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Transverse Mode Coupling in RHIC

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May 1990

Collider Accelerator Department
Brookhaven National Laboratory

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RHIC PROJECT

Brookhaven National Laboratory
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TRANSVERSE MODE COUPLING IN RHIC

E. RAKA

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INFORMAL REPORT

ALTERNATING GRADIENT SYNCHROTRON DEPARTMENT

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I. Introduction

In the Proceedings of the Workshop on the RHIC Performance,[1] it was stated that the transverse mode coupling instability[2,3] posed a potential intensity limitation for protons. This was based on the expression

$$I_b = \frac{4(E_t/qe)\ Q_s \qquad 4\sqrt{\pi}\,\sigma_\ell}{Im\ (Z_\perp)\ <\beta_\perp>\ R \qquad 3} \tag{1}$$

where E_t is the total energy, q the charge state, Q_s the synchrotron tune, $<\beta_\perp>$ the average beta function, R the machine radius, and σ_ℓ the rms bunch length of a Gaussian distribution in longitudinal phase space. For a $<\beta_\perp>$ of 55 m and 10^{11} protons/bunch, the allowed impedance $Z_\perp$ for protons at injection, where $Q_s = 0.11 \times 10^{-3}$, would be less than 1.2 MΩ/m. This would correspond to a longitudinal broad band impedance of 1.2 Ω assuming the simple relation

$$\left| \frac{Z}{n} \right|_{||} = \frac{\beta b^2}{2R}\ Z_\perp \tag{2}$$

is valid (b is the vacuum chamber radius). On page 116 of the RHIC CDR[4], a plot of the calculated $(Z/n)_{||}$ is shown. We see that for a copper plated vacuum chamber the imaginary part of the impedance is <1.0 Ω. However, not all of the circumference will contain the plating and the effect of all the vacuum chamber discontinuities may not be accounted for in the calculations used to obtain the Z/n plot. Hence the expressed concern of the study group.

The purpose of this report is to discuss the consequences of two factors that were omitted in applying Eq. 1, which comes from the ZAP[13] program, to RHIC. These are the space charge impedance and the incoherent tune spread of the beam.

II. General Description of Mode Coupling

Equation I is an expression for the bunch current required to produce a coherent frequency shift of the transverse mode m = 0 equal to the synchrotron frequency ω_s when the imaginary part of the transverse impedance $Z_\perp$ is assumed to be frequency independent. In general, the frequency of the m^{th} mode is given by[3]

$$\omega_m = \omega_\beta + m\omega_s + \Delta\omega_m \tag{3}$$

where ω_β is the incoherent single particle betatron frequency and the coherent frequency shift is

$$\Delta\omega_m = j \frac{\omega_o \qquad < \beta > I_o \qquad \Sigma\, Z_\perp\, h_m\, (\omega_p - \omega_\xi)}{(|m| + 1)\ 4\pi E/e\ \beta_o \qquad \Sigma\, h_m\, (\omega_p - \omega_\xi)} \tag{4}$$

with $h_m(\omega) = \lambda_m(\omega)\ \lambda_m(\omega)$. Here the λ's are the Fourier transforms of the oscillating part of the charge distribution. Now m can be 0, $\pm$ 1, $\pm$ 2, $\pm$ 3, etc., and mode coupling occurs if the frequency of a given mode approaches that of another mode n. In that case, a cross term $\Delta\omega_{mn}$ is involved in the stability analysis as well as $\Delta\omega_m$, $\Delta\omega_n$ (i.e., one must evaluate the sum over $Z_\perp$ with $h_{mn} = \lambda_m\ \lambda_n$.

Now in electron storage rings γ is very large so that only the broad band impedance contributes to $Z_\perp$. Because they require large rf voltages to make up for the energy losses due to synchrotron radiation $\omega_s/2\pi$ is large, i.e., 10 - 20 KHz. In addition, the bunches are very short so that the spectrum of the m = 0 and m = $\pm$ 1 modes sample $Z_\perp$ in the region where the broad band impedance peaks. This is shown in Fig. 1. As a result, the m = - 1 mode sees predominantly a capacitive impedance while the m = 0 mode sees an inductive $Z_\perp$ eff. Thus, their coherent frequencies merge as shown. Since the real part of $Z_\perp$ is largest where the imaginary part changes sign the $\Delta\omega_{01}$ term which depends upon Re $Z_\perp$ is at or near a maximum and consequently, the growth rate of the instability is largest for this situation. In this case, Eq. 1 is optimistic since the m = 0, - 1 modes would actually merge at a lower current than it predicts.

In the case of a proton storage ring like RHIC where the bunches are long, the $m = 0$ and ± 1 modes sample that part of the broad band impedance where $Z_\perp$ is inductive so that the $\Delta\omega_m$ move is the same direction with intensity but with a slope $\simeq$ to $(|m| + 1)^{-1}$ for a longitudinal phase space distribution $\approx (1 - r^2)$ (see Fig. 2). Hence the $m = 0$ and -1 modes would cross at twice the intensity given by Eq. 1 for a given $Z_\perp$. Actually, the two mode frequencies are given by the determinant of a 2 x 2 matrix:[5]

$$\begin{vmatrix} \lambda - m - M_{mm} & - M_{mn} \\ + M_{mn} & \lambda - n - M_{nn} \end{vmatrix} = 0 \tag{5}$$

$$\text{with } \lambda = \frac{\omega - \omega_\beta}{\omega_s} \; ; \quad M_{mn} = \Delta\omega_{mn}/\omega_s$$

so that for $m - n =$ odd (5a)

$$\lambda_{1,2} = 1/2 \; (m + M_{mm} + n + M_{nn}) \pm 1/2 \sqrt{(m + M_{mm} - n - M_{nn})^2 - 4M_{mn}^2}$$

Hence, when the two mode frequencies approach each other, one of the roots develops a negative imaginary part, i.e., one mode becomes unstable. The growth rate becomes a maximum when the uncoupled mode frequencies are equal. At even higher intensities, stability can be restored (Fig. 2). However, the other mode pairs can become unstable.[5]

These results are for a beam with no tune spread. The matrix elements are all real quantities for the broad band impedance and $\omega_\xi \approx 0$ i.e., when $n = m$, h_{mn} is an even function of ω_p, so only the imaginary part which is even in ω_p contributes to these elements. On the other hand, when $(n - m)$ is odd, h_{mn} is also an odd function of ω_p so that only the real part of $Z_\perp$ contributes to the off diagonal elements ($n - m = 1$). Hence, in the case of long bunches, the maximum growth rate which is proportional to M_{mn} is also quite small (at least for the lower order modes) but of course, ω_s is also small in proton machines. We shall discuss the effects of tune spread due to octupoles in a latter section.

The fact that the $\Delta\omega_{mm} = \Delta\omega_m$ are real for the broad band impedance is the reason why for low intensities a single bunch is stable if it is the only impedance present.

Even though the "coherent" frequency shift $\Delta\omega_m$ can put it outside of the incoherent band the net real impedance cannot produce growth if $\omega_\xi \geq 0$.

III. Mode Coupling in RHIC

The first point to be made in discussing transverse instabilities in RHIC at injection energies for Au ions and also for protons is that the space charge impedance will dominate the "broad band" impedance. It is given by

$$Z_\perp = \frac{-\, j\, Z_0 R}{\gamma^2\, \beta^2} \left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{6}$$

where $Z_0 = 377\ \Omega$, R is the machine radius and a and b are the beam and vacuum pipe radii. If we assume an emittance of 20 π mm mrad for protons and a $\bar{\beta} = 30$ m then at $\gamma = 31$, we obtain a beam radius of 4.5 mm. We take b = 36.5 mm and since R = 610 m we obtain a $Z_\perp = -\, j\ 11.6\ \mathrm{M}\Omega/\mathrm{m}$ for protons at injection. For gold we take the number quoted in the CDR[4] i.e., $-\, j\ 64\ \mathrm{M}\Omega/\mathrm{m}$. Now for the wide band impedance we assume a $(Z/n)_{||} = 2\Omega$ or twice the calculated value[4] so that $Z_\perp = j\ 2\mathrm{M}\Omega/\mathrm{m}$. Thus, the net $Z_\perp$ below the vacuum chamber cutoff frequency would be about eight times larger, but of opposite sign than the limit given by Eq. 1.

Now if we use Eq. 4 to calculate the allowed bunch current or $Z_\perp$

then we know that the modes $m = 0$, $|m| = 1$ will cross when $\Delta\omega_o = 2\omega_s$ (see Fig. 3). We can write

$$I_o = \frac{2\omega_s \; 4\pi \; E/e \; f_o \; \tau_\ell}{\omega_o < \beta > Z_\perp} = N_b e \; f_o$$

or

$$N_b = \frac{8\pi Q_s \; E/e \; \tau_\ell \; 10^{19}}{1.6 < \beta > Z_\perp} = 2 \times 10^{10}$$

for $E = 29$ GeV; $\tau_\ell = 11.0$ nsec, $\bar{\beta} = 30$M, $Z_\perp = 9.6$ MΩ/m. This is a factor of five less than the design value and thus potentially represents a serious intensity limitation! However, let us examine the terms under the square root in Eq. 5 keeping Fig. 3 in mind. If $M_{01} << M_{00}$ then there is only a narrow range of N_b in which the instability can occur. As M_{01} depends only upon the real part of the total impedance, it will indeed be small since for long bunches only the low frequency part of the broad band impedance will be sampled (see Fig. 2).

In order to calculate M_{01}, we must model the broad band impedance. We follow the argument of Zotter[6] and write

$$[Z_\perp \; (\omega)]_{BB} = \frac{\omega_r}{\omega} \quad \frac{R_T}{1 + jQ_T \left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} \tag{7}$$

where $Q_T \approx 1$ and one has replaced all the effects of discontinuities, bellows, etc. by a single broad band resonator with a $\omega_r \approx$ cutoff frequency of the round vacuum tube. Since R_T is the real part at "resonance" one can easily show that $I_m \; Z_\perp$ for $\omega << \omega_r$ is essentially jR_T. Hence, our choice of 2MΩ/m for the broad band impedance implies that R_T = 2MΩ/m here.

We take $\lambda_o \simeq \sin(\pi x/2)/(1 - x^2)$ and $\lambda_1 \simeq \cos(\pi x/2)/(4 - x^2)$ where $x = \omega\tau_\ell/\pi$.[7] For the real part of Eq. 7 we write $\mathrm{Re}(Z_\perp)_{BB} = yR_T/(y^4 - y^2 + 1)$ where $y = \pi x/\omega_r\tau_\ell$. Since the broad band part of $Z_\perp$ is a smooth function of ω, we can replace the summation in Eq. 4 by an integration over ω_p. With the aid of a partial fraction expansion, we obtain $8R_T/3\omega_r\tau_\ell$ for the effective $Z_\perp$ if $\omega_r\tau_\ell >> 1$,

Now we note that the imaginary part of $Z_\perp$ for long bunches is 9.6 MΩ/m or 4.3 R_T. Next we assume f_r = 2 GHz and with τ_ℓ = 11.8 nsec, we have $\omega_r\tau_\ell = 150$ so that $M_{01} << M_{00}$ or M_{11}. Thus, if $I_b(0, 1)$ is the current at which the modes 0, 1 cross then the unstable region is $\pm \Delta I_b = .0165\ I_b\ (0, 1)$ wide. Referring again to Fig. 3 which is drawn for the $\Delta\omega_m$ given by Eq. 4 for $M \geq 0$ and $I_m Z_\perp \simeq - j X$, we note that the next instability would arise for the m = 0, 3 mode intersection. Here the rate of change of the term $(M_{00} - 3 - M_{33})$ is 50% greater than the previous case while the M_{13} term would be smaller since there is very little overlap of the λ_1 and λ_3 spectra. Hence the unstable region should be even smaller than the zero, one case.

Next, let us consider the case for m = 2, 3 which would cross at $\approx 1.2 \times 10^{11}$ protons/bunch. We note that the rate of change of the term $(M_{22} - 1 - M_{33})$ with current is 1/6 that of the m = 0, 1 case while the effective real impedance in the M_{23} term is $48\ R_T/7\ \omega_r\tau_\ell$. One then finds that the width of the unstable region is $\pm \Delta I_b = .085\ I_b\ (2, 3)$ or $\pm 10^{10}$ protons/bunch since $M_{23} \simeq \mathrm{Re}\ Z_\perp/(1 + m)$(here m = 2). As the intensity is increased the mode numbers become larger and the rate of change of the term $(M_{mm} - 1 - M_{nn})$ which is $\simeq (m + 1)^{-1} - (n + 1)^{-1}$ becomes smaller. Although M_{mn} seems to decrease even though the real part of $Z_\perp$ effective increases the width of the unstable region will increase. Hence, this instability could potentially be a barrier to high current beams. One would obtain similar results for gold ions even though at injection $Q_s \simeq 1.4 \times 10^{-3}$ vs 0.11×10^{-3} for protons, since the space charge impedance is expected to be about six times greater than for protons at the same total charge per bunch. As noted above, these results are for bunches with no betatron or synchrotron frequency spread. Since both will be present in RHIC, we must discuss their effect.

IV. Tune Spread Considerations

There are three sources of tune spread within the bunch; spread in ω_s due to nonlinearities in the rf buckets; spread in ω_β due to the space charge forces or due to external octupoles or higher order multipoles. It has been shown that small spreads in ω_s only reduced the threshold current since it permits the overlap to take place sooner.[8] Since the spread in ω_s for the proton case is only about 6%, it will not contribute to stabilization of mode coupling.

The predominant source of tune spread will be due to the incoherent space charge tune shift ΔQ_{ic} given by:

$$\Delta Q_{ic} = \frac{-NRr_p}{\pi\gamma Q_o} \left[\frac{1}{\beta^2 B_o \gamma^2} \frac{1}{2a^2} + \frac{\epsilon_2}{g^2} \right] \qquad (8)$$

for a round beam in a round chamber. Here a is again the beam radius, Q_o is the tune, B_o the bunching factor, ϵ_2 is the incoherent magnetic image coefficient and g the magnet gap half height. For R/Q_o we use $< \beta > = 30$ m and with $a = 4.5$ mm, $g = 36.5$ mm, $\epsilon_2 = \pi^2/24$, we obtain $\Delta Q_{ic} = -1.3 \times 10^{-3}$ when $N = 10^{11}$, $\gamma = 31$. This shift is for a particle at the center of a bunch and with no momentum error. A particle at either end of the bunch and hence with no energy error will not experience any tune shift since the local density is $\approx$ zero. Thus, the incoherent space charge tune spread is essentially equal to the maximum tune shift. For B_o, we only considered the bunch width and not the peak to average ratio which for a parabolic line charge would be 1.5. In addition to the "linear" tune shift given by Eq. 8, there are also nonlinear space charge tune shifts that produce a reduction in ΔQ_{ic} for particles with finite amplitude.[9] This leads to the familiar necktie diagram for the incoherent tune spread shown in Fig. 4. Here one assumes zero chromaticity and we note that the shaded region is for particles at the center of the bunch.

Now there is also a coherent tune shift due to image fields in the vacuum chamber. Again, for a round beam in a round chamber we have[10]

$$\Delta Q_c = \frac{-Nr_p R}{\pi Q_o \gamma} \left[\frac{\xi_1}{\beta^2 \gamma^2 B_o b^2} + \frac{\epsilon_2}{g^2} \right] \tag{9}$$

where ξ_1 is one of two coherent image coefficients. The other, ξ_2, is not important when the ac magnetic fields do not penetrate the vacuum chamber. This is the change in tune that one would measure by kicking the beam and measuring Q as a function of N (assuming the contribution due to the broad band impedance and resistive wall impedance can be ignored). The coherent frequency shift given in Eq. 4 divided by ω_o is proportional to $(\Delta Q_c - \Delta Q_{ic})$ i.e., it is the shift of the coherent frequency away from the incoherent frequency $Q_{ic} = \omega_\beta/\omega_o$. If we make the usual identification $U = \omega_o\ (\Delta Q_c - \Delta Q_{ic})$ then since $\xi = 1/2$ here,

$$U = \frac{-Nr_p\ R\omega_o}{2\pi Q_o \gamma \beta^2 \gamma^3 B_o} \left[\frac{1}{a^2} - \frac{1}{b^2} \right] \tag{10}$$

Using the additional relation $Z_T = 4\pi j\ \gamma\ m_o c\ Q_o\ U/eI_o$ one then obtains Eq. 6.

If the simple criterion that stability is maintained as long as $(\Delta\omega_m/\omega_o)$ is less than the incoherent tune spread could be applied when the latter is due to the variation in space charge fields along the beam then many instabilities could never occur. However, this is certainly not the case in the AGS, CERN-PS, SPS, etc. Hence it cannot be assumed that this effect will help in the case of RHIC. It has been shown that the tune spread due to nonlinear space charge forces cannot of itself suppress instabilities. However, this effect in conjunction with external octupoles can provide the necessary Landau damping.[11] Hence, we shall discuss next the use of octupoles to control the mode coupling instability.

V. Stabilization With External Octupoles

In this section, we shall make extensive use of the results of Y.H. Chin[12]. Using a Hamiltonian formulation he derives specific relations for including the effect of external octupole fields and hence nonlinear betatron tune spread into the matrix eigenvalue problem for the coherent tune of a beam. The determinant[5] becomes

$$\begin{vmatrix} (F_{1m}^{-1} - M_{mm}) & - M_{mn} \\ M_{mn} & (F_{1n}^{-1} - M_{nn}) \end{vmatrix} = 0 \tag{11}$$

$$\text{or } (F_{1m}^{-1} - M_{mm})\ (F_{1n}^{-1} - M_{nn}) + M_{mn}^2 = 0 \tag{11a}$$

For a Gaussian like distribution in betatron phase space, one has

$$F_{1m} = \frac{Q_s}{2S} \left[1 - \tau_1 e^{\tau_1} E_1\ (\tau_1)\right] \tag{12}$$

where $E_1(\tau)$ is the exponential integral[13] and

$$\tau_1 = \frac{Q - mQ_s - Q_x}{2S} \tag{13}$$

with

$$S = \frac{-R}{8B_\rho} \frac{< \partial^3 B}{\partial x^3} \quad \beta_x^2 > I_{xo} \tag{14}$$

Here S is the tune spread due to the external octupole of strength $< \beta_x^2\ \partial^3 B/\partial x^3 >$ averaged over the ring. I_{xo} is the unperturbed action variable and the one in F_{1m} refers to dipole oscillations and Q_x is the unperturbed tune. The coherent tune is Q and it can be shown that the average unperturbed tune with octupoles is $Q_x = Q_x - 2S$.

unperturbed tune. The coherent tune is Q and it can be shown that the average unperturbed tune with octupoles is $\bar{Q}_x = Q_x - 2S$.

If one is interested in the stability of a single mode at intensities where mode coupling is not important, then the solution of the equation $(F_{1m}{}^{-1} - M_{mm}) = 0$ can be used to generate a stability diagram. For example, if m = 0, one obtains

$$1 = \frac{\lambda}{2S}\,[1 - \tau_1 e^{\tau_1} E_1\,(\tau_1)]$$

$$\text{with} \qquad \tau_1 = \frac{Q - Q_x}{2S} - 1 = \tau_2 - 1 \tag{15}$$

and $\lambda = Q_s M_{00} = (Q - Q_x)$ the tune shift without tune spread. If one maps the complex $\lambda/2S$ plane on to the complex τ_2 plane he obtains the usual stability diagram shown in Fig. 5. The curve $\mathrm{Im}(\tau_2) = 0$ is then the stability limit. Note that $\tau_1 = 0$ when $R_e(\tau_2) = 1$, $\mathrm{Im}(\tau_2) = 0$ and that $\mathrm{Im}(\lambda) = -\mathrm{Im}(M_{00})$ where $M_{00} = \Delta\omega_o/\omega_s$ and $\Delta\omega_o$ is given by Eq. 4. This is because Eq. 4 assumes oscillations of the form $e^{j\omega t}$ while Eq. 15 assumes $e^{-i\omega t}$ where $\omega = Q\omega_o$ is the coherent frequency. If the sign of S is reversed then the diagram is inverted about the x axis but the direction of increasing $Q\omega_o$ is still in the positive y direction. One has the relation sign $\mathrm{Im}(\tau_2)$ sign S = sign $\mathrm{Im}(\omega)$.[14]

Now it has been shown[1] that there will be transverse coupled bunch instabilities in RHIC at injection due primarily to the resistive wall impedance. It is the space charge impedance that overcomes the nominal tune spread and the wall impedance that determines the growth rate. This instability will be controlled by a feedback system described in Ref. 4, page 251. However, it will not be capable of suppressing a coupled mode instability.

Now, Eq. 11a has been solved for the following two cases $S = 0.3\ Q_s$

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and $S = -0.2 \cdot Q_s$ with $M_{22} = 4Q_s$ and $M_{33} = 3Q_s$ which corresponds to 1.2×10^{11} protons/bunch. In the first case, the τ_1's are > 0 while in the second case the τ_1's are < 0 since the M_{mm} are both > 0. We find that for $S = 0.3\ Q_s$, the instability threshold would occur at an $R_T = 0.94 \cdot 2M\Omega/m$ and $(Q - Q_x) = 4.96\ Q_s$. For $S = -0.2\ Q_s$ the threshold would occur at $R_T = 1.165 \cdot 2M\Omega/m$ and the frequency shift would be $(Q - Q_x) = 6.92\ Q_s$. Hence, we see that for $S < 0$ the amount of octupole required is a minimum. Since it is not known how the space charge octupole effects this instability, these results are not definitive. However, since $Q_s = 0.11 \times 10^{-3}$ for protons at 30 GeV while the available[15] ΔQ_x from octupoles is $> 10^{-2}$ there should be no difficulty in suppressing this instability at the design intensity and higher, should it arise.

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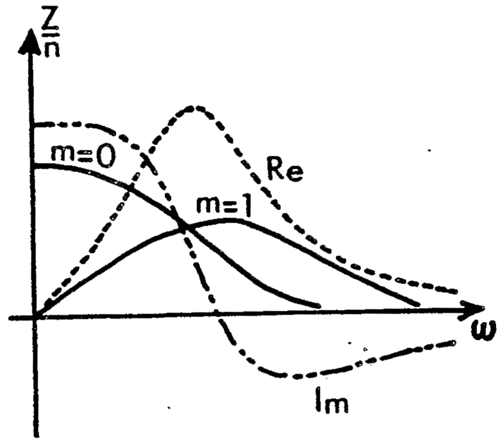


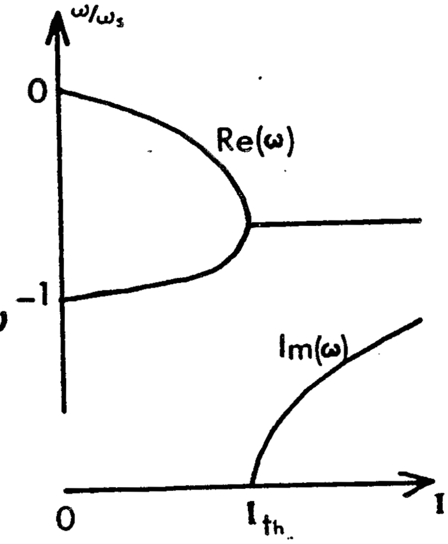
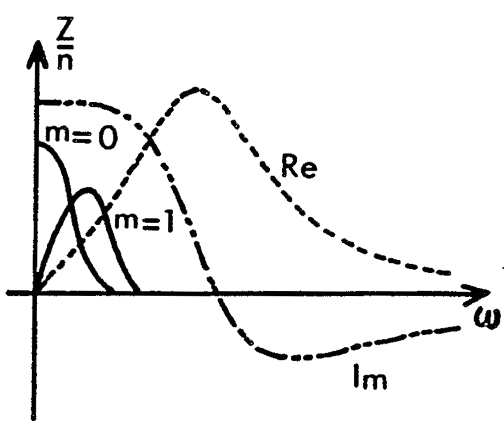


Fig 1 : Mode coupling instability
(short bunches)



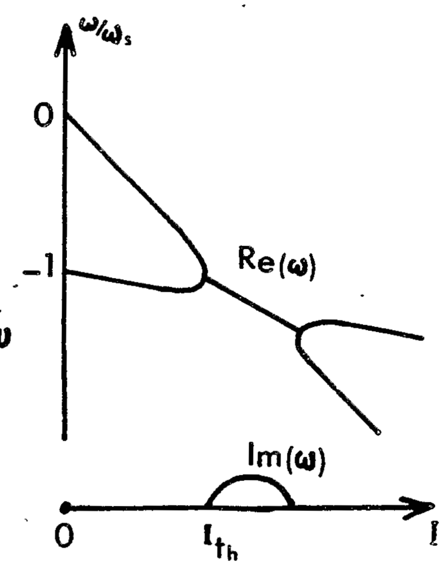


Fig. 2 Mode coupling instability
(long bunches)

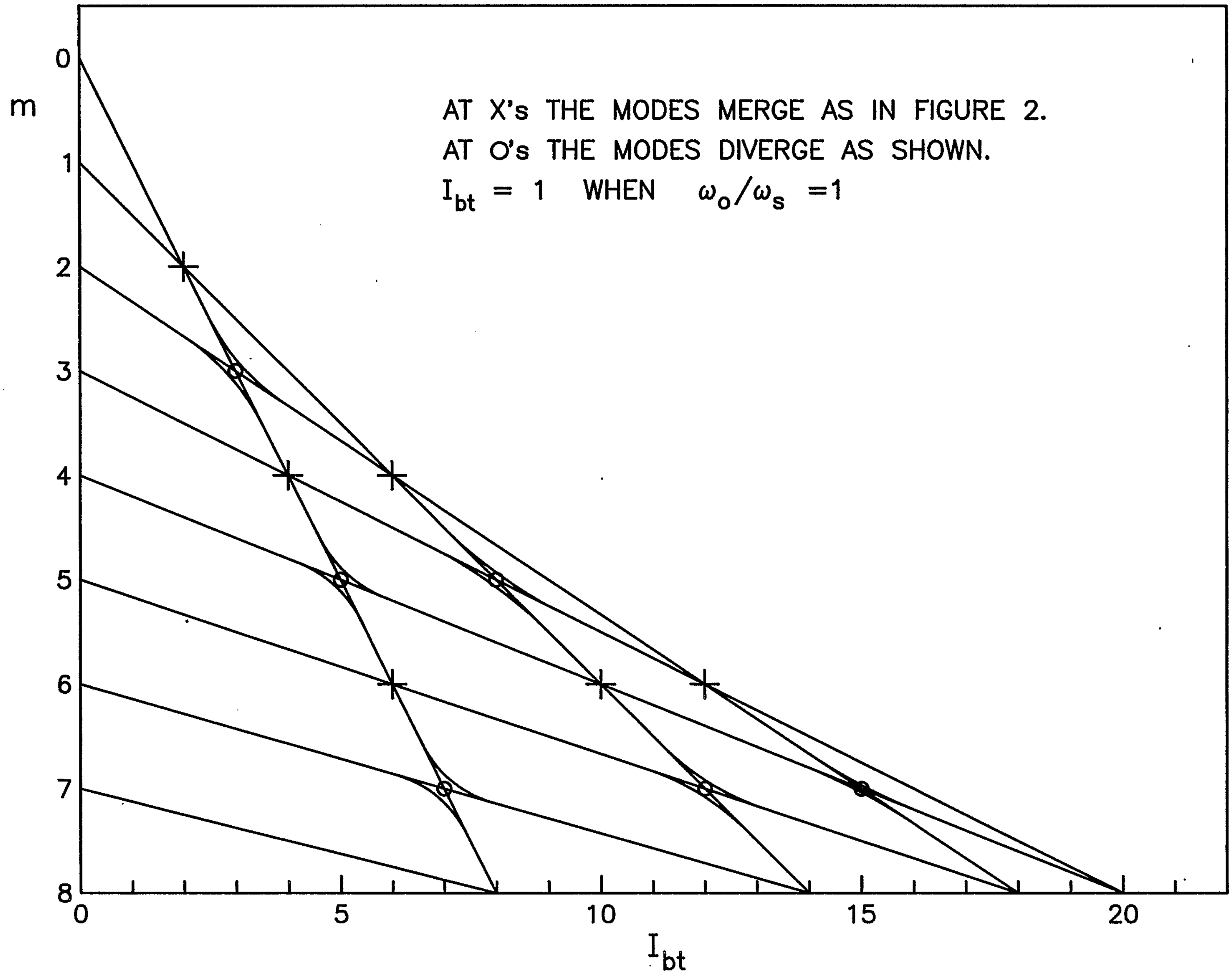
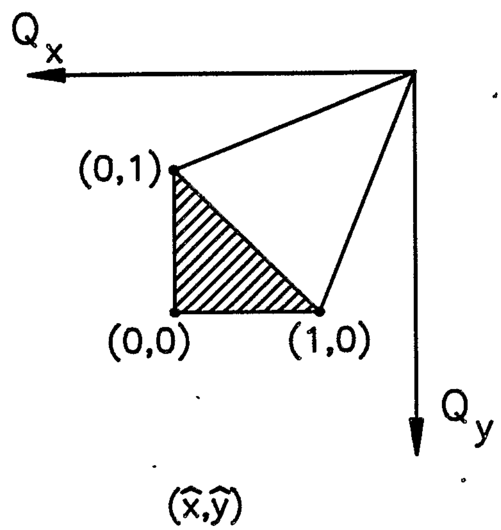

AT X's THE MODES MERGE AS IN FIGURE 2.
AT O's THE MODES DIVERGE AS SHOWN.
I_{bt} = 1 WHEN ω_o/ω_s =1
m
0
1
2
3
4
5
6
7
8
0
5
10
15
20
I_{bt}

Figure 3



SPACE CHARGE TUNE SHIFTS
Figure 4

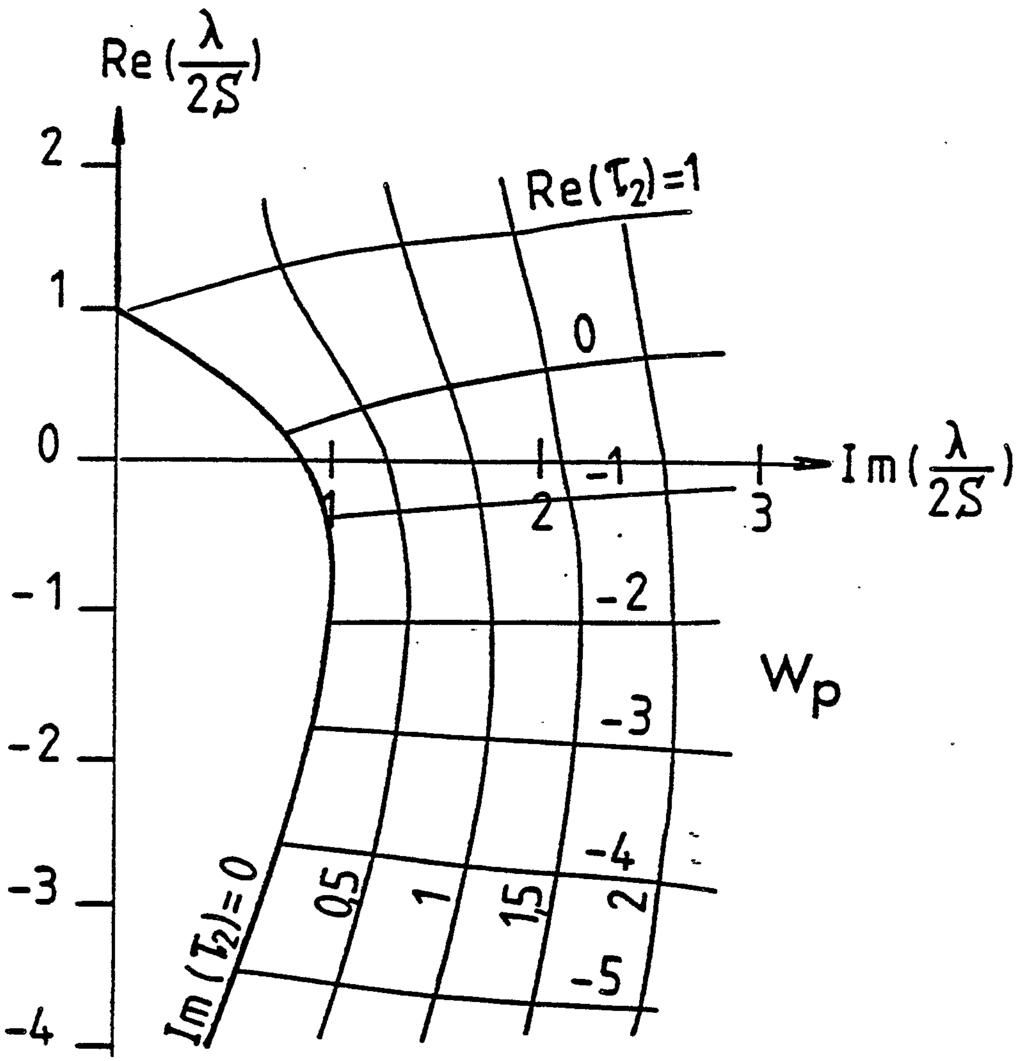


Fig. 5 The stability diagram. The thick solid line marks the stability limit in which the stable region is situated.