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Introduction

In this note the lifetime due to inelastic scattering of beam and residual gas ions is calculated in units of pressure (Torr). In addition, the transverse emittance growth due to elastic scattering is expressed in units of pressure. The definition of inelastic scattering includes both capture of an electron from a residual gas ion and central nuclear collisions between beam ion and gas atom. Emittance growth via elastic scattering is a simple consequence of multiple Coulomb scattering.

It is important to note that in an accelerator only the density of residual gas atoms is relevant to the machine operation. The measure of this density is the vacuum gauge, where this gauge is calibrated in pressure units at some known temperature T_G. The vacuum unit or pressure is of course temperature dependent, and thus when quoting vacuum requirements for RHIC it is vital to state the temperature at which the pressure is computed. It might be necessary to scale any computed gas density to the pressure appropriate for the measurement with the vacuum gauge. Typically, the vacuum gauge operates at room temperature $\sim 300^oK$. An explanation on how to rescale pressure as a function of temperature is given in the text.

This note assumes the residual gas density in the so–called warm section (300°K) of RHIC to be composed of 90% H_2, 5% CH_4 and 5% CO. The gas in the cold section (5°K) is assumed to be 100% He. The beam ions are taken to be $^{197}Au^{79+}$.

Variation of Pressure with Temperature

Vacuum requirements in an accelerator can be confusing, for the units of pressure vary with temperature. Furthermore, most vacuum gauges respond to a density of gas ρ_G, where this density is expressed in pressure units P_G at temperature T_G. Of course, both ρ_G and T_G can be different from the beam pipe values. In this note, T_W, ρ_W and P_W

correspond to thermodynamic variables in the warm section of RHIC, and T_c, ρ_c, P_c to the cold section.

The only quantity of interest to machine performance is the density of gas ions inside the beam pipe. From the ideal gas law PV = RT, the equivalent pressure P′ that this density produces at temperature T′ is simply

$$P' = (T'/T)\,P \qquad 1$$

However, if the gauge responds to a density ρ_G, and is calibrated at temperature T_G, we need Knudsens relations for gas flow between two distinct volumes connected by a small tube, i.e.,

$$P = (T/T_G)^{1/2}\,P_G$$

$$\rho = (T_G/T)^{1/2}\,\rho_G \quad . \qquad 2$$

Equating T′ with T_G, these relations give,

$$P' = (T_G/T)^{1/2}\,P_G \quad . \qquad 3$$

As a striking example, if the residual gas density is at 5°K, and the vacuum gauge is at room temperature (300°K), the equivalent pressure at 300°K is scaled by a factor of 300/5 over the pressure at 5°K. Moreover, this pressure is a factor of 7.74 higher than the pressure reading on the gauge.

Inelastic Scattering Lifetimes

The inelastic lifetime τ_I due to both electron capture and nuclear scattering is given by

$$\frac{1}{\tau_I} = c\rho\,(\sigma_C + \sigma_N) \qquad 4$$

where ρ is the residual gas density (atoms cm^{-3}), c is the velocity of light, σ_C is the capture cross section and σ_N is the nuclear collision cross section. Inserting Avogadro's number,

$N_A = 6.023 \times 10^{23}$ molecules/mole, at a standard volume of 22.414 litres/moles, and a standard temperature T_o of 273.15°K gives in pressure units of Torr,

$$\frac{1}{\tau_I} = 3.82 \times 10^{30}\ P_o\,(\mathrm{Torr})\left(\sigma_{\mathrm{C}}\left(cm^2\right) + \sigma_{\mathrm{N}}\left(cm^2\right)\right) \mathrm{hours}^{-1}$$

$$(T_o = 273.15°K)$$

$$= 3.48 \times 10^{30}\ P_G\,(\mathrm{Torr})\left(\sigma_{\mathrm{C}}\left(cm^2\right) + \sigma_{\mathrm{N}}\left(cm^2\right)\right) \mathrm{hours}^{-1} \qquad 5$$

$$(\mathrm{WarmSection})$$

$$= 2.69 \times 10^{31}\ P_G\,(\mathrm{Torr})\left(\sigma_{\mathrm{C}}\left(cm^2\right) + \sigma_{\mathrm{N}}\left(cm^2\right)\right) \mathrm{hours}^{-1}$$

$$(\mathrm{ColdSection})$$

Consider first the warm section of RHIC. The capture cross section σ_{C} is composed of three distinct mechanisms. Radiative electron capture is simply the inverse of the photoelectric effect. If Z_P and Z_T are the projectile and target atomic numbers σ_{REC} scales as

$$\sigma_{\mathrm{REC}} = a\ Z_P^5\ Z_T/\gamma \quad , \qquad 6$$

where γ is the beam Lorentz parameter and a is a constant that depends on the quantum dynamics of the capture process. Non–radiative electron capture, i.e., straight transfer without photon emission scales as

$$\sigma_{\mathrm{NREC}} = b\ Z_P^5\ Z_T^5/\gamma \quad , \qquad 7$$

where b is to be determined from quantum mechanical considerations.

In heavy ion colliders, the possibility of creating an electron–positron pair and subsequently capturing the electron results in the third capture mechanism called vacuum capture. This mechanism scales as

$$\sigma_{\mathrm{VAC}} = dZ_P^5\ Z_T^2\ ln\,(\gamma/\gamma_o) \qquad 8$$

where d and γ_o are constants to be determined from quantum mechanical calculations.

In Table I the constants a, b, d, γ_o are tabulated for H, C, O target atoms[1] and a $^{197}Au^{79+}$ projectile.

TABLE 1

Residual Gas Atom	a(cm^2)	b(cm^2)	d(cm^2)	γ_o
H	7.4×10^{-34}	8.4×10^{-43}	6.2×10^{-37}	7.91
C	7.4×10^{-34}	4.19×10^{-42}	6.2×10^{-37}	7.91
O	7.4×10^{-34}	4.60×10^{-42}	6.2×10^{-37}	7.91

Table I. Constants a, b, d, γ_o for $^{197}Au^{79+}$ projectile.

Using Table I, equations (4) – (5), and the appropriate gas composition percentages, the effective capture cross section is $\sigma_{\mathrm{C}} = 2.5 \times 10^{-25} cm^2$ at $\gamma = 30$ and $\sigma_{\mathrm{C}} = 1.1 \times 10^{-25} cm^2$ at $\gamma = 100$.

For central nuclear collisions, a simple energy independent "billiard ball" model is assumed.

In this model $\sigma_{\mathrm{N}} = \pi R_N^2$ where $R_N = 1.2\left(A_P^{1/3} + A_T^{1/3}\right)$fm.

With our gas composition for the warm section, it is calculated that $\sigma_{\mathrm{N}} = 4.65 \times 10^{-24}\mathrm{cm}^2$.

Comparing σ_{N} with σ_{C}, it can be seen that the energy independent central nuclear collisions dominate over capture at RHIC energies. This is in stark contrast to Booster or AGS energies. From Eq. (2), we find for the warm section of RHIC,

$$\tau_I = 6.04 \times 10^{-2} \quad [P_W\,(\mu\mathrm{Torr})]^{-1} \quad \text{hours} \qquad (T_W \;=\; 300^\circ K)\,, \qquad 9$$

where we assume $\mathrm{T}_G = \mathrm{T}_W$.

The lifetime quoted in Eq. (9) is for a gas density averaged around the whole ring. However, allowing for a 12 × 20 m warm section for the septum magnet at injection, and 12 × 43 m for the insertion warm sections gives the warm fraction of RHIC circumference as .2. Equation (9) is then modified to read,

$$\tau_I = .3 \; [P_W(\mu\text{Torr})]^{-1} \text{ hours} \quad (T_W = 300°K) \quad . \qquad 10$$

Hence for a pressure of 5×10^{-10} Torr, we expect a beam lifetime of 600 hours in the warm section.

Consider now the cold section of RHIC. With 100% He gas we find $\sigma_N = 2.48 \times 10^{-24}\text{cm}^2$. Taking into account the fraction of RHIC that is cold (.8), and ignoring contributions from capture we find

$$\begin{aligned}\tau_I &= 2.42 \times 10^{-3} \; [P_c(\mu\text{Torr})]^{-1} \text{ hours} \quad (T_c = 5°K) \\ &= 1.87 \times 10^{-2} \; [P_G(\mu\text{Torr})]^{-1} \text{ hours} \quad (T_G = 300°K)\end{aligned} \qquad 11$$

Hence for a pressure of 10^{-11} Torr, we expect a beam lifetime of 242 hours in the cold section.

Consider stated vacuum requirements in the LHC as an example. With their value for p– H_2 scattering of $\sigma_N = 10^{-29}m^2$ in Eq. (2), we get $P_o = 1.091 \times 10^{-7}$ Torr at 273.15°K. Using Eq. (1) to scale pressure with temperature gives their stated value of 1.2×10^{-7} Torr at 293°K.

Emittance Growth Due to Elastic Scattering

Multiple elastic Coulomb scattering will cause the transverse emittance of the beam to grow.

The rate of this growth is derived in Appendix A via the Fokker–Planck diffusion equation. The growth rate of the normalized emittance ϵ_N is

$$\frac{d\epsilon_N}{dt} = \frac{\gamma\beta}{2}\dot{\Theta}^2_{rms} \times F \qquad 12$$

where

$$F = \frac{1}{2}\sum_n C_n D_n \left(\frac{-1}{\lambda_n^2}\right) e^{-\lambda_n^2 \beta^2 \dot{\Theta}^2_{rms} \; t/4a^2} \quad , \qquad 13$$

$$C_n = \frac{1}{J_1(\lambda_n)^2}\int_o^1 f_o(z)\, J_o\left(\lambda_n\sqrt{z}\right) dZ \quad , \qquad 14$$

$$D_n = \int_o^{\lambda n} dy \;\, y^3 J_o(y) \quad , \qquad 15$$

λ_n is the root of Bessel function $J_o(\lambda_n) = 0$, and a is now defined to be the RHIC aperture. As explained in Appendix A, for an aperture that is large relative to a Gaussian beam parameter σ, F$\approx$ 1.0. Hence

$$\frac{d\epsilon_N}{dt} \simeq \frac{\gamma\beta}{2} \; \dot{\Theta}^2_{rms} \quad , \qquad 16$$

where β is the average RHIC beta function, and $\dot{\Theta}^2_{rms}$ is given by

$$\dot{\Theta}^2_{rms} \;\; = \;\; \left(\frac{15MeV}{m_p c^2 \gamma}\right)^2 \frac{Z_P^2}{A_P^2} \; \frac{c}{L_{RAD}} \quad , \qquad 17$$

where m_p is the proton mass, and L_{RAD} is the radiation length[2] defined by,

$$\frac{1}{L_{RAD}} = 2\alpha\frac{N_A}{A_T} \;\; Z_T^2 \;\, r_e^2 \;\, \rho \;\; ln(R_T/R_N) \qquad 18$$

where $\alpha = 1/137$, $r_e = 2.82 \times 10^{-13}$cm, and R_T is the Thomas–Fermi screening radius,

$$R_T = 1.4\,(\hbar c)\,/\alpha Z_T^{1/3} m_e c^2 \quad (cm) \quad . \qquad 19$$

Expressing ρ in terms of pressure, the effective radiation length $L^{\mathrm{EFF}}_{\mathrm{RAD}}$ for our warm section gas composition is calculated to be

$$\frac{1}{L^{\mathrm{EFF}}_{\mathrm{RAD}}} = \;\; 10^{-9} P_W\,(\mathrm{Torr})\,cm^{-1} \qquad (T_W = 300°K) \qquad 20$$

Hence for $^{197}Au^{79+}$ ions it is found

$$\frac{d\epsilon_N}{dt} = 112.5\frac{P_W\,(\mu\mathrm{Torr})}{\gamma}\mathrm{mm \;\; mrad \;\; hour}^{-1}$$
$$(T_W = 300°K) \qquad 21$$

Allowing for the fraction of the ring taken up by the warm section gives

$$\frac{d\epsilon_N}{dt} = 22.5 \frac{P_G\,(\mu\text{Torr})}{\gamma} \text{mm mrad hour}^{-1} \qquad (T_G = 300^\circ K) \tag{22}$$

Hence for Au beams at $\gamma = 100$, the normalized emittance grows at a rate of 1.13×10^{-4} mm mrad hour^{-1} for a pressure of 5×10^{-10} Torr in the warm section.

For the cold section of RHIC,

$$\frac{1}{L_{\text{RAD}}^{\text{EFF}}} = 2.13 \times 10^{-8} P_o\,(\text{Torr})\,cm^{-1} \quad (T_o = 5^o K) \tag{23}$$

Hence for $^{197}Au^{79+}$ ions,

$$\frac{d\epsilon_N}{dt} = 2.40 \times 10^3 \frac{P_c\,(\mu\text{Torr})}{\gamma} \text{mm mrad hour}^{-1} \qquad (T_c = 5^\circ K) \tag{24}$$

$$= 3.11 \times 10^2 \frac{P_G\,(\mu\text{Torr})}{\gamma} \text{mm mrad hour}^{-1} \qquad (T_G = 300^\circ K) \tag{25}$$

Hence for Au beams of $\gamma = 100$, the normalized emittance grows at a rate of 2.4×10^{-4} mm mrad hour^{-1} for a pressure of 10^{-11} Torr in the cold section.

References

1) R. Anholt and V. Becker, Phys. Rev. A36, 4628 (1987).

2) Y.S. Tsai, Rev. Mod. Phys. 46, 815 (1974).

Appendix A

Consider emittance growth via the Fokker–Planck equation,

$$\frac{\partial f}{\partial t} = D \frac{\partial}{\partial W}\left(W \frac{\partial f}{\partial W}\right) \qquad A.1$$

where f is the time–dependent distribution of beam particles in the transverse plane. If $\tau = Dt/W_a$ and $Z = W/W_a$ where D is the diffusion constant and $W_a = a^2/\beta$, where a is the aperture and β the beta function,

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial Z}\left(Z \frac{\partial f}{\partial Z}\right) \qquad A.2$$

However if $f(Z, \tau = 0) \equiv f_o(Z)$ then the standard solution to Eq.(A.1) is

$$f(Z,\tau) = \sum_n C_n J_o\left(\lambda_n \sqrt{Z}\right) e^{-\lambda_n^2 \tau/4} \quad ,$$

where

$$C_n = \frac{1}{J_1(\lambda_n)^2} \int_o^1 f_o(Z)\, J_o\left(\lambda_n \sqrt{Z}\right) dZ \quad , \qquad A.3$$

and $J_o(\lambda_n) = 0$

We assume a Gaussian for $f_o(Z)$,

$$f_o dZ = \frac{1}{2\pi\sigma_o^2}\, e^{-r^2/2\sigma_o^2} r\, dr\, d\theta = \frac{a^2}{2\sigma_o^2}\, e^{-a^2 Z/2\sigma_o^2}\, dZ \quad , \qquad A.4$$

$$\left(r = a\sqrt{Z}\right)$$

and are interested in how σ changes in time. Looking at the second moment $I(\tau = 0)$ of $f_o(Z)$ gives

$$I(\tau = 0) = 2\sigma_o^2 - 2\sigma_o^2\, e^{-\alpha}\left[\alpha + 1\right] \qquad A.5$$

where $\alpha = a^2/2\sigma_o^2$. Hence as $\alpha \to \infty$ I(τ=0)= $2\sigma_o^2$ as required. Similarly, looking at the second moment I(τ) of f(Z,τ) gives,

$$I(\tau) = 2\sigma_o^2 G(\tau) \qquad A.6$$

where

$$G(\tau) = 2\alpha \sum_n C_n D_n \frac{1}{\lambda_n^4}\, e^{-\lambda_n^2\, \tau/4} \quad , \qquad A.7$$

and

$$D_n = \int_o^{\lambda_n} dy\, y^3 J_o(y) \quad . \qquad A.8$$

These equations show how the Gaussian spreads as a function of the universal parameter τ, where τ is proportional to the diffusion constant.

Let us rewrite Eqs. (A.6) – (A.8) for emittance growth in real time. The diffusion constant for our problem is

$$D = \beta \dot{\Theta}_{rms}^2 \quad , \qquad A.9$$

hence

$$\tau = \frac{a^2}{\beta^2} \frac{1}{\dot{\Theta}_{rms}^2} \tau \quad . \qquad A.10$$

Thus, from Eq. (A.6)

$$\frac{dI}{dt} = 2\sigma_o^2 \frac{d\tau}{dt} \frac{dG(\tau)}{d\tau} \qquad A.11$$

$$= \frac{\beta^2 \dot{\Theta}_{rms}^2}{2} \sum_n C_n D_n \left(\frac{-1}{\lambda_n^2}\right) e^{-\lambda_n^2 \beta^2 \dot{\Theta}_{rms}^2\, t/4a^2} \qquad A.12$$

Using $\sigma(t) = \sqrt{\epsilon_N \beta/\gamma}$ and I = $2\sigma^2$(t) gives

$$\frac{d\epsilon_N}{dt} = \frac{\gamma\beta\dot{\Theta}^2_{rms}}{2} \sum_n \frac{1}{2} C_n D_n \left(\frac{-1}{\lambda_n^2}\right) e^{-\lambda_n^2 \beta^2 \dot{\Theta}^2_{rms}/4a^2} \qquad A.13$$

Evaluating the sum over n for large a gives a value equal to approximately unity (1.0019 for $\alpha = 10$ and 1.0741 for $\alpha = 30$) when α is large ($\geq$5).

Hence for large aperture,

$$\frac{d\epsilon_N}{dt} \simeq \frac{\gamma\beta\dot{\Theta}^2_{rms}}{2} \qquad A.14$$