BNL-101794-2014-TECH
AD/RHIC/RD/12;BNL-101794-2013-IR

Beam Position Monitor for RHIC

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September 1988

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U.S. Department of Energy
USDOE Office of Science (SC)

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Introduction

It has been proposed to use beam position monitors (BPM's), that are based on button electrodes (small capacitive pickup electrodes) in the cold sections of RHIC, mainly because they represent the ultimate in mechanical simplicity. Such electrodes represent signal sources with purely capacitive source impedances. I showed in a previous note that one may expect signal levels of the order of 10V per button, but also that the button capacitance is only of the order of 5pF. The cable that connects each button to its associated electronics loads it with its characteristic impedance, probably less than 250Ω, for signal pulses that are shorter than twice the time delay along its length $\tau_d = \ell\sqrt{LC}$ (where ℓ is the cable length and L and C its inductance and capacitance per unit length). This applies also for all other signals if the cable is characteristically terminated. If it is not, it acts for long pulses as a capacitance ℓC parallel to the terminating impedance, and it introduces also a switching transient at each change in signal level. The transient has a time structure on the scale of the delay time τ_d. The loading with the characteristic impedance introduces a time constant $\tau = C_{button} Z_o \approx$ 1nsec in the basic circuit. This results in signal voltages that are proportional to frequency in the frequency range up to 500MHz and constant beyond, the voltage of 10V quoted above applies only to this latter range. The rms lengths of the bunches circulating in RHIC will be ≈1m at injection, and ≈30cm when the experimenters are taking data, corresponding to typical frequencies of 50MHz and 160MHz. It is not easy to increase the critical time constant by the factor of ten one would like to see. $Z_o > 250\Omega$ seems impractical, while increasing both the button capacitance and its coupling to the beam (necessary to preserve signal levels) with a factor of ten simultaneously requires buttons that are physically so large, that they can no longer be regarded as small perturbations in a smooth vacuum pipe. An alternative is to keep the cable so short that the switching transients due to improper termination become small compared to the bunch signal. Then the button is loaded principally with the high input impedance of the electronics and the cable capacitance. This requires an electrical cable length which is rather less than half the rms lengths of the bunches to be measured, e.g., ℓ<10cm. This is too short to bridge the distance from a button to the outer surface of the dewar in which it is mounted, and probably short enough to introduce an inacceptable heatleak (notice that each BPM requires two cables and that there are some 250 BPM's of this type per ring). The cable can be made short if the electronics is mounted directly onto, or in the immediate vicinity of a button; this solution, if at all possible, is extremely unattractive because it implies that the electronics would have to operate at very low temperatures, mounted well out of reach, deep inside the dewars. Another solution exploits the fact that these buttons have no function but to serve as BPM's and that the longitudinal charge distribution in a bunch cannot change noticeably during a single revolution. BPM's in the ring respond therefore to the same charge distribution and their outputs are directly comparable if their electrical characteristics are the same. The calibration necessary to obtain absolute numbers (i.e., transverse beam locations with respect to the BPM centers in mm) can be done with information obtained from a single, more accessible, more elaborate diagnostic station mounted somewhere in one of the warm sections of the ring. It seems possible to produce the difference signal that contains the beam position information directly from the button signals by means of passive analogue equipment mounted close to the buttons; the difference signal can be carried away by a single, properly terminated cable that is not restrained in its length. We describe this arrangement in some more detail.

Model

Taking advantage of the slowness of the synchrotron motion I assume that the beam may be adequately approximated by a travelling wave with a lineal charge density $\lambda(s,t) = \lambda(s-vt)$, where s, periodic with the ring's circumference C, represents position along that circumference, $v=\beta c$ the synchronous velocity and t the time. A very similar image charge distribution is induced on the inside surface of the vacuum pipe, however, longitudinal structure in the beam on the scale of $\leq w$, where $w=\rho/\gamma<5\text{mm}$, with ρ the radius of the vacuum pipe (ρ=36.45mm in RHIC) and $\gamma=(1-\beta^2)^{-1/2}>8$ the Lorentz factor, is lost in its image. Disregarding this, one may approximate the total image charge on a ring of width ℓ located at s_o with $Q_{image}=-\int\lambda(s_o+x-vt)dx$, where $-0.5\ell\leq x\leq 0.5\ell$, and the net image current to it with

$$i_{image}= v[\lambda(s_o+0.5\ell-vt)-\lambda(s_o-0.5\ell-vt)] \approx d\lambda(t)/dt\cdot\ell\big|_{s=so},$$

if ℓ is small compared to the scale of the longitudinal structure in the beam. The circumferential image current density distribution depends upon the transverse charge density distribution in the beam relative to the vacuum pipe's cross section. The button electrodes are formed by insulating small sections of the vacuum pipe wall from its main body, e.g., as indicated in Fig 1, and the fraction of the total image current flowing to each electrode is determined by the beam/pipe configuration. Any deviation of the beam from the symmetry plane defined by the button/pipe geometry generates a difference in the button currents that can be used as a measure for that displacement. The image currents charge the capacitances of the buttons to the vacuumpipe, potential differences due to current differences can be measured via the transformer shown in Fig 1. The secundary of this transformer is loaded with the properly terminated cable to the electronics. The center of the primary is connected to the vacuum pipe via a small resistor, whose function is to control the longitudinal shunt impedance of the BPM by damping common modes. It does not affect the difference modes since it carries no difference current if the capacitance/inductance distribution is truly symmetric. The situation for the principal difference mode is crudely represented by the equivalent circuit in Fig 2, which shows a current source driving a resonant circuit. The current source produces a current that is proportional to the image current and to a scaling factor that depends on the beam/button/ pipe geometry. The capacitance of the resonant circuit represents the total capacitance between the two buttons, the inductance is that of the transformer primary and the damping resistor represents the transformed characteristic impedance of the cable leading to the electronics. The circuit is perhaps overly simplified, since it disregards the obvious capacitive coupling between the transformer's primary and secundary as well as its leakage inductance and the inductances of the leads between it and the buttons. However, a primary task is to choose the resonant frequency and the Q of this resonant circuit so, that the output voltage waveform of a bunch is maximized, while preventing perturbation by the previous bunch. I assume that the bunches have a $\cos^2$ distribution with lengths between 17nsec at injection and 6nsec while the experimenters are taking data. The smallest possible distance between bunch centers, as determined by the rf frequency of 342*78kHz, is 37nsec.

Calculations.

Using Laplace transforms I find that the response of the circuit in Fig. 2 to a $\cos^2$ pulse may be written as the sum of two waves, identical in magnitude and shape but of opposite polarities, the second wave delayed relative to the first one by the length of the pulse. The first wave may be described by:

$V_1(t) = 0$ for $t < 0$, while for $t \geq 0$:

$$V_1(t) = \frac{\Delta\Lambda}{C}\frac{1}{\{(\omega_o/\omega)^2-1\}^2+(1/\omega\tau)^2}\Bigg[\{(\omega_o/\omega)^2-1\}\cos(\omega t)+(1/\omega\tau)\sin(\omega t)-$$

$$-e^{-t/(2\tau)}\cdot\left\{\{(\omega_o/\omega)^2-1\}\cos\left[\omega_o t\sqrt{1-1/(2Q)^2}\right]+\frac{(\omega_o/\omega)^2+1}{\sqrt{(2Q)^2-1}}\sin\left[\omega_o t\sqrt{1-1/(2Q)^2}\right]\right\}\Bigg]$$

The second wave is then described by:

$V_2(t) = 0$ for $t < T$, while for $t \geq T$: $V_2(t) = -V_1(t-T)$. The composite wave is given by: $V(t) = V_1(t)+V_2(t)$. In these expressions: $\Delta\Lambda = -(G_1-G_2)\Lambda$, where Λ/ℓ represents the peak lineal charge density in the beam and $G_{1,2}$ the geometric coupling factors between the beam and buttons 1 and 2; $\omega T = 2\pi$, with T the length of the bunch, $\omega_o/2\pi$ the resonant frequency of the resonant circuit formed by the transformer primary and the capacitance C_s across that primary, $\tau = RC_s$ is the time constant of that circuit, with R the transformed characteristic impedance of the cable connected to the transformer's secundary, $C_s=C/2$ with C the capacitance per button, $Q = \omega_o\tau$. For $t < T$ only the first wave is present: it represents the forced oscillation of the system in response to the presence of a bunch. For $t > T$ the bunch has left the area and is no longer coupled to the system. Now the difference wave is of interest: it represents the free oscillation of a damped oscillator in response to initial conditions. Fig 3 shows the behaviour of $V(t)C/\Delta\Lambda$ for fixed bunchlength, thus fixed ω, and different values for ω_o. The first tic mark along the time axis indicates the instant of maximum voltage, the second and third ones indicate the instants that maximum voltage due to a bunch in the second or third bucket would occur. The voltage due to the first bunch adds to that due to the second one, and causes a measurement error when the second bunch's maximum is measured. It can be controled by choice of the Q factor, which was adjusted to make the error 0.01 of the principal maximum one, two or three rf periods after its occurrence. The error oscillates with the phase of the voltage at the critical instant. Therefore I varied that phase by changing ω_o (and Q) by small amounts relative to round values to find a local extremum. Fig 4 shows the response of $V(t)C/\Delta\Lambda$ and the limiting Q value under these constraints as functions of ω_o for the case that the $|$relative error$|$ ≤ 0.01 one rf period after the maximum. It may be seen that there are not very critical optimum values for ω_o and Q, which maximize the response: $\omega_o/2\pi \approx 42.44$MHz, $Q = 0.9772$ yield $\hat{V}C/\Delta\Lambda = -1.208$ for 17nsec bunches that pass every 37nsec, that is, every rf period. Similar optima can be established for the cases that there is a bunch every second or every third rf period. The results of such calculations are tabulated in Table I. The last line of this table gives the response of a system that was optimized for 17 nsec bunches to 4nsec bunches.

Table I

Buckets/bunch	1	2	3
$\omega_o/2\pi$ [MHz]	42.4	40.5	40.4
Q	0.977	1.966	2.969
$\hat{V}C/\Delta\Lambda$ (T_{bunch}: 17nsec)	1.208	1.808	2.100
$\hat{V}C/\Delta\Lambda$ (T_{bunch}: 4nsec)	1.52	1.71	1.77

The ω_o and Q of a BPM as described are fixed at the time of its construction. This means that the ratio $\omega_o/\omega = \nu_o T_{bunch}$ cannot remain optimum during a machine cycle because the bunchlength T_{bunch} decreases from 17nsec at injection to $\approx$ 6nsec later. The response changes accordingly, one might expect it to drop on the basis of Fig 4. However, the charge in the bunch remains preserved while it becomes shorter, thus its peak lineal charge density Λ increases. This effect is taken into account in Fig 5, which shows the response of a fixed BPM to bunches of equal charge but different lenghts. Fig 6 shows the actual waveforms for T_{bunch} = 17, 8 and 4nsec. It may be seen that the absolute maximum in the response curve, which occurs at the second peak for long bunches, shifts to the first one for the shorter ones.

A BPM with satisfactory behaviour for a bunch in every bucket requires apparently $\omega_o/2\pi \approx 42.4$MHz and Q $\leq$ 0.9772. Assuming a capacitance per button of 10pF, the inductance of the transformer primary becomes 2.8μH and the transformed cable impedance $Z_{0transformed}$ 733Ω. The transformer could consist of windings on a torus of hfr ferrite of the type used in TV antenna circuitry. The primary winding, whose ends are connected to the buttons, has a centertap which is grounded, possibly via a resistor; the secundary is connected to the (long) cable that leads to, and is properly terminated by, the electronics outside the dewar. Dimensionally the transformer can be very small, perhaps 2cm over all, and the connections between buttons and transformer can be short, no more than 10cm each, if it is placed according to Fig 1. A ferrite torus with a major radius of 7.5mm and a minor cross section of 5×5mm^2 would require $\mu_r n^2$= 4200 for L = 2.8μH, thus for μ_r=100 the number of primary turns would be n = $(42)^{1/2}$ = 6.5, for n = 10 one would need μ_r= 42. There are obviously many possibilities.

The G Factor

Provided that the dimensions of this BPM are small compared to the typical longitudinal dimensions of the bunch one may assume a purely electrostatic, 2-D model for the calculation of the coupling factors. Using an image technique I find that a charge filament λ parallel to, but at a distance $r\rho$, $|r|<1$, from the axis of a metallic circular cylindrical pipe of radius ρ induces an image charge density $\sigma(\varphi)$ on the inner surface of that pipe given by:

$$\sigma/\lambda = \frac{1}{2\pi}\cdot\frac{1-r^2}{1+r^2}\cdot\frac{1}{1-a\cdot\cos(\varphi-\vartheta)}, \qquad a = 2r/(1+r^2)$$

ϑ represents the angle of the radius to the filament relative to some reference direction, φ the angle of the radius to the testpoint on the pipe wall. Assuming that the buttons are sections of pipewall that stretch between $\pm\varphi_b$ and $\pi\pm\varphi_b$, $\varphi_b\leq\pi/2$, I obtain for their charges per unit length:

$$\frac{1}{\lambda}\cdot\frac{dQ}{d\ell} = 1/\pi\cdot\mathrm{atan}\left(\frac{1+r}{1-r}\tan\frac{\varphi-\vartheta}{2}\right)\Bigg|_{-\varphi_b}^{\varphi_b} \quad \text{and} \quad \frac{1}{\lambda}\cdot\frac{dQ}{d\ell} = 1/\pi\cdot\mathrm{atan}\left(\frac{1+r}{1-r}\tan\frac{\varphi-\vartheta}{2}\right)\Bigg|_{\pi-\varphi_b}^{\pi+\varphi_b}$$

$$= -1/\pi\cdot\mathrm{atan}\left(\frac{1-r}{1+r}\tan\frac{\varphi-\vartheta}{2}\right)\Bigg|_{-\varphi_b}^{\varphi_b}$$

The sum of these two functions is odd in r, but otherwise a complicated relation in r, φ_b and ϑ. This situation is much simplified if the shape of the buttons satisfies $\ell(\varphi)=\ell\cdot\cos(\varphi)$, $|\varphi|\leq\pi/2$, resp $\ell(\varphi)=-\ell\cdot\cos(\varphi)$, $\pi/2\leq\varphi\leq3\pi/2$. In that case the charge difference becomes:

$$\Delta Q/\lambda = \ell x/\rho, \qquad (x = r\rho\cdot\cos\vartheta, \text{ valid for } \ell\gg\rho/\gamma)$$

This expression is linear in displacement along the reference direction and independent of displacements orthogonal to it, very desirable characteristics for a BPM. This result is not surprising in view of our knowledge of $\cos\vartheta$ magnets. It follows that for this case $G2-G1 = \ell x/\rho$.

Shuntimpedance

There is another reason why it is desirable to avoid 'rectangular' buttons. The space between the sheetlike button and the locally modified vacuumpipe wall forms a cavity with mode wavelengths that are related to the dimensions of the cavity. These dimensions must be of the order of a few cm in order to provide sufficient sensitivity. Many of these eigen modes will be lossy, because their frequencies lie above the cut-off frequency of the vacuumpipe, but the lowest order ones could conceivably induce beam instability, particularly longitudinally. The mode pattern as well as shunt impedances for the individual modes are affected by the shape of the buttons, any deviation from rectangularity should reduce the impedances. The stipulation that $\ell(\vartheta)=\ell\cdot\cos\vartheta$ still leaves some freedom in the choice of button shape, this may be exploited as a meaens of affecting the shuntimpedance.

Performance

We have seen that the signal produced by this arrangement is more or less proportional to the electrical charge in the bunch, rather than to the lineal charge density, at least in the interval of bunch lengths considered. It is therefore convenient to define as sensitivity the peakvoltage per unit charge and per unit displacement. Taking as unit of charge the charge of 10^{11} protons, the nominal number of proton equivalents per bunch, I find for a $\cos\vartheta$ type BPM:

$$S = \frac{V}{Q_b x} = \frac{2}{C}\cdot\frac{\ell}{\ell_{br}}\cdot\frac{1}{\rho}\cdot F_{nb} 1.6022\cdot 10^{-8} \approx 0.41 [V Q_b^{-1} mm^{-1}]$$

where C is the capacitance of a single button, ℓ the button length, ℓ_{br} the length of the reference bunch (the quantity that determines the choice of ω_o), and ρ the inner radius of the circular cylindrical vacuumpipe and buttons. F_{nb} is the scaling factor that adjusts the sensitivity to the number of buckets per bunch, the factor 2 stems from the $\cos^2$ line charge density distribution in the bunch and Q_b represents the number of protoncharges per bunch in units of 10^{11}. Assuming that a ℓ = 2cm and C = 10pF represents a realizable combination for ρ = 37.5mm, that $\ell_{br} \approx$ 5m, and that we have to be prepared for the case that every bucket is filled, I find the number quoted. Note that the sensitivity is proportional to ℓ/C, it is thus important to keep the stray capacitance, i.e. the capacitance of feedthroughs and connections to the transformer small compared to the capacitances of the buttons. The voltage sensitivity is relatively independent of button size if this is realized because a button's capacitance tends to increase with its length. However, it is common knowledge that signal power is of more interest than signal voltage. One shows easily, that, at constant ω_o and $Q = R\omega_o C$, the power developed in the load increases proportionally to C, even though the voltage across it is constant, provided that the parasitic capacitance is negligeable. This means that, after transformation to the (fixed) input impedance of the signal processor, the signal voltage increases as $C^{1/2}$.

The resolution is determined by the sensitivity and the noise in the system. Regarding that system as a damped harmonic oscillator with a relative 3dB bandwidth of Q, I estimate for the rms noise signal $S_n = (4kTR\Delta\nu)^{1/2} = 22\mu V$. One may therefore expect a rms resolution of the order of $20\mu m$, provided that proper care is taken in the electronic equipment. Changes in the button length affect the noise signal, changing it as $C^{-1/2}$ if $Q = R\omega_o C$ is kept constant. The net result of changes in button length ℓ is then, that the signal voltage at the signal processor changes as $\ell^{1/2}$, while the signal to noise ratio, thus the resolution, changes as ℓ^{-1}. An increase in length will reduce the resonant frequencies of some of the eigen modes, and may have undesirable consequences for the behaviour of the shuntimpedance. The choice of length should be the subject of an experimental investigation, a purely theoretical choice would be difficult and perhaps not very reliable.

Discussion

I described the concept of a Beam Position Monitor that may be suitable for use in RHIC. It is mechanically a fairly simple device that requires only a few cm of longitudinal space and only one cable, which may be either coaxial, a length of twin lead or perhaps just a twisted pair, for connection to its associated electronic equipment. It can be designed to measure individual bunches separately and to provide nearly perfect linearity in one transverse direction with zero response in the other, for beams that are sufficiently relativistic, i.e., have γ large enough. It is adequate for relative measurements but requires calibration information from elsewhere for absolute measurements. This simplifies the design of the BPM and its signal processor. Since there are hundreds of these devices, all of them requiring the same calibration, it seems more economic and convenient to obtain the required information by means of a single diagnostic station, built for that purpose, than to provide each BPM individually with that capability. Its electrical characteristics were studied with the help of a perhaps somewhat over-simplified model; it seems clear that experimental work on a prototype is necessary to confirm present assumptions, estimates and conclusions. The signal produced consists of a damped wavetrain. The shape of that wave is predominantly determined by the BPM and the rms-bunchlength, and, presumably, little affected by the relative longitudinal charge distribution in the bunch. This has to be confirmed, the calculations were done assuming a $\cos^2$ distribution. The BPM signal must be converted into useful information by electronic equipment. This equipment must be capable of measuring the amplitude of the wave as well as its sign: the amplitude is proportional to the displacement of the beam, the sign gives the direction of the displacement. Ideally, the electronics would be gated to measure a particular bunch and contain an a/d converter, so that it produces a numerical result. A 10 bit or 12 bit converter which completes a conversion within 10μsec would be adequate; if it were faster by a factor n one could measure n bunches per revolution rather than one. Such devices are, apparently, available and rather cheap. 10 bits would yield a resolution of 1 in 1024, thus of $\rho/1024 \approx 37\mu m$, if $\rho = 37.5mm$, 12 bits would provide 10μm for the same ρ.

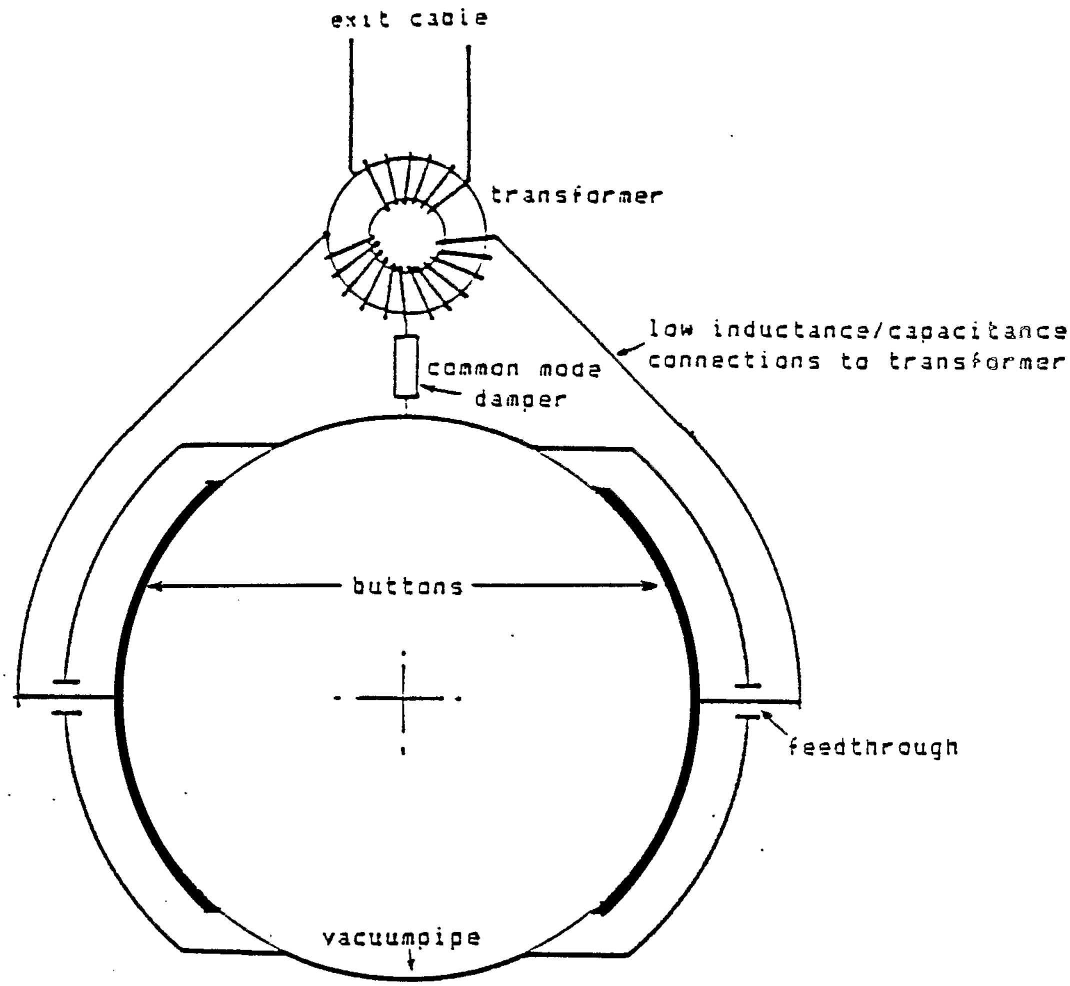


Fig 1. Beam Position Monitor, schematic and symbolic.

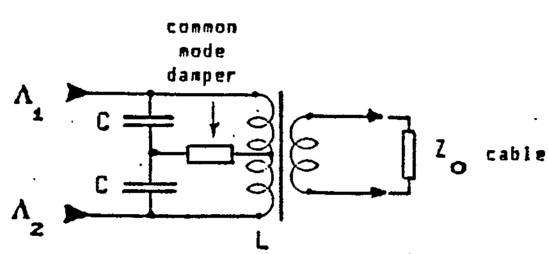


Fig 2. Equivalent circuit for BPM.

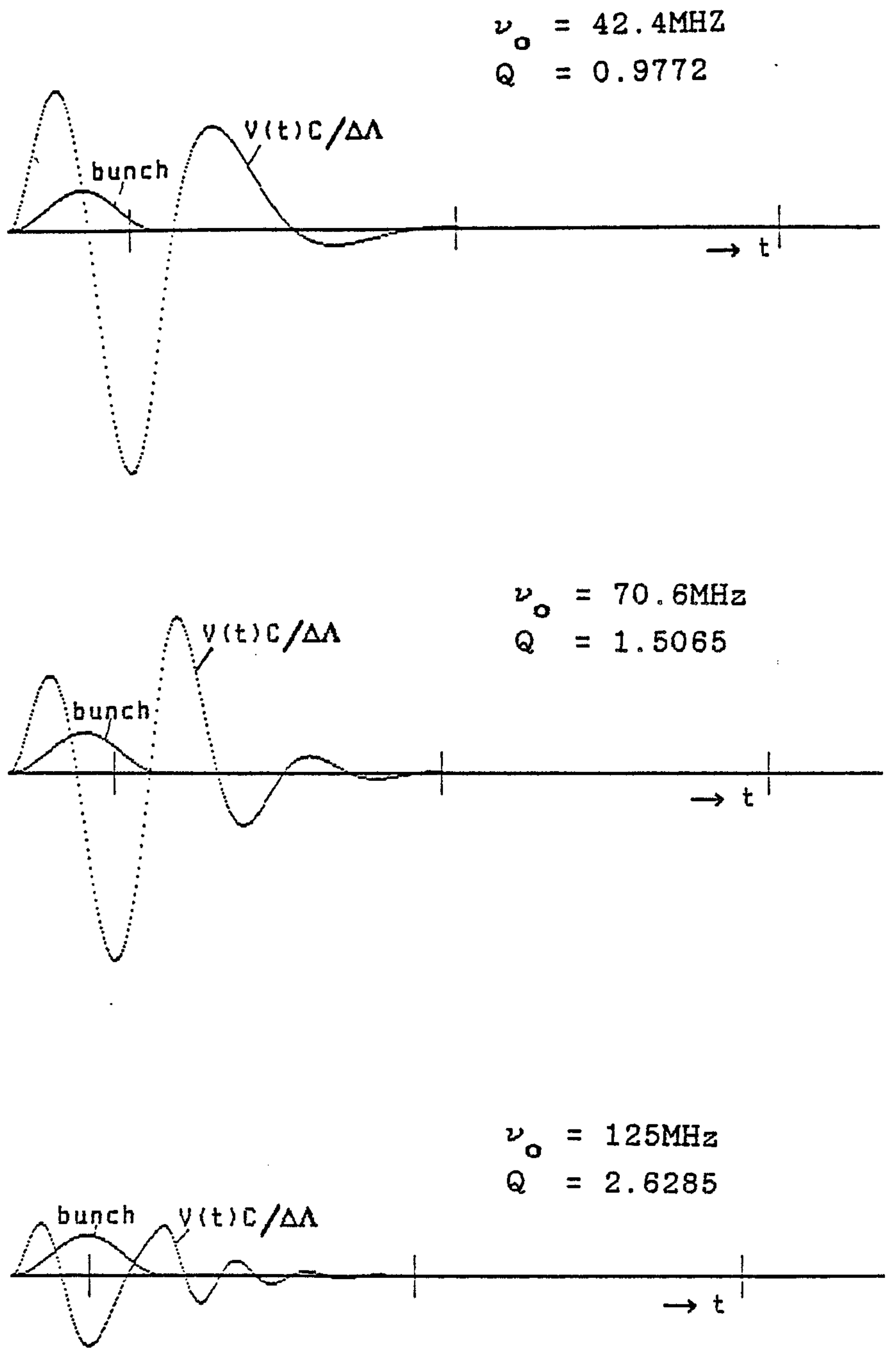
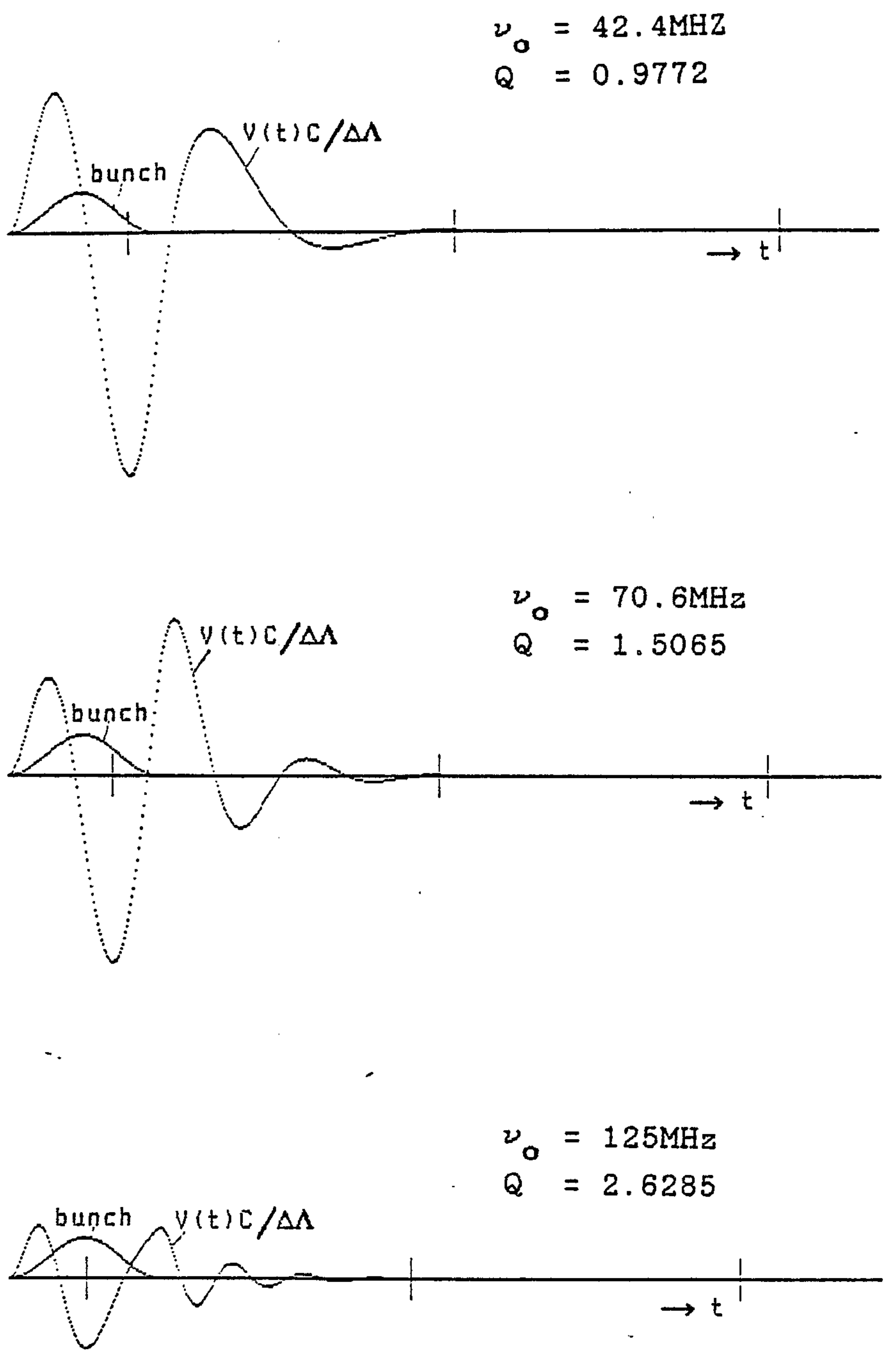


Fig 3. $(V(t)C/\Delta\Lambda)$ as function of resonant frequency for T_{bunch} = 17nsec.



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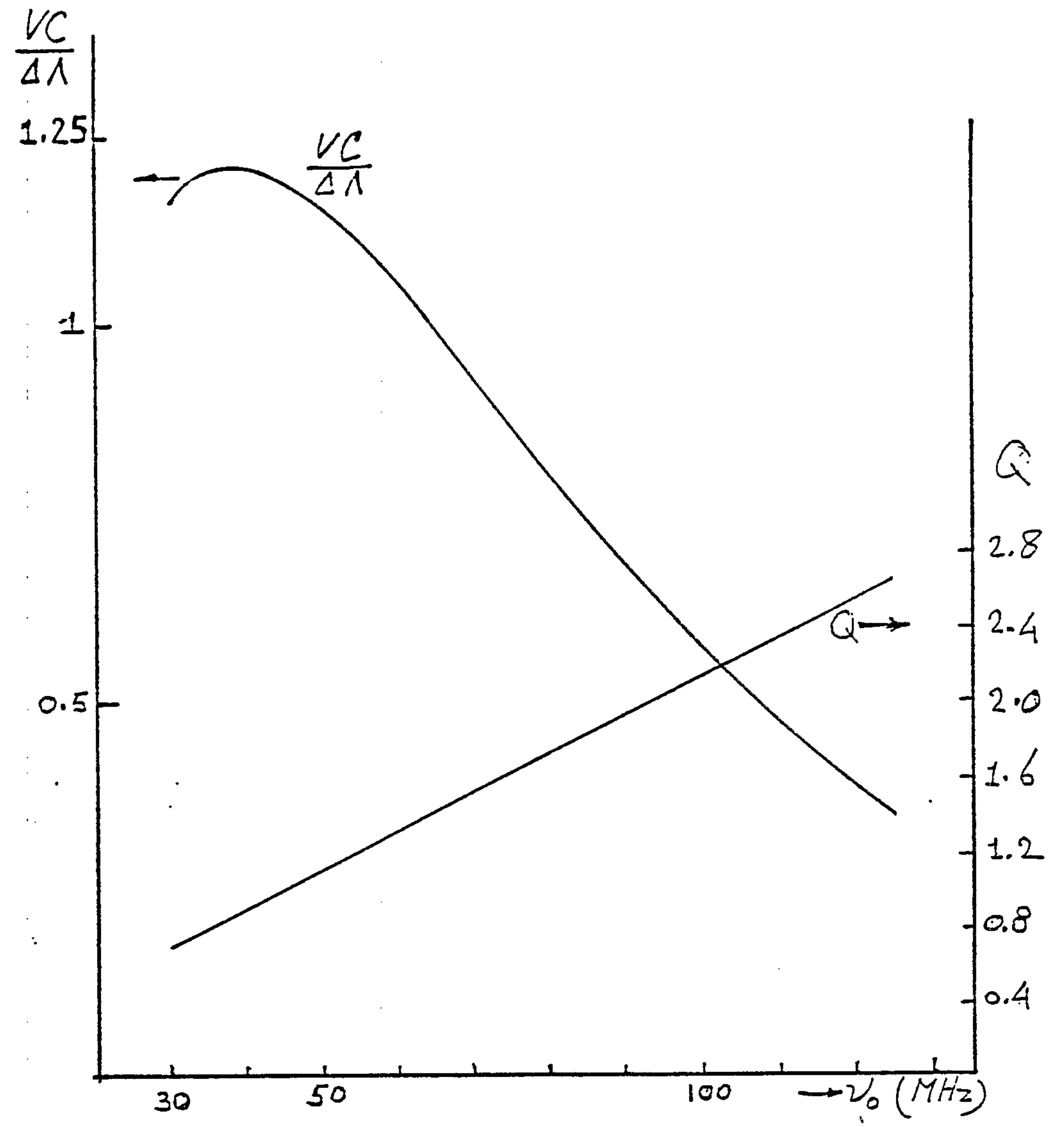

Fig 4. (VC/ΔΛ) and Q as functions of resonant frequency.

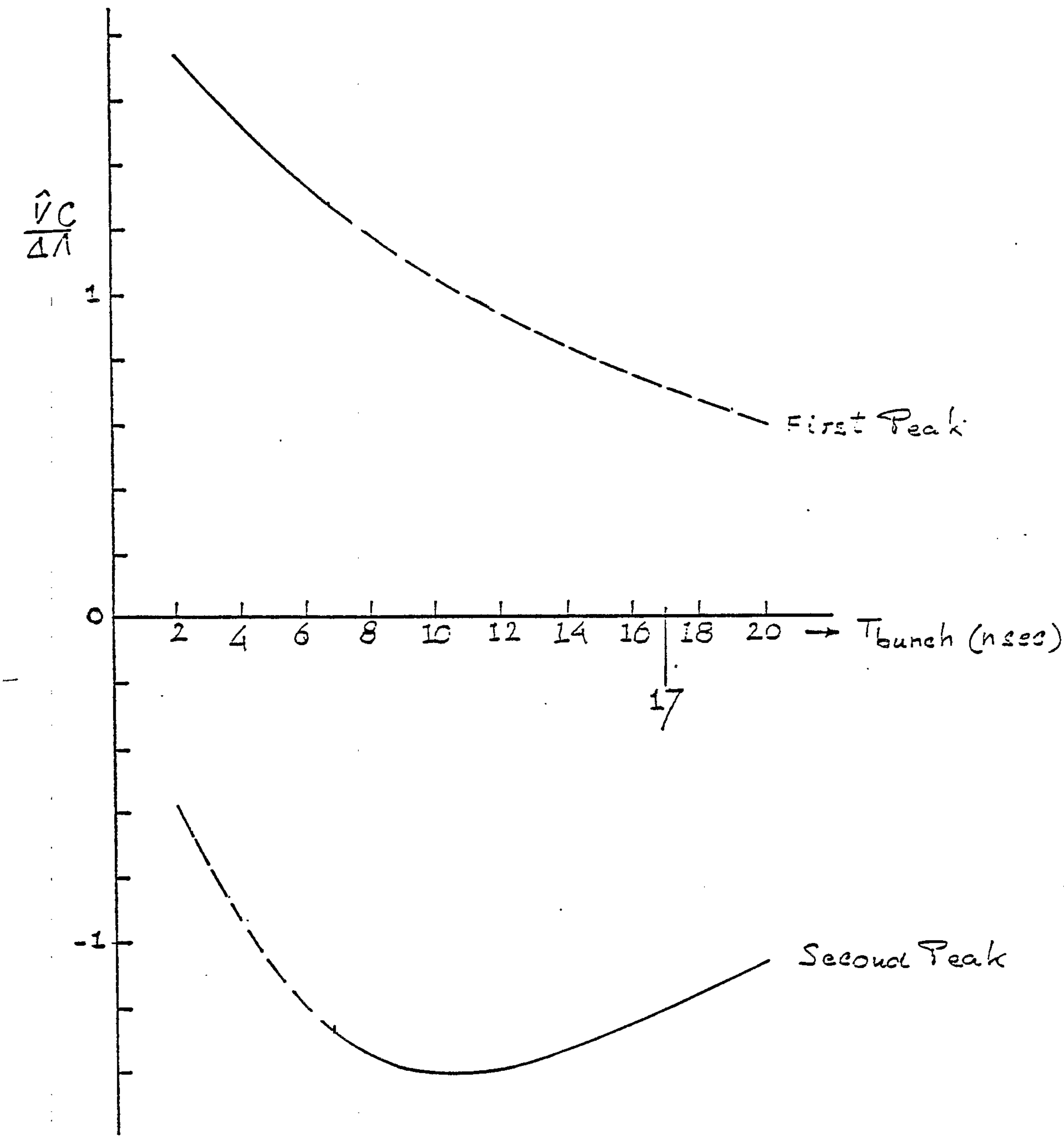

Fig 5. (VC/ΔΛ) as function of bunchlength.

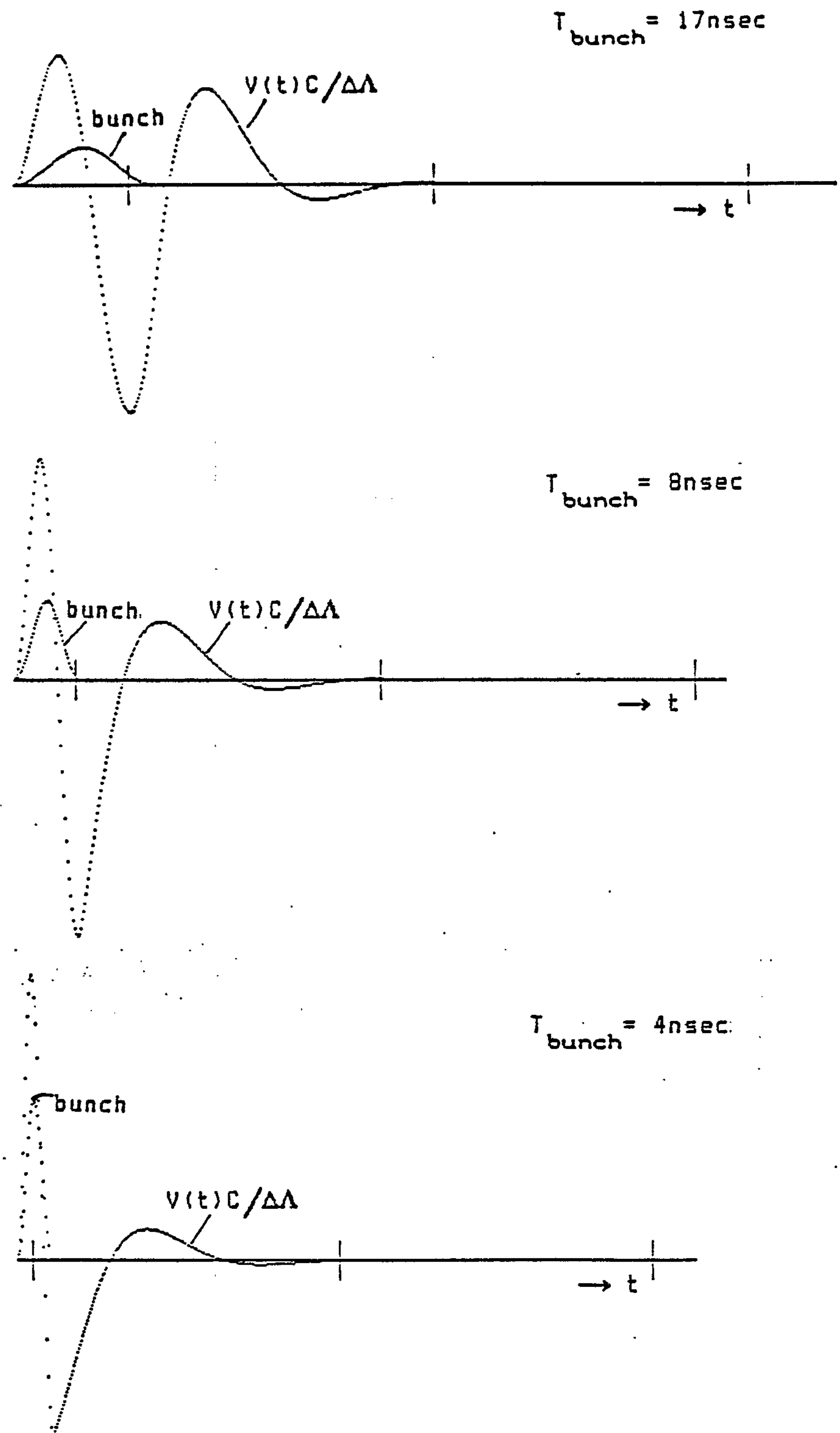

Fig 6. BPM waveform as function of bunchlength.
ν_o = 42.4MHz, Q = 0.9772