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Linear Coupling Correction and the Harmonics

G. Parzen

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Collider Accelerator Department
Brookhaven National Laboratory

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Brookhaven National Laboratory
Associated Universities, Inc.
Upton, NY 11973

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1. Effects Driven by the $\nu_x + \nu_y$ Harmonics

This note gives some of the reasons for why it is desirable to correct the "$\nu_x + \nu_y$ harmonics" of the driving terms, as well as the "zero harmonic" in correcting linear coupling. The expressions $\nu_x + \nu_y$ harmonics and zero harmonic are defined more accurately below. The following effects are driven by the $\nu_x + \nu_y$ harmonics.

1. The higher order tune shift. Most of the tune shift is produced by the zero harmonic. However, after the zero harmonic is corrected, there remains a tune shift which is roughly quadratic in the skew quadrupole field, a_1. For a RHIC lattice with 6 $\beta^* = 2$ insertions, the maximum higher order tune splitting that was found in simulation studies was[1] $|\nu_1 - \nu_2| = 20 \times 10^{-3}$, after the zero harmonic was corrected.
2. Beta function distortions. The beta functions of the normal modes may be considerably increased by the $\nu_x + \nu_y$ harmonics. For a RHIC lattice with 6 $\beta^* = 2$ insertions, the maximum beta function increase that was found[2] was about $\Delta\beta/\beta \simeq 100\%$, after the zero harmonic is corrected.
3. Emittance distortions. The emittance of the normal mode may be considerably larger than the particle emittances in the absence of coupling. For a RHIC lattice with 6 $\beta^* = 2$ insertions, the maximum increase in the total emittance ϵ_t that was found was $\Delta\epsilon_t/\epsilon_t \simeq 100\%$, after the zero harmonic is corrected.
4. Dynamic aperture loss. Effects 2 and 3 cause the particles to reach further out from the magnet center and thus reduce the dynamic aperture. For a RHIC lattice with 6 $\beta^* = 2$ insertions, the maximum loss in the dynamic aperture A_{SL} that was found[3] was $\Delta A_{SL}/A_{SL} \simeq 15\%$, after the zero harmonic is corrected.

5. The normal mode rotation angle. This may be loosely pictured as how much the plane of the normal mode is rotated by the skew quadrupole fields. The rotation angle is sometimes used as a measure of the coupling still present. For a RHIC lattice with 6 $\beta^* = 2$ insertions, the maximum rotation angle found was $\phi = 45^o$, after the zero harmonic is corrected.

It may be noted that in the simulation studies, less than half of the accelerators studied, having different distributions of field errors, showed appreciable amounts of the above effects. For RHIC, the probability may be better than 50% that the above listed effects, driven by the $\nu_x + \nu_y$ harmonics, may not be appreciable. Thus, it would not be surprising if HERA may not show these effects. If RHIC wants to gamble on the better than 50% probability, then it can omit the correction system for the $\nu_x + \nu_y$ harmonics. It is also not clear how damaging the $\nu_x + \nu_y$ effects are to RHIC's performance, which is another gamble RHIC can take.

It also may be noted that the suggestion to correct the $\nu_x + \nu_y$ harmonic was made by G. Guignard[7] for LEP because of the strong solenoids in LEP. Enough skew quadrupoles[8] were put in LEP to provide correction of the $\nu_x + \nu_y$ driving term.

2. The Driving Terms

The driving terms for the above $\nu_x + \nu_y$ effects,[4,5] to lowest order in a_1 are,

$$\begin{aligned} b_n &= \frac{1}{4\pi\rho} \int ds\ a_1 (\beta_x \beta_y)^{\frac{1}{2}} \exp\left[i (n - \nu_y) \theta_x + \nu_y \theta_y\right], \\ c_n &= \frac{1}{4\pi\rho} \int ds\ a_1 (\beta_x \beta_y)^{\frac{1}{2}} \exp\left[i (n - \nu_x) \theta_y + \nu_x \theta_x\right]. \end{aligned} \tag{1}$$

The important values of n for which the b_n and c_n need to be controlled are[4,5] $n = 0$ and $n \simeq \nu_x + \nu_y$. All the effects listed in section 1 have the same driving terms, to lowest order in a_1. Thus controlling the b_n, c_n for $n = 0$ and $n \simeq \nu_x + \nu_y$ will correct all the effects to a considerable extent. Simulation studies[6] indicate that controlling the relevant b_n, c_n will require about 4 or 5 families of skew-quadrupole correctors, if the betatron phases at the correctors are reasonably chosen.

In the above, it has been assumed that the tune is near the $\nu = \nu_y$ coupling resonance. If the tune is chosen to be near the $\nu_x - \nu_y = p$ resonance, then in the above the zero harmonic should be replaced by the p harmonic, or the $\nu_x - \nu_y$ harmonic.

3. References

1. G. Parzen, Tune Splitting in the Presence of Linear Coupling, BNL Report, AD/RHIC-93 (1991), Proc. of 1991 IEEE PAC, p. 1615.
2. G. Parzen, Beta Functions in the Presence of Linear Coupling, BNL Report, AD/RHIC-92 (1991), Proc. of 1991 IEEE PAC, p. 1872.
3. G. Parzen, Dynamic Aperture Effects due to Linear Coupling, BNL Report, AD/RHIC-94 (1991), Proc. of 1991 IEEE PAC, p. 1875.
4. G. Parzen, Theory of the Tune Shift due to Linear Coupling, BNL Report, AD/RHIC-100 (1991).
5. G. Parzen, Theory of the Beta Function Shift due to Linear Coupling, BNL Report, AD/RHIC-102 (1991).
6. G. Parzen, Correction of the Tune Shift and Beta Function Shift due to Linear Coupling, BNL Report AD/RHIC/AP-93 (1991), and Proc. of 5th ICFA Beam Dynamics Workshop (to be published, BNL-46800) (1991).
7. G. Guignard, Skew Quadrupole Schemes for the Latest LEP Design, LEP Note 199 (1979).
8. G. Guignard, L. Hand, J.P. Koutchouk and H. Moshammer, Measurement and Compensation of the Solenoid Effects in LEP, Proc. EPAC 90, p. 1432 (1990).