BNL-102002-2014-TECH

AD/RHIC/90;BNL-102002-2013-IR

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May 1991

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U.S. Department of Energy

USDOE Office of Science (SC)

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BNL–45515
AD/RHIC–90
Conference Paper

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RHIC PROJECT

Brookhaven National Laboratory
Associated Universities, Inc.
Upton, NY 11973

Under Contract No. DE–AC02–76CH00016 with the
UNITED STATES DEPARTMENT OF ENERGY

1991 IEEE Particle Accelerator Conference
San Francisco, CA
May 6-9, 1991 BNL-45515

BEAM LIFE–TIME WITH INTRABEAM SCATTERING AND STOCHASTIC COOLING*

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Abstract

A transport equation has been derived in terms of the longitudinal action variable to describe the time evolution of the longitudinal density distribution of a bunched hadron beam in the presence of intrabeam scattering and stochastic cooling. A computer program has been developed to numerically solve this equation. Both beam loss and bunch–shape evolution have been investigated for the $^{197}Au^{79+}$ beams during the 10–hour storage in the Relativistic Heavy Ion Collider currently under construction at the Brookhaven National Laboratory.

I. INTRODUCTION

During the 10–hour storage of the heavy–ion beams in the RHIC, one of the major concern is the particle loss and luminosity decrease caused by Coulomb interaction between the particles in the bunch. Most of the theories[1–4] developed on the subject of intrabeam scattering (IBS) concentrate on the growth of rms beam dimensions under the assumption that particle distribution remains Gaussian in both transverse and longitudinal phase space, thus disregarding boundary limitations and particle loss. Previous studies[4, 5] on RHIC using those theories indicate that the bunch area is in most cases comparable to the rf–bucket area during the storage. It is thus expected that particle loss through the edge of the rf buckets is appreciable. Under this circumstance, intrabeam–scattering calculation without taking into account the beam loss is inadequate to describe the particle motion.

This paper presents a new approach to the problem of beam life–time based on the Fokker–Planck equation for the density distribution function Ψ of the particles in the presence of intrabeam scattering and longitudinal stochastic cooling. Part A of section II introduces the transport equation in terms of the action variable J which describes the time evolution of Ψ. The coefficients of dynamic friction and diffusion due to IBS are obtained in part B, while the coefficients of coherent correction and diffusion due to stochastic cooling are evaluated in part C. Numerical method is addressed in section III to solve the transport equation for given initial distribution and boundary conditions. Loss and instantaneous distribution of the fully–stripped gold ions during the storage in the RHIC are investigated in section IV.

*Work performed under the auspices of the U.S. Department of Energy.

II. THEORETICAL APPROACHES

A. The Transport Equation

A transport equation in terms of angle–action variables Q and J describing the evolution of longitudinal particle distribution, can be obtained by averaging the 6–dimensional Fokker–Planck equation over all transverse variables. Because the time for intrabeam scattering and stochastic cooling to produce appreciable effect is typically much longer than the synchrotron–oscillation period, which is again much longer than the correlation time of the collision process, this equation can be further reduced by averaging over the angle variable Q for one synchrotron–oscillation period

$$\frac{\partial \Psi}{\partial t} = -\frac{\partial}{\partial J}(F\Psi) + \frac{1}{2}\frac{\partial}{\partial J}\left(D\frac{\partial \Psi}{\partial J}\right), \qquad (1)$$

where

$$F(J;t) = \int_0^1 dQ \, \frac{\langle \Delta J \rangle_{C,T}}{\Delta t}\bigg|_0, D(J;t) = \int_0^1 dQ \, \frac{\langle (\Delta J)^2 \rangle_{C,T}}{\Delta t}\bigg|_0. \qquad (2)$$

Here, $\langle \, \rangle_{C,T}$ denotes the average over both collision events and transverse variables. The subscript 0 in Eq. (2) implies that the integration over Q is performed along contours of particle motion in the absence of IBS and cooling. The boundary condition to this equation is

$$\begin{cases} J = 0: & -F\Psi + \dfrac{D}{2}\dfrac{\partial \Psi}{\partial J} = 0 \, , \\ J = \hat{J}: & \Psi = 0 \end{cases} \qquad (3)$$

B. Intrabeam Scattering

In terms of ϕ and W, the longitudinal equations of motion at storage are

$$\dot{W} = -\frac{C_\phi}{2}\sin\phi + U_W, \quad \dot{\phi} = 2C_W W, \qquad (4)$$

where

$$C_W = \frac{h^2\omega_0^2\eta}{2E\beta^2} \text{ and } C_\phi = \frac{qe\hat{V}}{\pi h}.$$

$\hat{V}$ is the peak voltage, $\eta = 1/\gamma_T^2 - 1/\gamma^2$, γ_T is the transition energy, $E = Am_0c^2\gamma$, βc is the synchronous velocity, ω_0 is the revolution frequency, and q and A are the charge and atomic number, respectively. Regarding the rate of energy increment U_W due to IBS and cooling as a perturbation, the unperturbed particle motion is

Hamiltonian. The constant of motion J can be obtained through a canonical transformation,

$$J = 8\sqrt{\frac{C_\phi}{C_W}}\left[\left(k^2-1\right)\mathrm{K}(k) + \mathrm{E}(k)\right], \quad k \le 1, \tag{5}$$

where k is[8] the normalized Hamiltonian, K and E are complete elliptical integrals of first and second kind. With the perturbation, the rate of increment in J can be expressed in terms of U_W

$$\frac{\Delta J}{\Delta t} = \left.\frac{\partial W}{\partial J}\right|_\phi^{-1} U_W, \tag{6}$$

where

$$\left.\frac{\partial W}{\partial J}\right|_\phi^{-1} = 8k\ \mathrm{K}(k)\cos 2\pi Q\ \left[1 - 4\xi\sin^2 2\pi Q + O\left(\xi^2\right)\right]. \tag{7}$$

Here, ξ becomes significant[8] only near the separatrix.

U_W may be evaluated in the rest frame of the synchronous particle where the motion of the particles is non-relativistic. Let $|\mathbf{u}| = |\bar{\mathbf{v}}_1 - \bar{\mathbf{v}}_2|$ denote the relative velocity between the test particle 1 and a media particle 2 in the rest frame. Rutherford's formula for the cross section of the Coulomb scattering is

$$\sigma(u,\theta) = \frac{q^4 e^4}{A^2 m_0^2 u^4 \sin^4\theta/2}, \tag{8}$$

where θ is the angle through which the velocity vector $\mathbf{u}$ undergoes a rotation during the collision. Integrating over both θ and the azimuthal scattering angle, the change in the longitudinal component of the velocity of the test particle and its square per unit rest-frame time[6–8] can be shown as

$$\langle\Delta\bar{v}_{z1}\rangle_\Omega = -2\Gamma\,\frac{u_z}{u^3}, \quad \text{and} \quad \langle(\Delta\bar{v}_z)_1^2\rangle_\Omega = \Gamma\,\frac{u_x^2 + u_y^2}{u^3}, \tag{9}$$

where

$$\Gamma \equiv \frac{4\pi q^4 e^4 \mathrm{Log}}{A^2 m_0^2}, \quad \mathrm{Log} \equiv -\ln\sin\frac{\theta_{min}}{2},$$

and θ_{min} is the minimum scattering angle. The Coulomb logarithm Log can be verified to be much larger than 1, which implies that the Fokker–Planck equation is a good approximation to describe the particle motion. A value of 20 is designated[3] to Log for simplicity.

Based on Eq. (9), the rate of average energy increment of the test particle in the laboratory frame is evaluated by integrating over all the velocity components of the media particles involved in the collision. If the distributions in horizontal and vertical phase space are assumed as Gaussians, F and D can be obtained by integrating over all the transverse components of the test particle,

$$F(J) = \int\frac{2dz}{\pi R}\int_0^{\frac{1}{4}} dQ\,\left.\frac{\partial W}{\partial J}\right|_\phi^{-1}(Q,J)\int_{J_{min}}^{\hat{J}}\left.\frac{\partial W}{\partial J}\right|_\phi\left(Q',J'\right) \times\left[A_F(\lambda_-) + A_F(\lambda_+)\right]\Psi\left(J'\right)dJ', \tag{10}$$

$$D(J) = \int\frac{2dz}{\pi R}\int_0^{\frac{1}{4}} dQ\left[\left.\frac{\partial W}{\partial J}\right|_\phi^{-1}(Q,J)\right]^2\int_{J_{min}}^{\hat{J}}\left.\frac{\partial W}{\partial J}\right|_\phi\left(Q',J'\right) \times\left[A_D(\lambda_-) + A_D(\lambda_+)\right]\Psi\left(J'\right)dJ', \tag{11}$$

where $\lambda_\mp = \dfrac{h\omega_0 a}{\gamma\beta^2 E}(W \mp W')$, $a = \frac{1}{2}\sqrt{\frac{6\beta\gamma\beta_{x,y}}{\epsilon_{Nx,y}}}$,

$$A_F(\lambda) = -\frac{2\Gamma E}{\beta^2\gamma^4 c^4}\frac{I_F(\lambda)}{4\pi\sigma_{x_\beta}\sigma_y}, \quad A_D(\lambda) = \frac{\Gamma E^2 R}{\gamma^3\beta c^5 h}\frac{I_D(\lambda)}{4\pi\sigma_{x_\beta}\sigma_y}. \tag{12}$$

Here, $\sigma_{x_\beta,y} = \sqrt{\beta_{x,y}\epsilon_{Nx,y}/6\beta\gamma}$ are the rms bunch widths that are time dependent. $\beta_{x,y}$ and $\alpha_{x,y}$ are the Courant–Snyder parameters. $\epsilon_{Nx,y}$ are the normalized emittances. The first integral in Eqs. (10&11) represents the average over the machine lattice; the second integral represents the average over synchrotron–oscillation period; while the third integral describes particles of different action J' involved in the collision. The integration over J' is performed such that $k(J')\sin 2\pi Q' \approx \sin[\phi(Q,J)/2]$, extending from J_{min} to the bunch edge $\hat{J}$, with $k(J_{min}) \approx [\sin\phi(Q,J)/2]$. For a round beam with $\beta_x x_p' + \alpha_x x_p \sim 0$,

$$\begin{aligned} I_F(\lambda) &= 2a^2\mathrm{sgn}(\lambda)\,\chi\left\{1 - \sqrt{\pi}|\lambda|e^{\lambda^2}\left[1 - \Phi(\lambda)\right]\right\} \\ I_D(\lambda) &= a\chi\left\{\sqrt{\pi}\left(1 + 2\lambda^2\right)e^{\lambda^2}\left[1 - \Phi(\lambda)\right] - 2|\lambda|\right\}, \end{aligned} \tag{13}$$

where Φ is the error function. Note that $\chi = e^{-\left(x_p\gamma\lambda/2a\sigma_{x_\beta}\right)^2}$ represents the damping of the dispersion x_p on the diffusion process.

C. Longitudinal Stochastic Cooling

The corresponding coefficients of Eq. (1) due to longitudinal stochastic cooling can be obtained by similar averaging procedures. The coherent correction F provided by the cooling system is[8–9]

$$F(J) = \frac{q^2 e^2\omega_0}{\pi}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\frac{l}{m}\mathrm{J}_l^2\left[m\omega_0\tau(J)\right] \times Re\left[G_F(m,+l) - G_F(m,-l)\right], \tag{14}$$

where

$$G_F(m,\pm l) = G\left[m\omega_0 \pm l\Omega(J)\right]\ e^{\pm il\Omega(J)\Delta\theta^{PK}/\omega_0}. \tag{15}$$

Here, $\tau(J) = \arccos\left(1 - 2k^2\right)/h\omega_0$ is the oscillation amplitude in time, $\Omega(J)$ is the synchrotron–oscillation frequency, and $\Delta\theta^{PK}$ is the azimuthal distance between the pick–up and the kicker. The summation on the revolution bands in Eq. (13) is performed over the system bandwidth, while the summation on the synchrotron sidebands is actually performed from $l = 1$ to $m\omega_0\hat{\tau}$. The factor $e^{il\Omega(J)\Delta\theta^{PK}/\omega_0}$ represents the phase slip that non– synchronous particles experience during their passage from the pick–up to the kicker. In order to minimize this "bad mixing", the distance between the pick-up and the kicker should be for any harmonic n in the bandwidth

$$\Delta\theta^{PK} n\hat{\tau}\Omega(0) \ll 1, \qquad \hat{\tau} \equiv \tau\left(\hat{J}\right). \tag{16}$$

The diffusion is contributed from both the particle Schottky noise and the system thermal noise. If the revolution bands are not overlapping within the system bandwidth, the Schottky coefficient D_{SH} is

$$D_{SH}(J) = \frac{Nq^4e^4\omega_0^2}{\pi} \sum_{l=1}^{\infty}\sum_{k=1}^{\infty} \frac{k^2\Psi(J')}{l\left|\frac{d\Omega(J')}{dJ'}\right|}\Bigg|_{\Omega(J')=k\Omega(J)/l}$$
$$\times \left\{|G(k,l)|^2 + |G(-k,-l)|^2 - 2Re\left[G(k,l)\,G(-k,-l)\right]\right\}, \tag{17}$$

where the double summation on l and k represents synchrotron sideband overlapping, and

$$G(k,l) = \sum_{m=1}^{\infty} \frac{G\left[m\omega_0 + l\Omega(J')\right]}{m} J_k\left[m\omega_0\tau(J)\right] J_l\left[m\omega_0\tau(J')\right]. \tag{18}$$

The thermal coefficient D_T is expressed in terms of the thermal temperature T^P at the pick-up,

$$D_T(J) = 2k_BT^Pq^2e^2 \sum_{m=1}^{\infty}\sum_{l=1}^{\infty} \frac{l^2}{m^2} J_l^2\left[m\omega_0\tau(J)\right] \tag{19}$$
$$\times\left[|G_T(m,+l)|^2 + |G_T(m,-l)|^2\right],$$

where G_T is the gain excluding the pick-up. The thermal contribution often appears negligible for the cooling of beams of high charge-state ions.

III. COMPUTER TECHNIQUES

A computer program has been developed to numerically solve Eq. (1) with given initial distribution and boundary conditions. The instantaneous rate of change of the transverse emittance is provided by standard IBS calculations.[4] The accuracy of this numerical approach is obtained by evaluating the zeroth moment of J and then by comparing to the expected value, which is equal to 1 in the case of proper cooling without particle loss.

IV. BEAM LIFE-TIME IN RHIC

Consider the beam of one of the highest charge-state ions $^{197}Au^{79+}$ in the RHIC where intrabeam scattering is expected to be the severest. Each bunch contains 10^9 particles. The initial bunch area is 0.3eV·s/u. In order to minimize beam growth and luminosity variation, the transverse emittance is initially blown-up to a normalized value of 60πmm-mrad. Thereafter, the growth in transverse emittance is small.

In the absence of cooling, particle loss can be minimized by keeping the peak rf voltage at a constant, maximally achievable value of 4.5MV. Figure 1 shows the time evolution of Ψ during the 10-hour period of operation. The beam loss becomes significant after about 3 hours when the bunch area is comparable to the bucket area. The total beam loss is about 20%. The asymptotic distribution in longitudinal phase space is found to be Gaussian-like, independent of initial conditions.

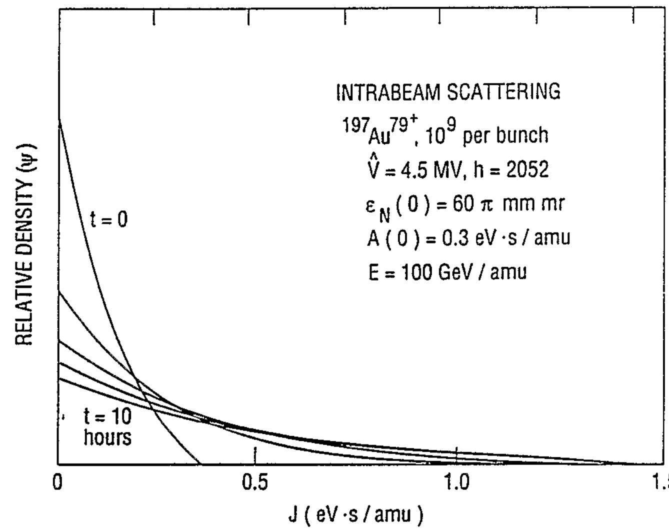

Figure 1: Evolution of the longitudinal distribution function Ψ as a function of longitudinal phase space area J. (t=0, 2.5, 5, 7.5, and 10 hour).

With a longitudinal stochastic cooling system of 4-8 GHz bandwidth, the bunch will reach a stationary state after about 3 hours. Beam loss is essentially eliminated. Because the mixing factor is much larger than 1, full-turn delay between pick-up and kicker is plausible. A factor of 3 increase in average luminosity can be achieved by applying the cooling without initially blowing up the transverse emittance.

V. ACKNOWLEDGMENT

The authors would like to thank Dr. G.Parzen for the valuable information on transverse beam growth, and Dr. J.Marriner for helpful discussion.

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