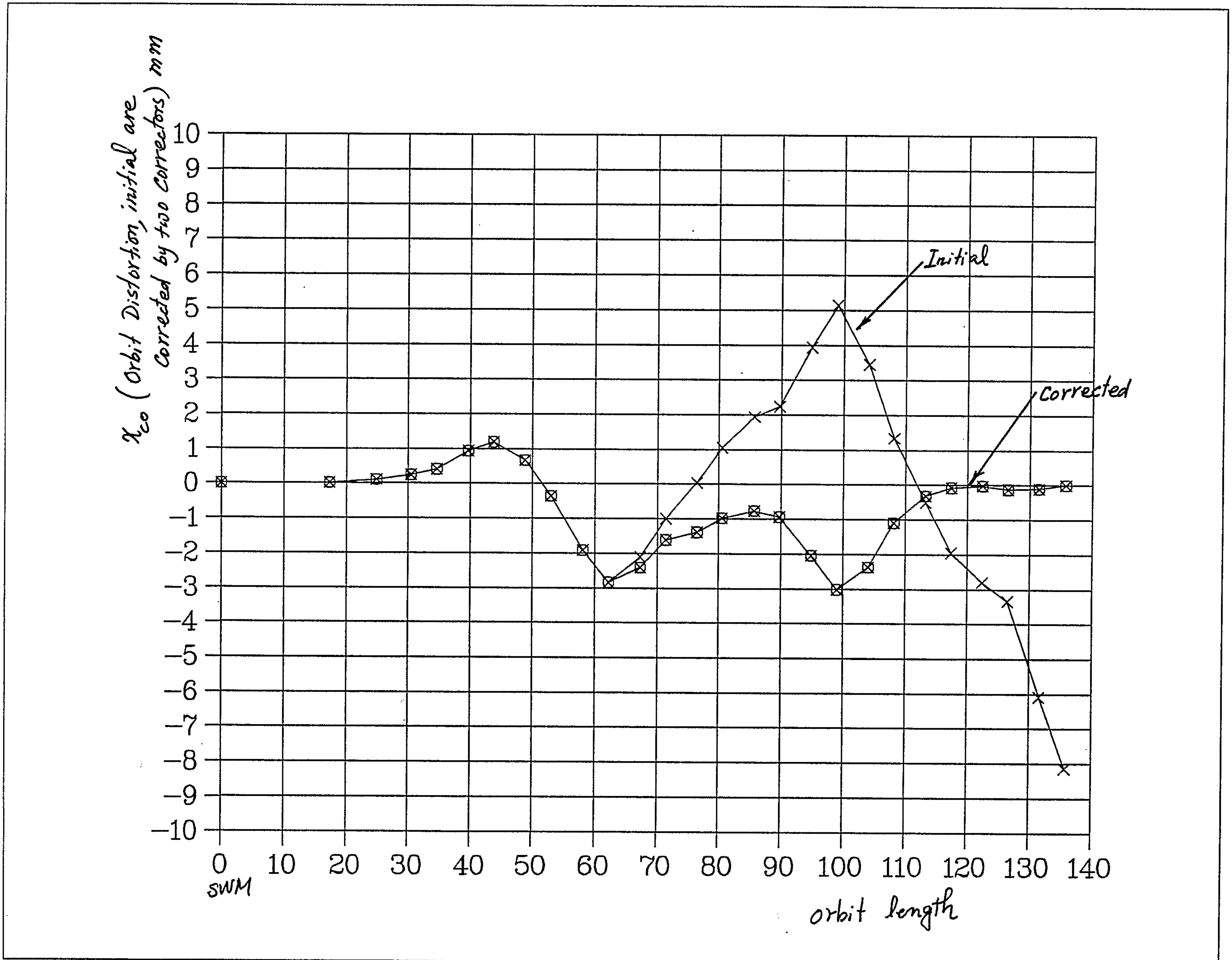


Figure 16 Initial and Corrected Orbit Distortion

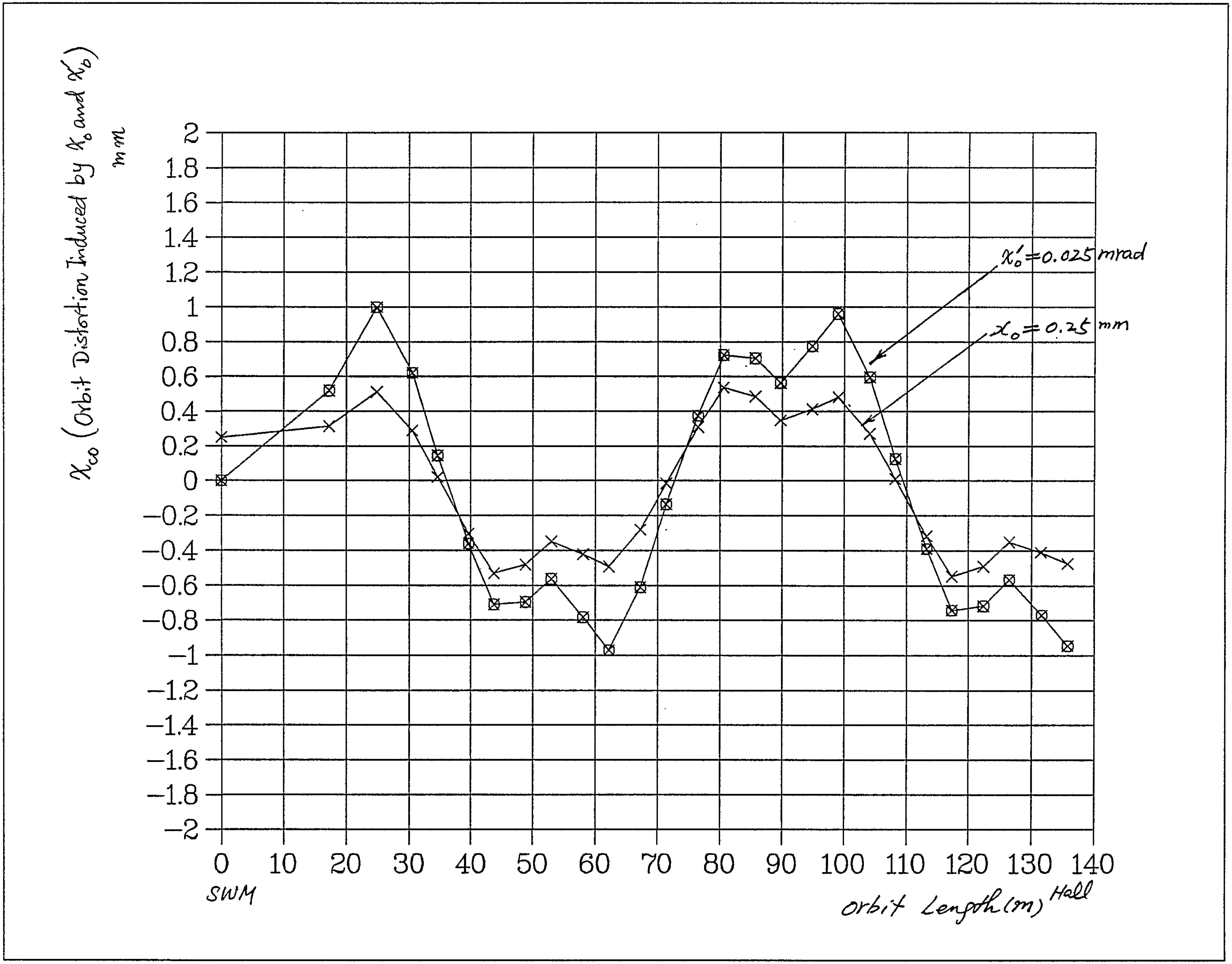


Figure 17. Orbit Distortion Induced by x_0 and x_0'

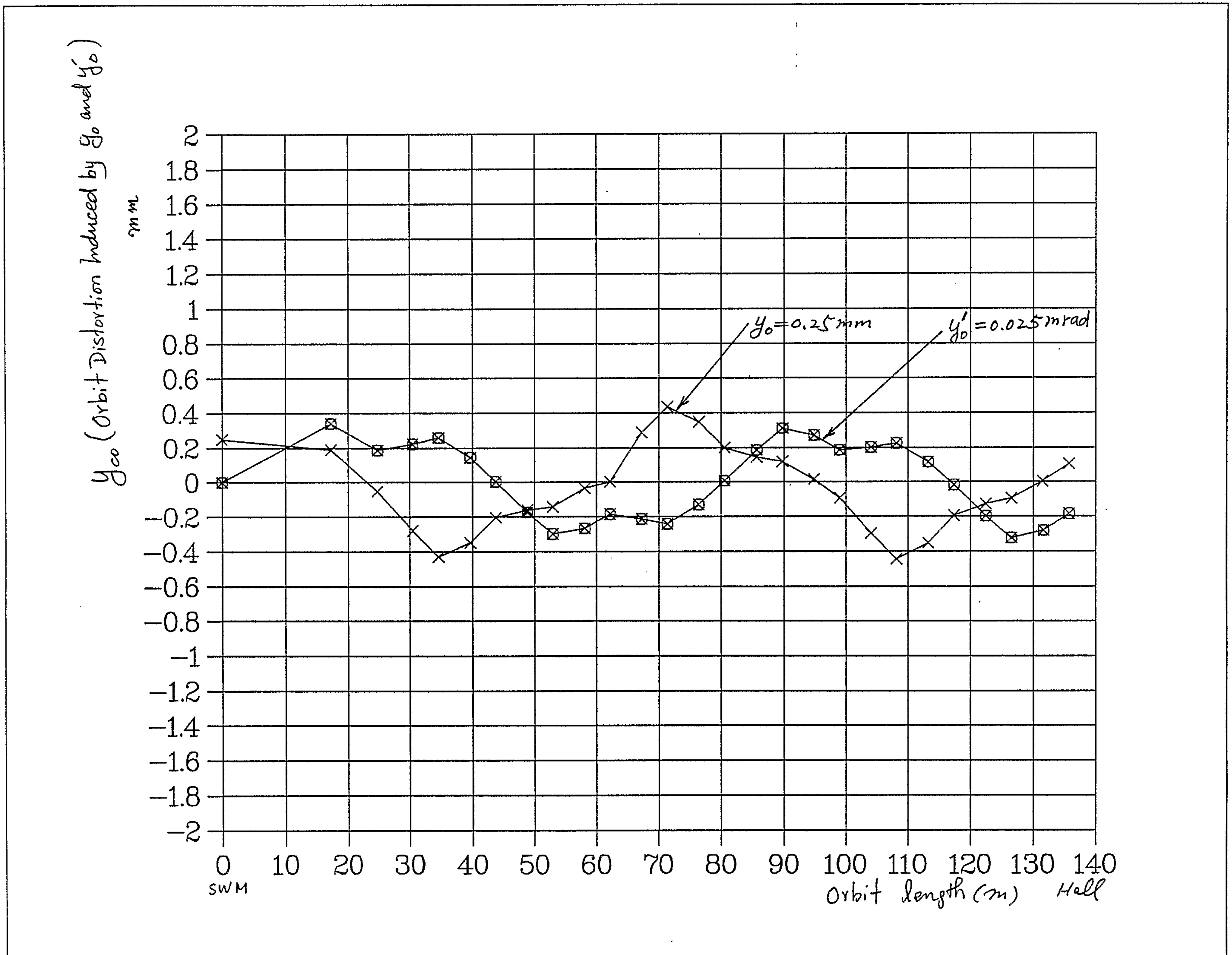


Figure 18. Orbit Distortion Induced by y_0 and y_0'