BNL-101436-2014-TECH

RHIC/RF- 17;BNL-101436-2013-IR

Longitudinal emittance blowup during damping of injection errors

D. P. Deng

January 1993

Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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RHIC/RF Technical Note No. 17

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D.-P. Deng
Brookhaven National Laboratory
Upton, NY 11973

January 12, 1994

Abstract

Having momentum and phase injection errors into *RHIC* from *AGS* , the longitudinal emittance in *RHIC* will blowup itself. A longitudinal damper can reduce the blowup, its power inversely depends on how much dilution is tolerable. This note studies gold ion (Au^{79+}) beam at injection, its emittance blowup at various kicker voltages.

1 Introduction

Rebucketing, transferring ion beams from accelerating cavities into storage cavities, is a major bottle neck in *RHIC* . Large longitudinal emittance beams will result large beam loss. It is important to have small longitudinal emittance beams in order to store as much beam as possible. One source of longitudinal emittance blowup is at injection, a bunch injected off the center of a bucket starts dipole oscillation which dilutes the particle distribution in the bucket due to synchrotron frequency spread within the bunch and hence leads to emittance blowup.

As an approximation for longitudinal emittance and small injection error in momentum (δp) or in phase ($\delta\phi$), the fractional change of longitudinal emittance $\frac{\Delta\epsilon}{\epsilon}$ can be written as

$$\frac{\Delta\epsilon}{\epsilon} = 2\frac{\delta\phi}{\Delta\phi} = 2\frac{\delta p}{\Delta p}, \tag{1}$$

where $\Delta\phi$ and Δp are the half bunch length and half momentum spread. Considering the case that the bunch length measured in the rf bucket is 128°, assuming the phase injection error is 10°, then the emittance dilution will be 31% if not corrected. Note throughout the note a bucket is assumed to be stationary.

The particle distribution of a matched bunch in a bucket is stationary. Whence the bunch is shifted relative to the center of the bucket, the distribution will evolve in time and eventually reach to another stationary distribution. One can define a decoherence time after which it's said the new distribution has lost any resemblance to its initial distribution, in other word the bunch has completely smeared out and the dilution process has finished. The decoherence time is the the time for the two particles on the opposite sides (in momentum error) or ends (in phase error) to overlap in phase which is (easily expressed in phase error) [1]

$$\tau_d = \frac{\pi}{\Delta\omega_\nu} = \frac{4\pi}{\omega_\nu\,\Delta\phi\,\delta\phi} \tag{2}$$

where $\Delta\omega_\nu = \frac{1}{4}\omega_\nu\,\Delta\phi\,\delta\phi$ is the synchrotron freqency difference of these two particles and ω_ν is the small angle synchrotron frequency (angular). It is easily seen that for the small bunch length or small phase errors or both, the decoherence time is long; for large bunch length or large phase errors or both, the decoherence time is short. Another way looking at it is that dilution is fast if the bunch experiences large part of the nonlinear focusing region of the bucket; slow if the bunch is mainly in the linear focusing region. Let's look at an example of gold ion (Au^{79+}) beam at injection. The parameters of the accelerating cavities are RF voltage $V_{rf} = 170\,kV$, RF frequency $f_{rf} = 26.66\,MHz$. Suppose for an emittance of $\epsilon = 0.2\,eVs/u$, the half bunch length is $\Delta\phi = 1.11\,rad$, the synchrotron frequency $f_\nu = \frac{\omega_\nu}{2\pi} = 92\,Hz$. Then for a phase error of $0.13\,rad$ ($7.6°$) or equivalent momentum error of $\frac{\delta p}{p} = 1.0 \times 10^{-4}$ the decoherence time is $\tau_d \approx 0.15\,s$ or roughly 14 synchrotron periods. Assuming fixed momentum $\frac{\delta p}{p}$, the decoherence time scales roughly as $\epsilon^{-\frac{1}{2}}$, from which one can see large longitudinal emittance dilute fast.

The longitudinal emittance of a bunch behaves like entropy in an isolated thermodynamical system, unless the bunch is being "cooled", it tends to increase. It is apparent that in order to have minimal emittance dilution, the time period to damp out the injection errors has to be significantly shorter than the decoherence time, which in turn depends on how much kicker voltage available in one turn. The next section will concentrate on the movement of the center of a bunch, effectively we assume that the bunch is a point charge.

2 Kicker voltage and bunch center motion

Since the injection errors differ from one bunch to another, damping has to be accomplished in a bunch-to-bunch mode. A longitudinal kicker which supplies energy to the bunch as a whole to reduce its oscillation amplitude is considered. Let's study the motion of the center of a bunch, which is assumed to be rigid.

$$(\frac{\delta E}{E})_n = (\frac{\delta E}{E})_{n-1} + \frac{eV_{rf}}{E}\sin\phi + k\frac{eV_k}{E} \tag{3}$$

$$\phi_n = \phi_{n-1} + \frac{2\pi h\eta}{\beta^2}(\frac{\delta E}{E})_{n-1} \tag{4}$$

where V_k is the maximum available voltage from the kicker per turn, k ($-1 \leq k \geq$) is a variable to control how the kicker supplies energy to the bunch. For simplicity, let's assume a constant voltage mode that is k only takes ± 1. If the energy of the bunch center is greater or equal than the synchronous energy, $k = -1$; otherwise $k = 1$. It is not physical to assume the bunch is rigid, for it will not show us how much emittance dilution has occurred, however it will guide us to determine a reasonable range of kicker voltages. The complete study the motion of the bunch and its dilution process relies on applying the set of equations above to all particles in the bunch, which is in the next section.

Take the same example given in last section, assuming the momentum injection error is $\frac{\delta p}{p} = 1.0 \times 10^{-4}$ and the kicker finishes damping when the center of the bunch sits in the phase space $|\frac{\delta p}{p}| \leq 10^{-5}$ and $|\delta\phi| \leq 1^\circ$, in which the injection errors give rise to negligible emittance dilution. Figure 1 shows the trajectory when the kicker voltage $V_k = 500V$, which takes about 9 synchrotron period. Figure 2 shows the trajectory when the kicker voltage $V_k = 1000V$, which takes close 5 synchrotron period. Figure 3 shows the trajectory when the kicker voltage $V_k = 5000V$, which takes just a little more than 1 synchrotron period. All these cases are summarized in Table 1

V_k (kV)	0.5	1.0	5.0
τ (τ_ν)	9	5	1

Table 1: Time (in synchrotron periods) required to damp injection error to negligible level

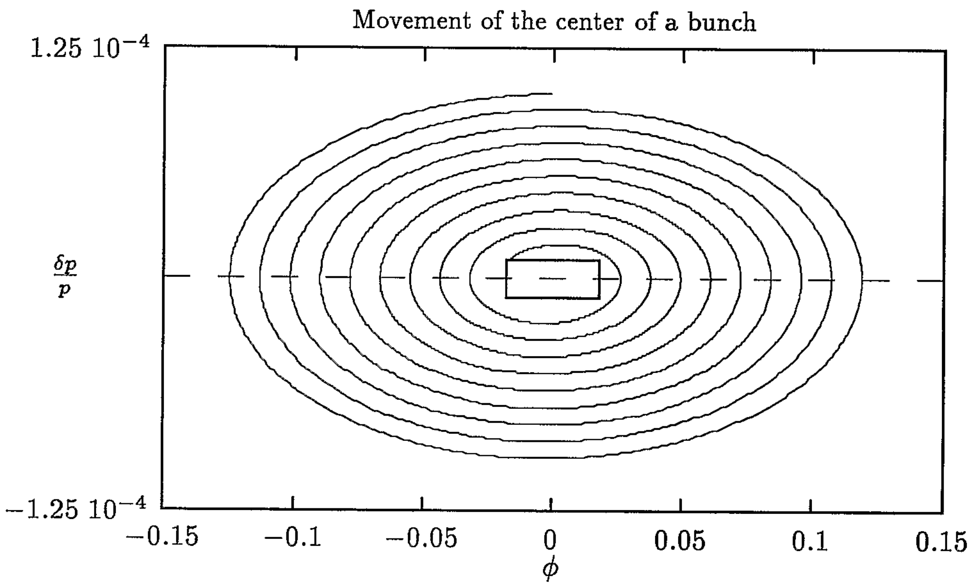

Figure 1: Movement of the center of the bunch when $V_k = 500V$

3 Emittance blowup

In this section, we simulate a bunch of particles under the action of the kicker. The equations of (4) become

$$(\frac{\delta E}{E})_{i,n} = (\frac{\delta E}{E})_{i,n-1} + \frac{eV_{rf}}{E}\sin\phi_i + k\frac{eV_k}{E} \quad (5)$$

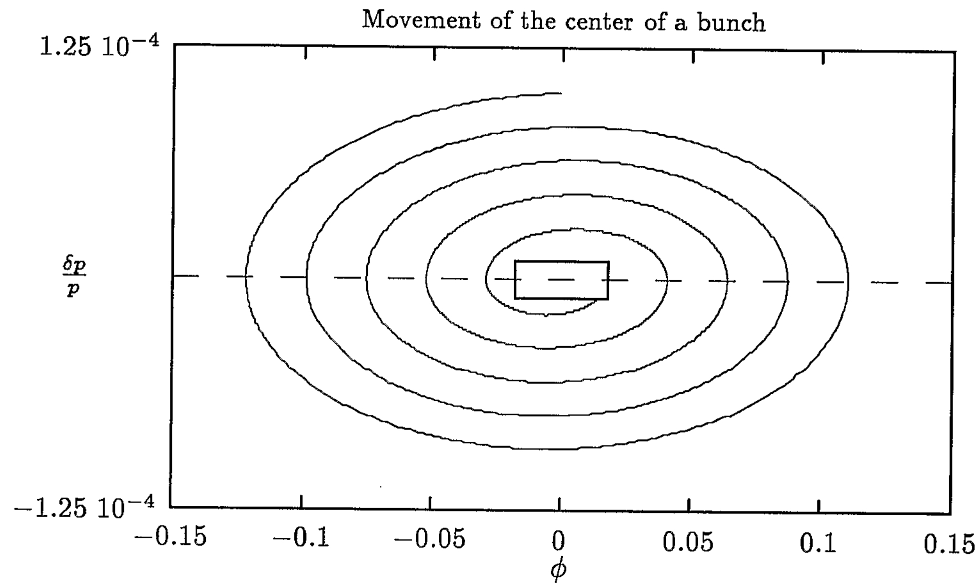

Figure 2: Movement of the center of the bunch when $V_k = 1000V$

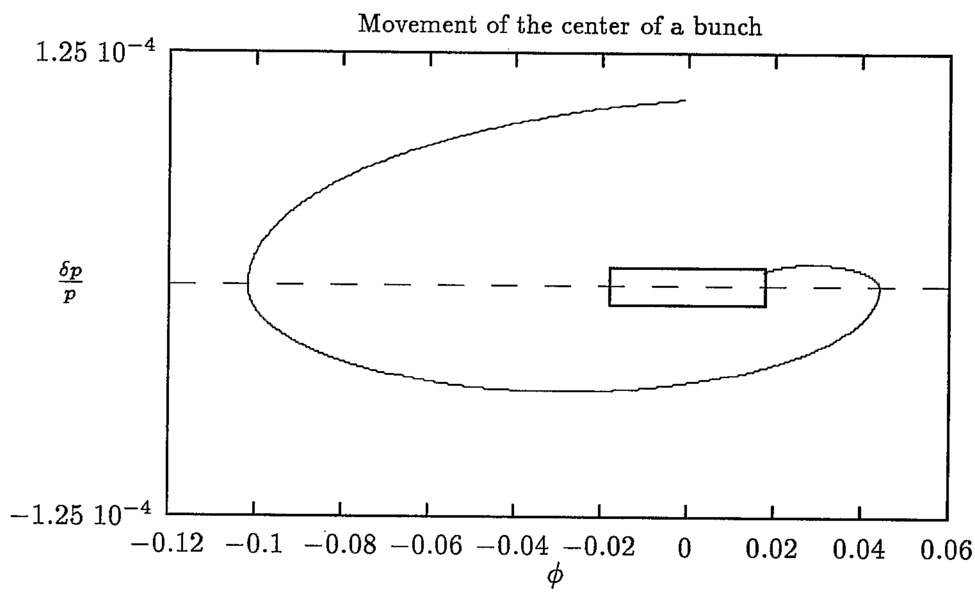

Figure 3: Movement of the center of the bunch when $V_k = 5000V$

$$\phi_{i,n} = \phi_{i,n-1} + \frac{2\pi h\eta}{\beta^2}(\frac{\delta E}{E})_{i,n-1} \qquad (6)$$

where index i denotes the ith particle.

As before, we assume the injection error is $\frac{\delta p}{p} = 10^{-4}$. We keep track of the emittance evolution of the bunch for three different kicker voltages $500V, 1000V$ and $5000V$. Figure 3 shows the results.

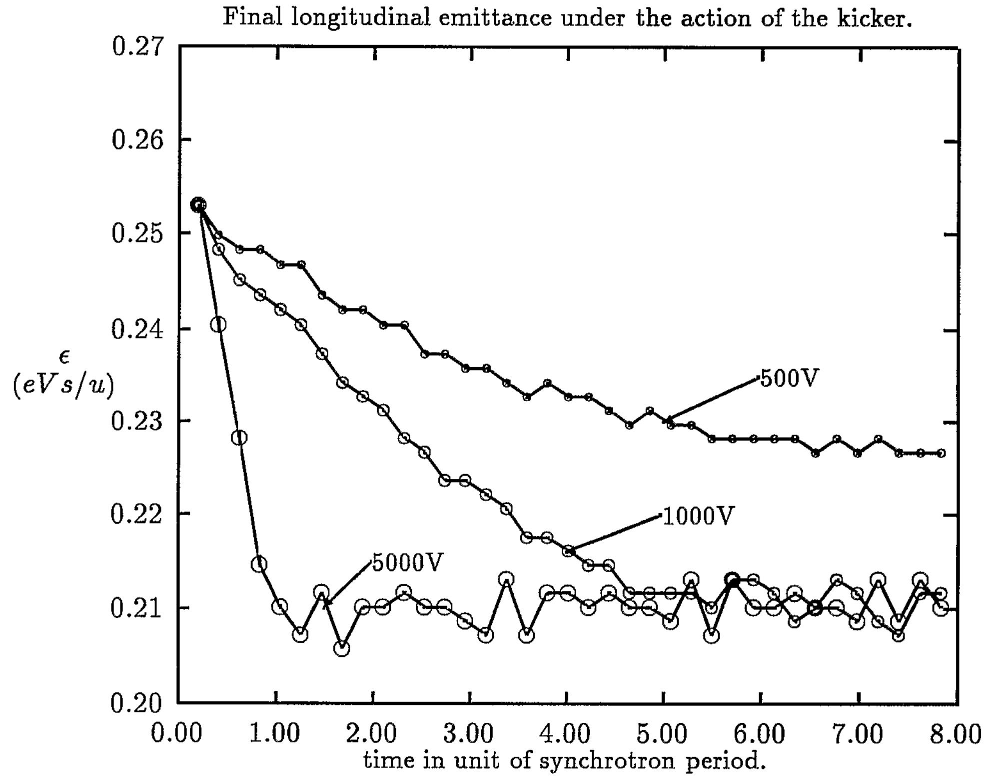

Figure 4: Final longitudinal emittance after the action of the kicker. Each point represents the final emittance after the kicker being turned off at time t.

As expected that the high voltage damps the error faster than low voltages. It's interesting to note that the $1000V$ and $5000V$ cases, though the damping rate differs by approximately 5 times, the final emittance blowup is very much the same. As explained in earlier that the emittance blowup is an inreversible process, any distortion at the beginning is reflected in the final emittance blowup. It's easy to calculate that if the kicker had $2MV$, then within one turn the injection error would be corrected and there's absolutely no emittance blowup.

4 Conclusions

In conclusion, we have studied the kicker's performance to damp the longitudinal emittance blowup due to injection error. Assume that we can tolerate less than 10% increase from injection error, we can project that the voltage requirement on the kicker should be $1000V$.

References

[1] Conceptual design of the relativistic heavy ion collider. BNL 52195, May 1989.