BNL-102156-2014-TECH
RHIC/AP/46;BNL-102156-2013-IR

Vacuum Pipe Heating in RHIC

A. G. Ruggiero

November 1994

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC)

DISCLAIMER

RHIC/AP/46
November 1994

Vacuum Pipe Heating in RHIC

A.G. Ruggiero, S. Peggs

Introduction

[Note by Peggs:] This RAP note is an almost completely literal transcription of an unpublished note with the same title that was originally written by A.G. Ruggiero in July 1987. My contribution has largely been to type the original into the computer, and to slightly modify the nominal parameters used in the sample calculations.

Vacuum Pipe Heating

Assume M equally spaced bunches of the same shape and population with gaussian longitudinal distribution. If N_b is the number of particles per bunch, σ_L is the rms bunch length, and P is the total power dissipated, then

$$P = \frac{1}{2}\sum_{n=1}^{\infty} R_{nM}\, I_{nM}^2 \tag{1}$$

where R_{nM} is the wall resistance at the nM harmonic and I_{nM} is the beam current at the nM harmonic. That is,

$$I_{nM} = 2\, I_{ave}\, exp\left(-\frac{1}{2}n^2\alpha^2\right) \tag{2}$$

where

$$\alpha = \frac{M\sigma_L}{R_0} \tag{3}$$

and the average radius $R_0 = 610.175$ m. The average current is given by

$$I_{ave} = \frac{ZN_b e\beta cM}{2\pi R_0} \tag{4}$$

where Z is the charge state.

The skin depth is normally given by

$$\delta = \frac{\delta_1}{\sqrt{n}} \tag{5}$$

where

$$\delta_1 = \sqrt{\frac{2\rho R_0}{M Z_0}} \tag{6}$$

and $Z_0 = 377\ \Omega$ is the impedance of free space.

Symbol	Units	Stainless Steel	Copper
ρ	$[\mu\Omega\ \mathrm{m}]$	.5	.00055
δ_1 (M = 57)	[mm]	.17	.0056
δ_1 (M = 114)	[mm]	.12	.0040
σ/l	$[10^{15}\Omega^{-1}\mathrm{m}^{-2}]$	1.37	1.54

Table 1: Electrical and skin depth parameters for stainless steel and copper, at 4.2 K cryogenic temperatures.

Table 1 also introduces the mean free path length for electrons l, the conductivity $\sigma = 1/\rho$, and the useful quantity σ/l, which is an invariant depending on the electronic density of the material.

For frequencies low enough that $l < \delta$ the normal skin depth equation 6 applies. For high frequencies, when $l > \delta$, the "anomalous skin depth" must be used to calculate the vacuum chamber resistance. The critical frequency at which anomalous behavior begins corresponds to a critical index number n_c given by

$$n_c \simeq \left(\frac{\sigma}{l}\right)^2 (\rho\delta_1)^2 \tag{7}$$

Putting this together gives

$$R = R_c \left(\frac{n}{n_c}\right)^{1/2} \qquad n < n_c \tag{8}$$

below the critical frequency, and

$$R = R_c \left(\frac{n}{n_c}\right)^{2/3} \qquad n > n_c \tag{9}$$

above the critical frequency. The critical resistance R_c used in these expressions is given by

$$R_c = \frac{R_0}{b} \rho^2 \left(\frac{\sigma}{l}\right) \tag{10}$$

where $b = 34.6$ mm is the vacuum chamber radius, assuming a circular geometry.

Symbol	Units	Stainless Steel	Copper
n_c (M = 57)		1.4×10^{10}	22
n_c (M = 114)		0.7×10^{10}	11
R_c	$[\Omega]$	6.04×10^6	8.2

Table 2: More parameters for stainless steel and copper, at 4.2 K cryogenic temperatures.

Table 2 shows that the anomalous skin depth effect is important for copper, but irrelevant for stainless steel.

The total power dissipated is found by combining equations 1 and 2

$$P = 2I_{ave}^2 \sum_{n=1}^{\infty} R_c \left(\frac{n}{n_c}\right)^p exp\left(-n^2\alpha^2\right) \tag{11}$$

where

$$p = 1/2 \qquad n \leq n_c \tag{12}$$

and

$$p = 2/3 \qquad n > n_c \tag{13}$$

The exponential function in equation 11 effectively cuts off contributions to the sum for n greater than

$$n_{cut-off} \sim \frac{1}{\alpha} = \frac{R_0}{M\sigma_L} = 10.7\left(\frac{57}{M}\right)\left(\frac{1.0}{\sigma_L}\right) \tag{14}$$

where the rms bunch length σ_L is measured in meters.

Results

The nominal parameters for protons and gold listed in Table 3, taken from the RHIC Design Manual [1], are assumed to be representative.

Species	Z	M	N_b [10^9]	I_{ave} [mA]
protons	1	57	100	71
gold	79	57	1	56

Table 3: Nominal beam parameters for protons and gold ions.

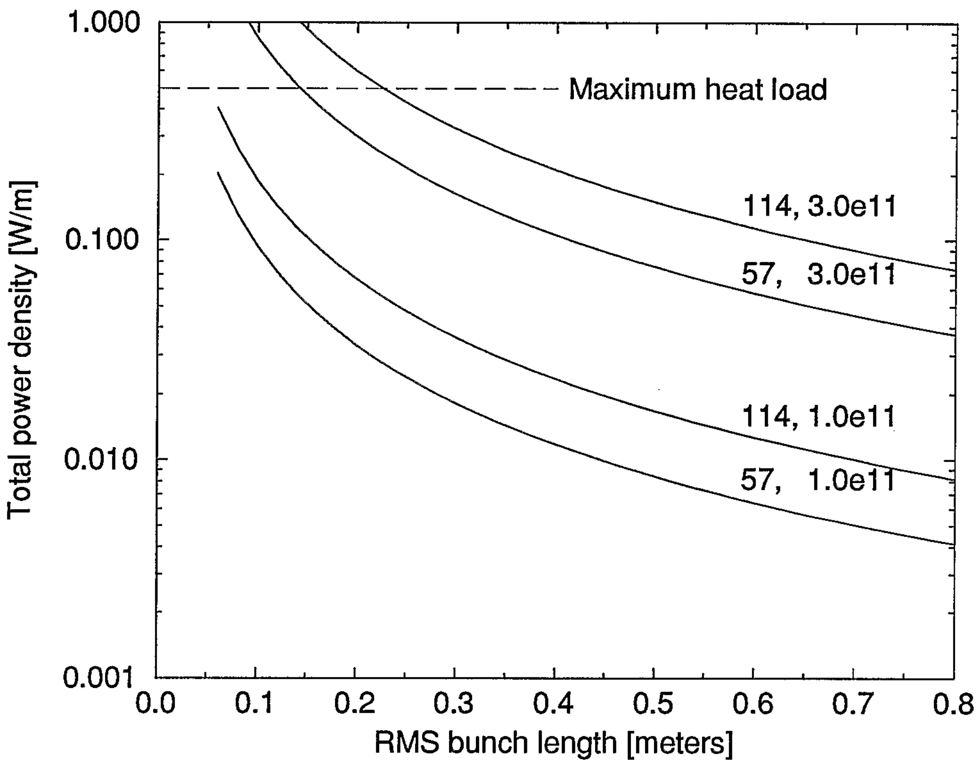

Figure 1: Total power density for a STAINLESS STEEL beampipe. The curve labels refer to 57 or 114 proton bunches, with 1.0×10^{11} or 3.0×10^{11} particles per bunch.

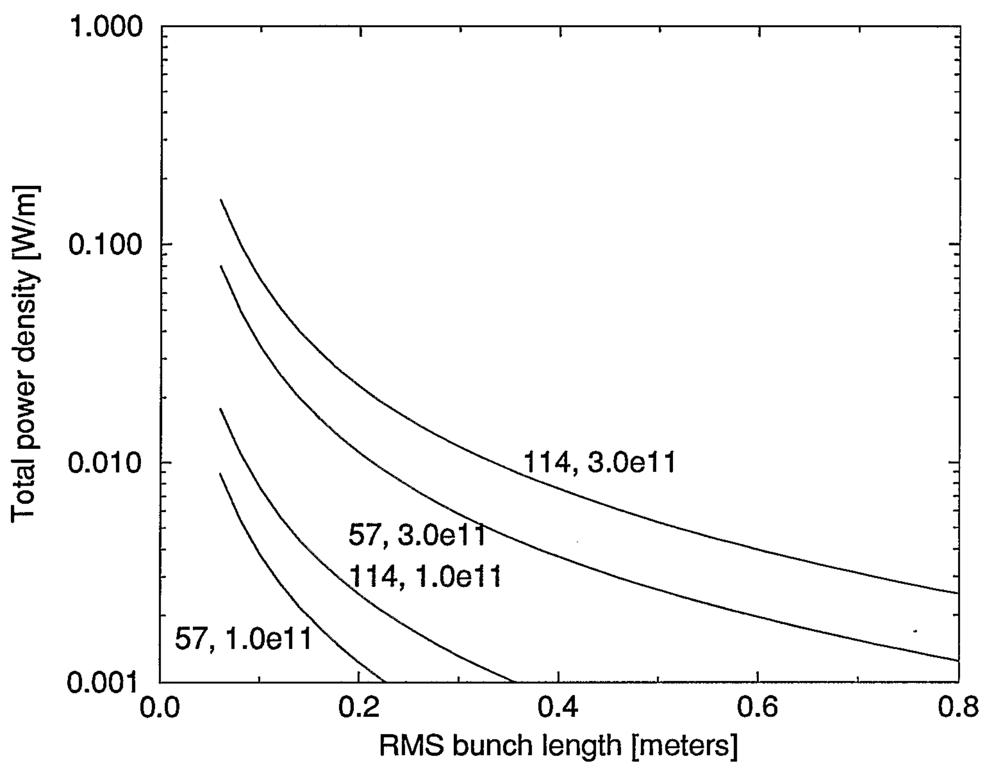

Figure 2: Total power density for a COPPER coated beampipe. The curve labels refer to 57 or 114 proton bunches, with 1.0×10^{11} or 3.0×10^{11} particles per bunch.

Protons have a slightly higher nominal average current. Future potential RHIC upgrades include doubling the number of bunches M from 57 to 114 bunches, and increasing N_b, the bunch population. In the extreme case, the number of protons per bunch might be increased to 3.0×10^{11}. Figure 1 shows that, with a stainless steel beampipe, the maximum tenable heat load of 0.5 Watts per meter is approached for short bunches in the high current upgrade scenario.

Species	phase	RMS bunch length σ_L [m]
protons	injection	.353
	storage	.072
gold	injection	.467
	start store	.119
	end store	.206

Table 4: Nominal rms bunch lengths for protons and gold ions.

Table 4 records the nominal rms bunch lengths [2]. It shows, in particular, that the rms proton bunch length of 0.072 m in storage is a potential problem in extreme upgrade scenarios. Nontheless, it is trivially possible to increase the longitudinal emittance and the bunch length of the protons, so long as σ_L remains much less than both the RF wavelength of 1.529 m, and the nominal value of $\beta^* = 1.0$ m.

Figure 2 shows, as expected, that the addition of a thin layer of copper to the inside of the beampipe makes a dramatic difference to the cryogenic heat load.

References

[1] "RHIC Design Manual", August 1993.

[2] "Collective Instabilities in RHIC", RHIC/AP/36, September 1994.