



BNL-101954-2014-TECH

AD/RHIC/42;BNL-101954-2013-IR

# Definitions of Terms Used in Intrabeam Scattering Computation and Tracking Studies

G. F. Dell

August 1988

Collider Accelerator Department  
**Brookhaven National Laboratory**

**U.S. Department of Energy**

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Accelerator Development Department  
BROOKHAVEN NATIONAL LABORATORY  
Associated Universities, Inc.  
Upton, New York 11973

AD/RHIC-42

RHIC TECHNICAL NOTE No. 42

Definitions of terms used in intrabeam scattering computation  
and tracking studies

F. Dell, H. Hahn and G. Parzen

August 4, 1988

*BNL*

Definitions of terms used in intrabeam scattering computations  
and tracking studies

F. Dell, H. Hahn and G. Parzen

Emittances

Intrabeam scattering computations express the results for transverse beam motion in terms of a single parameter, provided that full coupling is assumed. This parameter, the rms horizontal or vertical emittance  $\langle \epsilon \rangle$ , defines the Gaussian density distribution in 4-dimensional phase space  $(x, x', y, y')$  to be

$$\rho = \frac{N}{\pi^2 \langle \epsilon \rangle^2} \exp \left[ - \frac{x^2/\beta_H + x'^2 \beta_H + y^2/\beta_V + y'^2 \beta_V}{\langle \epsilon \rangle} \right]$$

with N the number of particles per bunch in the unlimited 4D phase space.

The customary BNL definition of the horizontal and vertical emittance of the beam (the so-called 95% emittance) is related to  $\langle \epsilon \rangle$  by

$$\epsilon_H = \epsilon_V = 3 \langle \epsilon \rangle$$

The rms betatron amplitude at QF/QD are given by

$$\sigma_{H,V} = \left[ \frac{\epsilon_{H,V}}{6} \beta_{H,V} \right]^{1/2}$$

Note the missing  $\pi$  in the emittance definition used here.

Stability limit and acceptance

In tracking studies, the stability of motion is usually tested for particles starting at QF/QD in the center of an arc with the initial conditions

$$x_o; \quad y_o = x_o \sqrt{\beta_V/\beta_H}; \quad x'_o = 0; \quad y'_o = 0$$

and the stability limit  $x_{SL}$  (used in Parzen's publications) is defined as the maximum stable  $x_o$ . It is fully equivalent to quote a machine acceptance (used in Dell's publications) which is related to the stability limit by

$$A = 2 x_{SL}^2 / \beta_H$$

Note that the factor  $\pi$  is again suppressed. Particles are expected to be stable within the 4-dimensional sphere given by

$$x^2/\beta_H + x'^2 \beta_H + y^2/\beta_V + y'^2 \beta_V = A$$

The dynamic aperture requirements (i.e., the stability limit or acceptance) for the magnets were based on the  $6\sigma$  rule which requires stable horizontal motion of particles with initial conditions

$$x_o > 6\sigma_H = (6 \varepsilon_H \beta_H)^{1/2}; \quad y_o = x'_o = y'_o = 0.$$

Because of coupling, the required stability limit as defined above becomes

$$x_{SL} > \frac{6}{\sqrt{2}} \sigma_H = (3 \varepsilon_H \beta_H)^{1/2}$$

It is also equivalent to require that the acceptance be

$$A > 6 \varepsilon_H = 18 \langle \varepsilon \rangle.$$

The fraction of particles contained within the stability limit/acceptance is given by

$$N_{SL}/N = 1 - \left[ 1 + \frac{A}{2\langle \varepsilon \rangle} \right] \exp \left[ - \frac{A}{2\langle \varepsilon \rangle} \right]$$

which assures that 99.9% of the particles will be stable.

#### References

- G. Parzen, RHIC-AP-55 (1988), RHIC-AP-58 (1988).  
H. Hahn, RHIC-AP-59 (1988).

Copies: 70

RHIC-##

RHIC-##  
(RHIC Tech Note)

---

460	N. P. Samios	911A	A. Stevens W. Weng
510A	*S. Aronson P. Bond M. Creutz *K. Foley H. Gordon *S. Kahana T. Kycia L. Leipuner S. Lindenbaum *F. Paige R. R. Rau H. Satz (see off-site)	911B	D. Barton G. Bunce *H. Foelsche P. Hughes (2 sets) *Y. Y. Lee D. Lowenstein J. Sanford
510F	R. Palmer	911C	AD Library (H. Martin - 2) E. Raka
535B	*V. Radeka	477B	(B. Orlowski - formal/informal)
555A	*L. Remsberg	1005	Post RHIC Tech Note File: 3
725B	NSLS Library		
725C	+H. Halama	Off-site: *for all below	
901A	*P. Thieberger *O. Hansen	H. Gutbrod L. McLerran H. Satz L. Schroeder W. Willis G. Young	GSI/CERN FNAL Bielefeld LBL (DOE) CERN ORNL
902A	+G. Cottingham +A. Greene P. Wanderer E. Willen		
		*Member of RHIC Task Force	(Ludlam)
		+Member of RHIC Machine Studies	(Hahn)
902B	+C. Goodzeit +G. Morgan *+R. Shutt +P. Thompson A. Blake (Magnet Authors)		
1005	M. Rhodes-Brown +J. Claus +E. Courant +G. Dell E. Forsyth *+H. Hahn D. Brown A. Flood P. Campbell	S. Y. Lee T. Ludlam G. Parzen A. Ruggiero Z. Parsa	

Dr. Larry McLerran  
Theory Group MS 106  
Fermilab  
P.O. Box 500  
Batavia, IL 60510

Dr. G. R. Young  
ORNL Bldg. 6003  
P.O. Box X  
Oak Ridge, TN 37831-6373

Dr. H. Gutbrod  
Ges. für Schwerionenforschung  
Postfach 11 05 41  
D-6100 Darmstadt  
Federal Republic of Germany

Dr. H. Gutbrod  
CERN EP Division  
CH 1211 Geneva 23  
SWITZERLAND

Prof. Dr. H. Satz  
Fakultät F. Physik  
Universität Bielefeld  
D-48 Bielefeld  
FRG

Dr. W. Willis  
NP Division  
CERN  
CH 1211 Geneva 23  
Switzerland

Dr. Lee Schroeder  
U.S. Department of Energy  
ER-23 GTN  
Washington, D.C. 20545