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Computer tracking of RF phase noise effects on the RHIC beam

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The RHIC rf system needs to hold the colliding beams for 10 hours in relatively small rf buckets. The rf phase noise the beam sees will gradually increase the beam longitudinal emittance and heavy beam loss can occur over a long period. It is therefore necessary to investigate the rf phase noise tolerance.

Much of the theoretical work has been done by G. Dome[1] and D. Boussard[2] of CERN. Some has been done by E. Raka[3] specifically for the RHIC and more studies are currently underway[4].

Computer tracking of rf noise effects is attractive as the numerical calculations carry out difficult non-linear problems efficiently. However, due to the small amplitude of noise and long integration time involved, interpretation of the results depends heavily on confidence of the calculation precision and the calculation models involved. This paper intends to show that with careful selection of parameters, both the above concern can be addressed without complicated analytical exercises.

PRECISION VERIFICATION

Rigorous analytical approach of mathematical error analysis can be extremely involved and unpractical to accomplish for real world problems. As physicists, however, we can convince ourselves that if the calculation conserves certain conservation laws, it should be correct. In our tracking, we check 1): the action variable J that is known to be constant for non-varying Hamiltonians and 2): the exact periodicity of motion in the phase space at the end of calculation, i.e., the particle motion contour in the phase space must not just preserve the area, but also need be tracing out the exact contour of particles that has identical initial conditions.

NOISE MODEL

While it is well known that random number generators make "white noise", the analysis of noise spectrum amplitude requires a good understanding of normalization between Fourier transform pairs and deviation from the real "white noise" due to use of finite computing resources.

In our calculation, a pseudo random number is generated every one tenth of the synchrotron oscillation period. We label this sampling period as t_s. This noise value is then added to the rf phase. The actual noise spectrum is therefore a series of "random steps" along the time axis. We introduce two mathematical functions to represent such a noise source.

The first one is called "sampling" function that is defined as:

$$s(t)=\sum_{k=-\infty}^{\infty} \delta(t-kt_s) \qquad \textbf{(1)}$$

The second function is a "gate" function that is defined as:

$$g(t)=1\Big|_{|t|\le\frac{t_s}{2}} \quad , \qquad g(t)=0\Big|_{|t|>\frac{t_s}{2}} \tag{2}$$

The series of random steps can be viewed as a convolution of the sampling function weighed by the random number and the gate function:

$$\begin{aligned} f(t) &= [f_{ran}(t)s(t)] * g(t) \\ &= \int_{-\infty}^{\infty} f_{ran}(\tau)s(\tau)g(t-\tau)d\tau \end{aligned} \tag{3}$$

The Fourier transform of the random function multiplied by the sampling function is an infinite series of summation of white noises that are shifted by the sampling frequency:

$$\begin{aligned} FT\,\{f_{ran}(t)s(t)\} &= \frac{1}{t_s}F_{ran}(j\omega) * \sum_{n=-\infty}^{\infty}\delta(\omega-\frac{2\pi n}{t_s}) \\ &= \frac{1}{t_s}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}F(j\eta)\delta(\omega-\eta-\frac{2\pi n}{t_s})d\eta \\ &= \frac{1}{t_s}\sum_{n=-\infty}^{\infty}F[j(\omega-\frac{2\pi n}{t_s})] \end{aligned} \tag{4}$$

The spectrum therefore still has equal amplitude everywhere. The Fourier transform of the gate function is a $\sin(x)/x$ shaped (Sync) function that has a zero at $x=\pi$:

$$\begin{aligned} FT\,[g(t)] &= \int_{-\frac{t_s}{2}}^{\frac{t_s}{2}} e^{-i\omega t}dt \\ &= t_s\,\frac{\sin(\frac{\omega t_s}{2})}{\frac{\omega t_s}{2}} \end{aligned} \tag{5}$$

Because our noise function is a convolution of sampled random number with the gate function, its spectrum is a simple multiplication of the two Fourier transforms. The actual noise spectrum is no longer "white", instead, it is a continuous spectrum with sin(x)/x shaped amplitude modulation along the frequency axis.

Now that we know the amplitude shape of the noise spectrum, we also need to know the absolute amplitude of the noise spectrum. A simpler approach is again to use physical concepts --- the law of energy conservation.

Since the amplitude squared is related to energy, let's examine the mean squared value of our noise function. Because our random noise generator has a uniform distribution of values in a given interval, the mean squared value should be identical to that of a linear function as the time sequence in mean squared value calculation does not matter:

$$
\begin{aligned}
<f_n^2(t)> &= \frac{1}{2T}\int_{-T}^{T} f_n^2(t)dt \\
&= \frac{1}{2T}\int_{-T}^{T} t^2 dt = \frac{T^2}{3}
\end{aligned}
\tag{6}
$$

Therefore, our noise function's mean squared value is simply its peak deviation divided by three. This power is distributed in the spectrum according to the [sin(x)/x]^2 shaped curve. Up to now we only used unity mathematical functions to represent both the random noise and its spectrum. To find out the amplitude relation between the random series and our noise spectrum, we let A_s^2 be the amplitude of the noise spectrum squared:

$$A_s^2\int_0^{\infty}\{\frac{\sin(\frac{\omega t_s}{2})}{\frac{\omega t_s}{2}}\}^2\,d\omega$$
$$=2\frac{A_s^2}{t_s}\frac{\pi}{2}=\frac{A_r^2}{3} \qquad (7)$$

where A_r is the peak deviation from mean of our random numbers. Thus:

$$A_s^2=\frac{A_r^2\,t_s}{3\pi} \qquad (8)$$

Therefore knowing the peak deviation of our random noise series, we can get the value of absolute power density at a given frequency.

A TRACKING EXAMPLE

In the following tracking example, particles are tracked for 12 x 10^6 synchrotron oscillation periods. For the RHIC colliding operation, the synchrotron oscillation is about 300 Hz so the equivalent real time is about 11.1 hours.

To verify the computation accuracy, no noise was introduced at first and three particles were tracked for 12 x 10^6 synchrotron oscillation periods. Any deviation from the original contour conflicts with known physical conservation and calculation algorithm and integration steps were modified till all three particles are on the original contour after 12 x 10^6 synchrotron oscillation periods. The result is shown in Fig.1.

Next 100 particles were placed uniformly in the phase space that is equivalent to 90% of the bucket height (the inner circle in Fig.2), as we expect this would be the bucket size for the RHIC colliding mode operation.

A random series with 0.0005 radian peak deviation and changes every 1/10th of synchrotron oscillation period was introduced. According to Equations 5 through 8, the phase noise has a continuous sync function shaped spectrum and the first zero is at 10 times the synchrotron oscillation frequency. According to G. Dome[1], in a synchrotron with sinusoidal rf voltage the diffusion of the rf phase space is caused by spectrum power densities at the synchrotron oscillation frequency and its multiples. According to the normalization relation given in Eq. 8, the peak values of phase noise at these frequencies are:

freq.in f_s	1	2	3	4	5	6	7	8	9	10
$A_{peak}/10^{-5}$ in rad/$\sqrt{2\pi Hz}$	7.17	6.81	6.25	5.51	4.64	3.68	2.69	1.71	0.81	0.00

Table 1. Peak phase noise value A_{peak} at multiples of synchrotron oscillation frequency.

Fig.2 shows the phase noise effects after 12 x 10^6 synchrotron oscillation periods. For this given noise, about 40 percent of the beam is lost (shaken out of the separatrix). About 40 percent of the original particles remain in the original phase space area --- the area enclosed by the inner circle.

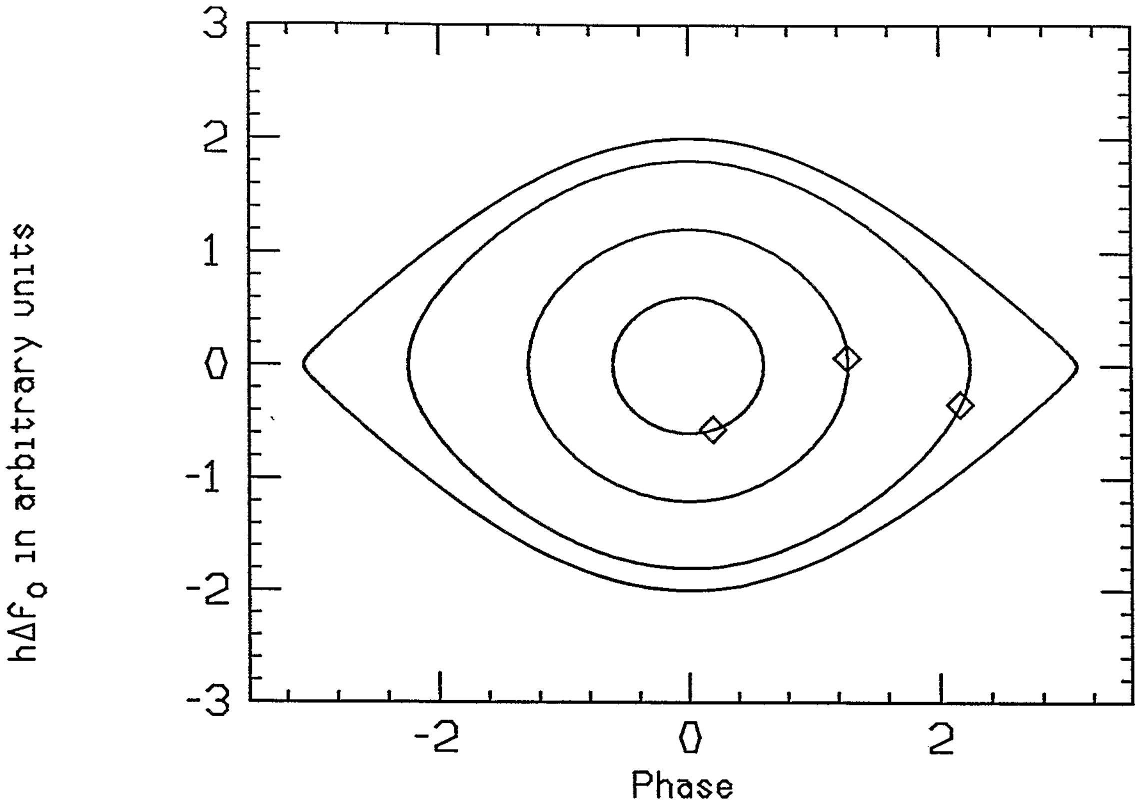

Figure 1. Accuracy check shows that all three particles are still on the original contour at the end of calculation.

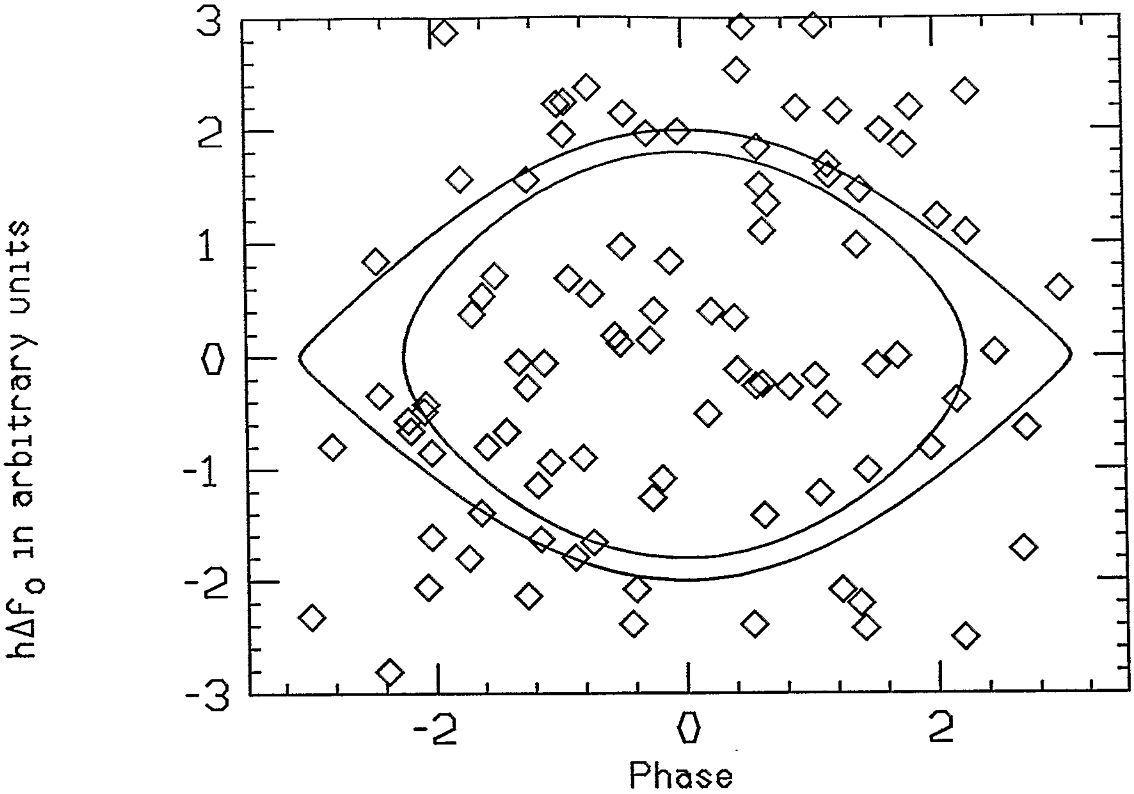

Figure 2. 100 particles were uniformly placed in the phase space enclosed by the inner circle. After 12 x 10^6 synchrotron oscillation with phase noise, about 40 percent of the particles were lost.

PRACTICAL CONSIDERATIONS

Now that we have obtained these numbers, a natural question is how small this noise is in terms of real world obtainability. To evaluate that, let's expand the small amplitude phase modulation to the first order. For an rf signal of amplitude *A* that has a sinusoidal phase modulation of frequency Ω and amplitude *m*, we have:

$$f(t)_{rf}=A\sin[\omega_o t+m\ \sin(\Omega t)]$$
$$=A\{\ \sin(\omega_o t)\cos[m\sin(\Omega t)]+\cos(\omega_o t)\sin[m\sin(\Omega t)]\} \tag{9}$$

where ω_o is the rf carrier frequency. Expand the above in the first order of m, we get:

$$f(t)_{rf}=A\sin\omega_o t+\frac{mA}{2}\sin(\omega_o-\Omega)t-\frac{mA}{2}\sin(\omega_o+\Omega)t \tag{10}$$

Therefore, a small simple harmonic phase modulation shows up as two sidebands located symmetrically about the carrier with offset Ω. The amplitude is $mA/2$ for each sideband. Because the value A is associated with the carrier amplitude, the value normalized to A gives half the amount of true phase modulation amplitude.

For practical measurements, a spectrum analyzer is used. Because of the large dynamic range involved, logarithmic vertical scale is usually used. A quantity normalized to the carrier frequency is called "dbc" --- or db's below carrier, defined as:

$$dbc\ =\ 20\ \log\ \frac{m}{2} \tag{11}$$

Most rf spectrum analyzers have a finite frequency resolution of 1 to 10 Hz. So the actual amplitude seen is the integration of the frequency interval. Substitute the above numbers into our equation and assume a 1 Hz resolution, we have, at the synchrotron oscillation frequency:

$$dbc\ =\ 20\ \log\ \frac{\sqrt{(7.17\ \times\ \sqrt{\frac{2\pi}{\mathrm{Hz}}}\ )^2\ \times\ 1\ Hz}}{2}\ =\ -80.9 \tag{12}$$

Considering the commercially available synthesized source of $20,000 range can generally have -80 dbc with 100 Hz offset, this is not a particularly stringent specification. For RHIC, the planned operation mode would have beam-rf phase lock. Rf source noise can be worse than this and the ultimate noise the beam sees within the bandwidth of the beam phase feedback loop[2] will depend on the phase discriminator. These issues will be discussed in future technotes.

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