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Effect of Systematic Sextupoles on the Beam Dynamics in the RHIC Lattice

S. Y. Lee

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Collider Accelerator Department

Brookhaven National Laboratory

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Brookhaven National Laboratory
Associated Universities, Inc.
Upton, NY 11973

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ABSTRACT

The effect of the systematic sextupoles in the RHIC lattice has been studied. Due to the 90^o phase advance and 12 FODO cells per arc, these systematic sextupoles in the dipole do not contribute to the half integer and third integer resonances. The effect of these systematic sextupoles on the chromaticity and tune shift with amplitude is analyzed.

1. Introduction

The sextupoles correction scheme discussed in Ref. 1 uses four families of sextupoles. These four families are needed to cancel the small systematic half integer stop–band at 28.5. The strengths of these four–family sextupoles are small variation of those of the two–family scheme.

Besides the chromatic correction sextupoles, the sextupole components in the dipoles are not negligible at the high field due to the magnetization and saturation respectively. Since the magnetization sextupole strength depends on the filament size, Cu/Sc, the copper to the superconductor ratio, and the operating current, the strength of the chromatic sextupole should be adjusted in the acceleration cycle. Besides, during the injection flat bottom, the magnetization sextupole strength will be time dependent. Thus the sextupole correction is time dependent and energy dependent. To carry out the analysis we shall derive some analytic formula to evaluate the effect of these sextupoles on the beam dynamics. In Section 2, we review briefly the systematic sextupole strength in the dipole. Section 3 discusses the effect of these sextupoles on the beam dynamics. Section 4 discusses the possible random error and conclusion.

2. Systematic Sextupole in the Dipole

Due to the magnetization of superconducting coil, the persistent sextupoles in the dipoles are not negligible. Recently, Thompson[2] analyzed the measured data of RHIC full length magnets DRA001–4, DRB005–6 and found that the sextupole strength due to persistent current in the superconducting coil is proportional to

$$F \equiv (\text{filament size}) \cdot /(1 + Cu/Sc) \tag{1}$$

where Cu/Sc is the copper to the superconductor ratio in the superconducting filament.

Figure 1a shows $10^4 \times b_2'$ at 25 mm vs current in the superconductor normalized with the factor F for DRB005. Indeed the scaling property discussed in Ref. 1 reflects well in the measured data. To obtain a first order estimate of the persistent current sextupoles, we shall average the b_2 of DRA and DRB magnets and scale with the factor F for the possible realistic RHIC magnets, i.e. filament size = 6 μm and $Cu/Sc = 2.25$. Figure 1b shows the b_2[m^{-2}] vs the current I in [kA]. Since the magnetic field B is related to the current by

$$B[T] = 0.711\ I[\text{kA}]$$

Assuming $B\rho = 95$ Tm from AGS, one obtains the injection field of $B = 0.39$ T, which corresponds to a current of 0.55 kA. Figure 1b indicates that the corresponding sextupole strength is $b_2 = 0.8$ m^{-2}. Using Thompson's parameterization, this sextupole b_2 coefficient [m^{-2}]is given by

$$b_2 = 0.123 - 1.42/I \; - \; 0.702/I^2 \; - \; \frac{4.49}{I} e^{-I/I_d}$$

where $I_d = 0.060$ kA, I is the current in kA.

At high field, the iron saturation gives rise to a sextupole component in the dipole magnets. Figure 2 shows the saturation sextupole strength vs the excitation current.[3] We obtain $b_2' = 7 \times 10^{-4}$ evaluated at 25 mm at $B\rho = 840$ Tm (or $I = 5.5$ kA).

The effective sextupole strength is a combination of persistent current at low field and saturation at high field. The total integrated sextupole strength is then defined to be

$$S \equiv \frac{B''\ell}{B\rho} = 2b_2\ell/\rho = 0.077783 b_2[\mathrm{m}^{-2}] \tag{2}$$

where ℓ and ρ are the length and the bending radius of the dipole respectively.

3. Effect of the Systematic Sextupoles on the Beam Dynamics

Since the arc of the RHIC lattice contains 12 FODO cells with 90^o phase advance per cell, the effect of these systematic sextupoles do not contribute in first order to the half integer stopband width. Their contribution to the 3rd resonances strength cancel also locally. The important effect to the beam dynamics are (i) chromaticity and (ii) the second order effect of sextupoles, i.e. the octupole effect, which gives rise to $2\nu_x - 2\nu_y = 0$ coupling resonance and the tune shift vs betatron amplitude.

(A) The Chromaticity

Let us denote the chromatic sextupole strength as S_F and S_D, i.e.

$$S_F = \frac{B''L_s}{B\rho} \quad \text{at focusing quad.} \tag{3a}$$

$$S_D = \frac{B''L_s}{B\rho} \quad \text{at defocusing quad.} \tag{3b}$$

The chromaticities C_x, C_y of the machine is given by

$$C_x = \frac{1}{4\pi}\int_o^C ds\beta_x \left(-\frac{B'(s)}{B\rho} - \frac{2}{\rho^2} + \frac{B''(s)}{B\rho}X_p\right) \tag{4}$$

$$C_y = \frac{1}{4\pi}\int_o^C ds\beta_y \left(+\frac{B'(s)}{B\rho} - \frac{B''(s)}{B\rho}X_p\right) \tag{5}$$

where X_p is the dispersion function and C is the circumference of the accelerator. Using the definition of the sextupole strengths in Eqs. (1) – (3) and the lattice functions of the FODO cells, we obtain

$$C_x = C_x^o + 313.3788S + 432.1116S_F + 42.2705S_D \tag{6}$$

$$C_y = C_y^o - 285.1942S - 83.8274S_F - 218.0549S_D \tag{7}$$

where S, S_F and S_D are in dimension of [m^{-2}]. Figure 3 shows the machine chromaticity of Eqs. (6) and (7) with $S_F = S_D = 0$ at various β^*.

Since the natural chromaticity of the machine depends on the β^* at the interaction point, therefore the sextupole strength depends also on the β^*. Setting the machine chromaticity $C_x = C_y = 0$, we can solve S_F and S_D as

$$S_F = -2.4046 \times 10^{-3} C_x^o - 4.6615 \times 10^{-4} C_y^o - 0.62062 S \tag{8}$$

$$S_D = 9.2442 \times 10^{-4} C_x^o + 4.7652 \times 10^{-3} C_y^o - 1.0693 S \tag{9}$$

The sextupole strengths S_F, S_D can be obtained easily from Eqs. (8) and (9). Depending on the sign of S, ($S < 0$ for persistent current and $S > 0$ for saturation), the strengths of the chromatic sextupoles, S_F and S_D shall be increased accordingly. Figure 4 shows the S_F and S_D strengths needed to obtain $C_x = C_y = 0$ as a function of energy for various β^* values. These chromatic sextupole strengths are well within the design capability.

(B) Tune Shift vs Amplitude

Because the sextupole strength is large in the hadron collider, the second order effect can be important as well. Using the second order perturbation expansion in the Hamiltonian, the betatron tunes of the machine can be expressed as

$$\nu_x = \nu_x^o + \alpha_{xx}\epsilon_x + \alpha_{xy}\epsilon_y \tag{10a}$$

$$\nu_y = \nu_y^o + \alpha_{xy}\epsilon_x + \alpha_{yy}\epsilon_y \tag{10b}$$

where α_{xx}, α_{xy} and α_{yy} are the second order integral of the sextupoles with the lattice functions. These coefficients can be integrated to become

$$\begin{aligned}
\alpha_{xx} &= -36900S^2 - 95000SS_F - 13100SS_D - 44000S_F^2 - 19600S_FS_D - 324S_D^2 \\
\alpha_{xy} &= 24200S^2 + 55800SS_F + 14600SS_D + 16200S_F^2 + 33900S_FS_D + 2380S_D^2 \\
\alpha_{yy} &= -36100S^2 - 21600SS_F - 54700SS_D - 1670S_F^2 - 19700S_FS_D - 8570S_D^2
\end{aligned}$$

Figure 5 shows the α_{ij} vs energy for various β^*. For example, at the normalized 95% emittance of 30 πmmmrad and 30 GeV/u with $\beta^* = 6$m, the tune spread is about 3×10^{-3} at 6σ. At top energy 100 GeV/u with six insertions of $\beta^* = 2$m, the tune spread will also be about 3×10^{-3} at 6σ. Normally, we will not operate $\beta^* = 2$m at six insertions. The tune spread can be considerably smaller. For the proton operation, the collider may have one or two insertions operating at $\beta^* = 0.5$ m, the tune spread should be less than 2×10^{-3} at 6σ amplitude due to small emittance for proton beams.

At injection with $\beta^* = 6$ m, S = 0.07778 b_2 = 0.0583 m^{-2}, where $b_2 = 0.75$ m^{-2} from Fig. 1b, we obtain $S_F = 0.165$ m^{-2} and S_D = -0.194 m^{-2} at $B\rho = 95$ Tm and

$$\alpha_{xx} = 54.8$$

$$\alpha_{xy} = -844.$$

$$\alpha_{yy} = -268.$$

The maximum tune shift for 6σ particles at 10πmm mrad 95% normalized emittance will be -4×10^{-3}, which is about a factor of 3 smaller than the space charge tune shift. It is worth pointing out the total tune spread of the beam will be smaller due to the fact that the space charge tune shift is important for small amplitude particles, while the tune shift due to the nonlinear elements are large for large amplitude particles.

4. Conclusion

We have analyzed the impact of the systematic sextupoles due to the persistent current and iron saturation on the beam dynamics. The chromaticity can be corrected within the capability of the RHIC chromatic sextupoles. These systematic sextupoles on the dipoles will not contribute to the half integer and third integer stopband due to the 90^o phase advance of FODO cell. The important contribution is the $2\nu_x - 2\nu_y$ coupling resonances and the tune shift. We found that the tune shift is of the order 3×10^{-3} at 6σ amplitude depending on the scenario of the RHIC collider.

Since the persistent current sextupoles scales with the factor F of Eq. (1), the sextupole field will be different with different coil configuration. A variation of F by 10%, the persistent current sextupole will drive the third integer stopband at 28.6667 and affect the half integer stopband at 28.5. Careful control of the factor F is important. Effects of these random sextupoles have been studied by particle tracking calculations.[4]

References

1. S.Y. Lee, G.F. Dell, H. Hahn, and G. Parzen, p. 1328, IEEE, Particle Accelerator Conference, 1987.
2. P. Thompson, RHIC–MD–92.
3. P. Thompson, private communication.
4. G.F. Dell and G. Parzen, private communication.

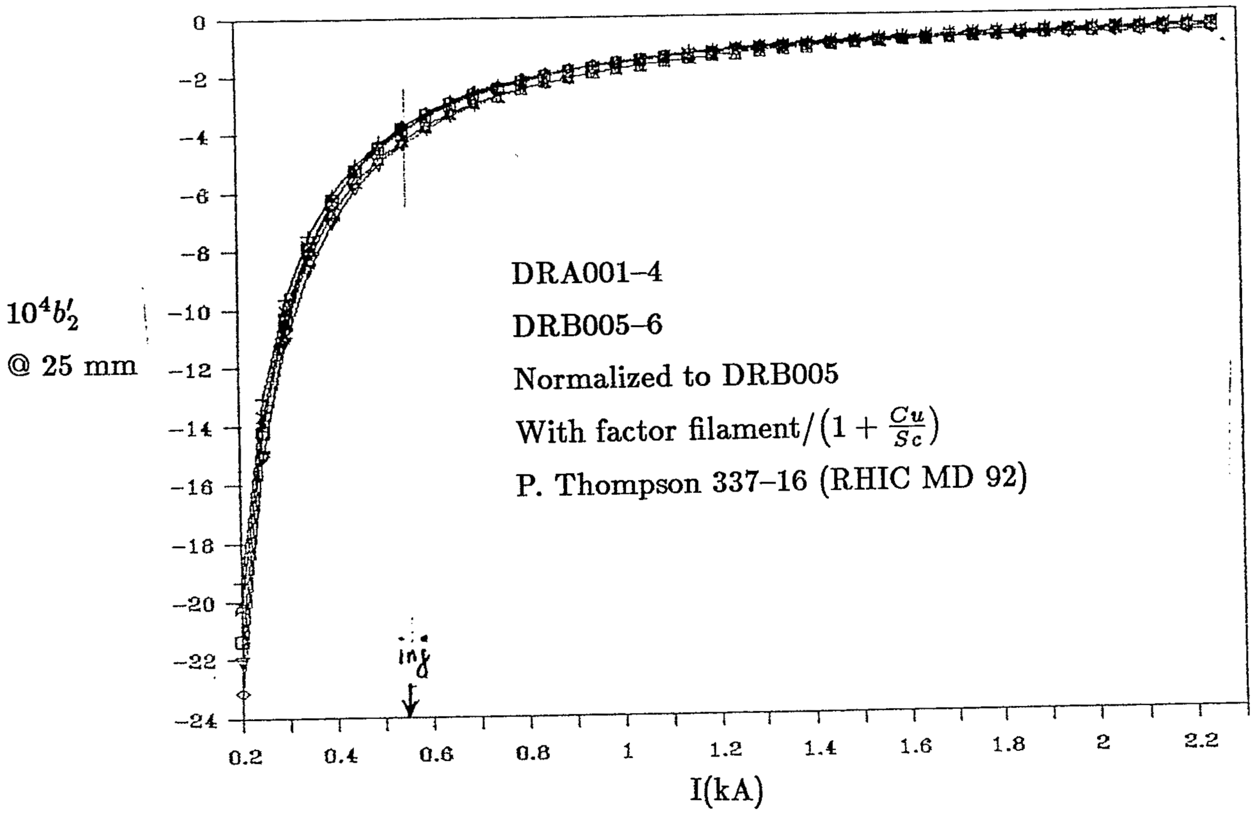

Fig. 1a. 10^4 b_2' (measured) for DRA and DRB magnets are shown with normalization factor F. Small dispersion in these magnets indicates that the scaling factor of Thompson works well.

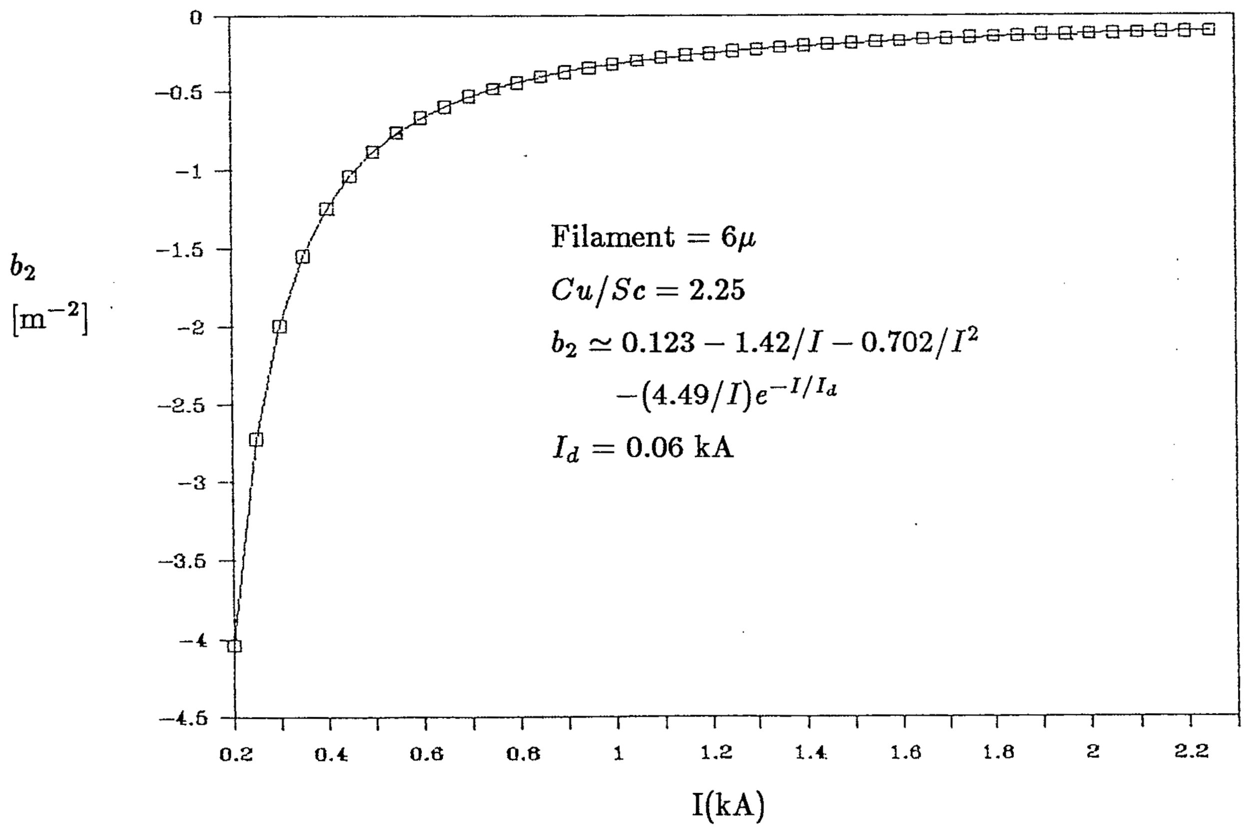

Fig. 1b. "b_2" is plotted vs I for possible RHIC coils.

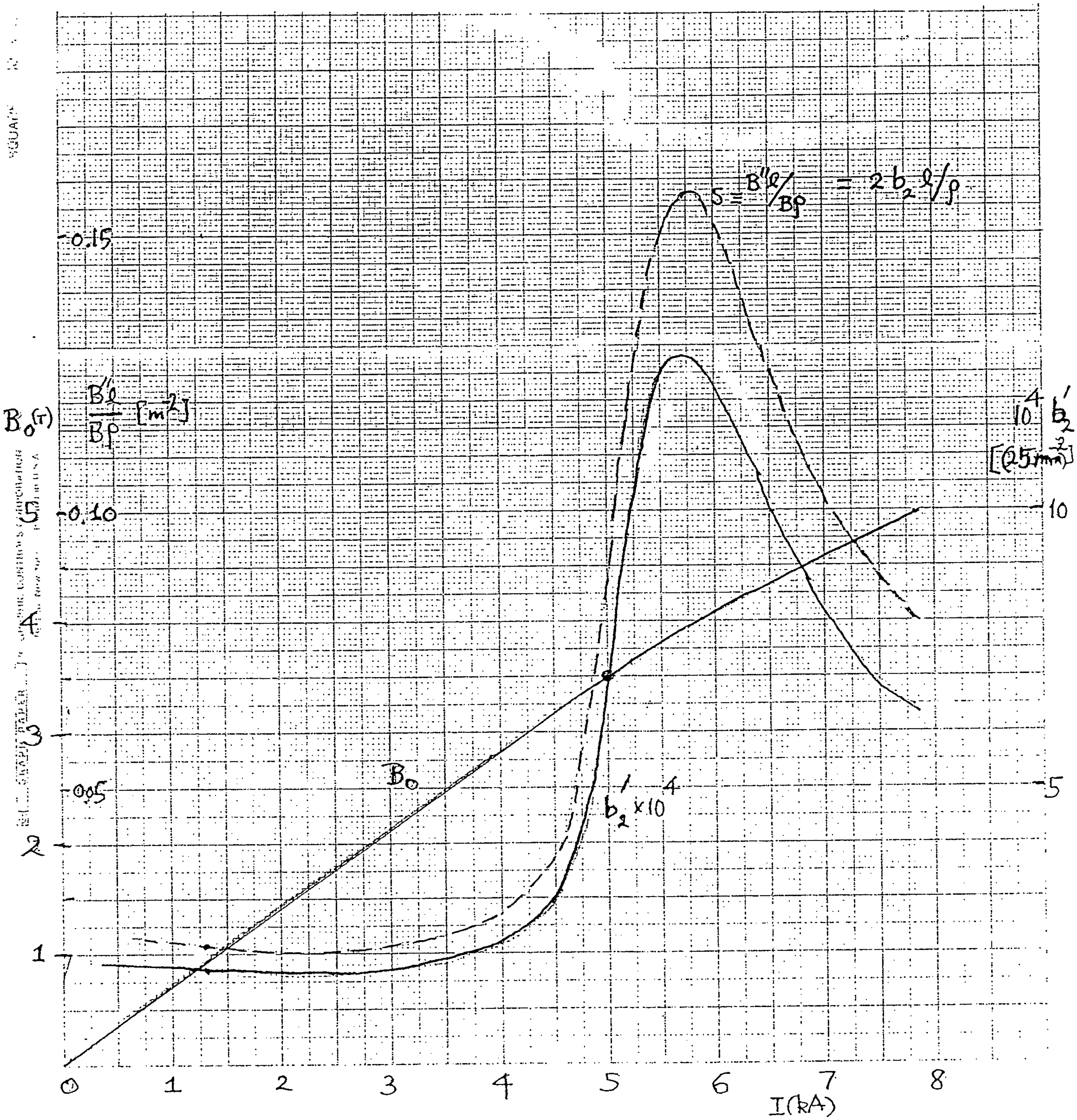
$S = B''\ell / B\rho = 2 b_2 \ell / \rho$
B_0(T)
$\frac{B'\ell}{B\rho}$ [m^{-2}]
0.15
0.10
0.05
$10^4\, b_2'$
[(25mm)$^{-2}$]
10
5
B_0
$b_2' \times 10^4$
I(kA)
0
1
2
3
4
5
6
7
8

Fig. 2.

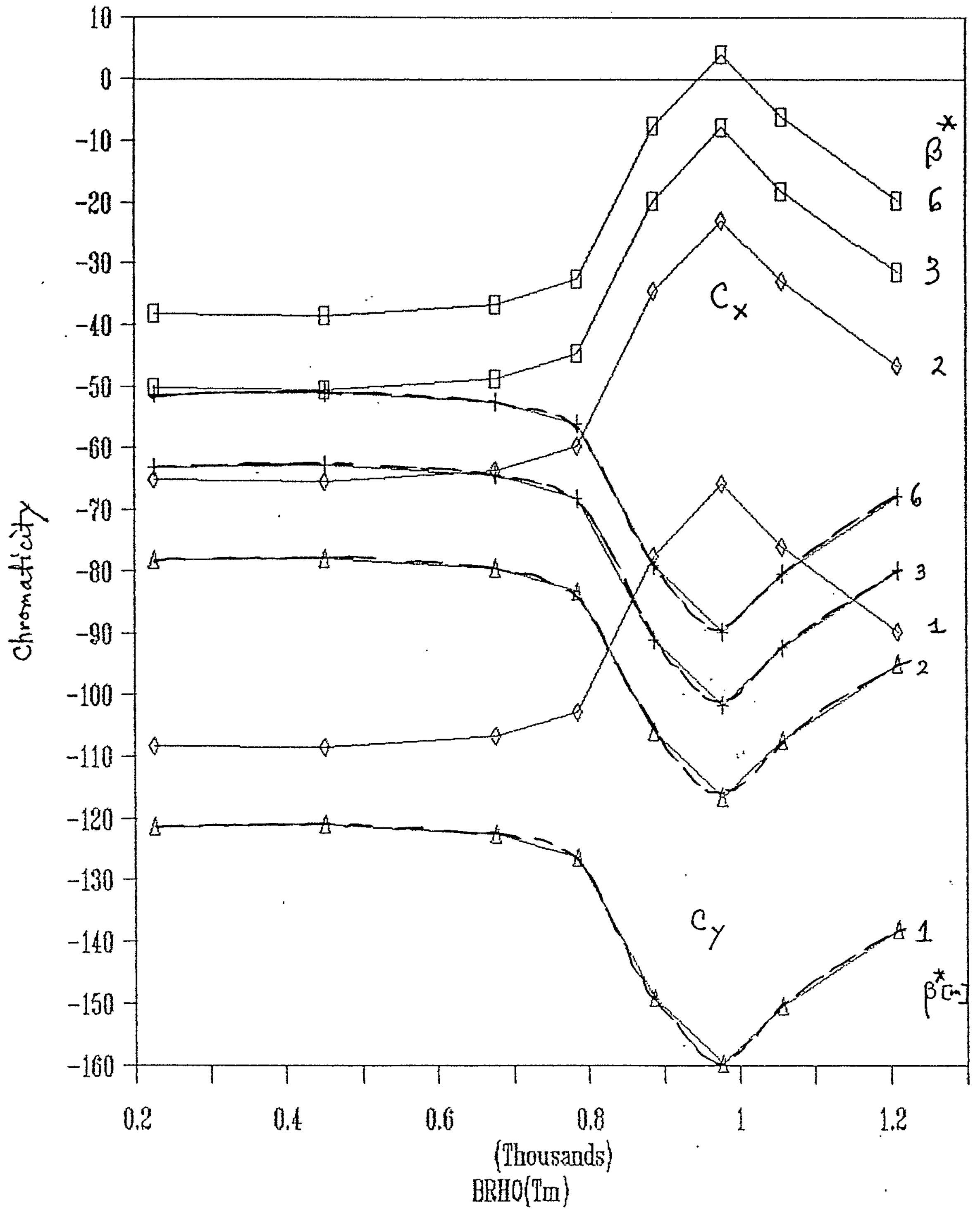

Fig. 3. Chromaticity with $S_F = S_D = 0$

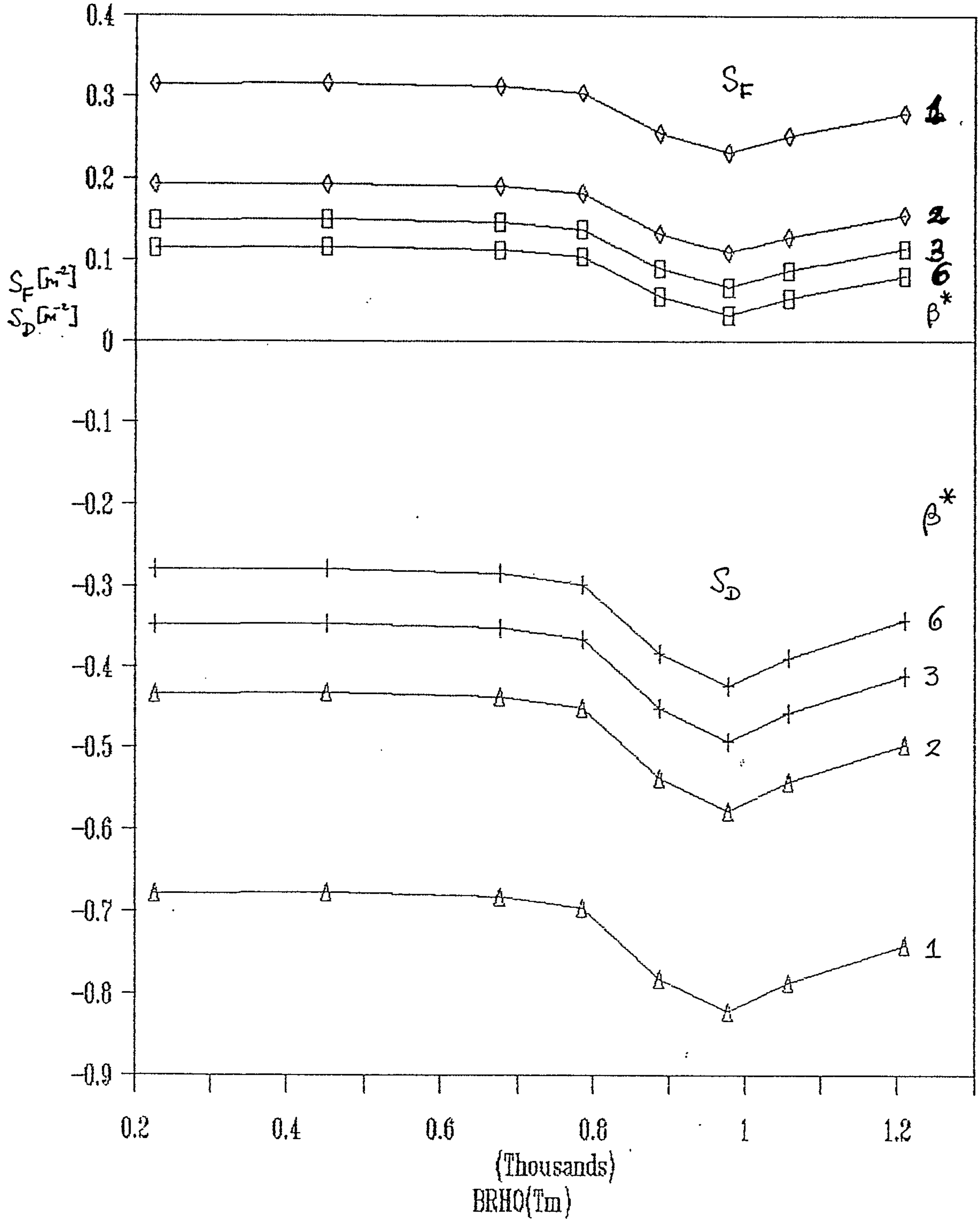

Fig. 4. Strength of chromatic sextupoles.

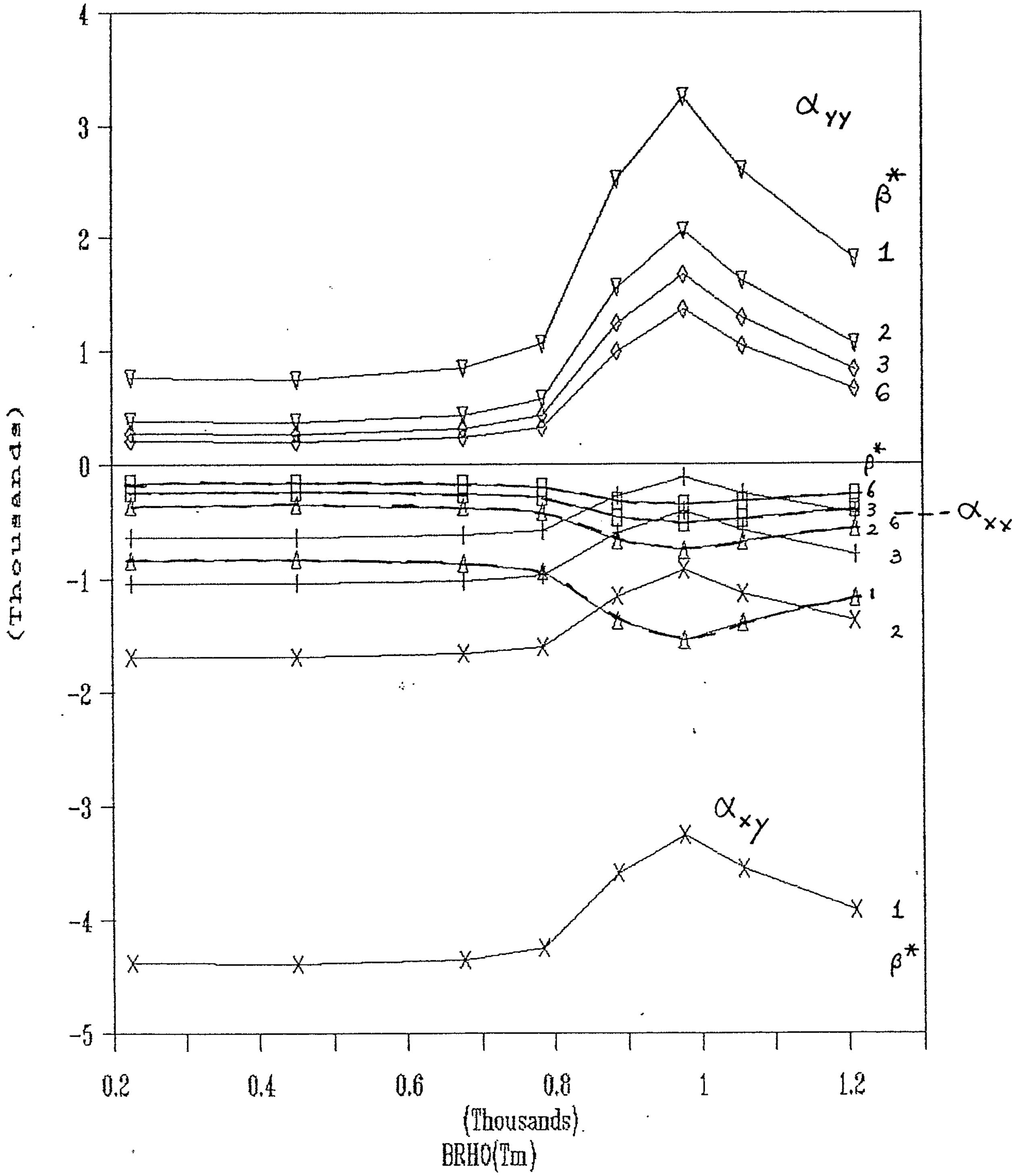

Fig. 5. Coefficients for tune vs amplitudes.