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The Influence of the Off Center of BPM on the Damper System

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In reference 1, the motion of the beam center and the required parameters of the damper system has been studied. In that study, we assume that the center of the BPM is just on the reference orbit. Now, the influence of BPM displacement will be studied. Let $-d$ be the transverse position of the BPM center relative to the reference orbit and $X_n \equiv (x_n, x'_n)$ be the vector representing the coherent oscillation at the BPM at the nth crossing. Then the vector at its (n+1)th crossing is:

$$X_{n+1} = M_{kp}\left(I - K_0 R M_{pk}^{-1}\right) M_{pk} X_n - M_{kp} K_0 R \begin{pmatrix} d \\ 0 \end{pmatrix} , \tag{1}$$

where M_{pk} and M_{kp} are the 2×2 transfer matrices respectively from BPM to kicker and from kicker to BPM and $K_0 R$ is the matrix representing the effect of the damper kicker. We have

$$K_0 R = K_0 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{2}$$

and

$$K_0 = \frac{ZeG\ell}{A\beta^2\gamma E_0 d_k} \frac{NeZ}{T} \frac{2R_p}{\sqrt{2} d_p} , \tag{3}$$

is the damper system parameter.[1]

We have

$$X_{n+1} = (M_0 - K_0 M_{kp} R) X_n - K_0 M_{kp} R \begin{pmatrix} d \\ 0 \end{pmatrix} \tag{4}$$

where $M_0 = M_{kp} M_{pk}$ is the undisturbed transfer matrix at the BPM. Let

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} . \tag{5}$$

Then

$$X_{n+1} = \begin{pmatrix} m_{11} - K_0 a & m_{12} \\ m_{21} - K_0 b & m_{22} \end{pmatrix} X_n - d \begin{pmatrix} K_0 a \\ K_0 b \end{pmatrix} \tag{6}$$

[1] A Feedback Device to Damp the Coherent Oscillations from Injection Errors in RHIC, J. Xu, J. Claus and A.G. Ruggiero, AD/RHIC–74, July 1990.

with

$$\begin{aligned} a &= \sqrt{\beta_k \beta_p} \sin \psi_{kp} \\ b &= \sqrt{\beta_k / \beta_p} (\cos \psi_{kp} - \alpha_p \sin \psi_{kp}) \; . \end{aligned} \tag{8}$$

The motion described by equation (6) can be interpreted as a motion $(\tilde{x}_n, \tilde{x}'_n)$ relative to a distorted closed orbit (x_c, x'_c).

Substituting $x_j = \tilde{x}_j + x_c$, $x'_j = \tilde{x}'_j + x'_c$ into equation (6), where $j = n, n+1, \ldots$, one obtains after some algebra

$$\tilde{X}_{n+1} = M X_n = \begin{pmatrix} m_{11} - K_0 a & m_{12} \\ m_{21} - K_0 b & m_{22} \end{pmatrix} \tilde{X}_n \tag{9}$$

$$X_c = -K_0 d \begin{pmatrix} 1 - m_{11} + K_0 a & -m_{12} \\ -m_{21} + K_0 b & 1 - m_{22} \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \end{pmatrix} \; . \tag{10}$$

Equation (9) is just the equation (8) of reference 1, which has already been analyzed. From equation (10) one gets

$$x_c = \frac{-Kd(\sin \psi_{kp} + \sin \psi_{pk})}{2(1 - \cos \mu_0) + K(\sin \psi_{kp} + \sin \psi_{pk})} \tag{11}$$

$$x'_c = \frac{-Kd[(\cos \psi_{kp} - \cos \psi_{pk}) - \alpha_p(\sin \psi_{kp} + \sin \psi_{pk})]}{\beta_p[2(1 - \cos \mu_0) + K(\sin \psi_{kp} + \sin \psi_{pk})]} \tag{12}$$

where $K = K_0 \sqrt{\beta_p \beta_k}$, μ_0 is the undisturbed phase advance per turn.

The coordinates of the distorted closed orbit (x_c, x'_c) are evidently proportional to the offset d of the BPM relative to the reference orbit and increase with increasing K, unless $\cos \mu_0 = 1$. This closed orbit distortion is undesirable, because its generation requires kicker power. The kicker maximum power is proportional to $(x_p + x_c)^2$, where x_p is the maximum coherent oscillation amplitude. It is therefore important to keep $d \simeq 0$. There are two reasons why $d \neq 0$. One is that the center of the BPM is offset relative to the reference orbit of the ring, the other that it is located at a point where the dispersion $\eta_p \neq 0$ and that the beam has an offset in momentum. The second cause can be made small by placing the BPM at a point where η_p is sufficiently small or by combining the properly delayed and scaled signals from two BPM's, located at points with approximately equal η_p and with a betatron phase advance of ideally $(2n-1)\pi$ rad, in such a way that their net signal is independent of the momentum offset. If the conditions are such that distortion of the closed orbit is unavoidable K should be varied adiabatically, e.g. when the damper is turned off, in order to avoid emittance blow up due to nonadiabatic changes in the closed orbit.

In order to understand the distortion of closed orbit by the damper system more clearly, we find A_c its action, or Courant–Snyder Invariant

$$A_c = (\gamma_p x_c^2 + 2\alpha_p x_c x_c' + \beta_p x_c'^2) = \left[\frac{Kd}{\beta_p^{1/2}} \frac{1}{2\sin\frac{\mu_0}{2} + K\cos(\frac{\mu_0}{2} - \psi_{pk})} \right]^2 . \tag{13}$$

which is of the order of $\frac{(Kd)^2}{\beta_p}$ unless $\cos\mu_0 = 1$.

In order to maximize the damping effect, one chooses the locations for BPM and kicker so that the phase advance ψ_{pk} is an odd integer times $\frac{\pi}{2}$, that is, $\sin\psi_{pk} = \pm 1$ or $\psi_{pk} = (2m+1)\frac{\pi}{2} + \delta$ with $|\delta| << 1$.

If the accelerator works near an integer resonance $\mu_0 = 2\pi n + \epsilon$ with $|\epsilon| << 1$, then

$$A_c = \left[\frac{Kd}{\beta_p^{1/2}} \frac{1}{(-1)^n \epsilon + (-1)^{n+m} K(\frac{\epsilon}{2} - \delta)} \right]^2 . \tag{14}$$

K should fulfil[1]

$$k_2 \leq K \leq k_1 \tag{15}$$

and

$$k_1, k_2 = \pm \frac{2\sin\mu_0}{1 \mp \cos\psi_{kp}} . \tag{16}$$

When $\mu_0 = 2n\pi + \epsilon$ and $\psi_{pk} = (2m+1)\frac{\pi}{2} + \delta$, one gets

$$k_1, k_2 \simeq \pm 2\epsilon$$

and

$$|K| \leq 2|\epsilon| . \tag{17}$$

In this case, neglecting second order terms ϵ^2 and $\epsilon\delta$,

$$A_c \simeq \left[\frac{Kd}{\beta_p^{1/2}} \frac{1}{\epsilon} \right]^2 \tag{18}$$

or

$$A_c \leq \frac{(2d)^2}{\beta_p} . \tag{19}$$

The above equation shows that when the accelerator works near an integer resonance, the distorted closed orbit may be much larger than when it does not work near integer resonance.

In the later case expression (13) must be used, with $K << 1$. Equation (17) shows that, when the accelerator works near an integer resonance, the damping rate which is proportional to K, is limited by the value of $|\epsilon|$.