BNL-101548-2014-TECH
RHIC/PG/5;BNL-101548-2013-IR

Definition Of Beam Emittances

A. G. Ruggiero

November 1983

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC)

DISCLAIMER

RHIC-PG-5

DEFINITION OF BEAM EMITTANCES

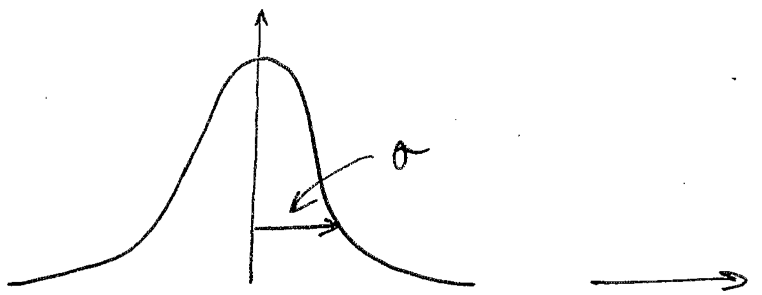
A.G. Ruggiero

(BNL, November 20, 1983)

Betatron Emittances

Assume that the beam has a gaussian distribution in both z and z' (H or V).

A beam ~~measurement~~ profile measurement will give the rms width σ



The rms emittance is then defined as

$$\varepsilon_{rms} = 2\pi \frac{\sigma^2}{\beta_z}$$

where β_z is the corresponding lattice amplitude value where the beam profile is taken.

The following emittances

$$\varepsilon = 4\pi \frac{\sigma^2}{\beta_z} \quad \text{and} \quad 6\pi \frac{\sigma^2}{\beta_z}$$

are commonly used in Europe and USA. The former define a 90% beam contour, the latter 95% of the beam.

For aperture requirement consideration I propose the definition that corresponds to 95% of the beam

$$\boxed{\varepsilon = 6\pi \frac{\sigma^2}{\beta_z}} \quad (1)$$

The factor π will always be left explicitly out.

Normalized Emittance

The emittance defined by (1) is not an invariant. The normalized emittance ε_N is defined so that the actual emittance ε at some beam energy described by the relativistic parameters β and γ is given by

$$\boxed{\varepsilon = \frac{\varepsilon_N}{\beta\gamma}} \quad (2)$$

For heavy ions and protons ε_N is a truly invariant provided dilution and diffusion are not introduced by external means.

We will use definition (2) for heavy ions as well as for protons.

Also observe that definition (2) is independent of the charge status of the ion. In fact also when the ion goes through a stripping foil, as long β and γ do not vary so its momentum will not vary and the emittance is not effected by the stripping (except by possible scattering which cause transfer from longitudinal to the transverse momentum).

Other Relations

Instead of the rms width σ we could measure the rms angle θ in z', then we also have

$$\begin{aligned} \varepsilon &= 6\pi \frac{\sigma^2}{\beta_z} \\ &= 6\pi\, \sigma\, \theta \qquad (3) \\ &= 6\pi\, \theta^2 \beta_z \end{aligned}$$

Longitudinal Beam Area

A. Bunched Beam -
Longitudinal Area of individual bunch

$$S = 6\pi \, \sigma_\tau \, \sigma_\varepsilon \qquad \text{in eV-sec} \qquad (4)$$

where

σ_τ , rms bunch length in time unit

$$\sigma_\tau = \sigma_\ell / \beta c$$

σ_ℓ , rms bunch length
βc , bunch velocity

σ_ε , rms energy spread

For heavy ion σ_ε could be measured in eV/A and then the bunch area will be in eV/A - sec units

Observe that here π is <u>not</u> left explicitely out but is included in the computation for S.
Eq. (4) defines the contour for 95% of the beam bunch -

B. Unbunched Beam -

Longitudinal Area of total Beam

$$B = 4\,\sigma_E \cdot T_0 \qquad \text{in eV-sec} \qquad (5)$$

where

T_0, revolution period
σ_E, rms energy spread

For heavy ion σ_E could be measured in eV/A and then the beam area will be in eV/A-sec -

Also eq. (5) defines a contour for 95% of the beam -

Observe that eqs(4) and (5) are <u>invariants</u> (aside from possible external sources of dilution and diffusion) and do not depend on the beam energy or on the particle charge status -