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## Tolerances on Systematic Field Errors in RHIC Magnets

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Collider Accelerator Department  
**Brookhaven National Laboratory**

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**RHIC Technical Note No. 58**

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# Tolerances on Systematic Field Errors in RHIC Magnets

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## I. Introduction

The goal of this report is to establish the systematic tolerances for the RHIC magnets such that the performance objectives of the accelerator may be achieved. The tolerances quoted in this report both summarize and expand the results obtained by several authors over the last few years.<sup>1,2,3,4,5</sup> The results in this report also complement the May 1989 edition of the RHIC Conceptual Design Report.<sup>6</sup>

The presence of magnetic nonlinearities in accelerator/storage rings is known to reduce dynamic aperture and beam lifetime. The design of the magnets must attempt to minimize the nonlinear harmonics without costly increase of the aperture, and tolerance criteria on systematic design-dependent errors must be established.

Specific experiments were performed on the CERN SPS<sup>7</sup> and the Fermilab Tevatron<sup>8</sup> intended to demonstrate a correlation between nonlinearities and beam behavior and to establish design criteria which would ensure the long-term beam stability in future machines.

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<sup>1</sup> H. Hahn, “*Tolerances on Systematic Magnet Errors*”, Report RHIC–AP–37 (1986).

<sup>2</sup> H. Hahn, “*RHIC Arc Magnet Field Quality Tolerances*”, Report AD/RHIC–22 (1987).

<sup>3</sup> A.G. Ruggiero, “*Analysis of the Decapole Systematic Error in the Dipoles and of the Correctors*”, Report AD/RHIC–AP–47 (1987).

<sup>4</sup> G.F. Dell, H. Hahn, G. Parzen, and A. Ruggiero, “*RHIC decapole Correction Magnet Requirements*”, Report AD/RHIC–28 (1987).

<sup>5</sup> G. Parzen, “ *$\nu$  Spreads Due to Systematic Field Errors Including  $b_3, b_4$  and  $a_1$* ”, Report AD/RHIC–AP–74 (1989).

<sup>6</sup> *Conceptual Design of the Relativistic Heavy Ion Collider RHIC*, Report BNL 52195 (1989).

<sup>7</sup> L. Evans, et al., “*The Nonlinear Dynamic Aperture Experiment in the SPS*”, Proc. European Particle Accelerator Conference, Rome, Italy (1988), p. 619.

<sup>8</sup> A. Chao, et al., “*Experimental Investigation of Nonlinear Dynamics in the Fermilab Tevatron*”, Phys. Rev. Lett. **61**, 2752 (1988) and FNAL Report FN-471 (1988).

The design of the CERN Large Hadron Collider requires that the magnet nonlinearities do not cause a tune spread of  $\Delta\nu > 5 \times 10^{-3}$  and a smear<sup>†</sup> of  $\sigma > 0.035$ .<sup>9,10,11</sup> The SSC Conceptual Design Report requires a sufficient linear aperture which is defined by the criteria that the smear is  $\sigma < 0.1$  and that the on-momentum tune shift with betatron amplitude  $\delta\nu < 5 \times 10^{-3}$ .<sup>12</sup>

Unfortunately, no experimental proof exists that long-term stability is assured if these criteria are satisfied. The tolerance criterion for smear is particularly vague whereas the limits on the tolerable tune spread are relatively firm. The tune spread within the beam depends mostly on the lower-order systematic harmonics and thus can be used to derive tolerances on the design field errors of the RHIC magnets.

The tune spread of the beam in RHIC is limited by the available tune range free of the 10th order and lower resonances, as shown in Fig. 1. The nominal tune is located between the 5th order systematic resonance at 28.800, and the 6th order resonance at 28.833. The working point of RHIC lies within a usable tune range of  $33 \times 10^{-3}$ . In reality the usable tune range available to RHIC might be reduced from this value. The beam-beam interaction and space charge effects, will cause a tune spread and reduce the tune range available for systematic magnet errors. Allowance must be made for other errors. For instance, exploratory calculations suggest a tune shift from random errors.<sup>13</sup>

In first approximation, the effect due to each field harmonic is additive. In order to limit the total tune spread contribution from systematic magnet errors, a tolerance on the tune shift due to an individual harmonic of about  $3 \times 10^{-3}$  will be imposed.

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<sup>†</sup> Note that in CERN publications and in the SSC-CDR, smear is defined as the rms fluctuation of the emittance whereas in the FNAL/SSC E778 publications as the rms fluctuation of the betatron amplitude.

<sup>9</sup> G. Brianti and K. Hübner, “*The Large Hadron Collider in the LEP Tunnel*”, Report CERN 87-5 (1987).

<sup>10</sup> J. Gareyte, et al., “*Dynamic Aperture and Long-Term Particle Stability in the Presence of Strong Sextupoles in the CERN PS*”, Proc. Particle Accelerator Conference, Chicago, 1989 and Report CERN SPS/89-2 (1989).

<sup>11</sup> F. Schmidt, “*Smear Calculations in the Presence of Linear Coupling for the 1988 Dynamical Aperture Experiment*”, Report CERN SPS/88-50 (1989).

<sup>12</sup> *Conceptual Design of the Superconductivity Super Collider*, Report SSC-SR-2020 (1986).

<sup>13</sup> G. Parzen, “ *$\nu$  - spread due to Random Multipoles*”, Report AD/RHIC-AP-84 (1989).

The tolerances on systematic field errors in RHIC dipoles and quadrupoles are obtained in this report by using analytic tune shift expressions discussed in the next section. A comparison with results from the tracking programs PATRICIA and ORBIT confirmed the adequacy of the analytical approach.

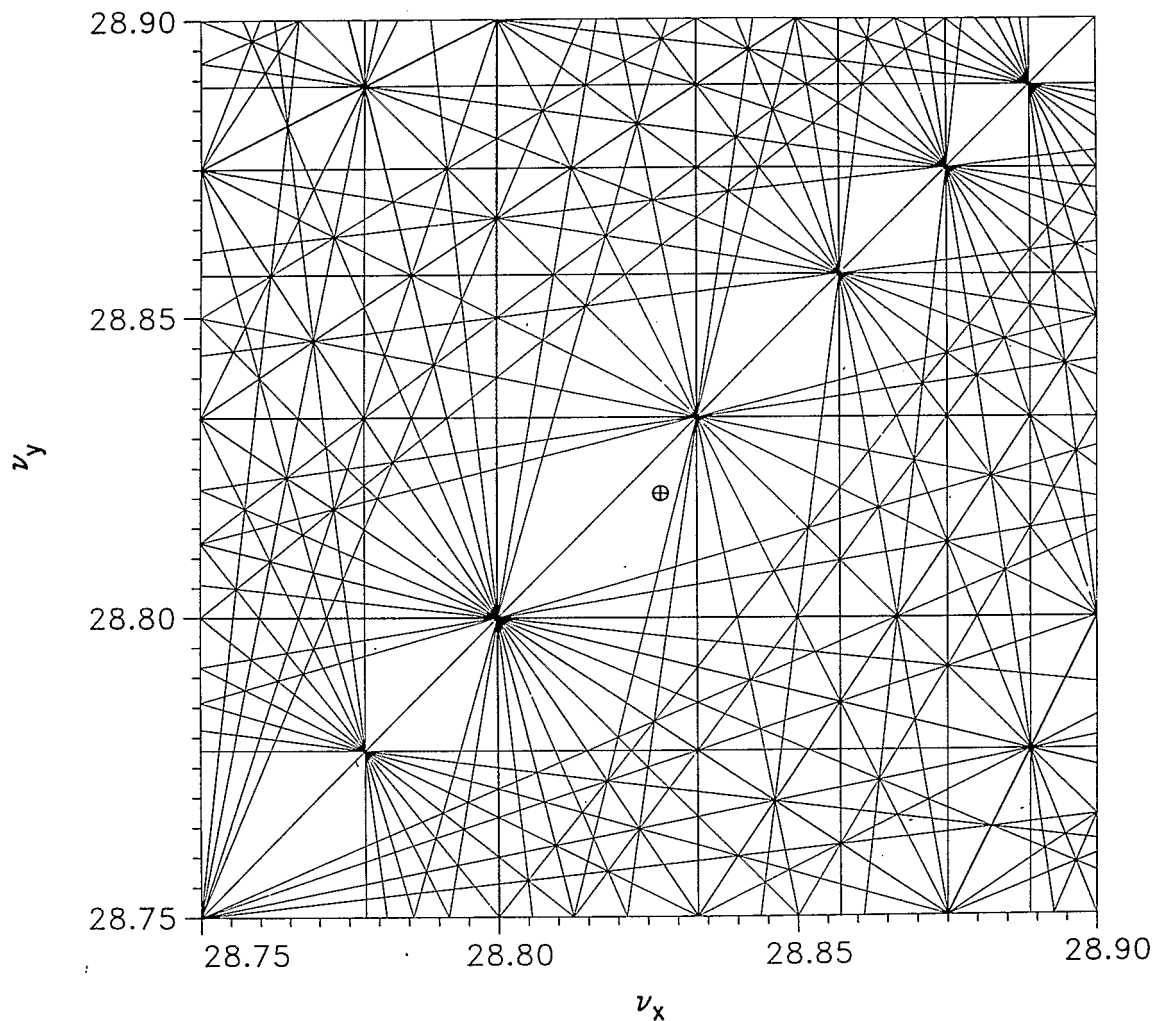


Fig. 1. Tune diagram showing the selected working point of RHIC at  $\nu_x = 28.826$ ,  $\nu_y = 28.821$  with neighboring sum and difference resonances.

## II. Betatron Tune Shift Expressions

The most important effect of systematic field errors is a spread in betatron tunes. In first order, only the normal field harmonics  $b_n$  contribute to the tune spread. The  $b_n$  are defined in terms of the vertical field on the median plane by

$$B_y(x) = B_o(1 + b_1x + b_2x^2 + \dots)$$

with  $B_o$  representing the nominal field in the arc dipoles.

The largest tune shift occurs for particles on the median plane having horizontal motion only. Using Jackson's results,<sup>14</sup> an analytical expression for the maximum tune shift can be written in the form

$$\Delta\nu_n = \sum_{i=0}^{(n-1)/2} \frac{C_{2i+1}^n C_{i+1}^{2i+2}}{2^{2i+2}} \epsilon_x^i \delta^{n-2i-1} \langle b_n \beta_x^{i+1} X_p^{n-2i-1} \rangle$$

where the summation goes from  $i = 0$  to  $i = \text{integer of } (n-1)/2$  and

$$C_j^k = \frac{k!}{j!(k-j)!}$$

$$\langle b_n \beta_x^k X_p^j \rangle = \frac{1}{2\pi\rho} \oint b_n \beta_x^k X_p^j ds$$

with the integral evaluated only where  $b_n \neq 0$ . Furthermore  $\delta = \Delta p/p$ ,  $\rho$  the bending radius in the arc dipoles,  $\epsilon_x$  the betatron invariant (defined by the maximum betatron amplitude,  $A$  as  $\epsilon_x = A^2/\beta_x$ ),  $\beta_x$  and  $X_p$  the usual lattice functions. Explicit expressions for the lower harmonics are given in Table I. Numerical results for the integrals over dipoles and quadrupoles are tabulated in the Appendix.

The values of maximum  $\delta$  and  $\epsilon_x$  to be used in the above tune shift expressions must be obtained for heavy ions from intrabeam scattering calculations. The results for gold beams in RHIC are given in the May 1989 Conceptual Design Report<sup>6</sup> and are here summarized in Figs. 2 and 3. These calculations assume a Gaussian distribution with an rms width of  $\sigma_p$  in the longitudinal plane and Gaussian distributions with 95% normalized emittances,  $\epsilon_H = \epsilon_V$  in the transverse planes.

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<sup>14</sup>A. Jackson, "Tune Shifts and Compensation from Systematic Field Components", Report SSC-107 (1987).

Table I. Maximum Horizontal Tune Shift for Multipole n

n	$\Delta\nu_n$
1	$\frac{1}{2}b_1\langle\beta_x\rangle$
2	$b_2\delta\langle\beta_x X_p\rangle$
3	$\frac{3}{2}b_3(\delta^2\langle\beta_x X_p^2\rangle + \frac{1}{4}\epsilon_x\langle\beta_x^2\rangle)$
4	$2b_4(\delta^3\langle\beta_x X_p^3\rangle + \frac{3}{4}\delta\epsilon_x\langle\beta_x^2 X_p\rangle)$
5	$\frac{5}{2}b_5(\delta^4\langle\beta_x X_p^4\rangle + \frac{3}{2}\delta^2\epsilon_x\langle\beta_x^2 X_p^2\rangle + \frac{1}{8}\epsilon_x^2\langle\beta_x^3\rangle)$
6	$3b_6(\delta^5\langle\beta_x X_p^5\rangle + \frac{5}{2}\delta^3\epsilon_x\langle\beta_x^2 X_p^3\rangle + \frac{5}{8}\delta\epsilon_x^2\langle\beta_x^3 X_p\rangle)$
7	$\frac{7}{2}b_7(\delta^6\langle\beta_x X_p^6\rangle + \frac{15}{4}\delta^4\epsilon_x\langle\beta_x^2 X_p^4\rangle + \frac{15}{8}\delta^2\epsilon_x^2\langle\beta_x^3 X_p^2\rangle + \frac{5}{64}\epsilon_x^3\langle\beta_x^4\rangle)$
8	$4b_8(\delta^7\langle\beta_x X_p^7\rangle + \frac{21}{4}\delta^5\epsilon_x\langle\beta_x^2 X_p^5\rangle + \frac{35}{8}\delta^3\epsilon_x^2\langle\beta_x^3 X_p^3\rangle + \frac{35}{64}\delta\epsilon_x^3\langle\beta_x^4 X_p\rangle)$
9	$\frac{9}{2}b_9(\delta^8\langle\beta_x X_p^8\rangle + 7\delta^6\epsilon_x\langle\beta_x^2 X_p^6\rangle + \frac{35}{4}\delta^4\epsilon_x^2\langle\beta_x^3 X_p^4\rangle + \frac{35}{16}\delta^2\epsilon_x^3\langle\beta_x^4 X_p^2\rangle + \frac{7}{128}\epsilon_x^4\langle\beta_x^5\rangle)$

The maximum  $\delta$  follows from the relation  $\delta = \sqrt{6}\sigma_p$  (95% of a Gaussian beam in 1-D) at injection energies and up to 30 GeV/u. However, voltage limitations of the rf system for RHIC at 100 GeV/u modify the definition of  $\delta$  to  $\delta = 2\sigma_p$ . Since transverse motion in the horizontal and vertical plane are fully coupled, the  $\epsilon_x$  which determines the maximum tune shift exceeds  $\epsilon_H$ . The maximum betatron invariant is given by  $\epsilon_x \approx (10/6)\epsilon_H/\pi$ , which corresponds to a  $\sqrt{10}\sigma_H$  betatron amplitude.<sup>15</sup>

In the following sections, the tolerances for the arc dipoles and arc/insertion quadrupoles are considered. These two lattice components are treated separately because the geometric symmetry of dipole and quadrupole magnets emphasize tolerance criteria for different magnetic field multipoles.

<sup>15</sup>G. Parzen, "Emittances and 4-dimensional Beam Surfaces in RHIC", Report AD/RHIC-AP-58 (1988).



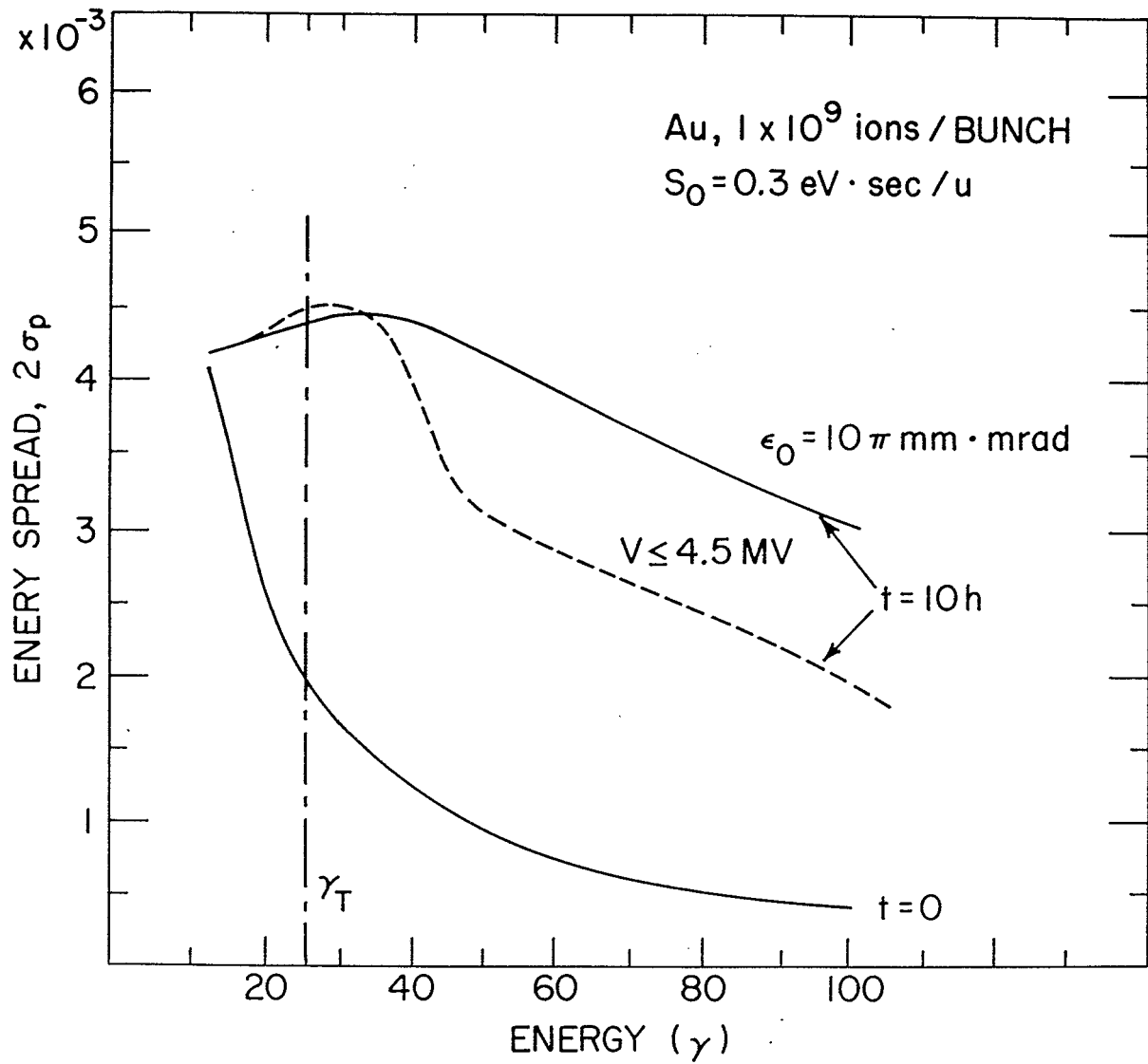


Fig. 2. Energy spread growth versus  $\gamma$  for Au ions. The dashed curve represents results for blown-up initial emittance.

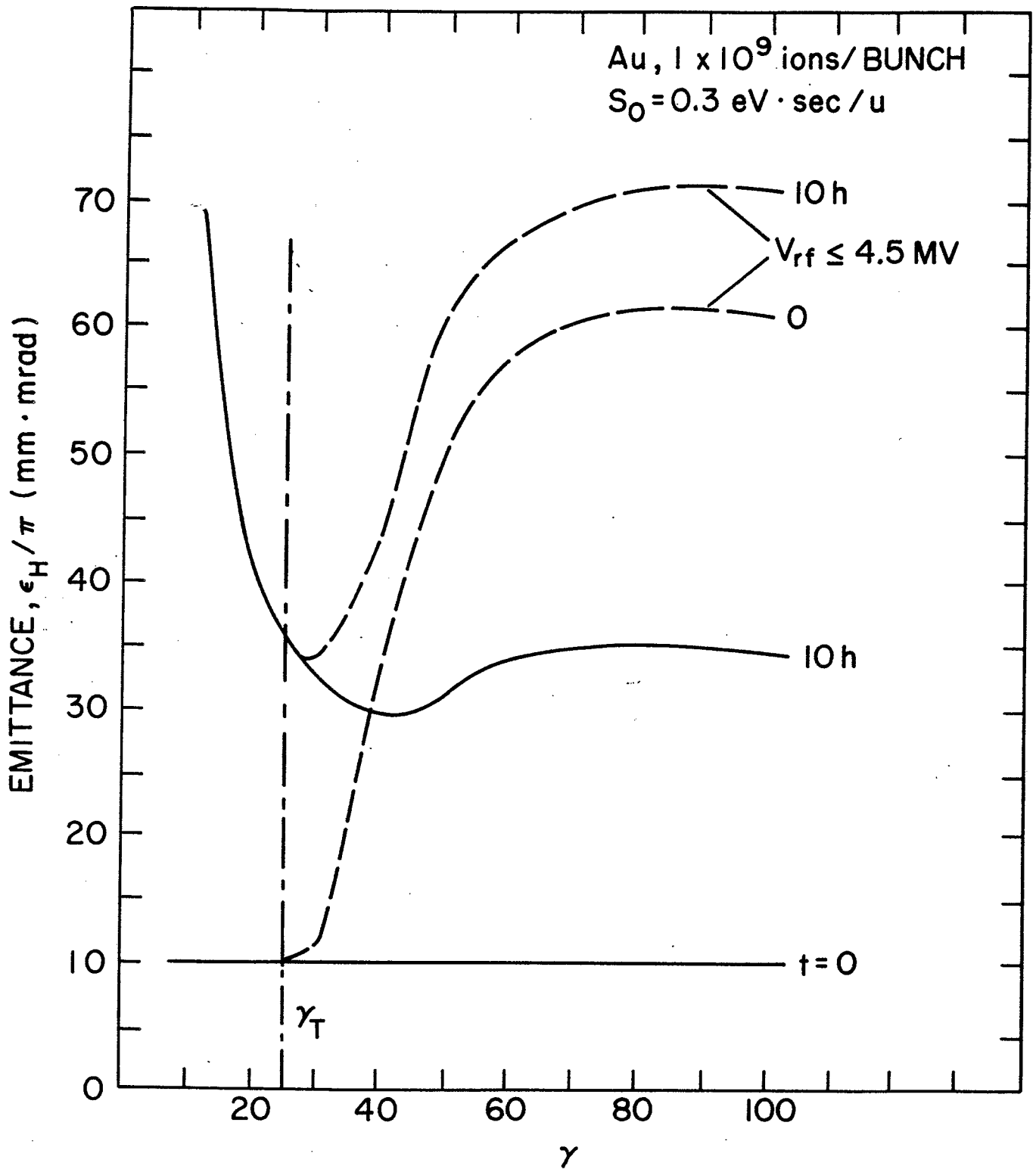


Fig. 3. Emittance growth versus  $\gamma$  for Au ions. The dashed curves represent results for blown-up initial emittance.

### III. RHIC Dipole Tolerances

Tolerances on the systematic harmonics in the arc dipoles are obtained by requiring that the maximum tune shift due to an individual harmonic remains below  $|\Delta\nu| = 3 \times 10^{-3}$ .

Calculated arc dipole tolerances are listed in Table II, using the worst-case emittance and momentum spread of gold beams at 10.4 GeV/u (injection), 30 GeV/u (10 h storage) and 100 GeV/u (top energy, 10 h storage). In this table the field harmonics are given in primed units

$$b'_n = 10^4 \times b_n \times (2.5\text{cm})^n$$

Also shown is the range of expected systematic fabrication errors. The theoretical design of a coil with 5 (and even 4) blocks can produce harmonics well below the tolerances quoted. However allowance for a 30% bias in the random fabrication errors must be made.<sup>6</sup> Not shown are expected saturation at top energy and magnetization effects at injection which depend on yoke design and conductor choice and are thus still subject to change.

The tolerances on the systematic harmonics in arc dipoles are depicted in Fig. 4 (solid curve). The dashed curve shows the more stringent tolerances based on the “ $6\sigma$ ” rule which has been used in the choice of magnet aperture

#### Quadrupole Tolerances

Systematic quadrupole (n=1) and sextupole (n=2) errors will be corrected by the regular quadrupole and chromaticity sextupole systems. These systems will be designed to control the working point (i.e. tune) and tune spread and tolerances apply to power supplies rather than magnets.

Controlling the tune to  $\Delta\nu < 1 \times 10^{-3}$  requires a current control in the main quadrupoles of

$$\left(\frac{\Delta I}{I}\right)_{b1} < \frac{p_{inj}}{p_{top}} \frac{\Delta\nu}{\nu} \approx 4 \times 10^{-6}$$

where  $p_{inj}$  and  $p_{top}$  is the ion momentum at injection and top energy.

#### Sextupole Tolerances

The natural chromaticity of RHIC with standard insertions is about  $\chi \approx -40$  and double that value for the low-beta configuration. Limiting the tune shift to  $\Delta\nu \approx 10^{-3}$  requires a control of the chromaticity to  $\Delta\chi < 0.2$  at 30 GeVc<sup>-1</sup>/u where  $\delta = \pm 0.55\%$ . The sextupole power supply accuracy follows

$$\left(\frac{\Delta I}{I}\right)_{b2} < \frac{30}{100} \frac{\Delta\chi}{\chi} \approx 1.5 \times 10^{-3}.$$

A systematic sextupole harmonic in the arc dipoles causes a chromaticity change of

$$\Delta\chi = \frac{2Nl_D}{2\pi\rho} \langle \beta_x X_p \rangle_D b_2$$

or in primed units  $\Delta\chi \approx 4b'_2$ . Although correctable, it is desirable to keep chromaticity changes due to the sextupole errors below 10% of the natural chromaticity. Taking into account the variations of beam parameters and lattice changes from low-beta insertions, one obtains the practical limits

$$b'_2 \lesssim \begin{cases} 1 & \text{@ mid-field} \\ 5 & \text{@ injection} \\ 5 & \text{@ top field} \end{cases}$$

Note that the strength of the chromaticity sextupoles is adequate to compensate the dipole sextupole errors.

The sextupole field in the arc dipoles is known to have a time dependence due to superconductor magnetization effects. Changes with time exceeding the tolerances of Table II will require adjustment of the chromaticity sextupole strength. The magnitude and magnet-to-magnet repeatability of time dependent effects has not been measured, and a full discussion of their implications has to wait. It is worthwhile pointing out that both LEP and HERA rely on reference magnets to compensate these effects.

It has been verified that the tune shift contribution from the insertion dipoles is negligible.

Table II. Arc Dipole Tolerances for RHIC

$T(\text{GeV/u})$	10.4	30	100	
$\delta(\%)$	$\pm 0.11$	$\pm 0.55$	$\pm 0.31$	
$\epsilon_H(\text{mm} \cdot \text{mrad})^*$	$10\pi$	$34\pi$	$35\pi$	
$A_\beta(\text{mm})^\dagger$	9.0	9.7	5.4	
n	$b'_n$	Tolerance		Expectation
1	0.07	0.07	0.07	$\pm 0.6$
2	0.76	0.14	0.24	$\pm 1.4$
3	1.4	0.28	0.90	$\pm 0.4$
4	7	0.56	3.1	$\pm 0.7$
5	19	1.1	11	$\pm 0.15$
6	75	2.0	36	$\pm 0.24$
7	230	3.7	120	$\pm 0.06$
8	760	6.6	370	$\pm 0.09$
9	2300	12	1100	$\pm 0.03$

\*Normalized emittance

†Maximum betatron amplitude at QF

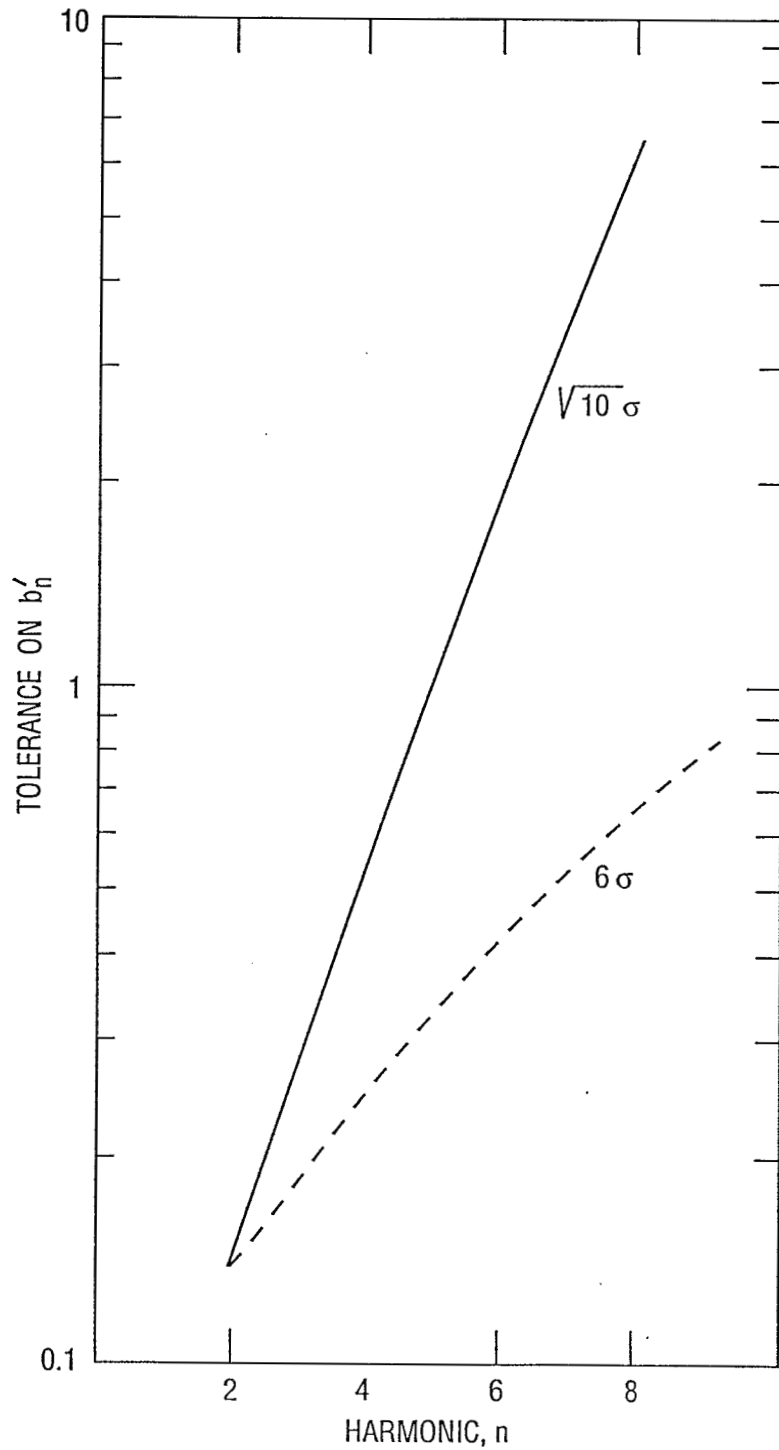


Fig. 4. Tolerances on field harmonics in arc dipoles based on coupled betatron motion with amplitude  $\sqrt{10}\sigma$  (solid curve) and the  $6\sigma$  rule (dashed curve).

Table III. Tune shift contribution from decapole errors

$T(\text{GeV}/u)$	30	100
$\delta(\%)$	$\pm 0.55$	$\pm 0.31$
$\epsilon_x(\text{mm} \cdot \text{mrad})$	1.71	0.539
$\beta^*(\text{m})$	6	2
Element	$10^3 \Delta\nu_4/b'_4$	
Arc Dipoles	5.38	0.97
BS	0.23	0.009
BC2	0.006	0.007

### Decapole Tolerances

The decapole error ( $b_4$ ) is an “allowed” harmonic and demands special attention because a correction system with only 2 independent variables (at QF and QD) is in principle insufficient and can only achieve a partial correction (error reduction by a factor of  $\sim 2$ ).

The importance of minimizing design and fabrication  $b_4$  errors has been previously recognized.<sup>4</sup> A reduction of the high-field saturation effects was achieved by increasing the coil-to-yoke spacing. A reduction of the mid-field error ( $\sim 2 \times 10^{-4}$  in dipoles DRB-006 and 007) can be achieved by a coil redesign. However, the tolerances on systematic  $b_4$  are such that they will drive the design and fabrication of arc dipoles.

The sensitivity of the tune shift to decapole errors in the insertion dipoles is lower by more than an order of magnitude (Table III) and consequently requires no special considerations.

Table IV. Tune shift contribution from octupole errors.

T(GeV/u)	30	100
$\delta(\%)$	$\pm 0.55$	$\pm 0.31$
$\epsilon_x(\text{mm} \cdot \text{mrad})$	1.71	0.539
$\beta^*(\text{m})$	6	2
Element	$10^3 \Delta\nu/b'_3$	
Arc dipoles	10.62	3.39
BS	0.57	0.10
BC2	0.78	1.8

### Octupole Tolerances

The octupole  $b_3$  harmonic is not allowed by the symmetry of dipole magnets. However, a bias in the fabrication errors must be expected which could exceed the tolerances given in Table II. Consequently, an octupole correction system consisting of two independent families at QF and QD must be provided.

The tune shift due to octupole errors contains a term independent of the dispersion. As a consequence,  $b_3$  errors in the insertion dipoles where  $\beta$  is large, will make a significant contribution as shown in Table IV.

The close proximity of the BC2 magnets of two rings could result in a larger  $b_3$  term. The installation of an octupole corrector next to BC2 seems indicated.



## IV. Quadrupole Magnet Tolerances

The tune shift contribution from unallowed harmonics ( $b_2, b_3, b_4, \text{etc.}$ ) in quadrupole magnets is negligible when compared to the dipole magnet contribution. However, the allowed harmonics  $b_5$  and  $b_9$  are a potential problem and require analysis.

The tune shift contribution due to  $n=5$  in various magnet elements is listed in Table V. Note that the  $n=5$  field error in quadrupoles is quoted in primed quadrupole units, defined by

$$q'_n = 10^4 \frac{G_o}{B_o} b_n r_C^{n-1}$$

where  $G_o$  is the quadrupole operating gradient, and  $r_C$  a reference radius taken as  $\approx 2/3$  coil radius. For RHIC  $r_C = 25$  mm in arc magnets and  $r_C = 40.6$  mm in the large aperture insertion quads.

Allowing for a maximum error in the dipoles of  $b'_5 < 0.15$  or a tune shift of  $\Delta\nu < 0.5 \times 10^{-3}$ , one can establish the quadrupole tolerances

$$q'_5 < \begin{cases} 100 & \text{@ injection} \\ 3.5 & \text{@ mid-field} \\ 15 & \text{@ top-field} \end{cases}$$

These tolerances can be met without the help of corrector magnets.

The tune shift contributions of  $b_9$  errors in quadrupoles are given in Table VI. This is an unallowed error in dipoles and any contribution from dipoles is negligible. With the results in Table VI one obtains the tolerance on the  $n=9$  errors in quadrupoles of

$$q'_9 < \begin{cases} 20 & \text{@ mid-field} \\ 600 & \text{@ top energy} \end{cases}$$

Table V. Tune shift contribution from n=5

T(GeV/u)	10.4	30	100
$\delta(\%)$	0.11	$\pm 0.55$	$\pm 0.31$
$\epsilon_x(\text{mm} \cdot \text{mrad})$	0.82	1.71	0.539
$\beta^*(\text{m})$	6	6	2
Element	$10^3 \Delta\nu/q'_5$		
Arc quad.	0.015	0.61	0.061
Ins.quad.Q4-Q8	0.003	0.06	0.061
Ins.quad.Q1-Q3	0.009	0.04	0.099
	$10^3 \Delta\nu/b'_5$		
Arc dip.		2.79	0.275
Ins.dip. BS		0.18	0.407
Ins.dip. BC2		0.23	0.009

Table VI. Tune shift contribution for n=9

T(GeV/u)	30	100
$\delta(\%)$	$\pm 0.55$	$\pm 0.31$
$\epsilon_x(\text{mm} \cdot \text{mrad})$	1.71	0.54
$\beta^*(\text{m})$	6	2
Element	$10^3 \Delta\nu/q'_9$	
Arc quad.	0.1340	0.00134
Ins.quad.Q4-Q8	0.0104	0.00013
Ins.quad.Q1-Q3	0.0024	0.00374

## APPENDIX

The contribution to the integral  $\langle \beta_x^{i+1} X_p^{n-2i-1} \rangle$  from the dipoles and the QF & QD quadrupoles in the arc cells have been evaluated using the SYNCH lattice functions. In the RHIC lattice,  $\beta_x$  and  $X_p$  is a function of momentum. However, it was found that the averaging integrals are essentially independent of  $\delta$  over the  $\pm 0.5\%$  range considered. For the arc dipoles a three-point Simpson-rule integration was used. For the arc quadrupoles a simple mid-point rule was utilized, taking into account the sign change of the systematic harmonics in QF and QD.

The Table VII gives the lattice function averages due to a single arc dipole of length  $l_D$

$$\langle \beta_x^j X_p^k \rangle_D = \frac{1}{l_D} \int \beta_x^j X_p^k ds$$

and also gives the lattice functions averaged over the focusing and defocusing arc quadrupoles each of length  $l_Q$ ,

$$\langle \beta_x^j X_p^k \rangle_Q = \frac{1}{2l_Q} \int \beta_x^j X_p^k ds$$

The total tune shift contribution from dipoles and quadrupoles in the N=72 arc cells is then obtained from

$$\langle b_n \beta_x^j X_p^k \rangle = \frac{2N}{2\pi\rho} (b_{nD} l_D \langle \beta_x^j X_p^k \rangle_D + b_{nQ} l_Q \langle \beta_x^j X_p^k \rangle_Q)$$

with the indices  $j = i + 1$  and  $k = n - 2i - 1$ .

The Table VIII gives selected lattice functions averaged over the entire ring. In this table the lattice function averages are defined by

$$\langle \beta_x^j X_p^k \rangle = \frac{1}{2\pi\rho} \oint \beta_x^j X_p^k ds$$

and are evaluated separately for different magnet elements.

Table VII. Lattice Function Averages  $\langle \beta_x^{i+1} X_p^{n-2i-1} \rangle_{D,Q}$ 

n	i=0	1	2	3	4	units
ARC DIPOLES						
1	23.6					m
2	27.4					m <sup>2</sup>
3	32.4	621				m <sup>3-i</sup>
4	39.0	753				m <sup>4-i</sup>
5	48.0	930	1.80 × 10 <sup>4</sup>			m <sup>5-i</sup>
6	60.2	1.17 × 10 <sup>3</sup>	2.28 × 10 <sup>4</sup>			m <sup>6-i</sup>
7	76.7	1.50 × 10 <sup>3</sup>	2.92 × 10 <sup>4</sup>	5.71 × 10 <sup>5</sup>		m <sup>7-i</sup>
8	99.4	1.95 × 10 <sup>3</sup>	3.81 × 10 <sup>4</sup>	7.45 × 10 <sup>5</sup>		m <sup>8-i</sup>
9	131	2.56 × 10 <sup>3</sup>	5.03 × 10 <sup>4</sup>	9.87 × 10 <sup>5</sup>	1.94 × 10 <sup>7</sup>	m <sup>9-i</sup>
ARC QUADRUPOLES						
5	155	3.09 × 10 <sup>3</sup>	6.11 × 10 <sup>4</sup>			m <sup>5-i</sup>
9	995	1.97 × 10 <sup>7</sup>	3.89 × 10 <sup>5</sup>	7.69 × 10 <sup>6</sup>	1.52 × 10 <sup>8</sup>	m <sup>9-i</sup>

Table VIII. Lattice Function Averages  $\langle \beta_x^{i+1} X_p^{n-2i-1} \rangle$ 

n	Magnet Type	i=0	1	2	units
STANDARD INSERTION $\beta^* = 6\text{ m}$					
3	arc dip.	28.8	553		$\text{m}^{3-i}$
	ins. dip.BS	0.8	84		
	ins. dip.BC2	0.2	226		
4	arc dip.	34.8	670		$\text{m}^{4-i}$
	ins. dip.BS	0.7	48.7		
	ins. dip.BC2	0.0	1.9		
5	arc dip.	42.8	838	$1.6 \times 10^4$	$\text{m}^{5-i}$
	ins. dip.BS	0.6	39.2	$0.4 \times 10^4$	
	ins. dip.BC2	0.0	19.6	$1.9 \times 10^4$	
	arc quad.	17.7	352	$0.7 \times 10^4$	
	ins. quad.Q4-Q9	0.9	34.9	$0.2 \times 10^4$	
	ins. quad.Q1-Q3	0.1	69.4	$5.7 \times 10^4$	
LOW-BETA INSERTION $\beta^* = 2\text{ m}$					
3	arc dip.	28.8	553		$\text{m}^{3-i}$
	ins. dip.BS	0.1	73		
	ins. dip.BC2	0.4	1751		
4	arc dip.	34.8	670		$\text{m}^{4-i}$
	ins. dip.BS	0.	16.4		
	ins. dip.BC2	0.	11.2		
5	arc dip.	42.8	828	$1.6 \times 10^4$	$\text{m}^{5-i}$
	ins. dip.BS	0.	4.1	$0.2 \times 10^4$	
	ins. dip.BC2	0.	92.9	$41.8 \times 10^4$	
	arc quad.	17.7	352	$0.7 \times 10^4$	
	ins. quad.Q4-Q9	1.0	32.8	$0.2 \times 10^4$	
	ins. quad.Q1-Q3	0.1	314	$170 \times 10^4$	