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Definitions of Terms Used in Intrabeam Scattering Computation and Tracking Studies

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Emittances

Intrabeam scattering computations express the results for transverse beam motion in terms of a single parameter, provided that full coupling is assumed. This parameter, the rms horizontal or vertical emittance $\langle\varepsilon\rangle$, defines the Gaussian density distribution in 4-dimensional phase space (x, x', y, y') to be

$$\rho = \frac{N}{\pi^2 \langle\varepsilon\rangle^2} \exp\left[- \frac{x^2/\beta_H + x'^2 \beta_H + y^2/\beta_V + y'^2 \beta_V}{\langle\varepsilon\rangle}\right]$$

with N the number of particles per bunch in the unlimited 4D phase space.

The customary BNL definition of the horizontal and vertical emittance of the beam (the so-called 95% emittance) is related to $\langle\varepsilon\rangle$ by

$$\varepsilon_H = \varepsilon_V = 3 \langle\varepsilon\rangle$$

The rms betatron amplitude at QF/QD are given by

$$\sigma_{H,V} = \left[\frac{\varepsilon_{H,V}}{6} \beta_{H,V}\right]^{1/2}$$

Note the missing π in the emittance definition used here.

Stability limit and acceptance

In tracking studies, the stability of motion is usually tested for particles starting at QF/QD in the center of an arc with the initial conditions

$$x_o; \quad y_o = x_o \sqrt{\beta_V/\beta_H}\,; \quad x'_o = 0; \quad y'_o = 0$$

and the stability limit x_{SL} (used in Parzen's publications) is defined as the maximum stable x_o. It is fully equivalent to quote a machine acceptance (used in Dell's publications) which is related to the stability limit by

$$A = 2\, x_{SL}^2 / \beta_H$$

Note that the factor π is again suppressed. Particles are expected to be stable within the 4-dimensional sphere given by

$$x^2/\beta_H + x'^2\, \beta_H + y^2/\beta_V + y'^2\, \beta_V = A$$

The dynamic aperture requirements (i.e., the stability limit or acceptance) for the magnets were based on the 6σ rule which requires stable horizontal motion of particles with initial conditions

$$x_o > 6\sigma_H = (6\, \varepsilon_H\, \beta_H)^{1/2}; \quad y_o = x'_o = y'_o = 0.$$

Because of coupling, the required stability limit as defined above becomes

$$x_{SL} > \frac{6}{\sqrt{2}}\, \sigma_H = (3\, \varepsilon_H\, \beta_H)^{1/2}$$

It is also equivalent to require that the acceptance be

$$A > 6\, \varepsilon_H = 18 \langle\varepsilon\rangle.$$

The fraction of particles contained within the stability limit/acceptance is given by

$$N_{SL}/N = 1 - \left[1 + \frac{A}{2\langle\varepsilon\rangle}\right] \exp\left[-\frac{A}{2\langle\varepsilon\rangle}\right]$$

which assures that 99.9% of the particles will be stable.

References

G. Parzen, RHIC-AP-55 (1988), RHIC-AP-58 (1988).

H. Hahn, RHIC-AP-59 (1988).

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