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# A Feedback Device to Damp the Coherent Oscillations from Injection Errors in RHIC

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**RHIC PROJECT**

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If the beam is injected with closed orbit errors $x_c$, $x'_c$ (or $y_c$, $y'_c$) at the injection point of a circular accelerator, the beam will execute coherent oscillations and will be diluted in betatron phase space within a time interval of about $1/\Delta\nu$ turns, even if it is properly matched to the focusing characteristics of the lattice, unless there is an effective damper system to prevent this. Here $\Delta\nu$ is the tune spread of the beam. Such a damper will not prevent dilution due to mismatches. Without such a damper the emittance of the beam will ultimately develop to a properly centered matched ellipse with an area in phase space that is larger than that of the injected one which is also matched but off–centered by $x_c$ and $x'_c$.

Let the equations of the centered ellipse and the injected off–centered but matched ellipse be

$$\epsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

and

$$\epsilon_0 = \gamma(x - x_c)^2 + 2\alpha(x - x_c)(x' - x'_c) + \beta(x' - x'_c)^2$$

respectively, where $\alpha$,$\beta$,$\gamma$ are the Twiss parameters at the injection point and $\epsilon_0$ is the injected beam emittance.

The dilution factor $\epsilon/\epsilon_0$ can be expressed as follows:

$$\frac{\epsilon}{\epsilon_0} = \left[1 + \sqrt{\frac{\epsilon_c}{\epsilon_0}}\,\right]^2$$

where $\epsilon_c = \gamma x_c^2 + 2\alpha x_c x'_c + \beta x'^2_c$. We have similar relations for the y–direction.

The allowable injection errors $x_c$, $x'_c$ and $y_c$, $y'_c$ in RHIC for dilution factor $\frac{\epsilon}{\epsilon_0}$ equal to 1.2, 1.4, 1.6, ... and 2.4 have been calculated and are shown in Figures 1 and 2. The lattice function values used at injection are:

Horizontal:

$\beta = 9.09$ meters

$\alpha = 0.00$

Vertical:

$\beta = 49.81$ meters

$\alpha = 0.00$

If we require an emittance growth less than 20% in both directions, the maximum allowable values are $x_c = 0.25$ mm, $x'_c = 0.03$ mrad, $y_c = 0.6$ mm and $y'_c = 0.012$ mrad. These tolerances may be difficult to meet and a feedback system to damp the coherent oscillations induced by the injection errors should be considered for RHIC.

We present an analysis of the requirements for a damper system to correct the coherent oscillations induced by injection errors. Coherent oscillations would be completely smeared after a number of turns which is the inverse of the tune spread $\Delta\nu$ in the beam. Due to space charge and other nonlinear effects we expect $\Delta\nu \sim 0.01$, so that the damper system is required to act fast enough to damp the oscillations within at most 100 turns.

The damper is made of a beam position monitor from where a signal proportional to the beam displacement is taken, amplified and transported to another location where it is applied across a kicker for the correction.

The induced voltages on the two electrodes of the beam position monitor are:

$$V_+ = \frac{NeZ}{T} R_p \left(1 + \frac{x_p}{d_p}\right) \tag{1}$$

$$V_- = \frac{NeZ}{T} R_p \left(1 - \frac{x_p}{d_p}\right) \tag{2}$$

where $N$ is the number of ions in a bunch, $Z$ is the charge state of the ion, $T$ is the revolution period of the bunch, $R_p$ is the effective impedance of the position monitor assumed to be made of strip line, $d_p$ is the distance between the two electrodes and $x_p$ is the position of the bunch center relative to the monitor center.

The combination of the two signals from the two electrodes $V_+$ and $V_-$ with a suitable power combiner is:

$$V_p = \frac{NeZ}{T} R_p \frac{x_p}{d_p} \frac{2}{\sqrt{2}} \ . \tag{3}$$

The voltage on the kicker $V_k$ is

$$V_k = GV_p \tag{4}$$

where $G$ is the amplification factor of the amplifier between the signal of the position monitor and the kicker.

The effect of the damper kicker on the ion motion can be expressed as follows:

$$A m_o \gamma \ddot{x} = \frac{ZeV_k}{d_k}\delta(s) \tag{5}$$

where $A$ is the atomic number of the ion, $m_o$ is the rest mass of nucleon and $d_k$ is the gap of the kicker, or,

$$\ddot{x} = \beta^2 c^2 \frac{d^2x}{ds^2} = \frac{ZeV_k\delta(s)}{Am_o\gamma d_k}$$

where $s$ is the kicker length along the ideal closed orbit.

$$x'' = \frac{d^2x}{ds^2} = \frac{ZeV_k\delta(s)}{A\beta^2\gamma E_o d_k}\ , \tag{6}$$

where $E_o = m_o c^2$. And

$$\Delta x'_k = \theta_k = \frac{ZeV_k\ell}{A\beta^2\gamma E_o d_k}\ . \tag{7}$$

Where $\theta_k$ is the kick (angular change) on the bunch when it passes through the kicker and $\ell$ is the length of the kicker.

We are interested in the determination of the damping rate. For this purpose we represent the effect of the kicker in matrix notation. Let $X_n \equiv (x_n, x'_n)$ be the vector representing the coherent oscillation at the pick–up location at the n–th crossing. Then the vector at the (n+1)–th crossing is

$$X_{n+1} = M_{kp}\ M_k\ M_{pk}\ X_n \tag{8}$$

where $M_{pk}$ and $M_{kp}$ are the $2 \times 2$ transfer matrices respectively from pick–up to kicker and from kicker to pick–up, and $M_k$ is the matrix representing the effect of the kicker. We have

$$\begin{aligned} M_k &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - k_0 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} M_{pk}^{-1} \\ &= I - k_0 R\ M_{pk}^{-1} \end{aligned} \tag{9}$$

where

$$k_0 = \frac{ZeG\ell}{A\beta^2\gamma E_0 d_k}\ \frac{NeZ}{T}\ \frac{2R_p}{\sqrt{2}d_p} \tag{10}$$

is a damper parameter that can be obtained by combining Eqs. (3,4 and 7).

We have

$$\begin{aligned} M &= M_{kp} M_k M_{pk} \\ &= M_0 - k_0 M_{kp} R \end{aligned} \tag{11}$$

where $M_0 = M_{kp} M_{pk}$ is the unperturbed transfer matrix at the pick–up location.

Let

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \tag{12}$$

and

$$M_{kp} R = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \tag{13}$$

with

$$\begin{aligned} a &= \sqrt{\beta_k \beta_p} \sin \psi_{kp} \\ b &= \sqrt{\frac{\beta_k}{\beta_p}} (\cos \psi_{kp} - \alpha_p \sin \psi_{kp}) \end{aligned} \tag{14}$$

where $\psi_{kp}$ is the phase advance from kicker to beam monitor. The damping rate is given by the eigenvalues of the total transfer matrix $M$

$$M = \begin{pmatrix} m_{11} - k_0 a & m_{12} \\ m_{21} - k_0 b & m_{22} \end{pmatrix} \tag{15}$$

the determinant of which

$$\text{Det}M = 1 - k_0 (m_{22} a - m_{12} b) \neq 1 \tag{16}$$

The eigenvalues are the solutions of the quadratic equation

$$\lambda^2 - \lambda(2 \cos \mu_0 - k_0 a) + \text{Det}M = 0 \tag{17}$$

where $\mu_0$ is the unperturbed phase advance per resolution. If, finally, we let

$$K = k_0 \sqrt{\beta_p \beta_k} \tag{18}$$

then

$$\lambda = \cos \mu_0 - \frac{k}{2} \sin \psi_{kp} \pm i \sqrt{(\sin \mu_0 + \frac{k}{2} \cos \psi_{kp})^2 - (\frac{k}{2})^2} \ . \tag{19}$$

There are two cases:

(i) $k_1 \leq k \leq k_2$ where $k_1$ and $k_2$ are the roots of the equation

$$(\sin \mu_0 + \frac{k}{2} \cos \psi_{kp})^2 - (\frac{k}{2})^2 = 0 \tag{20}$$

$$k_1 = -2 \sin \mu_0 / (1 + \cos \psi_{kp}) \tag{21}$$

$$k_2 = 2 \sin \mu_0 / (1 - \cos \psi_{kp}) \tag{22}$$

assuming $\sin \mu_0 > 0$. The roots are to be taken in the opposite order if $\sin \mu_0 < 0$.

In this case the eigenvalues of the system are complex quantities

$$\lambda = \sqrt{1 + k \sin \psi_{pk}} e^{\pm i\mu} \tag{23}$$

where $\psi_{pk}$ is the phase advance from monitor to the kicker. Apart from a phase factor $\mu$, the damping rate is $\sqrt{1 + k \sin \psi_{pk}}$ per turn. In order to provide effective damping it is required that $-1 < k \sin \psi_{pk} < 0$.

(ii) If

$$\left(\frac{k}{2}\right)^2 > (\sin \mu_0 + \frac{k}{2} \cos \psi_{kp})^2 \tag{24}$$

then the eigenvalues are real quantities. If the condition $-1 < k \sin \psi_{kp} < 0$ is satisfied, the damping rate of the coherent oscillations is given by $\frac{1}{2} k \sin \psi_{pk}$ per turn. In order to maximize the damping effect, one chooses locations for pickup and kicker so that the phase advance is an odd integer times $\pi/2$, that is

$$\sin \psi_{pk} \sim \pm 1 \tag{25}$$

in which case the damping rate is simply given by $k/2$ per turn. for RHIC we require $k \simeq 2\Delta\nu$ where $\Delta\nu \sim 0.01$.

In the case of RHIC, for gold beam, $N = 2 \times 10^9$, $Z = 79$, $A = 197$, $T = 12.8$ $\mu$sec, $R_p = 50\ \Omega$, $d_p = 76$ mm, $\sqrt{\beta_p \beta_k} = 50$ m. Then from Eq. (3) $V_p = 1.8 \times 10^{-3} x_p$ volts and from Eqs. (10,18)

$$K = \frac{3.3 \times 10^{-9} \ell G}{d_k} .$$

We get for the required voltage gain

$$G = \frac{6.7 d_k}{\ell} \times 10^6 \ . \tag{26}$$

If $d_k = 76$ mm, $\ell = 2000$ mm, the required $G$ is

$$G \simeq 2.3 \times 10^5 \ . \tag{27}$$

The peak power $P_k$ of the amplifier is

$$\begin{aligned} P_k &= \frac{V_k^2}{R_k} = \frac{(GV_p)^2}{R_k} \\ &= \frac{1.25 \times 10^8}{R_k} \left(\frac{d_k x_p}{\ell}\right)^2 \text{ watts} \ . \end{aligned} \tag{28}$$

If $x_{p,max} = 1$ mm, $R_k = 50\ \Omega$, $\ell = 2000$ mm, $d_k = 76$ mm, then the peak power is

$$P_{k,max} = 3.6 \times 10^3\ W \ .$$

The kicker power is inversely proportional to the square of the kicker length. If we increase $\ell$ from 2 meters to 4 meters and keep $R_k = 50\ \Omega$, then the peak power will be 900 W.

The rise and fall times of the damper–kicker system should be less than 100 nsec each to allow damping of the coherent motions of $2 \times 57$ bunches individually. The power requirement does not depend on the number of bunches in the RHIC, since the value given is the peak value. The power consumption can be reduced further by increasing the number of kickers. For instance with 4 sets of kickers each 4 meters long, the total power required is $P_{k,max} = 900\ \text{W}/4 \simeq 225$ W. We assume this value reasonable and attainable which thus set the maximum beam displacement to 1 mm. Otherwise the power requirement would increase quadratically with the beam displacement.

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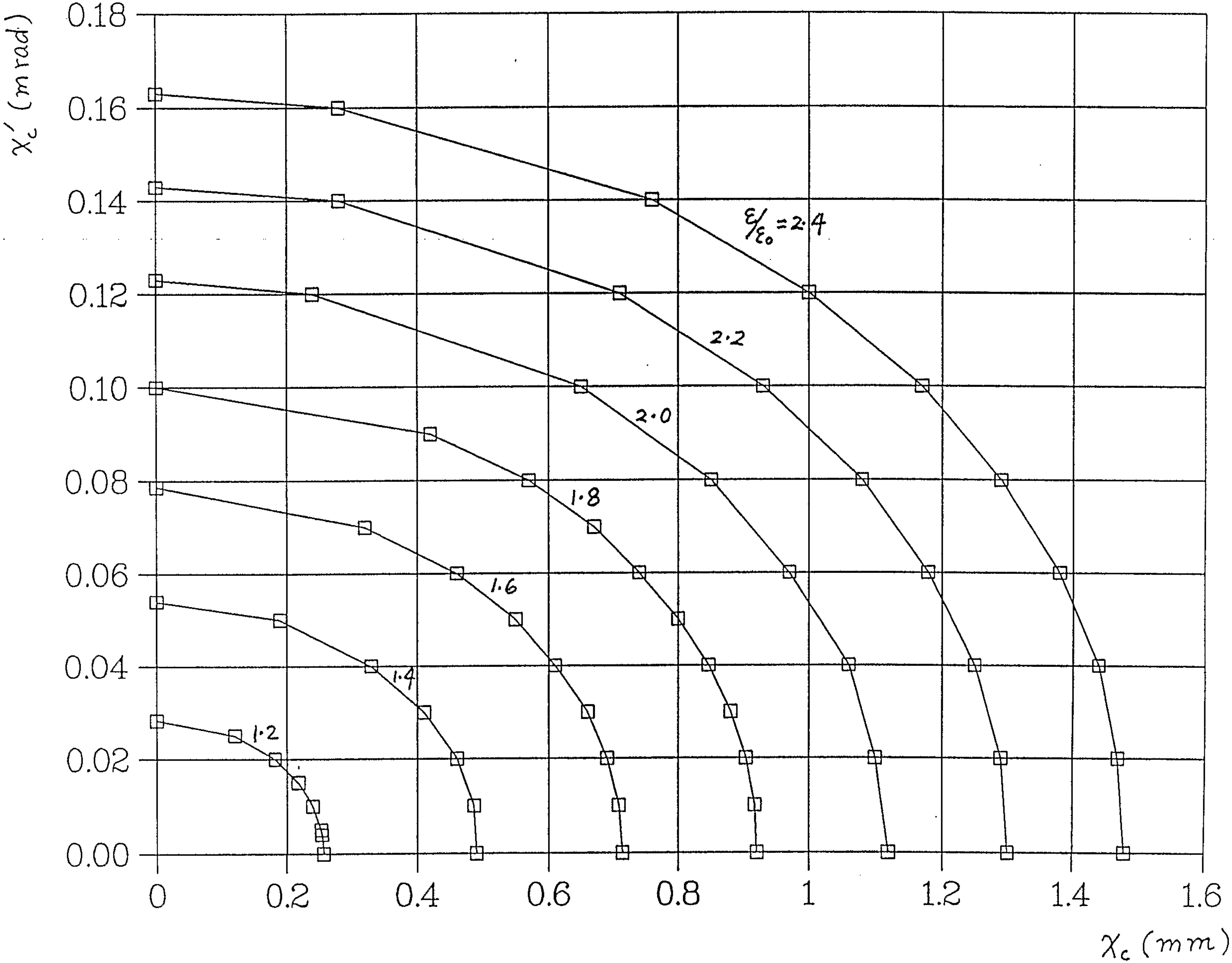

0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
$x_c'$ (mrad)
$x_c$ (mm)
$\varepsilon/\varepsilon_0 = 2.4$
2.2
2.0
1.8
1.6
1.4
1.2


Fig. 1

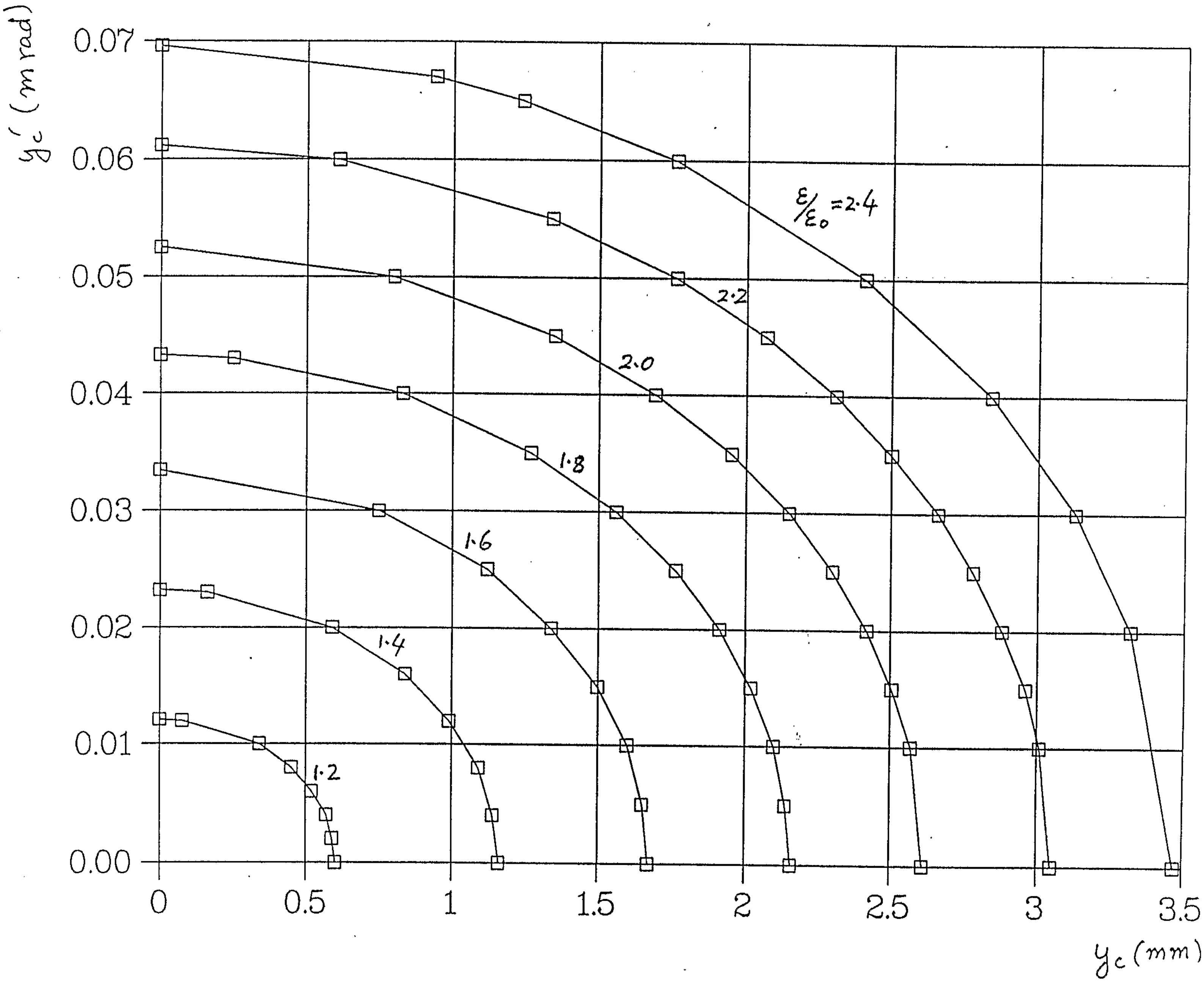
$y_c'$ (m rad)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
0.5
1
1.5
2
2.5
3
3.5
$y_c$ (mm)
$\varepsilon/\varepsilon_0 = 2.4$
2.2
2.0
1.8
1.6
1.4
1.2

Fig. 2