# Correction to AD/RHIC-47, Beam Transfer From AGS to RHIC 

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## RHIC Technical Note No. 47

Beam Transfer from AGS to RHIC
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## Introduction

RHIC, an acronym for Relativistic Heavy Ion Collider, is a facility for colliding heavy ions with each other, proposed for construction at Brookhaven National Laboratory. This facility, and the motivation for building it, have been described in [1]. It consists of two intersecting storage rings and the purpose of this note is to describe how these two rings are to be filled with beam.

The AGS will serve as the injector for RHIC. It will accelerate a number of bunches per pulse that may vary with the mode of operation. These bunches will be parked on an extraction orbit when they reach the extraction energy. There they circulate until they are transferred, one at a time, to one of the two RHIC rings, where they will be received in designated buckets. The option of transferring them all at once may also be available. The AGS extraction system exists already [2, 3], injection into RHIC occurs in its 6 o' clock insertion. The AGS extraction system works in the horizontal plane, RHIC's injection systems in the vertical one. The rings are filled one after the other. Filling RHIC will take initially about two minutes and later on up to, conceivably, fifteen minutes, per RHIC cycle of ten hours or more. Below we describe the system, sketch a mode of operation and consider tolerances.

## Relevant characteristics and parameters

The reference orbit of the AGS has a circumference of about 807.125 m and its harmonic number is 12, the circumference of each RHIC ring is 3833.845 m , (i.e., 19/4 AGS circumferences) and its harmonic number during injection is $342(=6 \times 12 \times 19 / 4)$. All of RHIC's six insertions are set for $\beta^{*}=6 \mathrm{~m}$ during the filling operation in order to provide sufficient aperture. The energy at which the bunches are transferred is $\gamma \approx 31.4 \cdot 2 / A$, with $Z$ and $A$ the charge and mass numbers, so that the magnetic rigidity at transfer is always the same, regardless of the particle species: Bp $\approx 98.224 \mathrm{Tm}$. Nominally the bunches will be about $17 n s e c$ long, i.e., less than 0.075 of the distance between bucket centers (224.3nsec) in the AGS, but 0.45 of that in RHIC ( 37.4 nsec ); proton bunches may be somewhat shorter. Their nominal longitudinal emittance is 0.3 eVsec ,
their nominal radial and axial emittances are $20 \pi m m$-mrad each for protons and $10 \pi m m-m r a d$ each for all other particles. When the bunches are transferred one at a time, it is always the last bunch of the train circulating in the AGS that is extracted and it is always injected to form, or, occasionally, to replace, the last bunch of the train being built up in RHIC, so that the fall times for the extraction and injection kickers can be relatively long. The rise time of the injection kicker determines the best obtainable filling factor of RHIC; we propose to make it 95 nsec , so that every third bucket may be filled (filling every bucket would require it to be $\leq 20 \mathrm{nsec}$ ). This may be contrasted with the rise time required for AGS extraction kicker, which must be $\leq 207 \mathrm{nsec}$ (always assuming bunch lengths $\leq 17 \mathrm{nsec}$ ).

## Equipment

The bunches pass through a transfer line in moving from the AGS to RHIC. A layout of this line is shown in Fig. 1. It begins as the existing AGS's Fast Extracted Beam (FEB) line, which has been in operation for many years for the neutrino physics program. A switching magnet, placed in this line, will define the beginning of a spur which will lead, via an also already existing tunnel, to RHIC. This arrangement allows normal operation of the AGS program as long as the switching magnet is off, and interrupts it only when RHIC is being filled. The spur contains a second switching magnet, the ring selector, where it splits in two branches, one leading to the RHIC ring in which the particles rotate clockwise, the other to the counter-clockwise ring. The design is very similar to the one made for the CBA project, and nearly all of the components already available or designed for that project are used (see Figure 2 and ref [4]). The deviations from that design were dictated by the differences in geometry and lattice functions between RHIC and CBA, and consist of a change in beam level and minor adjustments at the downstream end.

The first switching magnet drives the beam into the first section of the spur which deflects the beam both horizontally and vertically, such that, at the entrance to the ring selector, its axis runs along the intersection of a horizontal plane, 40 cm above RHIC's median plane and the vertical plane through the machine center and the crossing point at 6 o'clock. This plane is a plane of reflection symmetry: reflecting one ring in it yields the other; the same is true for the two branches of the transfer line.

The horizontal deflection in this first section is $20^{\circ}$ in an arc with an average radius of 405.82 m , the change in vertical level is 1.4 m . The horizontal deflection and the change in level are mixed: the former is performed by a
string of 8 gradient magnets in an AG focusing arrangement, the latter by a pair of pitching dipoles, the first one of which is located between the second and third horizontal deflectors. The first horizontal deflector functions as switching magnet. The gradient magnets are each about 3.66 m long, produce a radius of orbit curvature of 83.83 m and carry therefore a field of 1.17 T , they are separated from each other by drift spaces that are 14.05 m long. The second pitching dipole is placed between the second and third quadrupoles of a string of six between the last horizontal deflector and the ring selector. These, together with the quadrupoles in the fast beam line upstream of the switching magnet, provide for flexibility in the choice of focusing parameters at the entrance of the ring selector. At that point the beam is nominally free of horizontal and vertical dispersion ( $x_{p}=x_{p}^{\prime}=y_{p}=y_{p}^{\prime}=0$ ); however, the system may be reset to compensate for the vertical dispersion generated by the final drop in beam level of 40 cm in the injection region. As it stands, the design of this section does not preserve the vertical polarization of polarized beams. A different configuration separating the horizontal and vertical bends would have to be adopted to preserve polarization in this section.

The next section consists of the two branches which begin at the entrance to the ring selector and end in the injection halls. Since they are mirror-symmetric we shall describe only one. The ring selector guides the beam via a dispersion match into one of these two 'big bends', strings of gradient magnets which guide the beam along arcs with average radii of 96.333 m and deflection angles of 1.04 rad each. The 24 gradient magnets in each big bend are arranged in six cells in a OFoFOODoDO pattern with horizontal and vertical betatron phase advances of about $\pi / 2$ rad per cell, and maximum values for the amplitude and dispersion functions of $\hat{\beta}=26.5 \mathrm{~m}, \hat{x}_{p}=2.2 \mathrm{~m}$. The 0 's stand for drift spaces of 0.715 m each, the o's for drift spaces of 0.45 m each while the F's and D's represent focusing and defocusing bending magnets with lengths of about 3.66 m , fields of about 1.27 T and gradients $\left|\mathrm{B}^{\prime} / \mathrm{B}\right|$ of $2.9381 \mathrm{~m}^{-1}$. The 1.43 m long 00 spaces are available for collimators, beam monitors and correction magnets.

Each big bend ends in its associated injection hall. There it is followed by a string of five horizontally deflecting dipoles which are located and excited to make the projection of the transfer line axis onto RHIC's median plane coincide with the reference orbit of the appropriate ring down stream of the last dipole. These magnets will be about 2.95 m long, deflect the beam through about 0.049rad each and must therefore have fields of about 1.61T. Six quadrupoles for adjustment of the match between big bend and RHIC
are imbedded in this string, one at each end and one in each of the four drift spaces between the dipoles.

Injection occurs at end of the last part of the transfer system. This section contains four special magnets, the last one being the injection kicker, which guide the approaching beam from its initial level, 40 cm above RHIC's median plane down onto the injection closed orbit. Their arrangement is shown in Fig. 3 in elevation and in plan. The first magnet, SIO, is located directly above ring magnets BS 2 and Q 70 ; it must have a bending strength of 5.46 Tm , thus with a length of 4.77 m a field of 1.14 T . It sends the beam downward, via the last quadrupole of the transfer line, QMI7, to septum magnet SI1. This magnet has a bending strength of 4.74 Tm , a length of 4.15 m and a field strength of 1.14 T . Its current septum lies between the incident beam and the circulating one. Leaving SII the beam passes off axis through the aperture of vertically defocusing ring quadrupole Q80, then through septum magnet SI2, and is finally bent into the median plane by injection kicker KI. SI2 must have a bending strength of 0.275 TM . It is 1.4 m long and has therefore a field strength of 0.196 T . The kicker will have to provide a bending strength of 0.132 Tm . Its length is 3.25 m , its aperture at least 40 mm vertical $\times 20 \mathrm{~mm}$ horizontal, its field 0.04T.

The behavior of the beam downstream of SI2 is may be judged from Fig. 4. The elevation and plan admittance envelope traces in this figure are drawn for an admittance of $3.2 \pi \mathrm{~mm}$-mrad for the incident beam and $3.2 \pi m m-m r a d$ and $6 \pi \mathrm{~mm}-\mathrm{mrad}$ for the circulating one. The septa of SI1 and SI2 must fit between the admittance envelopes for the incident and the circulating beams. We determine the available space, assuming that the apertures in the transfer line should provide everywhere for at least twice the local beam dimensions, and that the ring admittance must be $6 \pi m m$-mrad. The transfer line admittance must then be at least $3.2 \pi \mathrm{~mm}$-mrad, since the largest real ( $2.5 \sigma$ ) incident beam emittance expected is $0.8 \pi m m$-mrad; the quoted ring admittance provides ample space for the increases in emittance from intra beam scattering. Using this, one finds that the spaces available for the septa in question are 15 mm for SI1 and 1 mm for SI2 at their downstream ends. This severe constraint may be relaxed to a much more reasonable 20 mm for SI1 and 4.5 mm for SI2 if one allows vertical displacement of these magnets over a distance of 5 mm immediately after the ring has been filled. They are placed to leave a ring admittance at least equal to that of the transfer line, namely $3.2 \pi m m-m r a d$, during the filling interval and they are moved to increase it to at least $6 \pi m m-m r a d ~ a f t e r ~$ the ring is filled. In doing so one takes advantage of the fact that a smaller
ring admittance is quite acceptable during the filling period, since the emittance increase of the circulating beam occurs only on a time scale that is long compared to that period. The dimensions given for magnets SIO, SI1 and SI2 are only rough estimates. Detailed design work, in particular for the latter two, may produce configurations that offer the nominal admittance of $6 \pi \mathrm{~mm}$-mrad also during the filling operation. Their physical withdrawal after that operation is complete would then not be necessary.

## Operation

The mode of operation proposed here is based on the assumption that the conditions in RHIC represent the standard of reference for all activities. In a setup procedure[1] RHIC is made ready for filling and then kept stationary in that state until it is filled. Similarly, nearly all magnets in the beam transfer system are excited and also kept stationary. Exceptions are the extraction and injection kickers and the pulsed septum magnets. The case considered in the following is the most general one: the transfer of individual bunches, one by one, from whatever bunch train has been accelerated in the AGS, into the waiting rf buckets of RHIC.

When used for filling RHIC the AGS may accelerate trains of from 1 to 12 bunches. Just prior to the events leading to extraction the closed orbit control system is set to hold the circumference of the closed orbit at 4/19- $\delta$ of the circumference of the injection orbit in RHIC (19/4 = 4.75 is the ratio between the circumferences of the reference orbits, $\delta$ is a small number, yet to be determined). The acceleration continues and the rf frequency keeps increasing until its sixth harmonic approaches the rf frequency in RHIC. When the difference has become small enough a phase locking loop between the two rf systems takes over from the radial position loop as controller of the AGS rf frequency and fixes it at exactly the $1 / 6$ sub-harmonic of the frequency in RHIC with predetermined relative phases of the charge distributions in the two rings. The bunch that is to be extracted from the AGS must arrive in the bucket in RHIC that trails the last one filled by a predetermined number of buckets, in order to accommodate the injection kicker rise time, while maximizing the filling factor.

Acceleration continues because the AGS guiding field is still increasing. It will be stopped, by blocking further changes in the guiding field, when the synchronous particles reach the extraction energy. The strength of the guiding field is taken as a measure for the synchronous energy and its change is stopped at the instant that it reaches a reference value. Use of

RHIC's guiding field as reference makes the beam transfer process relatively insensitive to small variations in that field. The closed orbit, which had been too short until the phase lock took over, expands with the continuing increase of the AGS field. The conditions are chosen such that it will coincide with the extraction orbit at the instant that the field increase is terminated. At that moment the phase of the AGS rf relative to that of RHIC isjumped so as to place stationary buckets around the bunches and the rf voltage is adjusted to preserve the match between bucket and bunch. These measures help to prevent dilution of the bunches in synchrotron phase space. It may be useful to reduce the rate of acceleration and the stable phase angle during the preceding operations in order to gain time for them, to reduce the changes in the guiding field that are associated with changes in the ( $\dot{B}-d r i v e n$ ) eddy current distribution, and to lend them a measure of adiabaticity.

The AGS functions as a storage ring from this point onwards. The bunches in it have the proper synchronous energy and circulate on the proper extraction orbit, the last bunch in approximately proper phase relationship to the charge distribution in the ring to be filled. Any remaining small phase error is now corrected by adiabatic adjustment. The rate at which such an adjustment can be made depends upon the synchrotron period in the AGS and on the amount of dilution in synchrotron phase space considered acceptable; it is rather low: $2 \mathrm{rad} / \mathrm{sec}$ for protons, $0.15 \mathrm{rad} / \mathrm{sec}$ for gold for a $1 \%$ blow up in bunch area. The rate must be so low because the bunches are so short, relative to the rf period; their dilution per synchrotron period increases with the square of their fractional displacement.

With its phase now as correct as it can be made to be the bunch keeps circulating until its transfer to RHIC is initiated by excitation of the extraction and injection kickers. Each is triggered by the rf of the ring it serves after some form of coincidence circuit has flagged the fact that the charge distributions have reached appropriate azimuthal positions in both rings. This waiting period may last up to 19 revolution periods in the AGS and up to 4 in RHIC, thus up to about $50 \mu \mathrm{sec}$, which time represents a superperiod' in the AGS/RHIC cycle. The bunch moves out of the AGS, through the transfer line and arrives in the bucket in RHIC which it finds waiting upon passing through the injection kicker. The kickers are reset to zero excitation immediately thereafter, and preparation for the next transfer begins.

If the AGS is now empty it is returned to injection energy and a new cycle begins. Matters are more complex if it contains more bunches. Unless one can shift these bunches in the AGS with respect to the raster defined by the
buckets in RHIC one would now face the following situation: The choice of harmonic numbers, 12 in the AGS and 342 in RHIC implies that a particular bunch in the AGS could only be placed in one of 19 equally spaced buckets in RHIC after the two rf systems are synchronized. Calling the bucket which houses this bunch bucket 1 , one finds easily that bunches in buckets 4 , 7 and 10 could only reach these same 19 buckets in RHIC. Another set of 19 is available to bunches 2, 5, 8 and 11 and still another to bunches 3, 6, 9, and 12. It follows that this method makes only $57(=3 \times 19)$ buckets in RHIC available for filling. One finds also that one cannot invert the order of the bunches, as is necessary for proper injection. If the counting above goes upstream it gives the proper sequence of extraction: first the bunch from bucket 1 , then the one from bucket 2, etc. But the bucket in RHIC that is available to bunch 2 and closest to the one occupied by bunch 1 lies upstream, rather than downstream of that bucket; using it would damage bunch 1 due to the fall time of the injection kicker. In contrast, one of the buckets available to bunch 3 lies one AGS bunch spacing downstream from the bucket for bunch 1. One must therefore move the bunches that stay in the AGS one bunch spacing upstream after each extraction. We shall discuss a few methods for doing this. ${ }^{1}$

We showed earlier that, if the AGS yields a single bunch per cycle, the number of buckets that can be filled is only restricted by the rise time of the injection kicker because that single bunch can be phased arbitrarily relative to the bucket pattern in RHIC. There is therefore no problem in filling every $\mathrm{n}^{\text {th }}$ of the 342 available buckets, with $n$ any integer, provided that the rise time of the injection kicker is less than $37.4 n-17$ nsec, with 37.4 and 17 the distance between successive bucket centers and maximum bunch length in nsec, and that a few 'fillable' buckets are left unoccupied near the end of the filling cycle to accommodate the injection kicker's fall time. (This is no real loss since a hole of the order of $1 \mu \mathrm{sec}$, the length of some 27 buckets, must be left in any case to accommodate the rise time of the beam disposal kicker.) The only objection to this procedure, apart from its slight economic cost, is that it takes time, about 320 AGS pulses, or about 450 sec. This is time is no longer short in terms of the time constants for longitudinal intra beam scattering in heavy ion beams of the proposed brightness at this energy and one fears that, in the absence of a stochastic cooling system, considerable degradation might result.

[^1]The parking time and the beam degradation can be reduced by accelerating more than one bunch per AGS pulse, because filling RHIC then requires fewer AGS pulses. Such a procedure requires separate phasing of the AGS bunches relative to the bucket pattern in RHIC if more than 57 buckets are to be filled per ring.

There are at least two methods for manipulating bunch phases in the AGS, besides the adiabatic phase adjustment mentioned before. Both leave its guide field undisturbed and use rf gymnastics. According to one method, suitable for shifts of several bucket lengths or more, one decelerates the beam in the AGS somewhat, in order to move it to an orbit of smaller circumference. Its angular velocity increases accordingly, since its energy is above the AGS transition energy, and it begins to advance relative to the bucket pattern in RHIC. This means that the rf frequency must be increased slightly while the phase and the voltage must be adjusted to keep the bunches centered on the synchronous fixed points and their shapes matched to the bucket shape. The change in frequency can be obtained by modulating the reference signal of the phaselocked loop mentioned above, or by returning the frequency control temporarily to the radial position loop. Coming close to the desired configuration one returns the beam to the extraction closed orbit by reduction of the rf frequency. The process is relatively fast: shifting a bunch by a bucket length takes about $n=1 /(h \eta \Delta E / E)$ revolutions. Here is $\Delta E / E$ the energy reduction relative to the extraction energy, $\boldsymbol{\eta}$ the phase slip factor and $h$ the harmonic number. For protons with $\gamma=31.4, \Delta E / E=10^{-2}$ and $\mathrm{h}=12$ : $\eta \approx 0.013$ and $n \approx 641$ revolutions or 1.7 msec ; for the heaviest ions: $\gamma \approx 12.59, \eta \approx 0.0077$ thus $n \approx 1080$ revolutions or 2.9 msec . The $\Delta E / E$ chosen displaces the orbit by about $x_{p} \Delta E / E \approx 2.5 \mathrm{~cm}$ in the points where ${\underset{p}{p}}^{\text {has }}$ its maximum value of 2.5 m . How large $\Delta E / E$ may be chosen depends on the dynamic acceptance of the AGS at this energy. The process will cause dilution in synchrotron phase space, and the phase and the amplitude of the rf voltage must be tightly controlled to minimize this effect. It may be difficult to realize the desired bunch phase relative to RHIC's bucket pattern exactly and adiabatic trimming of that phase may be unavoidable. The displacements of the closed orbit must appear adiabatic to the betatron motion in order to avoid dilution in horizontal betatron phases pace. In particular one cannot jump instantaneously from one orbit to another or back by jumping the rf frequency from one value to another without doing serious damage.

The other method seems more suitable for smaller shifts, although it may be applied repeatedly to obtain large ones. It is only superficially different from the first one since it exploits the same physical mechanism. It is
slower, but logistically simpler and therefore, perhaps, less likely to cause dilution. Making use of the fact that the bunches are so small in comparison with the bucket, one induces a coherent synchrotron oscillation by jumping the phase of the rf voltage by half the amount one wants to displace the bunch. At the instant the bunch has completed half an oscillation the rf phase is jumped again, by the same amount and in the same direction. This stops the coherent synchrotron motion because the stable fixed points are centered once more on the bunches. The second phase jump is applied at the instant that the bunch(es) cross the extraction closed orbit, as detected by a precision BPM. This scheme is slower than the previous one: small shifts take half a synchrotron period ( 2.7 msec for protons, decreasing with the root of the mass number for ions, down to 1.7 msec for the heaviest ions, and increasing with the amplitude of the forced oscillation in the familiar nonlinear manner). The shift per half oscillation is restricted to perhaps $1 / 2$ the distance between bucket centers by the dilution caused by the spread in synchrotron frequencies over the particles in the bunch. Beyond this there is little chance of error: the needed phase references can be obtained, essentially error less, from RHIC's rf system, the rf frequency and voltage remain untouched, there is no consideration of adiabaticity, either in synchrotron or in betatron phase space. The non-linearity of the synchrotron half period as function of amplitude will cause an increase in energy spread. The effect is small since the phase spread over the bunch is small: in the time that the central particle requires for the half oscillation the one with the smallest phase offset performs slightly more, the one with the largest slightly less than a half oscillation, this shows up as energy errors of those boundary particles at the instant that the oscillation is terminated.

The effect, a skewing of the bunch shape, increases with the size of the bunch, the amplitude of the coherent oscillation and the number of half oscillations performed. The bunch dilution caused by the various ways for moving bunches azimuthally will have to be studied in more depth than has been done so far.

## Tolerances

Transfer of a beam from the AGS to a RHIC ring will in general affect its quality adversely. The deterioration may be the result of the mislocation of a bunch's center of charge relative to its intended synchronous fixed point in 6D phase space, and/or of a mismatch between the 6D emittance of a bunch and the lattice functions of the ring in which it is to circulate. Tolerances
must be established to ensure that such deterioration can be kept within accepted bounds. Some of these tolerances can be relaxed if means are provided for error correction after injection. Dipole errors, i.e., errors in position or energy and direction or phase, can be corrected to some extent by transverse or longitudinal dampers, which have to be present anyway for the control of beam instabilities. Normally such dampers are not particularly powerful, since they are intended for damping of coherent motion with infinitesimal small amplitude. That power has to increase in proportion to the error if the errors to be corrected are not very small, because the correction must be completed in a time that is small compared to the decoherence time of the error motion. The betatron decoherence, controlled by the spread in betatron frequencies due to nonlinearities, and chromaticity and momentum spread, may be estimated at a few hundred revolutions; the synchrotron decoherence, controlled by the nonlinearity in the synchrotron motion, takes a time of the order of a synchrotron period, thus several thousand revolutions, depending on the rf voltage. Betatron errors must therefore be corrected within several tens of revolutions, while synchrotron corrections can be allowed to take several hundreds. Assuming that the correction per turn is proportional with the instantaneous error, one finds that the betatron correctors must provide each turn a kick that is a non-negligible fraction of the absolute error. E.g., correction of a $1 \%$ error in the injection kicker, corresponding to a $\Delta \mathrm{B} \ell$ of $13.2 \cdot 10^{-4} \mathrm{Tm} \approx$ $13.4 \mu \mathrm{rad}$, would require a corrector capability of about

$$
2[(-\ln 0.01) / 20] \cdot 13.4 \cdot 10^{-6} \cdot \mathrm{Bp} \approx 6 \cdot 10^{-4} \mathrm{Tm} \text { per turn }
$$

in order to have it reduced to $1 \%$ of the original error in 20 turns; the factor two accounts for the fact that the average correction per revolution is only one half of the nominal one in betatron phase space. The correction system would require a bandwidth that goes well beyond the bunch frequency of about $(27 / k) M H z$, with $k$ the number that specifies the number of buckets per bunch in RHIC. Similarly, reduction of energy errors of the order of $10^{-4}$, or $3.1 \mathrm{MeV} / \mathrm{u}$, to $10^{-5}$ or $0.31 \mathrm{MeV} / \mathrm{u}$ in 200 revolutions would require a wide band accelerating capacity of the order of $36 \mathrm{keV} / \mathrm{turn} / \mathrm{u}$.

An exact discussion of the consequences of errors is rather complex, principally because most errors tend to modify the distribution functions in phase space qualitatively, e.g., Gaussian beams do not remain Gaussian. For the order of magnitude type of estimates necessary for setting tolerances, we limit ourselves to the simplest approximations. In order to judge closed orbit errors, in either one of the two transverse phase spaces or in the longitudinal one, we estimate the increase in emittance of an elliptical beam, injected
into a ring with a closed orbit error, though properly matched to its focusing characteristics. The emittance that ultimately develops is the area of an properly centered matched ellipse that circumscribes the injected one. One finds then for the relative increase in emittance

$$
\Delta \varepsilon / \varepsilon=(\bar{\varepsilon}-\varepsilon) / \varepsilon \approx 2\left[\left\{x_{c}^{2}+\left(\alpha x_{c}+\beta x_{c}^{\prime}\right)^{2}\right\} /(\varepsilon \beta)\right]^{1 / 2}
$$

where the various symbols are generic ones. $\alpha$ and $\beta$ stand for the linear focusing parameters, $x_{c}$ and $x_{c}{ }^{\prime}$ for the orbit errors, $\varepsilon$ and $\bar{\varepsilon}$ for the old and new emittance.

We distinguish between systematic and random errors. Systematic errors are errors that are constant, or, at most, drift slowly during a filling cycle. Among them are errors in the transfer line magnet excitations, and in the coordinates of the nominal extraction and injection closed orbits. Their consequences can be made unobservably small. Random are the errors that change randomly on successive beam transfer operations. Principal among them are the excitations of the extraction and injection kickers, the pulsed septum magnets, and errors in rf voltage amplitudes and phases. We consider only random errors in what follows.

It is easily seen that an absolute deflection error ${ }_{c} x^{\prime}$ causes $\Delta \varepsilon / \varepsilon \approx$ $2 x_{c}^{\prime} \sqrt{\beta / \varepsilon}$, if the location where the error is introduced has an amplitude function value $\beta$. The principal sources of such errors in the filling system under discussion are the kickers for AGS extraction and RHIC injection. Other important sources are the pulsed septum magnets. The magnets associated with AGS extraction affect the horizontal emittance because extraction occurs horizontally, those associated with RHIC injection affect the vertical emittance because injection occurs vertically.

Random mislocation of the extraction closed orbit represent another source of emittance blowup. We make use of the design, or reference, orbit of the transfer line, defined for surveying purposes. This reference orbit intersects with the nominal extraction closed orbit in the AGS and with the nominal injection closed orbit in RHIC. The extraction and injection kickers are located at these intersection points. One verifies during the set up procedure that this situation is realized by checking that beams that are free of coherent motion in 6D phase space before extraction from the AGS are, on average, free of such motion after injection into RHIC. Random closed orbit misalignments, relative to the nominal configuration, will still occur in consequence of variation of the AGS guiding field while the beam is circulating on the extraction closed orbit, misexcitation of or changes in the closed orbit bump for extraction and of noise in the BPM that is used to announce the arrival of
the beam on the extraction closed orbit. Using the expression above we obtain for the contribution to $\Delta \varepsilon / \varepsilon$ by closed orbit errors:

$$
\Delta \varepsilon / \varepsilon \approx 2 x_{c}\left[\left(1+\alpha^{2}\right) /(\varepsilon \beta)\right]^{1 / 2}
$$

where $\alpha$ and $\beta$ represent lattice parameters for the horizontal plane at the kicker locations. We may now determine tolerances for the various contributors by permitting each of them an equal fractional increase of the emittance, determining the fraction from the total fractional increase considered acceptable for the coordinate under consideration (horizontal, vertical or longitudinal) and the number of contributors, assuming statistical independence.

The longitudinal motion can be treated in similar fashion if we let $x_{c}$ stand for synchronous energy error $\delta E, x_{c}^{\prime}$ for the (longitudinal) displacement from the synchronous fixed point $\delta t=\delta \varphi /\left(h \omega_{2}\right), \alpha=0$ and $\beta=\beta_{s}$ for:

$$
\begin{aligned}
\beta_{\mathrm{s}} & =\mathrm{h} \omega_{\mathrm{s}}\left[-B^{2} \gamma \mathrm{E}_{0} \operatorname{ecos} \varphi_{\mathrm{s}} /(2 \pi \mathrm{~h} \eta)\right]^{1 / 2}\left[1+\pi h \eta \mathrm{eV} \cos \varphi_{\mathrm{s}} /\left(2 \mathrm{~B}^{2} \gamma \mathrm{E}_{0}\right]^{1 / 2}[\mathrm{eVsec}]\right. \\
& \approx \mathrm{h} \omega_{\mathrm{s}}\left[\gamma \mathrm{E}_{0} \mathrm{eV} /(2 \pi \mathrm{~s}|\eta|)\right]^{1 / 2}
\end{aligned}
$$

where the approximation is valid for our purpose. The symbols are the familiar synchrotron ones: e electrical charge, eV peak energy gain per turn, $\mathrm{E}_{0}$ rest energy, $\gamma$ Lorentz factor, $h$ harmonic number, $\omega_{s}$ angular velocity and $\eta$ phase slip factor. The emittance $\varepsilon$ stands now for the bunch area (in eVsec).

It must be noted that an error in synchronous energy causes not only an increase in bunch area, but also one in horizontal emittance, because the dispersion parameter, $\alpha_{p} \neq 0$ at the location of the extraction kicker in the AGS. It is therefore accompanied by a misalignment and a misdirection of the closed orbit. Since any horizontal deviation from the extraction closed orbit represents an energy error, and vice versa, the coherent horizontal betatron amplitude at the extraction kicker is given by $\hat{x}_{\delta}=x_{p} \Delta E / E$. This motion is not correlated to any other horizontal betatron motion; it contributes a term $\Delta \varepsilon / \varepsilon$ $=2 x_{p} \Delta E / E /(\varepsilon \beta)^{1 / 2}$ rms. There are no further effects, provided that the transfer line is properly matched on both sides.

Working out a particular case we choose protons, transferred at $\gamma=31.4$, with a bunch area of 0.3 eVsec and invariant transverse emittances of $20 \times 10^{-6} \pi$ m-rad. A peak energy gain per turn of 300 kV and $\gamma_{t r}=8.447$ yield $\beta_{\mathrm{s}}=2.66 \cdot 10^{15}$ [ $\mathrm{eV} / \mathrm{sec}$ ], thus the half height and half length of the bunch are $\Delta \mathrm{E}=16 \mathrm{MeV}$ and 6 nsec , respectively. In order to calculate the relationship between errors in the coordinates of a bunch's center and the volume enclosed in its ultimate envelope we scale the coordinates such that the bunch appears spherical with radius $r$. The errors cause its center to be located at a distance dr from the origin. The sphere that is centered on the origin and that contains the bunch has than a radius $r+d r$. Calling the volume of the bunch its emittance $\varepsilon$ one
finds for the increase in emittance as a consequence of the displacement of the bunch center: $\mathrm{d} \varepsilon / \varepsilon=\mathrm{n} \mathrm{dr} / \mathrm{r}$, where n represents the dimensionality of the problem. The displacement depends upon the individual coordinate errors $\mathrm{dc}_{\mathrm{n}}$ as $(d r)^{2}=\sum n\left(d c_{n}\right)^{2}$, thus the expectation for the relative increase in emittance due to relative coordinate errors $d c / r$ with equal expectations is then $(d \varepsilon / \varepsilon)_{r m s}=n^{3 / 2}(d c / r)$. It follows that a rms blowup with a factor $\leq 1.1$ of the individual transverse and longitudinal emittances ( $n=2$ ) requires ( $\mathrm{dc} / \mathrm{r}$ ) ${ }_{\mathrm{rms}}$ $\leq 0.07$ for all coordinates, preservation of the transverse brightness to within $10 \%$ requires $(\mathrm{dc} / \mathrm{r})_{r m s} \leq 0.0125$ for the transverse coordinates while preservation of the luminosity to within $10 \%$ requires ( $\mathrm{dc} / \mathrm{r})_{r m s} \leq 0.025$, because it is proportional to the root of the transverse 4D emittance. Preservation of the 6 D brightness to within $10 \%$ would require $(\mathrm{dc} / \mathrm{r})_{r m s} \leq 0.0068$. Tolerances for the critical magnets, calculated under the assumption that the luminosity should be preserved to within $10 \%$, are shown in Table I. The first few rows give values for the focusing parameters at the locations of the kickers and septum magnets, the next one the deflection angles $\vartheta$ in these magnets and the last one the calculated relative rms accuracies $\Delta \vartheta / \vartheta$ for these angles.

Table I

|  | $\begin{gathered} \text { AGS H5 } \\ \text { Ext Kckr } \end{gathered}$ | AGS H10 <br> Spt Mgn | Transfer line |  |  | RHIC IKI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | SIO | SI1 | SI2 |  |
| $\beta$ [m] | 15 | 23 | 33 | 39 | 23 | 45 |
| $x_{p}[\mathrm{~m}]$ | 1.5 | 2.2 |  |  |  |  |
| $\vartheta[\mathrm{mrad}]$ | 1.1 | 22 | 55.5 | 48.2 | 2.8 | 1.3 |
| $\Delta \vartheta / \vartheta$ | $\begin{aligned} & 1.9 \cdot 10^{-3} \\ & 2.1 \cdot 10^{-3} \end{aligned}$ | $\begin{aligned} & 7.7 \cdot 10^{-5} \\ & 8.5 \cdot 10^{-5} \end{aligned}$ | $2.6 \cdot 10^{-5}$ 2.0 | $\begin{aligned} & 2.7 \cdot 10^{-5} \\ & 10^{-4} \end{aligned}$ | $\begin{aligned} & 6.0 \cdot 10^{-4} \\ & 6.6 \cdot 10^{-4} \end{aligned}$ | $\begin{aligned} & 9.3 \cdot 10^{-4} \\ & 1.0 \cdot 10^{-3} \end{aligned}$ |

Two tolerances are shown for each magnet, those in the first row are based on the assumption that they have mutually independent power supplies, those in the second row assume that SIO and SI1 are electrically in series on a common power supply, or coupled in some other manner that guarantees a 1 to 1 relation between their fields to within $10^{-5}$. The relaxation of the tolerances in that case results from the fact that these magnets deflect the beam by about the same amount but in opposite directions, so that they more or less cancel each other as far as their net deflection angle is concerned.

Accepting also a longitudinal blowup with a factor 1.1 we will tolerate rms uncertainties in synchronous energy and location of the bunch center of 1.12 MeV , thus $\delta E / E=3.8 \cdot 10^{-5}$, and $\delta T=0.42 \mathrm{nsec}$, corresponding with an rms rf
phase error in the AGS of $\delta \varphi=11.8 \mathrm{mrad}=0.7^{\circ}$. The energy error causes a radial bunch displacement in the $\hat{\beta}$ points of $x_{p} \Delta E / E=0.0836 \mathrm{~mm}$, and contributes a $(\Delta \varepsilon / \varepsilon)_{x}=0.03$ which we neglect at this point.

It is evident that the tolerances on the fields in the magnets are rather small. They can be relaxed by introducing a transverse damping system, as mentioned earlier. The field tolerances and the strengths of the error correctors in a ring can be traded: stronger correctors allow larger tolerances; finding the best arrangement requires detailed engineering.

The synchronous energy may be obtained from the strength of the guiding field if the rf frequency, thus the synchronous revolution period, is fixed. The relation between synchronous energy and magnetic field for fixed rf frequency may be described by:

$$
\Delta \gamma / \gamma=\left(\gamma^{2}-1\right) /\left(\gamma^{2}-\gamma_{t r}^{2}\right)(\Delta B / B)
$$

Since $\gamma_{t r}=8.447$ in the AGS one finds for protons at $\gamma=31.4: \Delta \gamma / \gamma=1.077 \cdot \Delta \mathrm{~B} / \mathrm{B}$ and for ions with $A / Z=2.5: \Delta \gamma / \gamma=1.828 \cdot \Delta B / B$. It follows, that the tolerance on a guiding field measurement as a measure for the synchronous energy is $2.0 \cdot 10^{-5}$ if the energy must be determined to within $3.8 \cdot 10^{-5}$.

It was mentioned earlier that the bunches in the AGS may be shifted relative to the raster imposed by RHIC by inducing a (sequence of) coherent half synchrotron oscillation(s). The completion of each half oscillation was to be detected by a precision BPM. An error by that monitor leaves a coherent energy error given by $\Delta \gamma / \gamma=\left(1-1 / \gamma^{2}\right)\left(\Delta x / x_{p}\right) \approx \Delta x / x_{p}$. Placing the BPM at a point of maximum dispersion ( $x_{p} \approx 2.2 \mathrm{~m}$ ), one finds that detecting $|\Delta \gamma / \gamma|=3.8 \cdot 10^{-5}$ requires a resolution $\Delta x \approx 80 \cdot 10^{-6} \mathrm{~m}$ of the BPM. It seems likely that the BPM will be used as a null-detector (comparing the orbit locations at the beginning and the and of the half oscillation), this would require a resolution of $59 \mu \mathrm{~m}$ since the two measurements are statistically independent. In RHIC the rf frequency is not imposed but controlled by a radial loop. There a BPM resolution $\leq x_{p} \Delta \gamma / \gamma=5.7 \cdot 10^{-5}=57 \mu \mathrm{~m}$ would be acceptable.

We noted before that the guiding field in the AGS must be established to within about $4 \cdot 10^{-5}$ and assumed that it would remain constant at that level with that tolerance for as long as the AGS functions as a storage ring. It may be difficult to satisfy this tolerance requirement, and the longitudinal damper necessary for correcting such errors may have to be rather large. It is convenient, but not strictly necessary, to demand constancy. The fundamental requirement is not that the synchronous energy be fixed, but that the angular velocities of particles and the synchronous energies remain matched in the AGS
and in RHIC. A field error at fixed rf frequency changes both the energy and the closed orbit such that the revolution period remains unchanged. On the other hand, a change in frequency in a fixed field causes the energy and the closed orbit to change. It appears possible to change the 'common' rf frequency (i.e., the frequency in the AGS and its sixth harmonic in RHIC) in response to a known field error in the AGS in such a manner that the two rings remain matched: the two closed orbit circumferences change in the same proportion.

The variation in revolution period in any synchrotron may be described by

$$
\Delta \mathrm{T} / \mathrm{T}=\left(\gamma_{t r}^{-2}-\gamma^{-2}\right) \Delta \mathrm{p} / \mathrm{p}-\gamma_{\mathrm{tr}}^{-2} \Delta \mathrm{~B} / \mathrm{B} .
$$

Keeping the angular velocity and the synchronous energy in the two rings matched implies equal values for $\Delta T / T$ and $\Delta p / p$ in them. This is achieved, regardless of deviations of the guiding fields, if one imposes:

$$
\left[\gamma_{t r}^{-2} \Delta \mathrm{p} / \mathrm{p}-\gamma_{\mathrm{tr}}^{-2} \Delta \mathrm{~B} / \mathrm{B}\right]_{\mathrm{AGS}}=\left[\gamma_{\mathrm{tr}}^{-2} \Delta \mathrm{p} / \mathrm{p}-\gamma_{\mathrm{tr}}^{-2} \Delta \mathrm{~B} / \mathrm{B}\right]_{\mathrm{RHIC}}
$$

or

$$
\Delta \mathrm{p} / \mathrm{p}=\frac{\gamma_{\mathrm{tr}_{\mathrm{AGS}}}^{-2} \Delta \mathrm{~B} / \mathrm{B}_{\mathrm{AGS}}-\gamma_{\mathrm{tr}_{\mathrm{RHIC}}}^{-2} \Delta \mathrm{~B} / \mathrm{B}_{\mathrm{RHIC}}}{\gamma_{\mathrm{tr}_{\mathrm{AGS}}}^{-2}-\gamma_{\mathrm{tr}_{\mathrm{RHIC}}}^{-2}}=\frac{\gamma_{\mathrm{tr}_{\mathrm{RHIC}}}^{2}}{\gamma_{\mathrm{tr}_{\mathrm{RHIC}}}^{2}-\gamma_{\mathrm{tr}_{\mathrm{AGS}}}^{2}} \Delta \mathrm{~B} / \mathrm{B}_{\mathrm{AGS}}
$$

since $\Delta B / B_{\text {rhic }}=0$ if $B_{\text {rhic }}$ is the field reference. It is not difficult to enforce this relation fairly accurately. One measures $\dot{B}_{A G S}$, e.g., as a voltage across a pickup coil and integrates it from the instant that $\mathrm{B}_{\mathrm{AGS}}$ crosses the reference field onward. The $\dot{B}$ voltage is proportional to the rates with which the particles must be accelerated in each ring in order that the synchronism condition above be met, its integral is proportional with the deviation of the synchronous frequencies from their reference values. The $\dot{B}$ signal can be used to control the stable phases of the rf systems relative to the bunches to establish the necessary rates of acceleration or deceleration, itsintegrated value is supplied as a correction signal to the frequency control loop of RHIC. The rf voltage amplitudes should in principle be adjusted to match the phase shifts, but since the changes in stable phase angle will be small, such adjustment may well fall within the noise.

Though this approach reduces the consequences of AGS field errors in longitudinal phase space it tends to cause dilution in transverse phase space because it displaces the actual closed orbit from the reference closed orbit in each ring ${ }^{2}$ : the field error at transfer is now expressed as a coherent

[^2]betatron motion. The deviation is given by
\[

$$
\begin{aligned}
& \delta \mathrm{x}=x_{\mathrm{p}} \Delta \mathrm{p} / \mathrm{p}=x_{\mathrm{p}} \frac{\gamma_{\mathrm{tr}}^{\mathrm{RHIC}}}{\gamma_{\mathrm{tr}_{\text {RHIC }}^{2}}^{2}-\gamma_{\mathrm{tr}}^{2}} \Delta \mathrm{~B} / \mathrm{B}_{\mathrm{AGS}} \\
& \delta x^{\prime}=x_{p}^{\prime} \Delta \mathrm{p} / \mathrm{p}=x_{\mathrm{p}}^{\prime} \frac{\gamma_{\mathrm{tr}_{\text {RHIC }}}^{2}}{\gamma_{\mathrm{tr}}^{2}-\gamma_{\mathrm{RHIC}}^{2}} \Delta \mathrm{~B} / \mathrm{B}_{\mathrm{AGS}}
\end{aligned}
$$
\]

Numerically one finds for $\Delta B / B=10^{-4}$ in the AGS ( $x_{p} \approx 2.5 \mathrm{~m}$ ) $\delta x \approx 0.28 \mathrm{~mm}$ and in RHIC ( $x_{\mathrm{p}} \approx 1.5 \mathrm{~m}$ ) $\delta x \approx 0.17 \mathrm{~mm}$, no longer negligible compared to the local betatron half widths of 3.78 mm and 5.64 mm , respectively. The relative effect is smaller in betatron phase space than in synchrotron phase space primarily because the longitudinal emittance is rather smaller than the betatron emittance. This coherent betatron motion can be dampened by active dampers in RHIC; one could also prevent it from occurring by forcing the reference orbit in the transfer line to track the extraction orbit in the AGS. This, however, would require modification of existing equipment at the upstream end of the transfer line to an extend that seems unwarranted in view of the expected benefit.

## Remarks

A number of important issues have still to be considered in the future. First among them is the conception and design of the injection septum magnets, particularly SI1 and SI2. Detailed study and prototype work on these magnets is essential and may make it possible to re-optimize the configuration of the
 filling period. This would obviate the need for their physical withdrawal after the rings have been filled. The overriding consideration is the magnitude and shape of their stray fields in the space reserved for the circulating beam. These fields present high order, off-axis multipoles to the circulating beam and vary adiabatically with time as far as the betatron motion is concerned. The stray dipole field, for example, causes a closed orbit distortion which may be compared with the distortions caused by ring quadrupole misalignments. A quadrupole displaced by 0.25 mm presents (at injection) a dipole with $\Delta y \int B^{\prime} d \ell=0.002 \mathrm{Tm}$. Quadrupole misalignments of that order require significant dipole correction; this suggests that the stray field dipole have $\int B d \ell<2 \cdot 10^{-3} \mathrm{Tm}$, implying that the stray field of SI1 should be less than $4 \cdot 10^{-4}$ of its main field. The next multipole is a quadrupole, it causes a (time-dependent) tune shift. We will not discuss its implications here, nor
those of the multipoles of higher order. Clearly the design of SI1 and SI2 is a critical task.

Means for adjusting the coordinates of the axis of the incident beam at the exit of the injection kickers must be provided. Four properly located steering magnets can satisfy this requirement and locations and strengths have to be specified. These functions could also be provided by vernier controls on the extraction and injection kickers and septum magnets.

Tolerances on the strengths and the locations of the components in the transfer line must be determined in order to judge the need for and the size of a dipole corrector scheme. The transverse locations of the bending magnets are important, since they are gradient magnets, not dipoles.

Diagnostic equipment with adequate resolution and accuracy which responds to the passage of a single bunch is required in the transfer line.

The dispersion matches between the entrance of the beam selector and the entrances of the big bends are fairly critical, but no trimming quadrupoles for adjustment of those matches have been proposed. Mismatches are inconvenient, but, within limits, not lethal because they can, presumably, be compensated with the 7 quadrupoles at the exit end of the big bends. This approach may cause some loss of horizontal acceptance of the transfer line, its acceptability must be checked.

The transfer system is designed and its component values have been chosen to provide a match between the linear focusing parameters of the AGS and RHIC. It contains enough quadrupoles which can be adjusted to obtain such a match experimentally. They can also be used to obtain deliberate mismatches which constitute one way for controlling the distribution in the emittances of the beams circulating in RHIC. Means and methods for measuring the quality of the match have not yet been considered.

The possible need for sextupoles for the correction of chromatic effects has been mentioned. That need depends upon the desired accuracy of the match. The betatron phase advance along the transfer line is about $11 \pi r a d$ in both reference directions. It varies with momentum, in first estimate proportional to the inverse square root of the relative momentum deviation, causing, in the first place an equal rotation of the emittance ellipses at the exit of the injection magnet. The eccentricities will also change. These two effects represent mismatches relative to the nominal situation and cause dilution. They must be estimated.

Finally the exact balance of tolerance and damper requirements must be resolved in the context of other dampers in RHIC. It seems clear that both
strict equipment tolerances and dampers will be required if serious growth of the 6 D emittance is to be avoided.

## Acknowledgment

The beam transfer system we described is in large measure a development of the system designed for the CBA Project, closed some years ago. A large group of people contributed to that design, among them W.T. Weng, who was involved in the general layout and beam dynamics, and R.E. Thern, who was concerned with magnet design and testing. Thern worked also on matters of geometry and survey. He produced the spread sheet, mentioned in the appendix, that is used for determining the surveying coordinates of the components of the previous and present designs.

## References

1 Conceptual Design of the Relativistic Heavy Ion Collider RHIC, May 1989, BNL 52195

2 W.T. Weng: The New AGS Fast Extraction System, BNL 51310, September 15, 1980
3 W.T. Weng: The AGS New Fast Extraction System and the Single Bunch Extraction Test, Trans. Nucl. Sci., Vol. NS-30, No.4, p. 2956, August 1983
4 R.E. Thern: Bending Magnets for the CBA Beam Transport Line, Trans. Nucl. Sci., Vol. NS-30, No.4, p. 2950, August 1983

The tables that follow describe the principal parameters of the transfer line between AGS and RHIC for a particular choice of settings. They are extracted from a Synch output file which can be found in the RHIC data base on the BNLDAG VAX under the name: [rhic.main.lat]rhinj.out. The Synch input file that generated that file is called: [rhic.main.lat]rhinj.dat. It must be noted that the program was run on BNL's IBM 3090, not on any VAX. A further three files: [rhic.main.lat]rhinj.geo1, [rhic.main.lat]rhinj.geo2 and [rhic.main.lat]rhinj.geo3 describe the geometry of the system in an absolute way as well as relative to BNL's established surveying coordinate systems. These last three files were generated by a PC spread sheet program that takes its input from the Synch output file. They were used as input for the CAD program that produced the component layout drawings Figs. 1 and 2. The geometry program should be rerun whenever the input file for Synch is changed and it will be necessary to check the consistency of the various files whenever decisions are to be made.

The tables show the betatron phase advance $Q$ (in units of $2 \pi r a d$ or $360^{\circ}$ ), the Twiss and Frank parameters $\alpha$ and $\beta$ for each of the two transverse reference directions $x$ and $y$, and the dispersions $x_{p}$ and $y_{p}$, all as functions of the distance from the extraction septum magnet at $H$ in the AGS along the nominal trajectory. 'The initial values for the $\alpha$ 's, $\beta$ 's and the dispersions are experimental values derived from beam size measurements in the U-line ${ }^{1}$, since a believable calculation of the transfer function between the extraction kicker's entrance plane to the exit of the extraction septum magnet's exit plane is not yet available. That calculation requires a path integration through the stray fields of the AGS main magnets. It will be performed and the gradients of the quadrupoles in the transfer line will be adjusted to match the results. The tables show also the lengths, deflection angles, radii of orbit curvature and gradients of the various components.

[^3]| Pos Name | S [m] | $\mathrm{Q}_{\mathrm{x}}$ | $\begin{gathered} \beta_{x} \\ \lceil\mathrm{ml} \end{gathered}$ | $\alpha_{x}$ | $[\mathrm{m}]$ | Q ${ }_{\text {y }}$ | $\begin{gathered} \beta_{y} \\ {[\mathrm{~m}]} \end{gathered}$ | $\alpha_{y}$ | $\begin{gathered} y_{\mathrm{p}} \\ {[\mathrm{~m}]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.000 | 0.000 | 37.59 | -4.778 | -2.960 | 0.000 | 8.05 | 1.053 | 0.000 |
| ks | 5.000 | 0.012 | 101.21 | -7.947 | -4.435 | 0.169 | 4.06 | -0.256 | 0.000 |
| 2 u 01 | 5.374 | 0.013 | 107.25 | -8.184 | -4.545 | 0.183 | 4.30 | -0.354 | 0.000 |
| 3 uq 1 | 6.326 | 0.014 | 91.50 | 23.042 | -4.151 | 0.213 | 6.79 | -2.520 | 0.000 |
| 4 u 12 | 6.821 | 0.015 | 70.10 | 20.162 | -3.605 | 0.223 | 9.56 | -3.057 | 0.000 |
| 5 uq2 | 7.774 | 0.018 | 51.95 | 0.693 | -3.038 | 0.236 | 12.70 | 0.077 | 0.000 |
| 6 u231 | 8.252 | 0.020 | 51.30 | 0.680 | -2.983 | 0.242 | 12.64 | 0.039 | 0.000 |
| $7 \mathrm{ud1}$ | 10.332 | 0.027 | 48.59 | 0.621 | -2.714 | 0.268 | 12.82 | -0.122 | 0.000 |
| 8 u232 | 10.792 | 0.028 | 48.03 | 0.608 | -2.648 | 0.274 | 12.95 | -0.158 | 0.000 |
| 9 ud 2 | 12.872 | 0.035 | 45.62 | 0.548 | -2.328 | 0.298 | 13.94 | -0.319 | 00 |
| 10 u233 | 13.332 | 0.037 | 45.12 | 0.535 | -2.251 | 0.304 | 14.25 | -0.355 | 0.000 |
| 11 ud3 | 15.412 | 0.044 | 43.02 | 0.476 | -1.879 | 0.326 | 16.06 | -0.515 | 0.000 |
| 12 u234 | 15.667 | 0.045 | 42.78 | 0.469 | -1.831 | 0.328 | 16.32 | -0.534 | 0.000 |
| 13 uq3 | 16.124 | 0.047 | 42.30 | 0.577 | -1.742 | 0.332 | 16.85 | -0.619 | 0.000 |
| 14 u34 | 31.264 | 0.114 | 32.05 | 0.100 | 1.230 | 0.415 | 54.42 | -1.862 | 0.000 |
| 15 uq4 | 31.990 | 0.118 | 33.90 | -2.686 | 1.411 | 0.417 | 53.83 | 2.648 | 0.000 |
| 16 u45 | 38.914 | 0.138 | 82.72 | -4.365 | 3.527 | 0.449 | 24.30 | 1.617 | 0.000 |
| 17 uq5 | 39.641 | 0.140 | 85.52 | 0.567 | 3.672 | 0.453 | 23.02 | 0.174 | 0.000 |
| 18 u56 | 46.574 | 0.154 | 78.40 | 0.460 | 4.316 | 0.502 | 22.76 | -0.137 | 0.000 |
| 19 uq6 | 47.300 | 0.155 | 74.45 | 4.8 | 4.291 | -0.507 | 23.98 | -1.561 | 0.000 |
| 20 u67 | 61 | 0. | 2.98 | -0.000 | 1. | -0.556 | 100.07 | -3.653 | 0.000 |
| 21 uq7 | 62.620 | 0.411 | 3.34 | -0.512 | 1.872 | -0.557 | 99.32 | 4.672 | 0.000 |
| 22 u78 | 75.219 | 0.555 | 76.16 | -5.267 | 1.820 | -0.605 | 18.07 | 1.776 | 0.000 |
| 23 uq8 | 75.946 | 0.557 | 80.61 | -0.774 | 1.778 | -0.612 | 16.32 | 0.672 | 0.000 |
| 24 u891 | 79 | 0. | 86. | -0.844 | 1.391 | -0 | 12.68 | 0.358 | 0.000 |
| 25 u4f | 83.138 | 0.571 | 71.79 | 4.482 | 0.898 | -0.697 | 14.35 | -0.851 | 0.000 |
| 26 u892 | 83.748 | 0.572 | 66.43 | 4.303 | 0.804 | -0.703 | 15.43 | -0.924 | 0.000 |
| 27 u5d | 87.406 | 0.582 | 51.86 | 0.012 | 0.384 | -0.736 | 19.00 | 0.032 | 0.000 |
| 28 u193 | 88.015 | 0.584 | 51.85 | -0.000 | 0.335 | -0.741 | 18.98 | 0.000 | 0.000 |
| 29 u 293 | 88.625 | 0.586 | 51.86 | -0.012 | 0.286 | -0.746 | 19.00 | -0.032 | 0.000 |
| 30 u6d | 92.283 | 0.597 | 66.43 | $-4.303$ | 0.083 | -0.779 | 15.43 | 0.924 | 0.000 |
| 31 u894 | 92.892 | 0.598 | 71.79 | -4.482 | 0.063 | -0.786 | 14.35 | 0.851 | 0.000 |
| 32 u7f | 96.550 | 0.605 | 86.33 | 0.844 | 0.000 | 0.831 | 12.68 | -0.358 | 0.000 |
| 33 u895 | 113.641 | 0.642 | 63.28 | 0.505 | 0.000 | 0.949 | 50.90 | -1.878 | 0.000 |
| 34 uq9 | 114.367 | 0.6 | 64.56 | -2.294 | 0. | 51 | 52.02 | 0.348 |  |
| 35 u910 | 125.063 | 0.663 | 124.73 | -3.331 | 0.000 | 0.986 | 47.04 | 0.117 | 0.000 |
| 36 uq10 | 125.790 | 0.664 | 106.49 | 26.803 | 0.000 | 0.988 | 56.52 | -13.995 | 0.000 |
| 37 u011 | 126.896 | 0.666 | 55.43 | 19.326 | 0.000 | 0.990 | 91.77 | -17.850 | 0.000 |
| 38 uq11 | 127.623 | 0.669 | 37.96 | 6.106 | 0.000 | 0.992 | 101.52 | 5.222 | 0.000 |
| 39 u112 | 141.851 | 1.124 | 68. | -8.243 | 0.00 | 1.068 | 9.29 | 1.260 | 0.000 |
| 40 uq12 | 142.577 | 1.125 | 72.47 | 2.800 | 0.000 | 1.081 | 8.57 | -0.229 | 0.000 |
| 41 w01 | 160.101 | 1.228 | 11.80 | 0.663 | 0.000 | 1.232 | 54.29 | -2.381 | 0.000 |
| 42 w1d | 163.759 | 1.281 | 11.80 | -0.663 | 0.082 | 1.243 | 54.29 | 2.381 | 0.000 |
| 43 w12 | 177.811 | 1.374 | 54.50 | -2.376 | 0.728 | 1.337 | 11.64 | 0.655 | 0.000 |
| 44 w2f | 181.469 | 1.384 | 54.50 | 2.377 | 0.853 | 1.391 | 11.64 | -0.655 | 0.000 |


| Pos Name | S [m] | $Q_{x}$ | $\begin{gathered} \beta_{\mathrm{x}} \\ {[\mathrm{~m}]} \end{gathered}$ | $\alpha_{x}$ | $\begin{gathered} \boldsymbol{x}_{\mathbf{p}} \\ {[\mathrm{m}]} \end{gathered}$ | $Q_{y}$ | $\begin{gathered} \beta_{y} \\ {[\mathrm{~m}]} \end{gathered}$ | ${ }_{\mathbf{y}}$ | $\begin{gathered} y_{p} \\ {[\mathrm{~m}]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 45 w231 | 187.580 | 1.408 | 30.00 | 1.631 | 0.981 | 1.450 | 24.23 | -1.406 | 000 |
| 46 wp 1 | 189.409 | 1.419 | 24.44 | 1.409 | 1.019 | 1.461 | 29.79 | -1.631 | -0.009 |
| 47 w232 | 195.520 | 1.478 | 11.78 | 0.663 | 1.146 | 1.485 | 54.30 | -2.381 | -0.070 |
| 48 w3d | 199.178 | 1.531 | 11.77 | -0.661 | 1.494 | 1.495 | 54.30 | 2.381 | -0.094 |
| 49 | 213.230 | 1.624 | 54.4 | -2.377 | 3.940 | 1.590 | 11. | 0.655 | 32 |
| 50 w 4 f | 216.888 | 1.635 | 54.48 | 2.373 | 4.017 | 1.643 | 11.63 | -0.655 | -0.164 |
| 51 | 230.940 | 1.728 | 11.82 | 0.663 | 2.153 | 1.738 | 54.28 | -2.380 | -0.374 |
| w5d | 234.597 | 1.781 | 11.83 | -0.664 | 2.071 | 1.748 | 54.28 | 2.380 | -0.369 |
| 53 w56 | 248.649 | 1.874 | 54.54 | -2.376 | 3.292 | 1.842 | 11.63 | 0.655 | -0.116 |
| 54 w6f | 252.307 | 1.884 | 54.52 | 2.380 | 3.167 | 1.896 | 11.64 | -0.655 | -0.065 |
| 55 w67 | 266.359 | 1.978 | 11.77 | 0.662 | 1.007 | 1.991 | 54.31 | -2.382 | 0.083 |
| 56 w7d | 270.017 | 2.030 | 11.76 | -0.661 | 0.659 | 2.001 | 54.31 | 2.382 | 0.107 |
| 57 w78 | 284.068 | 2.125 | 54.46 | -2.378 | 0.078 | 2.095 | 11.64 | 0.655 | 0.136 |
| 58 w | 287.726 | 2.135 | 54.48 | 2.373 | 0.0 | 2.149 | 11.63 | -0.655 | 0.166 |
| 59 v01 | 301.698 | 2.227 | 11.93 | 0.672 | 0.000 | 2.243 | 53.91 | -2.371 | 0.370 |
| 60 vq1d | 302.438 | 2.237 | 11.76 | -0.442 | 0.000 | 2.245 | 53.85 | 2.444 | 0.369 |
| 61 v12 | 316.896 | 2.344 | 45.77 | -1.911 | 0.000 | 2.351 | 10.25 | 0.572 | 0.102 |
| 62 vq 2 f | 317.636 | 2.347 | 45.60 | 2.132 | 0.000 | 2.362 | 10.12 | -0.397 | 0.091 |
| 63 v231 | 325.850 | 2.392 | 18.78 | 1.133 | 0.000 | 2.450 | 24.37 | -1.337 | 0.009 |
| 64 wp2 | 327.679 | 2.409 | 15.04 | 0.91 | 0. | 2.461 | 29.6 | -1.546 | 0.000 |
| 65 v232 | 333. | 2.498 | 8.49 | 0. | 0. | 2.485 | 52. | -2.231 | . 000 |
| 66 vq3d | 334.411 | 2.512 | 8.90 | -0.745 | 0.000 | 2.487 | 51.83 | 2.810 | 0.000 |
| 67 v34 | 348.484 | 2.612 | 64.48 | -3.204 | 0.000 | 2.623 | 6.73 | 0.395 | 0.000 |
| 68 vq 4 f | 349.224 | 2.618 | 64.72 | 2.883 | 0. | 2.641 | 6.70 | -0.344 | 0.000 |
| 69 v 45 | 368. | 2. | 6.9 | 0.066 | 0. | 2. | 84. | -3.613 | 0.000 |
| 70 vq5d | 369.545 | 2.817 | 7.40 | -0.649 | 0.000 | 2.797 | 84.27 | 3.499 | 0.000 |
| 71 v56 | 387.683 | 2.937 | 94.06 | -4.129 | 0.00 | 2.911 | 9.04 | 0.649 | 0.000 |
| vq6f | 388.42 | 2.939 | 93.14 | 5.343 | 0. | 2.924 | . 81 | -0.330 | 0.000 |
| 73 v60 | 388.955 | 2.940 | 87.55 | 5.174 | 0.00 | 2.934 | 9.20 | -0.398 | 0.000 |
| 74 swm | 392.612 | 2.948 | 53.80 | 4.045 | 0.0 | 2.986 | 13.79 | -0.858 | 0.000 |
| 75112 | 402.568 | 3.049 | 5.24 | 0.83 | 0.55 | 3.053 | 43.35 | -2.111 | 0.000 |
| 76 g1d | 406.225 | 3.188 | 5.13 | -0.795 | 0.98 | 3.066 | 36.81 | 3.583 | 0.000 |
| 77113 | 410.850 | 3.265 | 19.29 | -2.268 | 1.8 | 3.102 | 11.71 | 1.845 | 0.000 |
| 78 g2f | 413.796 | 3.284 | 26.66 | 0.0 | 2.18 | 3.161 | 6.23 | 0.216 | 0.000 |
| 79114 | 415.081 | 3.292 | 26.6 | -0. | 2. | 3.195 | 5.95 | -0.000 | - |
| 80 hol | 415.796 | 3.297 | 26.62 | -0.027 | 2.180 | 3.214 | 6.03 | -0.120 | 0.000 |
| 81 g3f | 419.454 | 3.323 | 15.77 | 2.473 | 1.73 | 3.287 | 13.52 | -2.259 | 0.000 |
| 82 os | 419.904 | 3.328 | 13.63 | 2.269 | 1.625 | 3.292 | 15.64 | -2.463 | 0.000 |
| $83 \mathrm{g4d}$ | 423.561 | 3.399 | 6.13 | 0.119 | 1.213 | 3.318 | 26.47 | 0.027 | 0.000 |
| 84 hol | 424.276 | 3. | 6.0 | 0.000 | 1. | 3.322 | 26. | -0.000 | 0.000 |
| 85 hol | 424.991 | 3.437 | 6.13 | -0.118 | 1.213 | 3.327 | 26.47 | -0.027 | 0.000 |
| 86 g5d | 428.649 | 3.508 | 13.62 | -2.268 | 1.626 | 3.353 | 15.64 | 2.463 | 0.000 |
| 87 os | 429.099 | 3.513 | 15.75 | -2.470 | 1.732 | 3.358 | 13.52 | 2.259 | 0.000 |
| $88 \mathrm{g6f}$ | 432.756 | 3.539 | 26.60 | 0.027 | 2.182 | 3.431 | 6.04 | 0.120 | 0.000 |
| 89 hol | 433.471 | 3.544 | 26.58 | 0.000 | 2.182 | 3.450 | 5.95 | 0.000 | 0.000 |


| Pos Name | S $\mathrm{m}]$ | $\mathrm{Q}_{\mathrm{x}}$ | $\begin{gathered} \beta_{x} \\ {[\mathrm{~m}]} \end{gathered}$ | ${ }_{x}$ | $[\mathrm{m}]$ | $Q_{y}$ | $\begin{gathered} \beta_{y}^{y} \\ {[\mathrm{~m}]} \end{gathered}$ | $\alpha_{y}$ | $\begin{gathered} y_{\mathrm{p}} \\ {[\mathrm{~m}]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90 hol | 434.186 | 3.548 | 26.60 | -0.027 | 2.182 | 3.469 | 6.04 | -0.120 | 0.000 |
| 91 g7f | 437.844 | 3.574 | 15.75 | 2.470 | 1.733 | 3.541 | 13.52 | -2.259 | 0.000 |
| 92 os | 438.294 | 3.579 | 13.62 | 2.268 | 1.627 | 3.546 | 15.64 | -2.462 | 0.000 |
| 93 g8d | 441.952 | 3.651 | 6.13 | 0.118 | 1.214 | 3.572 | 26.47 | 0.027 | 0.000 |
| 94 hol | 442.667 | 3.669 | 6.05 | -0.000 | 1.214 | 3.577 | 26. | 0.000 | 0.000 |
| 95 hol | 443.382 | 3.688 | 6.13 | -0.119 | 1.214 | 3.581 | 26.47 | -0.027 | 0.000 |
| 96 g9d | 447.039 | 3.760 | 13.63 | -2.269 | 1.627 | 3.607 | 15.64 | 2.462 | 0.000 |
| 97 os | 447.489 | 3.765 | 15.77 | -2.473 | 1.733 | 3.612 | 13.52 | 2.259 | 0.000 |
| 98 g10f | 451.147 | 3.791 | 26.62 | 0.027 | 2.183 | 3.685 | 6.04 | 0.120 | 0.000 |
| 99 hol | 451.862 | 3.795 | 26.60 | -0.000 | 2.183 | 3.704 | 5.95 | -0.000 | 000 |
| 100 hol | 452.577 | 3.799 | 26.62 | -0.027 | 2.183 | 3.723 | 6.04 | -0.120 | 0.000 |
| $101 \mathrm{gl1f}$ | 456.234 | 3.825 | 15.77 | 2.473 | 1.733 | 3.796 | 13.52 | -2.259 | 0.000 |
| 102 os | 456.684 | 3.830 | 13.63 | 2.269 | 1.627 | 3.800 | 15.64 | -2.463 | 0.000 |
| $103 \mathrm{gl2d}$ | 460.342 | 3.902 | 6.13 | 0.119 | 1.213 | 3.827 | 26.47 | 0.027 | 0.000 |
| 104 hol | 461.057 | 3.921 | 6.05 | 0.000 | 1.213 | 3.831 | 26.45 | -0.000 | 0.000 |
| 105 hol | 461.772 | 3.939 | 6.13 | -0.118 | 1.213 | 3.835 | 26.47 | -0.027 | 0.000 |
| 106 g13d | 465.430 | 4.011 | 13.62 | -2.268 | 1.626 | 3.862 | 15.64 | 2.463 | 0.000 |
| 107 os | 465.880 | 4.016 | 15.75 | -2.470 | 1.732 | 3.867 | 13.52 | 2.259 | 0.000 |
| $108 \mathrm{g14f}$ | 469.537 | 4.042 | 26.60 | 0.027 | 2.181 | 3.939 | 6.04 | 0.120 | 0.000 |
| 109 hol | 470.252 | 4.046 | 26.58 | 0.000 | 2.181 | 3.958 | 5.95 | 0.000 | 00 |
| 110 hol | 470.967 | 4.051 | 26.60 | -0.027 | 2.181 | 3.977 | 6.04 | -0.120 | . 000 |
| 111 g 15 f | 474.625 | 4.077 | 15.75 | 2.470 | 1.731 | 4.050 | 13.52 | -2.259 | 0.000 |
| 112 os | 475.075 | 4.082 | 13.62 | 2.268 | 1.625 | 4.055 | 15.64 | -2.462 | 0.000 |
| 113 g16d | 478.732 | 4.153 | 6.13 | 0.118 | 1.213 | 4.081 | 26.47 | 0.027 | 0.000 |
| 114 hol | 479.447 | 4.172 | 6.05 | -0.00 | 1. | 4.085 | 26. | 0.0 | 00 |
| 115 ho | 480.162 | 4.191 | 6.13 | -0.119 | 1.212 | 4.090 | 26.47 | -0.027 | 0.000 |
| 116 g17d | 483.820 | 4.262 | 13.63 | -2.269 | 1.625 | 4.116 | 15.64 | 2.462 | 0.000 |
| 117 os | 484.270 | 4.267 | 15.77 | -2.473 | 1.731 | 4.121 | 13.52 | 2.259 | 0.000 |
| $118 \mathrm{gl8f}$ | 487.928 | 4.293 | 26.62 | 0.027 | 2.180 | 4.194 | 6.04 | 0.120 | 0.000 |
| 119 hol | 488. | 4.298 | 26.60 | -0.000 | 2.18 | 4.213 | 5.95 | -0.000 | 0.000 |
| 120 hol | 489.358 | 4.302 | 26.62 | -0.027 | 2.180 | 4.232 | 6.04 | -0.120 | 0.000 |
| 121 g 19 f | 493.015 | 4.328 | 15.77 | 2.473 | 1.731 | 4.304 | 13.52 | -2.259 | 0.000 |
| 122 os | 493.465 | 4.333 | 13.63 | 2.269 | 1.625 | 4.309 | 15.64 | -2.463 | 0.000 |
| 123 g20d | 497.123 | 4.405 | 6.13 | 0.119 | 1.213 | 4.336 | 26.47 | 0.026 | 0.000 |
| 124 hol | 497.838 | 4.423 | 6.05 | 0.000 | 1.213 | 4.340 | 26.45 | -0.000 | 0.000 |
| 125 hol | 498.553 | 4.442 | 6.13 | -0.118 | 1.213 | 4.344 | 26.47 | -0.027 | 0.000 |
| 126 g21d | 502.210 | 4.514 | 13.62 | -2.268 | 1.626 | 4.370 | 15.64 | 2.463 | 0.000 |
| 127 os | 502.660 | 4.519 | 15.75 | -2.470 | 1.732 | 4.375 | 13.52 | 2.259 | 0.000 |
| 128 g22f | 506.318 | 4.545 | 26.60 | 0.027 | 2.182 | 4.448 | 6.04 | 0.120 | 0.000 |
| 129 hol | 507.033 | 4.549 | 26.5 | 0.000 | 2.182 | 4.467 | 5.95 | 0.000 | 0.000 |
| 130 hol | 507.748 | 4.553 | 26.60 | -0.027 | 2.182 | 4.486 | 6.04 | -0.120 | 0.000 |
| 131 g 23 f | 511.406 | 4.580 | 15.75 | 2.470 | 1.733 | 4.559 | 13.52 | -2.259 | 0.000 |
| 132 os | 511.856 | 4.584 | 13.62 | 2.268 | 1.627 | 4.563 | 15.64 | -2.462 | 0.000 |
| 133 g24d | 515.513 | 4.656 | 6.13 | 0.118 | 1.214 | 4.590 | 26.47 | 0.027 | 0.000 |
| 134 hol | 516.228 | 4.675 | 6.05 | -0.000 | 1.214 | 4.594 | 26.45 | 0.00 | 0.000 |


| Pos Name | S [m] | $\mathrm{Q}_{\mathrm{x}}$ | $\begin{gathered} \beta_{x} \\ {[\mathrm{~m}]} \end{gathered}$ | ${ }_{\mathbf{\alpha}}$ | $\begin{gathered} \boldsymbol{x}_{\mathbf{p}} \\ {[\mathrm{m}]} \end{gathered}$ | $\mathrm{Q}_{\mathbf{y}}$ | $\begin{gathered} \beta_{\mathrm{y}} \\ {[\mathrm{~m}]} \end{gathered}$ | $\alpha_{y}$ | $\begin{gathered} y_{\mathrm{p}} \\ {[\mathrm{~m}]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 135 hol | 516.943 | 4.693 | 6.13 | -0.119 | 1.214 | 4.598 | 26.46 | -0.027 | 0.000 |
| 136 g 25 d | 520.601 | 4.765 | 13.63 | -2.269 | 1.627 | 4.625 | 15.64 | 2.462 | 0.000 |
| 137 os | 521.051 | 4.770 | 15.77 | -2.473 | 1.733 | 4.630 | 13.52 | 2.259 | 0.000 |
| 138 g 26 f | 524.708 | 4.796 | 26.62 | 0.027 | 2.183 | 4.702 | 6.04 | 0.120 | 0.000 |
| 139 hall | 525.226 | 4.799 | 26.60 | 0.007 | 2.183 | 4.716 | 5.96 | 0.033 | 0.000 |
| 140 qmil | 525.906 | 4.803 | 26.38 | 0.306 | 2.174 | 4.734 | 6.04 | -0.155 | 0.000 |
| 141 m121 | 527.494 | 4.813 | 25.52 | 0.240 | 2.131 | 4.774 | 6.96 | -0.424 | 0.000 |
| 142 bmi | 530.441 | 4.832 | 24.46 | 0.118 | 2.124 | 4.828 | 10.91 | -0.915 | 0.000 |
| 143 m 122 | 531.755 | 4.840 | 24.22 | 0.064 | 2.152 | 4.846 | 13.61 | -1.137 | 0.000 |
| 144 qmi2 | 532.435 | 4.845 | 22.45 | 2.482 | 2.090 | 4.853 | 16.32 | -2.944 | 0.000 |
| 145 m 231 | 533.593 | 4.854 | 17.13 | 2.113 | 1.852 | 4.862 | 23.93 | -3.630 | 0.000 |
| 146 bmi | 536.540 | 4.896 | 7.45 | 1.173 | 1.319 | 4.876 | 50.38 | -5.338 | 0.000 |
| 147 m 232 | 537.179 | 4.912 | 6.08 | 0.969 | 1.219 | 4.878 | 57.45 | -5.712 | 0.000 |
| 148 qmi3 | 537.859 | 4.931 | 5.63 | -0.282 | 1.191 | 4.880 | 57.59 | 5.524 | 0.000 |
| 149 m 341 | 539.392 | 4.970 | 6.95 | -0.576 | 1.304 | 4.885 | 41.94 | 4.685 | 0.000 |
| 150 bmi | 542.339 | 5.022 | 12.00 | -1.141 | 1.592 | 4.901 | 19.01 | 3.086 | 0.000 |
| 151 m342 | 542.921 | 5.030 | 13.40 | -1.253 | 1.664 | 4.907 | 15.61 | 2.764 | 0.000 |
| 152 qmi4 | 543.601 | 5.037 | 13.41 | 1.240 | 1.640 | 4.914 | 13.87 | -0.094 | 0.000 |
| 153 m 451 | 544.492 | 5.049 | 11.35 | 1.071 | 1.470 | 4.924 | 14.09 | -0.159 | 0.000 |
| 154 bmi | 547.438 | 5.104 | 6.68 | 0.514 | 0.979 | 4.956 | 15.63 | -0.361 | 0.000 |
| 155 m 452 | 550.499 | 5.190 | 5.31 | -0.065 | 0.542 | 4.985 | 18.51 | -0.583 | 0.000 |
| 156 qmi5 | 551.179 | 5.210 | 5.72 | -0.546 | 0.456 | 4.991 | 18.54 | 0.549 | 0.000 |
| 157 m561 | 552.579 | 5.243 | 7.69 | -0.864 | 0.300 | 5.003 | 17.14 | 0.451 | 0.000 |
| 158 bmi | 555.525 | 5.288 | 14.75 | -1.533 | 0.044 | 5.033 | 15.05 | 0.256 | 0.000 |
| 159 m562 | 563.088 | 5.333 | 50.93 | -3.251 | -0.431 | 5.116 | 15.23 | -0.280 | 0.000 |
| 160 qmi6 | 563.768 | 5.335 | 51.80 | 2.003 | -0.460 | 5.123 | 16.75 | -1.997 | 0.000 |
| 161 m671 | 564.811 | 5.338 | 47.73 | 1.902 | -0.476 | 5.131 | 21.24 | -2.308 | 0.000 |
| 162 sio | 568.899 | 5.354 | 33.69 | 1.527 | -0.545 | 5.153 | 45.07 | -3.525 | -0.103 |
| 163 m781 | 570.237 | 5.361 | 29.78 | 1.395 | -0.568 | 5.157 | 55.04 | -3.924 | -0.171 |
| 164 qmi7 | 570.917 | 5.365 | 30.59 | -2.608 | -0.605 | 5.159 | 55.38 | 3.439 | . 197 |
| 165 m782 | 571.917 | 5.369 | 36.06 | -2.864 | -0.700 | 5.162 | 48.73 | 3.208 | -0.223 |
| 166 si1 | 575.417 | 5.381 | 59.14 | -3.726 | -1.028 | 5.177 | 29.12 | 2.397 | -0.238 |
| 1670782 | 576.417 | 5.384 | 66.85 | -3.978 | -1.122 | 5.183 | 24.56 | 2.165 | -0.221 |
| 168 q8o | 577.034 | 5.385 | 69.77 | -0.713 | -1.162 | 5.187 | 22.67 | 0.934 | -0.213 |
| 169 q8o | 577.651 | 5.387 | 68.57 | 2.637 | -1.168 | 5.191 | 22.21 | -0.184 | -0.212 |
| 1700891 | 578.651 | 5.389 | 63.41 | 2.521 | -1.150 | 5.198 | 22.62 | -0.230 | -0.216 |
| 171 si2 | 580.051 | 5.393 | 56.58 | 2.359 | -1.124 | 5.208 | 23.36 | -0.296 | -0.219 |
| 1720892 | 594.414 | 5.483 | 12.75 | 0.693 | -0.855 | 5.284 | 41.45 | -0.964 | -0.229 |
| 173 ki | 597.664 | 5.531 | 9.48 | 0.316 | -0.794 | 5.296 | 48.21 | -1.115 | -0.229 |
| 1740893 | 598.164 | 5.539 | 9.19 | 0.258 | -0.785 | 5.298 | 49.34 | -1.139 | -0.229 |
| 175 q9o | 598.576 | 5.546 | 9.09 | 0.000 | -0.781 | 5.299 | 49.81 | -0.000 | . 228 |


| Pos Name | length <br> [m] | bend <br> [mrad] | $\begin{gathered} \rho \\ {[\mathrm{m}]} \end{gathered}$ | $\begin{gathered} \mathrm{B}^{\prime} /(\mathrm{B} \rho) \\ {[\mathrm{m}]^{-2}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 5.374005 |  |  |  |
| 3 uq1 | 0.952500 |  |  | 0.32885802 |
| 4 | 0.495300 |  |  |  |
| 5 uq 2 | 0.952500 |  |  | -0.31874932 |
| 6 | 0.477256 |  |  |  |
| 7 ud1 | 2.080006 | 24.725 | 84.127764 |  |
| 8 | 0.459994 |  |  |  |
| 9 ud2 | 2.080006 | 24.725 | 84.127764 |  |
| 10 | 0.459994 |  |  |  |
| 11 ud3 | 2.080006 | 24.725 | 84.127764 |  |
| 12 | 0.255397 |  |  |  |
| 13 uq3 | 0.457200 |  |  | 0.0062707454 |
| 14 | 15.139416 |  |  |  |
| 15 uq4 | 0.726440 |  |  | -0.11450203 |
| 16 | 6.924040 |  |  |  |
| 17 uq5 | 0.726440 |  |  | 0.081090116 |
| 18 | 6.933184 |  |  |  |
| 19 uq6 | 0.726440 |  |  | 0.081090116 |
| 20 | 14.593062 |  |  |  |
| 21 uq7 | 0.726440 |  |  | -0.11450203 |
| 22 | 12.599619 |  |  |  |
| 23 uq8 | 0.726440 |  |  | 0.080325355 |
| 24 | 3.534764 |  |  |  |
| 25 u4f | 3.657600 | 34.906 | 104.789606 | 0.018823143 |
| 26 | 0.609600 |  |  |  |
| 27 u5d | 3.657600 | 34.906 | 104.789606 | -0.018823143 |
| 28 | 0.609600 |  |  |  |
| 29 | 0.609600 |  |  |  |
| 30 u6d | 3.657600 | 34.906 | 104.789606 | -0.018823143 |
| 31 | 0.609600 |  |  |  |
| 32 u7f | 3.657600 | 34.906 | 104.789606 | 0.018823143 |
| 33 | 17.090476 |  |  |  |
| 34 uq9 | 0.726440 |  |  | -0.060001498 |
| 35 | 10.695975 |  |  |  |
| 36 uq10 | 0.726440 |  |  | 0.36507553 |
| 37 | 1.106874 |  |  |  |
| 38 uq11 | 0.726440 |  |  | -0.32883725 |
| 39 | 14.227735 |  |  |  |
| 40 uq12 | 0.726440 |  |  | 0.21564016 |
| 41 | 17.524174 |  |  |  |
| 42 w1d | 3.657600 | 43.633 | 83.833108 | -0.023528529 |
| 43 | 14.051700 |  |  |  |
| 44 w2f | 3.657600 | 43.633 | 83.833108 | 0.023528529 |


| Pos | Name | $\begin{gathered} \text { length } \\ {[\mathrm{m}]} \end{gathered}$ | bend <br> [mrad] | $\begin{gathered} \rho \\ {[\mathrm{m}]} \end{gathered}$ | $\begin{gathered} B^{\prime} /(B \rho) \\ {[m]^{-2}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 45 |  | 6.111450 |  |  |  |
| 46 | wp1 | 1.828800 | 10. | 182.78 |  |
| 47 |  | 6.111450 |  |  |  |
| 48 | w3d | 3.657600 | 43.633 | 83.833108 | -0.023528529 |
| 49 |  | 14.051700 |  |  |  |
| 50 | W4f | 3.657600 | 43.633 | 83.833108 | 0.023528529 |
| 51 |  | 14.051700 |  |  |  |
| 52 | w5d | 3.657600 | 43.633 | 83.833108 | -0.023528529 |
| 53 |  | 14.051700 |  |  |  |
| 54 | w7f | 3.6576 | 43.633 | 83.833108 | 0.023528529 |
| 55 |  | 14.051700 |  |  |  |
| 56 | w7d | 3.657600 | 43.633 | 83.833108 | -0.023528529 |
| 57 |  | 14.051700 |  |  |  |
| 58 | W8f | 3.657600 | 43.633 | 83.833108 | 0.023528529 |
| 59 |  | 13.972064 |  |  |  |
| 60 | vq1d | 0.740000 |  |  | -0.1204328 |
| 61 |  | 14.457299 |  |  |  |
| 62 | vq 2 f | 0.740000 |  |  | 0.11939398 |
| 63 |  | 8.214856 |  |  |  |
| 64 | wp2A | 1.828800 | -10. | 182.87 |  |
| 65 |  | 5.991919 |  |  |  |
| 66 | vq3d | 0.740000 |  |  | -0.13049325 |
| 67 |  | 14.072442 |  |  |  |
| 68 | vq 4 f | 0.740000 |  |  | 0.12684975 |
| 69 |  | 19.581019 |  |  |  |
| 70 | $v q 5 d$ | 0.740000 |  |  | -0.11365668 |
| 71 |  | 18.138067 |  |  |  |
| 72 | $v q 6 f$ | 0.740000 |  |  | 0.13605879 |
| 73 |  | 0.532168 |  |  |  |
| 74 | SWm | 3.657600 | 47.165 | 77.5775756 | 0. |
| 75 |  | 9.955836 |  |  |  |
| 76 | g1d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 77 |  | 4.624169 |  |  |  |
| 78 | g2f | 2.946400 | 38.199 | 77.1367914 | 0.0380886 |
| 79 |  | 1.284570 |  |  |  |
| 80 |  | 0.715 |  |  |  |
| 81 | g3f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 82 |  | 0.450000 |  |  |  |
| 83 | g4d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 84 |  | 0.715000 |  |  |  |
| 85 |  | 0.715000 |  |  |  |
| 86 | g5d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 87 |  | 0.450000 |  |  |  |
| 88 | g6f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 89 |  | 0.715000 |  |  |  |


| Pos | Name | length <br> [m] | bend <br> [mrad] | $\begin{gathered} \rho \\ {[\mathrm{m}]} \end{gathered}$ | $\begin{gathered} \mathrm{B}^{\prime} /(\mathrm{B} \rho) \\ {[\mathrm{m}]^{-2}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 90 |  | 0.715000 |  |  |  |
| 91 | g7f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 92 |  | 0.450000 |  |  |  |
| 93 | g8d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 94 |  | 0.715000 |  |  |  |
| 95 |  | 0.715 |  |  |  |
| 96 | g9d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 97 |  | 0.450000 |  |  |  |
| 98 | g10f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 99 |  | 0.715000 |  |  |  |
| 100 |  | 0.715000 |  |  |  |
| 101 | g11f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 102 |  | 0.450000 |  |  |  |
| 103 | g12d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 104 |  | 0.715000 |  |  |  |
| 105 |  | 0.715000 |  |  |  |
| 106 | g13d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 107 |  | 0.450000 |  |  |  |
| 108 | g14f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 109 |  | 0.715000 |  |  |  |
| 110 |  | 0.715000 |  |  |  |
| 111 | g15f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 112 |  | 0.450000 |  |  |  |
| 113 | g16d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 114 |  | 0.715000 |  |  |  |
| 115 |  | 0.715000 |  |  |  |
| 116 | g17d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 117 |  | 0.450000 |  |  |  |
| 118 | g18f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 119 |  | 0.715000 |  |  |  |
| 120 |  | 0.715000 |  |  |  |
| 121 | g19f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 122 |  | 0.450000 |  |  |  |
| 123 | g20d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 124 |  | 0.715000 |  |  |  |
| 125 |  | 0.715000 |  |  |  |
| 126 | g21d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 127 |  | 0.450000 |  |  |  |
| 128 | g22f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 129 |  | 0.715000 |  |  |  |
| 130 |  | 0.715000 |  |  |  |
| 131 | g23f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 132 |  | 0.450000 |  |  |  |
| 133 | g24d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 134 |  | 0.715000 |  |  |  |


| Pos Name | length <br> [m] | bend <br> [mrad] | $\begin{gathered} \rho \\ {[\mathrm{m}]} \end{gathered}$ | $\begin{gathered} \mathrm{B}^{\prime} /(\mathrm{B} \rho) \\ {[\mathrm{m}]^{-2}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 135 | 0.715000 |  |  |  |
| 136 g25d | 3.657600 | 47.419 | 77.1392391 | -0.0380886 |
| 137 | 0.450000 |  |  |  |
| 138 g26f | 3.657600 | 47.419 | 77.1392391 | 0.0380886 |
| 139 | 0.517663 |  |  |  |
| 140 qmil | 0.680000 |  |  | 0.018017221 |
| 141 | 1.588065 |  |  |  |
| 142 bmi | 2.946400 | 48.507 | 60.7471175 |  |
| 143 | 1.314723 |  |  |  |
| 144 qmi2 | 0.680000 |  |  | 0.15645349 |
| 145 | 1.158010 |  |  |  |
| 146 bmi | 2.946400 | 48.507 | 60.7471175 |  |
| 147 | 0.639683 |  |  |  |
| 148 qmi3 | 0.680000 |  |  | -0.28441136 |
| 149 | 1.533050 |  |  |  |
| 150 bmi | 2.946400 | 48.507 | 60.7471175 |  |
| 151 | 0.582034 |  |  |  |
| 152 qmi4 | 0.680000 |  |  | 0.27595845 |
| 153 | 0.890697 |  |  |  |
| 154 bmi | 2.946400 | 48.507 | 60.7471175 |  |
| 155 | 3.061489 |  |  |  |
| 156 qmi5 | 0.680000 |  |  | -0.092385216 |
| 157 | 1.399065 |  |  |  |
| 158 bmi | 2.946400 | 48.507 | 60.7471175 |  |
| 159 | 7.563012 |  |  |  |
| 160 qmi6 | 0.680000 |  |  | 0.15006942 |
| 161 | 1.043211 |  |  |  |
| 162 sio | 4.088028 | 50.407 | 81.0309163 |  |
| 163 | 1.337782 |  |  |  |
| 164 qmi7 | 0.680000 |  |  | -0.19498627 |
| 165 | 1.000000 |  |  |  |
| 166 si1 | 3.500000 | 43.193 | 81.0309163 |  |
| 167 | 1.000000 |  |  |  |
| 168 q8o | 0.617102 | -1.545 |  | 0.07867868 |
| 169 q8o | 0.617102 | -1.545 |  | 0.07867868 |
| 170 | 1.000000 |  |  |  |
| 171 si2 | 1.400000 | -2.800 | 500. |  |
| 172 | 14.362796 |  |  |  |
| 173 ki | 3.250000 | -1.324 | 2454.5 |  |
| 174 | 0.500000 |  |  |  |
| 175 q9o | 0.411734 |  |  | -0.056282546 |



Figure I


Fig. 2a Cross-section of gradient magnet for BIG BENDS.


Fig. 2b Photograph of gradient magnet for BIG BENDS.


RHIC INJECTIDN
Fig. 3a.


Fig. 3b

Fig. 3a,b Layout of east injection area, plan view and elevation.


FIG 4 a


FIG $4 b$


Fig. 4a,b,c Details of admittance profiles for incident and circulating beams in east injection area.

Horizontal scale: 1m between large divisions, vertical scale: 1 cm between divisions. Each diagram shows the exit face of ringmagnet BS2, the entrance and exit faces of ringnagnets Q70 and Q8O and the entrance face of ringmagnet Q90. Also shown are the entrance and exit faces of the injection line magnets SIO, QMI7, SI1, SI2 and injection kicker KI. The curves labeled '1' represent the boundaries of the $3.2 \pi$ mm-mrad for the incident beam, ' 2 ' represents the $6 \pi$ mm-mrad admittance for the circulating beam, '3' represents the admittance for the circulating beam temporarily reduced to $3.2 \pi \mathrm{~mm}-\mathrm{mrad}$ during the filling period. '4' represents the inner surface of the nominal beam pipe. Fig. $4 a$ shows an elevation, 4 b a plan view and 4 c an elevation at a larger vertical scale.


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[^1]:    ${ }^{1}$ An alternative is to fill only odd numbered buckets in the AGS. However, this is of little help if more than 57 buckets are to be filled in RHIC.

[^2]:    2 and because the deflection angles in the magnets of the transfer line change.

[^3]:    1 R.E. Thern: Measurement of Twiss Parameters and Emittance in the U Line. AGS Studies Report 193, October 12, 1985

