BNL-102009-2014-TECH
AD/RHIC/97;BNL-102009-2013-IR

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May 1991

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U.S. Department of Energy
USDOE Office of Science (SC)

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BNL–45522
AD/RHIC–97
Conference Paper

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RHIC PROJECT

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Upton, NY 11973

Under Contract No. DE–AC02–76CH00016 with the
UNITED STATES DEPARTMENT OF ENERGY

1991 IEEE Particle Accelerator Conference
San Francisco, CA
May 6-9, 1991
BNL-45522

BEAM–BEAM INTERACTION AND HIGH ORDER RESONANCES*

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Abstract

There is experimental evidence from SPS[1], that very high order resonances (an order > 10) can cause particle loss during the beam–beam interaction. These results can be simulated by tracking a single particle against a round beam. If the round beam has an rms size of σ and the particle's initial amplitude is 2σ then no effects are observed due to high order resonances. However, if the particle's initial amplitude is 5σ the beam-beam 16th order resonances are strong. Furthermore, if we add tune modulation (i.e. power supply ripple) then the 5σ particle's motion becomes chaotic (when it is near a 16th order resonance) and can result in particle loss. This is in agreement with the beam–beam experiments performed at the SPS. Implications on RHIC and SSC will be discussed.

I. INTRODUCTION

Beam–beam experiments on the SPS have shown that when colliding a proton beam ($\approx 26\pi$mm·mrad) against an anti-proton beam ($\approx 13\pi$mm·mrad), there is an increase in background proton radiation when the tunes cross through the 16th order resonance. One surprise is that the loss occurred to the larger beam. The reason for this will be made clear as we continue.

Beam–beam effects have been studied both experimentally and theoretically.[2-8] However, most of these studies dealt with the low order resonance effects (an order < 10). In order to study the high order resonance effects we tracked a single particle, with a given initial amplitude, through a collision point against a round beam of rms radius σ. The low order resonances were avoided by choosing the operating tunes of RHIC.[9]

II. THE SIMULATION

The main accelerator is modeled as a linear machine with the following transfer matrix

$$\begin{pmatrix} \cos 2\pi\nu_x & \beta_x^* \sin 2\pi\nu_x & 0 & 0 \\ -\frac{1}{\beta_x^*}\sin 2\pi\nu_x & \cos 2\pi\nu_x & 0 & 0 \\ 0 & 0 & \cos 2\pi\nu_y & \beta_y^* \sin 2\pi\nu_y \\ 0 & 0 & -\frac{1}{\beta_y^*}\sin 2\pi\nu_y & \cos 2\pi\nu_y \end{pmatrix}$$

where β_x^* and β_y^* are beta values at the crossing point and ν_x, ν_y are horizontal and vertical betatron tunes respectively.

After the particle circles the accelerator, it will encounter the round beam at the crossing point. This beam–beam interaction is modeled with the following potential

$$U(r) = \frac{Nr_o}{2\gamma\sigma} V(r) = \frac{Nr_o}{2\gamma\sigma} e^{-r^2/2\sigma}$$

where N is the number of particles, $r_o = q^2e^2/Am_oc^2$ is the classical radius of the particle with charge qe, mass number A, m_o is the atomic mass unit, γ is the Lorentz factor and σ is the rms beam size. From the above potential, the kicks to the slope become:

$$\Delta x' = 4\pi\,\xi f(r)\frac{x}{\beta^*}$$
$$\Delta y' = 4\pi\,\xi f(r)\frac{y}{\beta^*}$$

where $\xi = Nr_o\beta^*/4\pi\gamma\sigma^2$ is the linear beam–beam parameter, $\beta^* = \beta_x^* = \beta_y^*$ and

$$f(r) = \frac{2\sigma^2}{r^2}\left[1 - e^{-r^2/2\sigma^2}\right] \underset{r\to 0}{\sim} 1.$$

A particle with an initial maximum amplitude of $x_o = y_o = 2\sigma$ will have an rms amplitude over betatron motion of $< x >_{rms} = < y >_{rms} = \sqrt{2}\sigma$. Hence, the particle is most likely to be found in a box of area $8\sigma^2$. A particle with the same probabilities in a circle has a radius, $r = \sqrt{\frac{8}{\pi}}\sigma$, which occupies the same area. Expanding the potential term into a multipole expansion leaves

$$V(r) = \sum_{k=0} \frac{1}{k!}\left(-\frac{r^2}{2\sigma^2}\right)^k$$

The multipoles in this case are very small. However, when the initial maximum amplitude is $x_o = y_o = 5\sigma$, the corresponding radius is $r = 5\sqrt{\frac{2}{\pi}}\sigma$. This leads to large multipole terms, in particular the 8th term (driving the 16th order resonance) grows to ~ 400 and the 11th term (driving the 22nd order resonance) then drops to ~ 200. This is confirmed with tracking discussed in section III.

When two beams are colliding with different beam sizes, particles of the larger beam are more likely to be at the 5σ position than those of the smaller beam, then the losses will be seen in the larger beam. Additionally,

*Work performed under the auspices of the U.S. Department of Energy.

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only even order resonances are excited by this multipole expansion. However, when two beams are colliding off center, the odd order resonances will become important.

The final ingredient in the simulation is to include the linear effects of the beam–beam potential. This affects both the betatron tunes and the β^* at the crossing point. When these effects are taken into account, we observe the smear goes to zero as the initial amplitude goes to zero as it should. The simulation code was checked by reverse tracking to reproduce the initial conditions. In 10^4 revolutions, the error in reproducing the initial conditions is $\sim 10^{-13}$, increasing to $\sim 10^{-7}$ in 10^7 revolutions (in double precision on an IBM 3090).

Finally, we also study the effect of tune modulation on the particle motion. The betatron tunes are assumed to be $\nu_x = \nu_x^o + \Delta\nu_x \sin[n\overline{f}]$ and $\nu_y = \nu_y^o + \Delta\nu_y \sin[n\overline{f} + \varphi]$ for the n'th revolution and a tune modulation of $\Delta\nu_x$ and $\Delta\nu_y$. Furthermore, $\overline{f} = 2\pi C f / v$ where f is the frequency of the tune modulation and φ sets up a phase difference between the modulations.

III. THE RESULTS

The simulation, described above, was used to study the beam–beam interaction. In particular the tunes were varied from $\nu_x^o = 28.809$ to $\nu_x^o = 28.830$ in steps of 0.0001 and $\nu_y^o = \nu_x^o - 0.004$. Furthermore, $\xi = 0.02$, $\beta^* = 2$ m, $\sigma = 5.5 \times 10^{-4}$ m, $\gamma = 100$, $C = 3833.845$ m, $x_o = 5\sigma$, $x'_o = 0$, $y_o = 5\sigma$ and $y'_o = 0$. Figure 1 shows the maximum $E_x + E_y$ and minimum $E_x + E_y$ versus the resulting ν_x, (where E_x and E_y are the emittances in πmm·mrad). In this figure we observe 3 major regions: Region (I) shows larger emittance to the 16th order resonances at harmonic 461. Additionally, there are 8 peaks in $(E_x + E_y)_{max}$ and the 8 troughs in $(E_x + E_y)_{min}$ corresponding to the 9 resonances $16\nu_x = 461$, $14\nu_x + 2\nu_y = 461$, $12\nu_x + 4\nu_y = 461, \ldots, 16\nu_y = 461$. Also the parametric $16\nu_x$ and $16\nu_y$ resonances are harder to be recognized. Region II shows no resonances; Region III shows some peaks due to the 22nd order resonances with harmonic at 634.

Although we see resonance effects in Fig. 1, long term tracking will not reveal any instabilities or chaotic behavior. When tune modulation effects are included, new effects appear. Consider a tune modulation of $\Delta\nu_x = \Delta\nu_y = 0.001$, $\varphi = 0$ (i.e. tune modulation parallel to the coupling line) and at a frequency of 60 Hz. Note, it takes about 1,300 revolutions for the particle to go through one full cycle of the tune modulation, thus, long term tracking is necessary.

Figure 2 plots $(E_x + E_y)_{max}$ and $(E_x + E_y)_{min}$ versus ν_x with tune modulation on (each point tracks through 10^5 revolutions). In region I, the emittances have greatly increased due to the 16th order resonances. Region III shows no significant effect due to the 22nd order resonance.

The points A and B shown on Fig. 2 are the tunes where we performed long term tracking. A particle at point A can cross a 16th order with results shown in Fig. 3. In 10^6 revolutions the resonance can be crossed 1500 times. The motion seems chaotic with large emittance growth. The total emittance is outside the available aperture. Detailed phase space indicates that the particle is locked on a sum resonance line and then is transported through different resonance to another sum resonance. Through this mechanism the total emittance is increased irreversibly. For comparison, Fig. 4 shows the tracking in Region II at point B (i.e. point B on Fig. 2). Here, the motion appears to be quite regular and bounded.

IV. CONCLUSION

In all large colliders such as RHIC, SSC, SPS, Tevatron, etc. it is very likely that these machines will collide beams of unequal size. In this case, the larger beam can have a significant number of particles at the position of 5σ or larger relative to the smaller beam. Since, these particles may also have tunes near high order resonances, they can get lost leading to increased background radiation which the detectors must be able to handle. However, this will not lead to total beam loss, only losses to particles at the fringes of the beam.

This is especially important to machines such as RHIC which will collide beams of different species. When colliding a proton beam against a gold beam, the gold beam will be larger. As the tune of the machine crosses a high order resonance, there will be an increase in the background of gold ions that the detectors must contend with.

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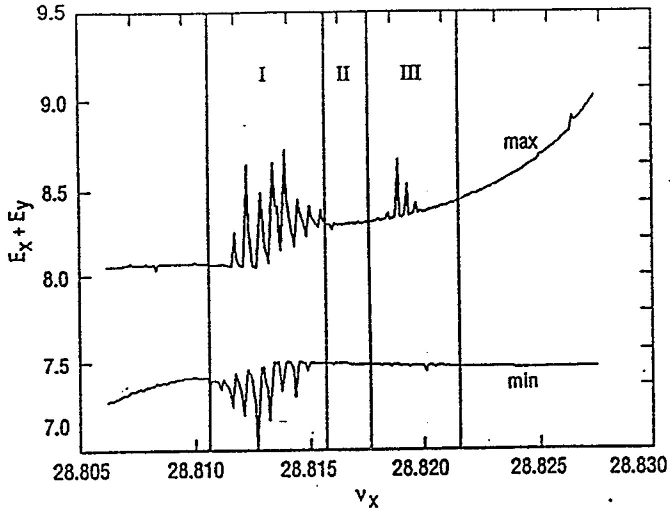

Fig. 1: Plot of $(E_x + E_y)_{min}$ and $(E_x + E_y)_{max}$ versus the horizontal tune ν_x with no tune modulation. Regions I, II, and III refer to the particle crossing different resonances, see text.

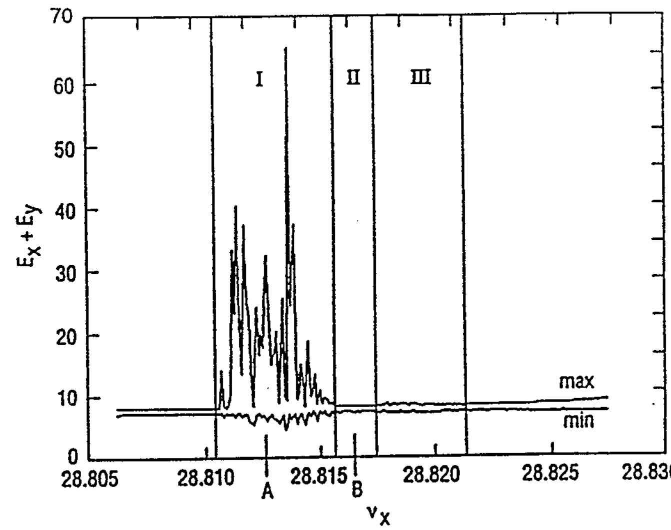

Fig. 2: Plotting $(E_x + E_y)_{min}$ and $(E_x + E_y)_{max}$ versus ν_x with tune modulation. Point A and B refer to Figs. 3 and 4 respectively. Note the expanded scale of the total emittance.

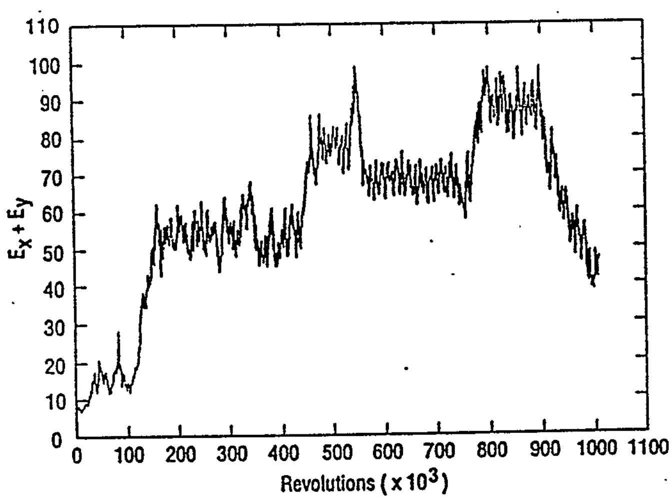

Fig. 3: Long term tracking of particle on a 16th order resonances. The tunes are shown at point A on Fig. 2.

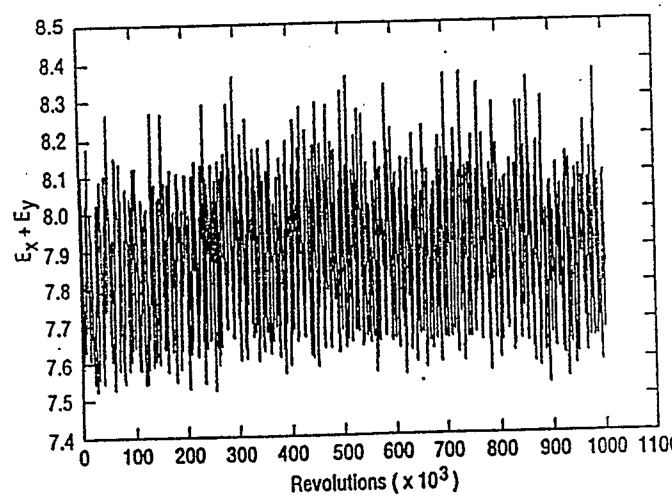

Fig. 4: Long term tracking of a particle away from resonances. The tunes are shown at point B on Fig. 2.