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Symmetric Design for Helical Spin Rotators at RHIC

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1 Introduction

The design of Siberian Snakes and spin rotators based on helical dipole magnets has been adopted for RHIC collider. The snake scheme has a symmetry that restores the closed orbit to the reference orbit at the exit and sets the vertical component of snake spin rotation axis to be equal to zero. Unlike this approach, the previous spin rotator design is asymmetric. It consists of 3 helical dipoles. In order to provide the orbit restoration and a proper spin direction at the rotator exit, just three parameters are needed. For the given scheme these are three values of magnetic field of the three dipoles. Each magnet in this scheme can have a various number of periods and a various helicity sign. Thus a variety of possible schemes exists. The best asymmetric variant from the point of view of RHIC requirements has been reported elsewhere [1,2]. In this variant the middle helical dipole is two periods long. Thus the scheme really consists of four modules.

This paper analyses another approach to a spin rotator design. Snake-like symmetric schemes based on four helical modules are considered and compared to the present design with the point of view of RHIC requirements.

2 Proposed rotator designs

Let us consider a spin rotator which consists of four helical dipole magnets. Each magnet has its own value of magnetic field B and its own helicity S. The length of the helix period, λ, will be considered to be the same for all magnets and equal to 2.4 meters as it is presently defined by the helical magnet design [3]. In principle each individual helical dipole can be several periods long but for use at RHIC magnets with just one period will be considered in this paper because of the demand that a whole rotator should be placed in one cryostat. Also the direction of magnetic field at the entrance of each modules can be different and will be characterized by the angle α encountered from the vertical direction.

As the next step we want to impose some symmetry conditions relating the magnets in the scheme. These conditions should provide the automatic restoration of a particle orbit after the rotator. Let us remember that in the paraxial field approximation a particle entering a helix parallel to the axis goes out the magnet again parallel to the axis, but with a displacement shifted along the direction defined by the magnetic field vector at the entrance [1]:

$$\begin{aligned} x &= x_0 + \sin\alpha \cdot \frac{p}{\gamma} \cdot S \cdot \lambda, \\ y &= y_0 - \cos\alpha \cdot \frac{p}{\gamma} \cdot S \cdot \lambda, \end{aligned}$$

where $S = k/|k|$ is the helicity of the helical magnet, $p = q_0 B/(c|k|)$ is the dimensionless field value and $|k| = 2\pi/\lambda$. Then for the rotator including 4 magnets the orbit restoration after the rotator implies:

$$\begin{aligned} \sum_{i=1}^{4} \sin\alpha_i \cdot p_i \cdot S_i &= 0 \\ \sum_{i=1}^{4} \cos\alpha_i \cdot p_i \cdot S_i &= 0 \qquad (1) \end{aligned}$$

Following these expressions a possible way to introduce symmetry into the scheme is to combine the helical magnets in pairs and require that the orbit shifts caused by the magnets of one couple compensate each other. Asserting that magnets of each couple have the same field direction angle α, it follows from (1) that these helical dipoles must have the same field and opposite helicities or the same helicity and opposite fields. Combining the rotator magnets in the pairs does not mean that only two consecutive magnets are connected to each other. For example, we can relate by the symmetry conditions the first helical module with the third, and the second module with the fourth.

After introducing the symmetry conditions the rotator scheme depends on two values of magnetic field. These values must be chosen to satisfy spin conditions. Specifically, to have after the rotator the particle spin put in the horizontal plane and to have a desired spin orientation in this plane. For RHIC the required spin orientation in the horizontal plane after the rotator will depend on the energy, due to the presence of dipole magnets inserted between the rotator and the interaction point, where the longitudinal beam polarization is required. If one characterizes the spin direction after the rotator by a ϕ angle encountered from the longitudinal axis, then $\phi = 10.2°$ corresponds to the lowest RHIC energy

γ	ϕ	Var.1		Var.2		Var.3	
		B_1	B_2	B_1	B_2	B_1	B_2
27	10.19	2.13	2.77	-2.92	2.80	3.04	2.74
50	18.88	2.38	2.65	-2.83	2.67	2.85	2.61
100	37.75	2.87	2.47	-2.66	2.31	2.41	2.48
150	56.63	3.22	2.51	-2.57	1.81	1.88	2.47
200	75.50	3.41	2.78	-2.54	0.04	0.70	2.54
250	94.38	3.50	3.11	-2.54	-1.69	-1.84	2.65

Table 1: Required magnetic field (in Tesla) at various beam energies.

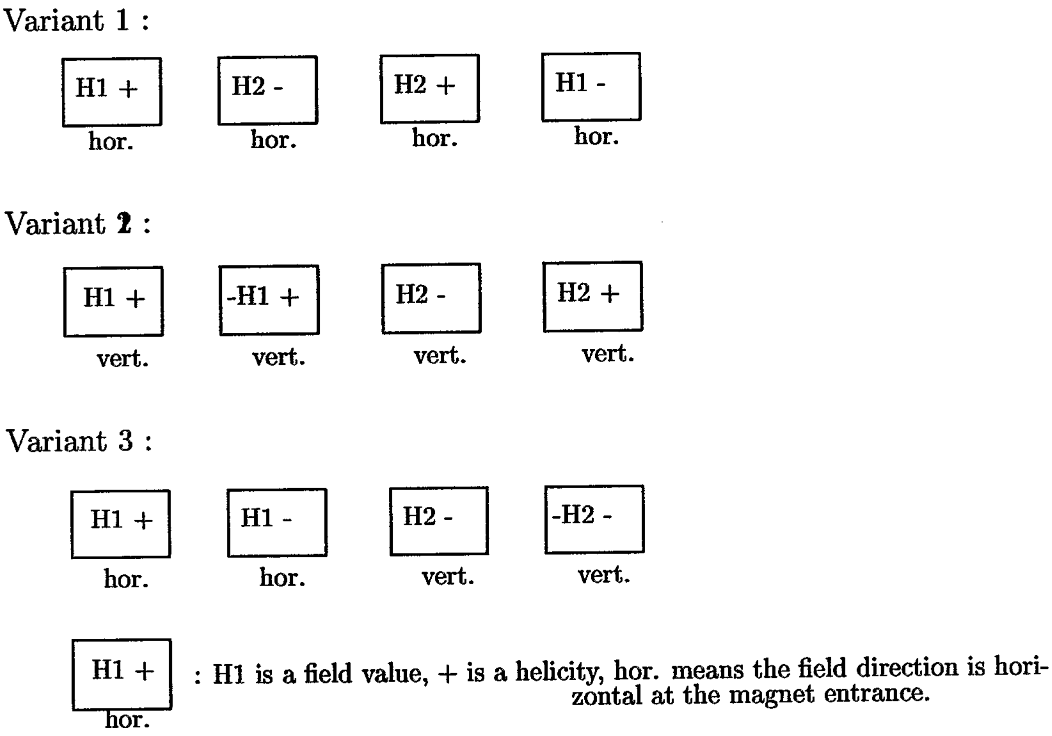
$\gamma = 27$, and $\phi = 101.2°$ corresponds to the highest RHIC energy $\gamma = 268$. Because the spin transformation matrix for helical dipoles has sufficiently complex non-linear dependence on the magnetic field [4] the analysis of possible schemes has been performed with the use of the specially written ROT4 code. Only the on-axis magnetic field was taken into account. Some additional requirements for RHIC scheme are listed below:

1. The length of period is 2.4 m;
2. The maximum value of magnetic field to be analyzed is restricted to 4 T;
3. It follows from two preceding items that the parameter p must be less than 0.5;
4. The orbit deviation inside the rotator must be less than 4 cm;
5. The rotator has to provide the full required range of ϕ angle from 10.2° to 101.2°.

For simplicity only the variants with the two possible field orientations (horizontal or vertical) at a module entrance were considered.

About ten schemes have been found to be better or at least not worse than the nominal asymmetric design with the point of view of the maximum orbit excursion at the lowest energy ($\gamma = 27$). On first glance three variants of them can be selected for further consideration. These schemes are shown in Figure 1. Table 1 lists for the each variant the magnetic field values (in Tesla) necessary to obtain longitudinal spin vector in the interaction region at various beam energies.

Variant 1 provides the best orbit excursion at $\gamma = 27$, only 2.3 cm, to be compared to 3.3 cm in the asymmetric design. At energies higher than $\gamma = 90$ the orbit deviation becomes worse than in the asymmetric scheme but the loss just 1-2 mm. The maximum orbit excursion dependence on the ϕ angle is drawn in Figure 2 for Variant 1 and for the nominal asymmetric scheme for a



: H1 is a field value, + is a helicity, hor. means the field direction is horizontal at the magnet entrance.

Figure 1: Three proposed variants of symmetric rotator design.

Variant	Max.orbit (cm) at $\phi = 10.2°$	Max.orbit (cm) at $\phi = 90°$
1	2.31	0.44
2	3.18	0.34
3	3.31	0.32
Assym.	3.34	0.34

Table 2: Maximum orbit deviations for 3 symmetric variants and nominal asymmetric scheme.

comparison. Figures 3 and 4 show the possible range of spin directions after the rotator and the connection between the two field values. When considered together, these two figures provide the magnetic fields needed to obtain the proper spin at the rotator exit.

The other remarkable feature of the Variant 1 is the cancelation of the longitudinal field integral along the beam orbit, at least to first-order. It results from the internal design symmetry or more strictly speaking from the presence of the helical magnets having the same field but opposite helicities.

Nevertheless, if one wants a closed orbit that is not worse at the highest RHIC energy than in the asymmetric scheme the Variants 2 or 3 can be used. Figure 5 shows the maximum orbit deviation dependence on the final spin angle for all 3 proposed designs and for the nominal asymmetric design at a fixed energy of $\gamma = 100$. One can see that the Variants 2 and 3 have the better values for particle orbit almost at the whole range of the final spin angles. Unlike Figure 2 Figure 5 does not of course give the correct values for orbit deviation because of the above mentioned connection between the spin direction after the rotator and the particle energy. Table 2 lists the correct values for the maximum orbit deviations for $\phi = 10.2°$ and $\phi = 90°$.

In Figure 6 the dependence of the maximum required dipole field on the energy is shown for all considered designs.

3 Conclusion

In concluding we want to point out again that the main advantage of the symmetric proposed schemes in comparison with the nominal asymmetric one is that their symmetry makes easier not only the theoretical description of the schemes, but also the control and adjustment of the operating conditions. This is because just two independent parameters, the two values of magnetic field of helical dipoles, exist instead of three parameters in the nominal design. It is more profitable also from the point of view of the number of power supplies needed.

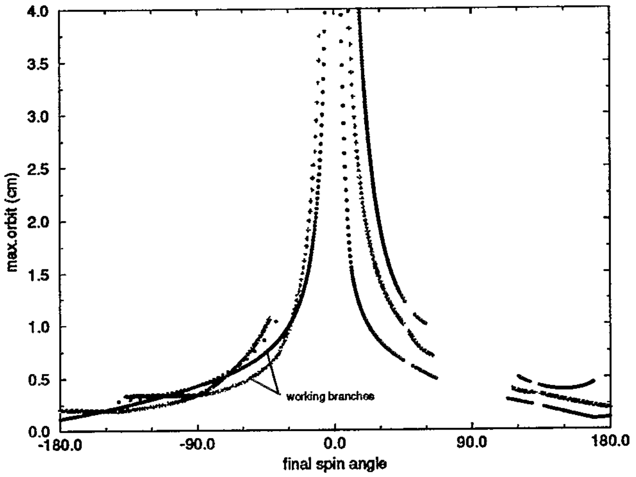


Figure 2: The maximum orbit deviation dependence on the spin direction angle after the rotator for asymmetric scheme (crosses) and for Variant 1 (circles).

4 Acknowledgment

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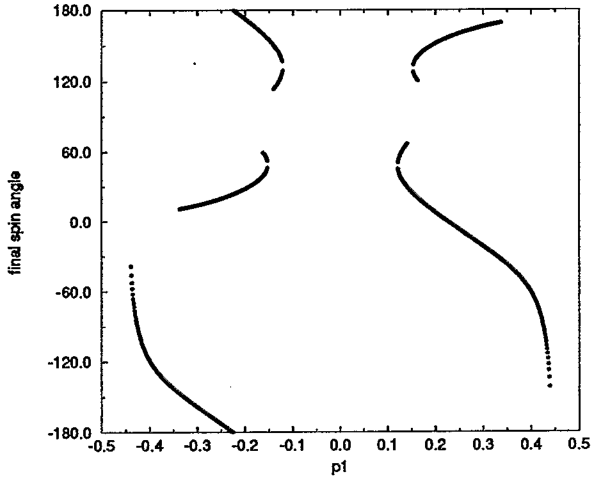

Figure 3: The relationship between the spin direction angle after the rotator and the field p_1 of first magnet.

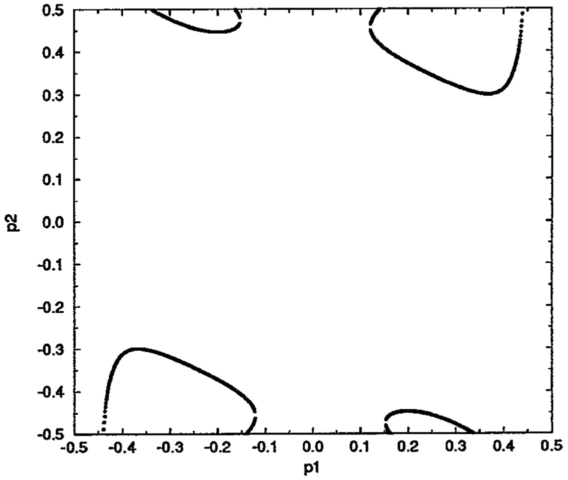

Figure 4: The relationship between the fields, p_1 and p_2, of first and second helical magnets.

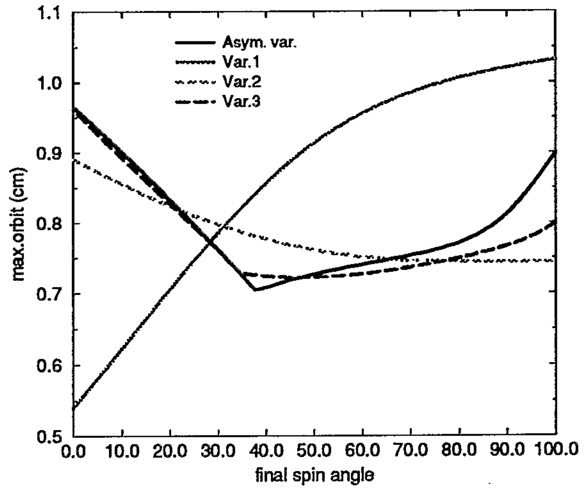

Figure 5: The maximum orbit deviation values on the spin direction angle after the rotator for all considered designs at fixed energy $\gamma = 100$.

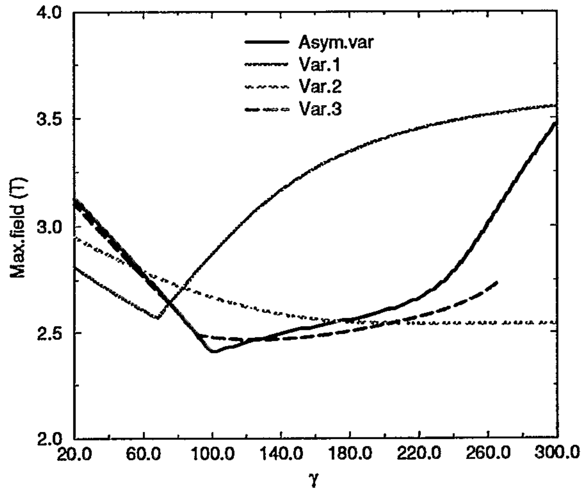

Figure 6: The maximum value of magnetic field required at given energy to provide the longitudinal polarization in IP.