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Multipole expansion for the eddy current correction coils

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Abstract

A closed form for the multipoles is derived for the eddy current correction coils and its images. These multipole coefficients are about the same as that obtained from the coils and their nearest images.

1. Introduction

Magnet field due to current filament in two parallel iron plates appears often in the particle accelerators, for example, the eddy current on the vacuum chamber in the low energy fast cycling synchrotron and the multipole correction coils in the magnet. In this note, we shall derive simple expression for the multipole coefficients for a single coil and apply the method to the eddy current correction coil in AGS Booster.

2. Field Due to a Single Current Filament in Two Sheets of Parallel Plates

The magnetic field due to a single filamet at (x_m, y_m) shown on Fig. 1 in two sheets of infinite permeability iron can be expressed as the linear combination of fields due to the currents and its images. Using the complex variable formulation of Beth,[1] $H = H_y + iH_x$, the field at the observation point $z = x + iy$ is

$$H = \frac{I}{2\pi}\left\{\frac{1}{z - z_m} + \sum_{\substack{k=1\\ odd}}^{\infty}\left(\frac{1}{z - z_m^* - ikg} + \frac{1}{z - z_m^* + ikg}\right) + \sum_{\substack{k=2\\ even}}^{\infty}\left(\frac{1}{z - z_m - ikg} + \frac{1}{z - z_m + ikg}\right)\right\} \tag{1}$$

where $z_m = x_m + iy_m$ is the location of current filament and $z_m^* = x_m - iy_m$, g is the gap of the iron. Eq. (1) can be summed to give

$$H = \frac{I}{4g}\left(\tanh\frac{\pi(z - z_m^*)}{2g} + \coth\frac{\pi(z - z_m)}{2g}\right) \tag{2}$$

The analytic form can be expanded onto the multipole as

$$H = \frac{I}{4g}\sum_n \frac{1}{n!}(\alpha_n + \beta_n)\left(\frac{\pi}{2g}\right)^n z^n \tag{3}$$

where

$$\alpha_n = \left.\frac{\partial^n \tanh(s - \frac{\pi z_n^*}{2g})}{\partial s^n}\right|_{s=0} \tag{4}$$

$$\beta_n = \left.\frac{\partial^n \coth(s - \frac{\pi z_n}{2g})}{\partial s}\right|_{s=0} \tag{5}$$

are listed in Table 1.

Table 1. Multipole Coefficients of a Single Wire

n	α_n	$t = \tanh(-\frac{\pi z_m^*}{2g})$	$c \pm \cosh(-\frac{\pi z_m^*}{2g})$
0	t		
1	c^{-2}		
2	$-2tc^{-2}$		
3	$-2c^{-4}+4t^2c^{-2}$		
4	$16\ tc^{-4}-8t^3c^{-2}$		
5	$16\ c^{-6}-88t^2c^{-4}+16t^4c^{-2}$		
6	$-272tc^{-6}+416t^3c^{-4}-32t^5c^{-2}$		
7	$-272c^{-8}+2880t^2c^{-6}-1824t^4c^{-4}+64t^6c^{-2}$		
8	$7936tc^{-8}-24576t^3c^{-6}+7680t^5c^{-4}\ -128t^7c^{-2}$		
n	β_n	$\tilde{t} \equiv \coth(-\frac{\pi z_m}{2g})$	$s \equiv \sinh(-\frac{\pi z_m}{2g})$
0	$\tilde{t}$		
1	$-s^{-2}$		
2	$2\tilde{t}s^{-2}$		
3	$-2s^{-4}-4\tilde{t}^2s^{-2}$		
4	$16\tilde{t}s^{-4}+8\tilde{t}^3s^{-2}$		
5	$-16s^{-6}-88\tilde{t}^2s^{-4}-16\tilde{t}^4s^{-2}$		
6	$272\ \tilde{t}s^{-6}+416\tilde{t}^3s^{-4}+32\tilde{t}^5s^{-2}$		
7	$-272s^{-8}-2880\tilde{t}^2s^{-6}-1824\tilde{t}^4s^{-4}-64\tilde{t}^6s^{-2}$		
8	$7936\tilde{t}s^{-8}+24576\tilde{t}^3s^{-6}+7680\tilde{t}^5s^{-4}+128\tilde{t}^7s^{-2}$		

The expansion coefficients in Eq. (3) involves $(\pi z/2g)^2$, one may expect that the multipole expansion would require $|\pi z/2g| < 1$. In reality, the above statement is not necessary true. As an example, we assume two current sheets fill the gap. Let $\lambda = I/g$ be the current per unit length. The result is the same as two infinite parallel current sheets in the free space. The resulting field is given by

$$H = \lambda \ .$$

All multipole coefficients vanishes except $n = 0$. The multipole expansion is valid irrespect of the height of the gap.

The radius of convergence for the multipole expansion depends on the pole of Eq. (2). At the location of the current filament, the magnetic field has a simple pole singularity shown in Eq. (2). The radius of convergence is the distance to the nearest current filament. In the following, we shall see realistic examples on the radius of convergence for the multipole expansion.

3. Application to AGS Booster

The eddy current correction coil is shown in Fig. 2, i.e., the current I at z_m and z_m^* and $-I$ at $-z_m$ and $-z_m^*$. The total field is a linear combination of these filaments and their multipole images. The multipole expansion can be expressed at sums of the coefficients of section 2.

$$\begin{aligned} H =& \frac{1}{4g}\sum_n \frac{I_m}{n!}\Big(\alpha_n(z_m) + \alpha_n(z_m^*) - \alpha_n(-z_n) - \alpha_n(-z_m^*) \\ &+ \beta_n(z_m) + \beta_n(z_m^*) - \beta_n(-z_m) - \beta_n(-z_m^*)\Big)\left(\frac{\pi}{2g}\right)^n z^n \end{aligned} \tag{6}$$

The perturbation to the main dipole magnetic flux density is then given by

$$\frac{\Delta B}{B} = \frac{\mu_0 H}{B} = \sum_{n=0}^{\infty} b_n z^n \tag{7}$$

with

$$b_n = \frac{\mu_0}{4gB}\frac{1}{n!}\sum_m 2\Big(\alpha_n(z_m) + \alpha_n(z_m^*) + \beta_n(z_m) + \beta_n(z_m^*)\Big)\left(\frac{\pi}{2g}\right)^n I_m \tag{8}$$

$$b_{n=odd} = 0$$

where I_m is the current at the m–th filament position. For the configuration of Fig. 2, the summation is $m = 1$ and 2 and $I_m = m \cdot I$. The left–right and up–down symmetries are already included in obtaining Eq. (8).

Table 2. The Multipole Coefficient for Obtaining the Sextupole Component $B_2 = -0.785$ m^{-2} to Compensate the Eddy Current Sextupoles

n	$b_n(m^{-n})$
0	6.2728×10^{-3}
2	-0.785
4	-314.8
6	9.911×10^{4}
8	8.9017×10^{7}
10	-2.0831×10^{11}

Table 2 shows the result of multipole coefficients. It is worthwhile to point out that these b_n coefficients do not differ appreciably from the calculation with only nearby images.[2] It is thus concluded that nearby image approximation is not bad to obtain simple approximation.

Figure 3 shows the B/B_0 seen by the particle in a perfect dipole, where the dipole and sextupole components in the flux density of the correction coil has been removed due to perfect compensation. At the distance of 1 inch, the decapole component b_4 contributes about 4×10^{-4}. The effect would be more important at the tune of 4.8, which is a systematic 5th order resonance for the AGS Booster.

Figure 4 compares exact B/B_0, where b_0 and b_2 are compensated, the multipole expansion,

$$\frac{\Delta B}{B} = \sum_{n=4}^{N} b_n x^n \quad .$$

Note that multipole expansion is indeed valid only up to about 1.5″. The exact field is well behaved within 3×10^{-4}, while the multipole becomes unrealistically large.

On the other hand, when the first coil is moved to 2.5″ and the second coils to 5″, the multipole expansion gives good field description up to 2.5″ from $n = 4$ to $n = 10$. The multipole expansion is indeed a good approximation to the exact expression up to the coil location.

4. Conclusion

We have derived analytic expression for the multipole coefficients due to current coils in the parallel iron plates. The radius of convergent for the multipole expansion depends on the location of the coils but not on the gap of the iron. The method is then applied to the eddy current correction coils on the vacuum chamber. The results indicate that the multipole obtained from the current carrying filaments and their nearby images is a good approximation up to the location of the nearest filament.

The multipole expression obtained in this study can however be used in the calculation of multipole coefficients due to the eddy current as well.

References

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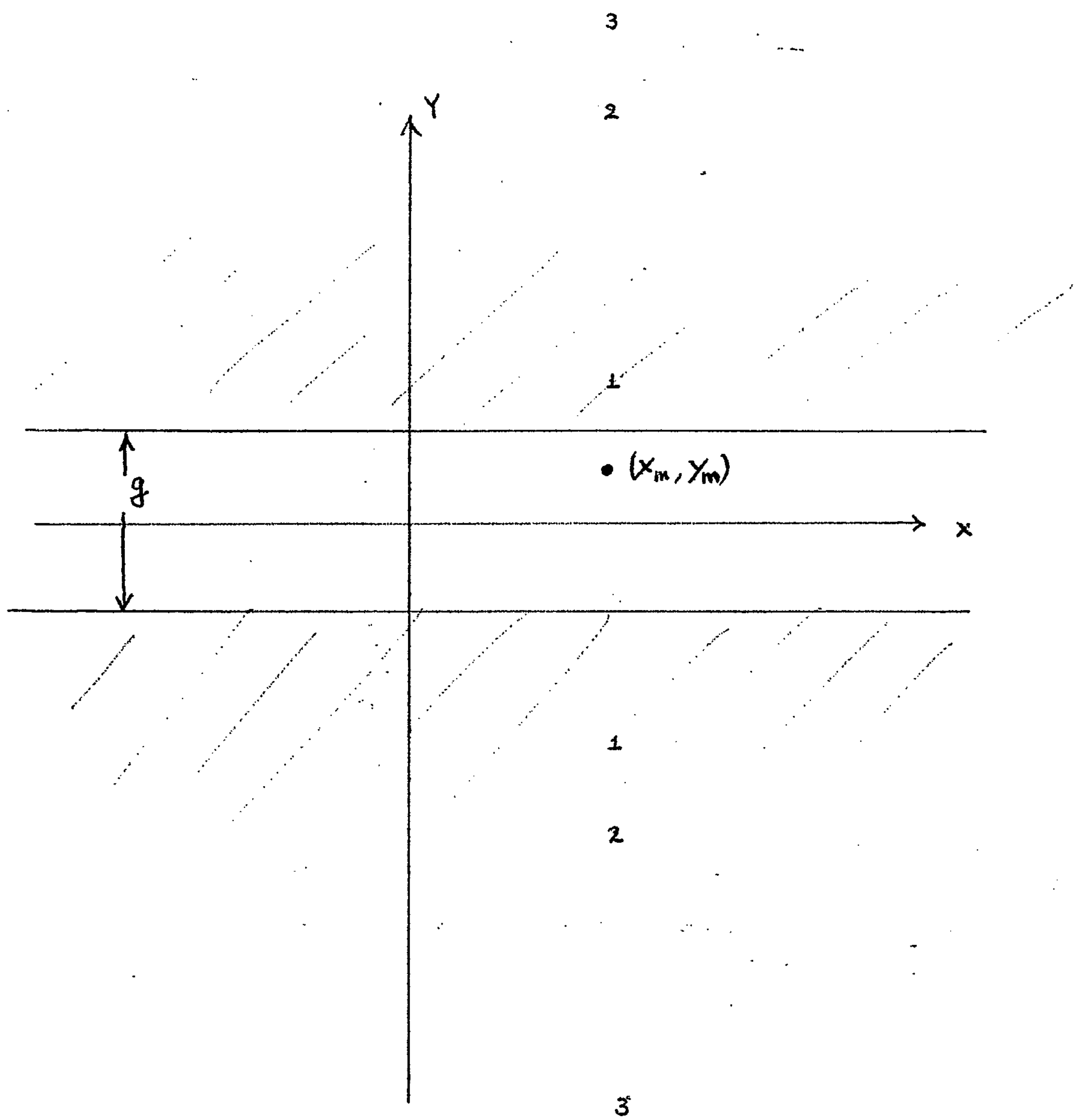
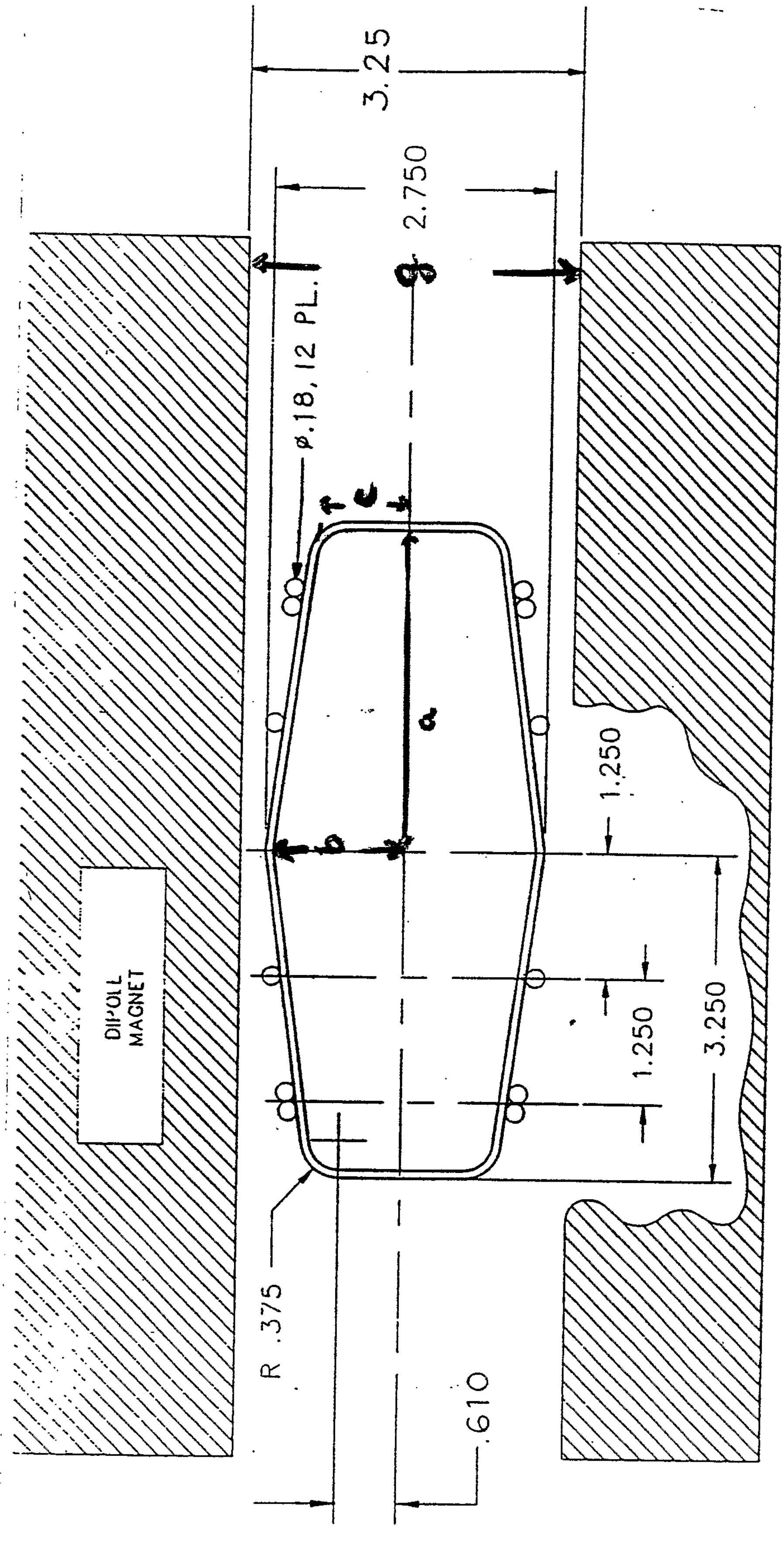


Figure 1. The current filament location (x_m, y_m) and its images marked 1,2,3,...etc. These images are summed in Eq. (1) to obtain the field H.


3.25
2.750
ø.18, 12 PL.
c
a
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1.250
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DIPOLE MAGNET
R .375
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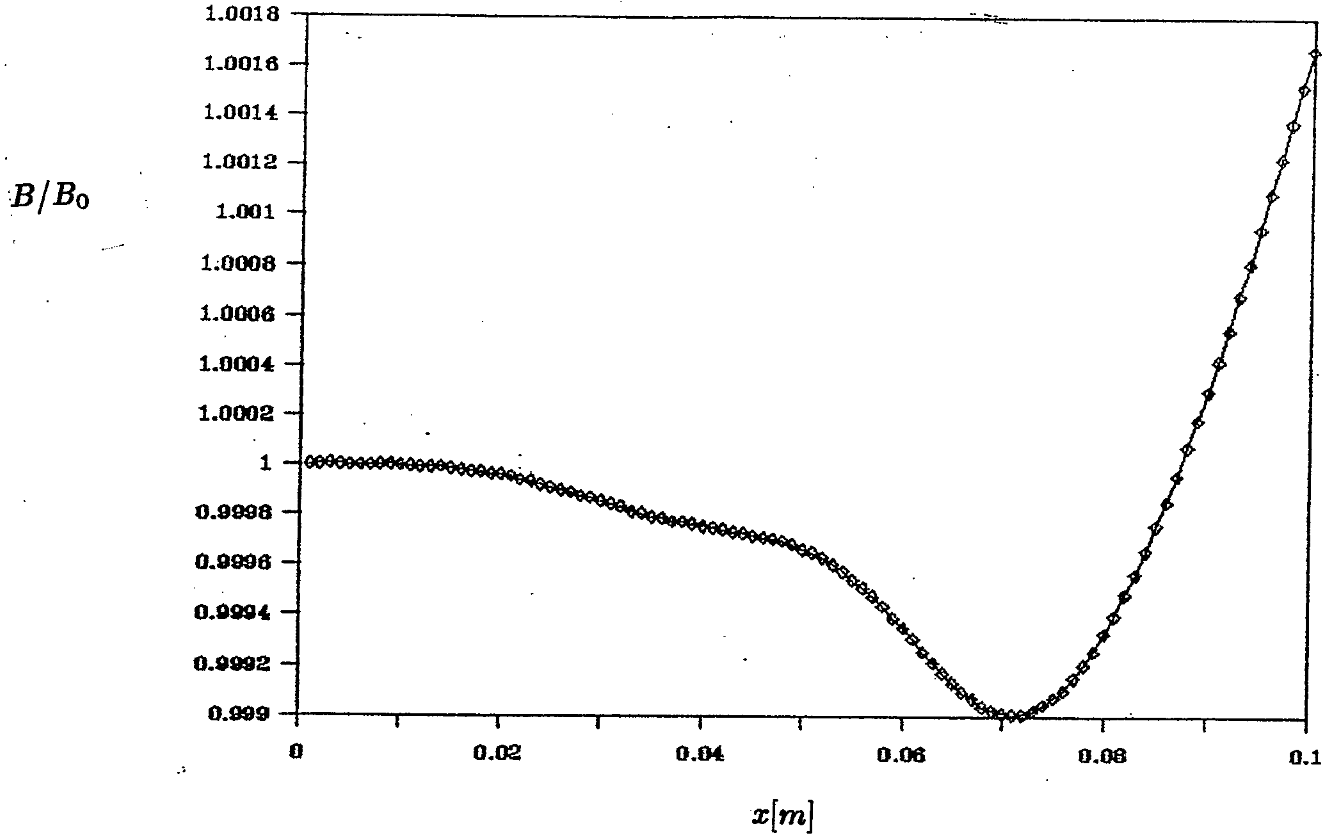

Figure 3. B/B_0 of a perfect dipole, B_0 and the flux density due to eddy current correction coils are plotted as a function of x.

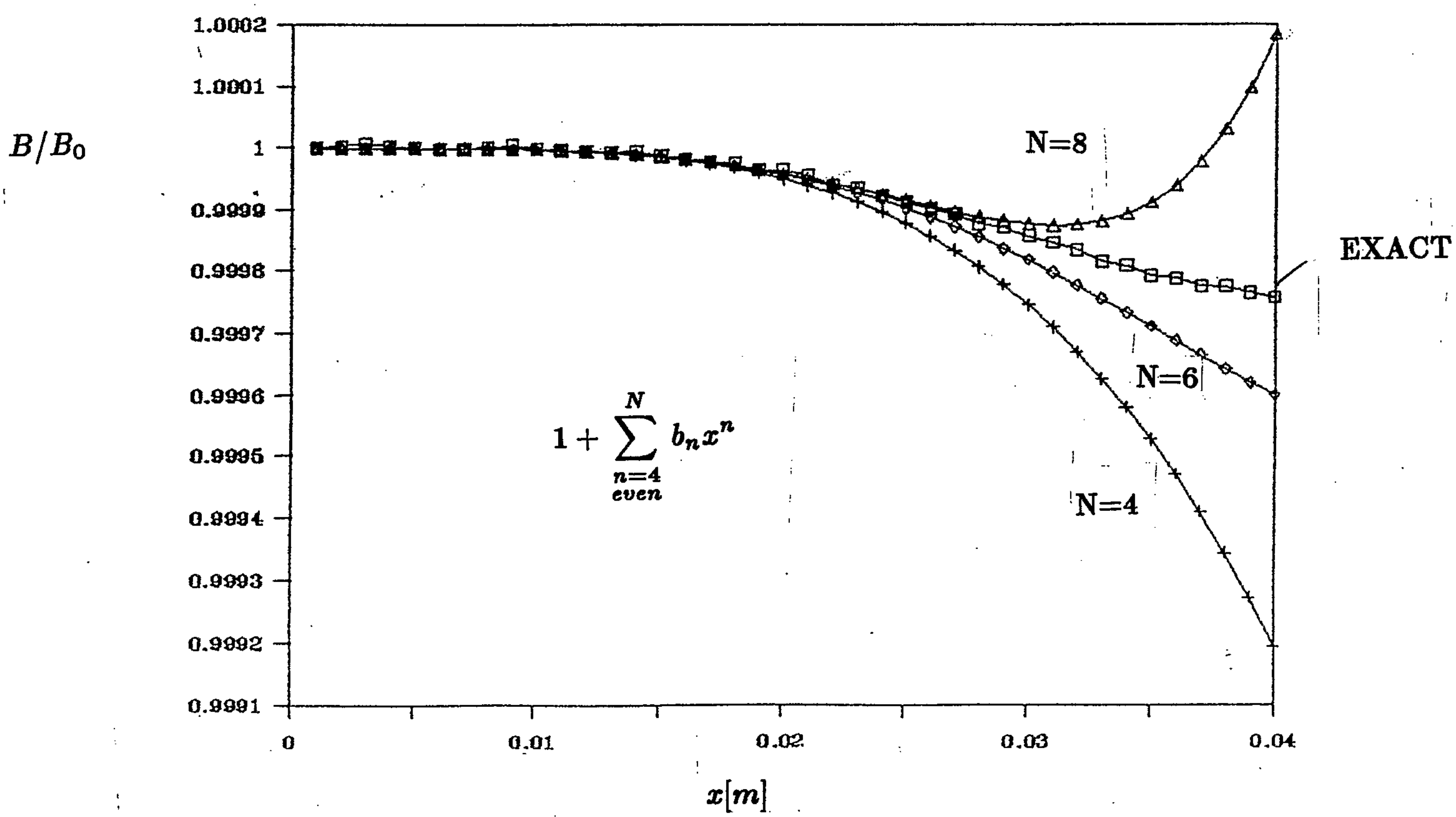

Figure 4. The multipole expansion would deviate from the exact B seen by the particle at $x > 30$ mm.