

Definition Of Beam Emittances

A. G. Ruggiero

November 1983

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No.DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DEFINITION OF BEAM EMITTANCES

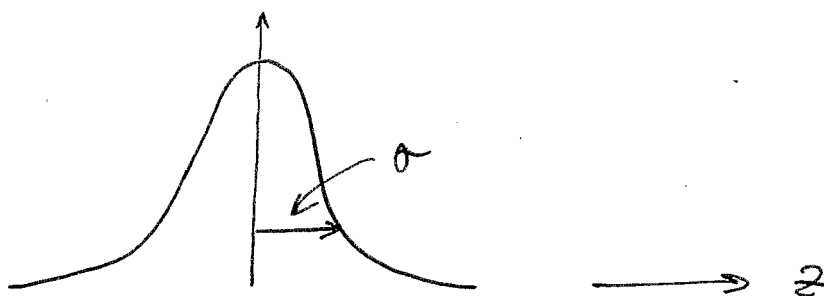
A.G. Ruggiero

(BNL, November 20, 1983)

Betatron Emittances

Assume that the beam has a gaussian distribution in both z and z' (H or V) -

A beam ~~measurement~~ profile measurement will give the rms width σ



The rms emittance is then defined as

$$E_{\text{rms}} = 2\pi \frac{\sigma^2}{\beta_z}$$

where β_z is the corresponding lattice amplitude value where the beam profile is taken -

The following emittances

$$E = 4\pi \frac{\sigma^2}{\beta_z} \quad \text{and} \quad 6\pi \frac{\sigma^2}{\beta_z}$$

are commonly used in Europe and USA -
 the former define a 90% beam contour,
 the latter 95% of the beam.

For aperture requirement consideration
 I propose the definition that corresponds to
 95% of the beam

$$\boxed{\varepsilon = 6\pi \frac{\sigma^2}{\beta_z}} \quad (1)$$

The factor π will always be left explicitly
 out -

Normalized Emittance

The emittance defined by (1) is not an
 invariant - The normalized emittance ε_N is
 defined so that the actual emittance ε at
 some beam energy ~~is~~ described by the rela-
 tivistic parameters β and γ is given by

$$\boxed{\varepsilon = \frac{\varepsilon_N}{\beta\gamma}} \quad (2)$$

(3)

For heavy ions and protons E_N is a truly invariant provided dilution and diffusion are not introduced by external means -

We will use definition (2) for heavy ions as well as for protons -

Also observe that definition (2) is independent of the charge status of the ion - In fact also when the ion goes through a stripping foil, as long β and γ do not vary so its momentums will not vary and the emittance is not effected by the stripping (except by possible scattering which cause transfer from longitudinal to the transverse momentum)

Other Relations

Instead of the rms width σ we could measure the rms angle θ in z' , then we also have

$$E = 6\pi \frac{\sigma^2}{\beta_z}$$

$$= 6\pi \sigma \theta$$

$$= 6\pi \theta^2 \beta_z$$

(3)

Longitudinal Beam Area

A. Bunched Beam -

Longitudinal Area of individual bunch

$$S = 6\pi \sigma_z \sigma_E \quad \text{in eV-sec} \quad (4)$$

where

σ_z , rms bunch length in time unit

$$\sigma_z = \sigma_e / \beta c$$

σ_e , rms bunch length

βc , bunch velocity

σ_E , rms energy spread

For heavy ion σ_E could be measured in eV/A and then the bunch area will be in eV/A-sec units

Observe that here π is not left explicitly out but is included in the computation for S .

Eq. (4) defines the contour for 95% of the beam bunch -

B. Unbunched Beam -

Longitudinal Area of total Beam

$$B = 4 \sigma_E \cdot T_0 \quad \text{in eV-sec} \quad (5)$$

where

T_0 , revolution period

σ_E , rms energy spread

For heavy ion σ_E could be measured in eV/A and then the beam area will be in eV/A-sec -

Also eq. (5) defines a contour for 95% of the beam -

Observe that eqs (4) and (5) are invariants (aside from possible external sources of dilution and diffusion) and do not depend on the beam energy or q or the particle charge states -