

## Definition Of Beam Emittances

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DEFINITION OF BEAM EMITTANCES

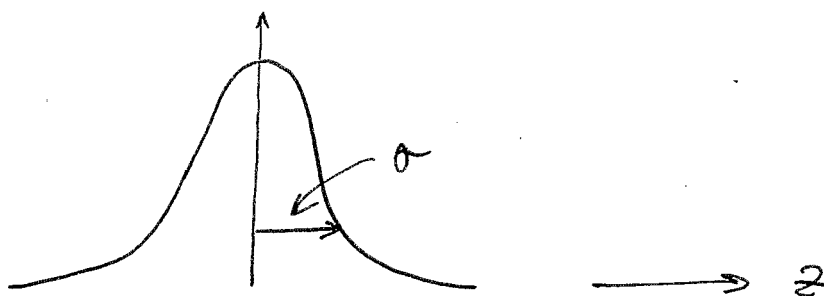
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## Betatron Emittances

Assume that the beam has a gaussian distribution in both  $z$  and  $z'$  (H or V) -

A beam ~~measurement~~ profile measurement will give the rms width  $\sigma$



The rms emittance is then defined as

$$E_{\text{rms}} = 2\pi \frac{\sigma^2}{\beta_z}$$

where  $\beta_z$  is the corresponding lattice amplitude value where the beam profile is taken -

The following emittances

$$E = 4\pi \frac{\sigma^2}{\beta_z} \quad \text{and} \quad 6\pi \frac{\sigma^2}{\beta_z}$$

are commonly used in Europe and USA -  
 the former define a 90% beam contour,  
 the latter 95% of the beam.

For aperture requirement consideration  
 I propose the definition that corresponds to  
 95% of the beam

$$\boxed{\varepsilon = 6\pi \frac{\sigma^2}{\beta_z}} \quad (1)$$

The factor  $\pi$  will always be left explicitly  
 out -

### Normalized Emittance

The emittance defined by (1) is not an  
 invariant - The normalized emittance  $\varepsilon_N$  is  
 defined so that the actual emittance  $\varepsilon$  at  
 some beam energy ~~is~~ described by the rela-  
 tivistic parameters  $\beta$  and  $\gamma$  is given by

$$\boxed{\varepsilon = \frac{\varepsilon_N}{\beta\gamma}} \quad (2)$$

(3)

For heavy ions and protons  $E_N$  is a truly invariant provided dilution and diffusion are not introduced by external means -

We will use definition (2) for heavy ions as well as for protons -

Also observe that definition (2) is independent of the charge status of the ion - In fact also when the ion goes through a stripping foil, as long  $\beta$  and  $\gamma$  do not vary so its momentums will not vary and the emittance is not effected by the stripping (except by possible scattering which cause transfer from longitudinal to the transverse momentum)

### Other Relations

Instead of the rms width  $\sigma$  we could measure the rms angle  $\theta$  in  $z'$ , then we also have

$$E = 6\pi \frac{\sigma^2}{\beta_z}$$

$$= 6\pi \sigma \theta$$

$$= 6\pi \theta^2 \beta_z$$

(3)

## Longitudinal Beam Area

A. Bunched Beam -

Longitudinal Area of individual bunch

$$S = 6\pi \sigma_z \sigma_E \quad \text{in eV-sec} \quad (4)$$

where

$\sigma_z$ , rms bunch length in time unit

$$\sigma_z = \sigma_e / \beta c$$

$\sigma_e$ , rms bunch length

$\beta c$ , bunch velocity

$\sigma_E$ , rms energy spread

For heavy ion  $\sigma_E$  could be measured in eV/A and then the bunch area will be in eV/A-sec units

Observe that here  $\pi$  is not left explicitly out but is included in the computation for  $S$ .

Eq. (4) defines the contour for 95% of the beam bunch -

B. Unbunched Beam -

Longitudinal Area of total beam

$$B = 4 \sigma_E \cdot T_0 \quad \text{in eV-sec} \quad (5)$$

where

$T_0$ , revolution period

$\sigma_E$ , rms energy spread

For heavy ion  $\sigma_E$  could be measured in eV/A and then the beam area will be in eV/A-sec -

Also eq. (5) defines a contour for 95% of the beam -

Observe that eqs (4) and (5) are invariants (aside from possible external sources of dilation and diffusion) and do not depend on the beam energy or  $q$  or the particle charge status -