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Simulation of Transition and Transfer

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USDOE Office of Science (SC)

DISCLAIMER

AD/RHIC-RD-5

Simulation of Transition and Transfer

(Mini-Workshop on RHIC RF Systems)

July 11-15, 1988
Collider Center

J. Wei
BNL

by J. Wei, 1988.7

SIMULATION OF TRANSITION & TRANSFER

I. Transition Energy Crossing

- $\hat{V} = 100$ kV crossing transition
- Scheme of $\dot{\gamma}$ increase or γ_t jump

S. Y. Lee, A. G. Ruggiero
J. Claus

II. Transfer to High Frequency R.F. System

- Top energy bunch rotation
- Bunch rotation with 0.3 eV·s/a bunch
- Switch over near transition

J. M. Brennan, E. Raka
S. Y. Lee ...

I. Transition Energy Crossing

- $\hat{V} = 1.2$ MV ?

  momentum spread too large, nonlinear

- $\hat{V} = 100$ kV crossing :

  much better,

  space charge + nonlinear effect. $\gamma_t\left(\frac{\Delta p}{p}\right)$

  ⇒ phase space area blow up

- γ_t jump, or $\dot{\gamma}$ increase ?

  Good, "clean" crossing

$$\begin{cases} \dot{w} = \dfrac{\gamma e \hat{V}}{2\pi} (\sin\phi - \sin\phi_s) + \Delta_{s.c.} + \Delta_Z \\ \dot{\phi} = \dfrac{h \Omega_0}{p_0 R_0} \cdot w \cdot \eta(w) \qquad \left(w = \dfrac{\Delta E}{\Omega_0}\right) \end{cases}$$

* Kinematic mismatching

$$\eta\left(\frac{\Delta p}{p}\right) = \eta_0 + \eta_1 \cdot \frac{\Delta p}{p} + \cdots$$

* Low freq. impedance

  space charge, inductive, capacitive

  ⇒ bunch mismatching

* High freq. impedance

  resistive, inductive, capacitive (s.c.)

  ⇒ microwave instability

$$\underline{\eta = \eta\left(\frac{\Delta p}{p_0}\right)}$$

$$\Omega R = \beta c \quad , \qquad \frac{\Delta\Omega}{\Omega_0} = \frac{\beta}{\beta_0}\frac{R_0}{R} - 1$$

$$\frac{R}{R_0} = 1 + \alpha_1 \frac{\Delta p}{p_0} + \alpha_2 \left(\frac{\Delta p}{p_0}\right)^2 + \cdots$$

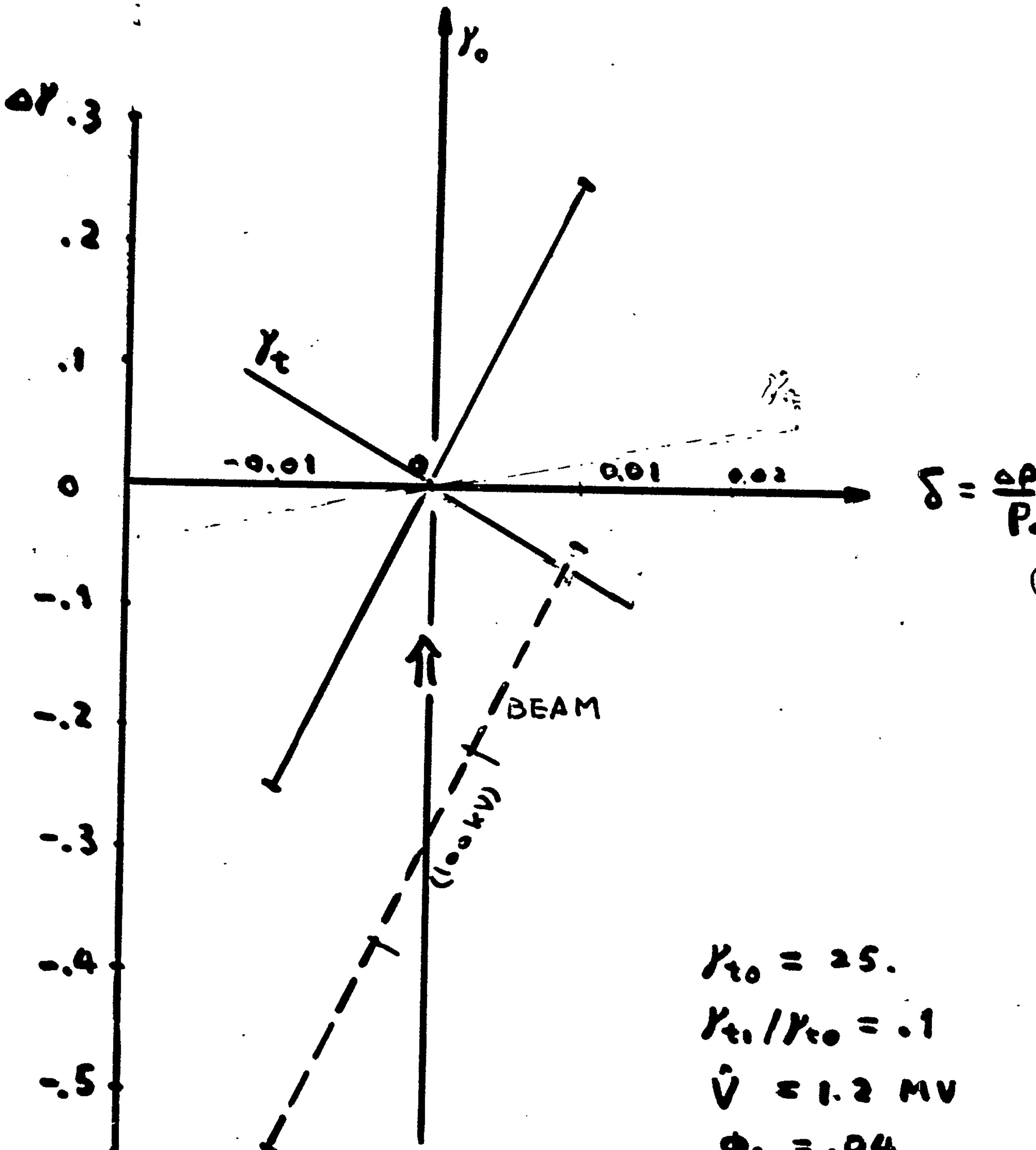
$$\frac{\beta}{\beta_0} = 1 + \frac{1}{\gamma_0^2}\cdot\frac{\Delta p}{p_0} - \frac{3\beta_0^2}{2\gamma_0^2}\cdot\left(\frac{\Delta p}{p}\right)^2 + \cdots$$

$$\underline{\eta = \frac{\Delta\Omega}{\Omega} \Big/ \frac{\Delta p}{p_0} \cong \eta_0 + \left[\frac{3}{2}\frac{\beta_0^2}{\gamma_0^2} - \frac{\gamma_{t1}}{\gamma_{t0}^2}\right]\cdot\frac{\Delta p}{p_0}}$$

$$\left(\frac{1}{\gamma_0^2\gamma_t^2} - \frac{1}{\gamma_t^4}\right)$$

$$\delta = \frac{\Delta\gamma}{\gamma} = \left[1 + \frac{1}{\beta_0^2\gamma_0^2}\right]\cdot\frac{\Delta p}{p_0}$$

Mismatch at transition (RHIC)



$\gamma_{t0} = 25.$

$\gamma_{t1}/\gamma_{t0} = .1$

$\hat{V} = 1.2$ MV

$\Phi_s = .04$

$\Delta\gamma = 2.07 \times 10^{-5}$/turn

$\dot{\gamma} = 1.62$/sec.

$\tau_0 = 12.79\ \mu s$

Estimate of δ growth.

$$\ddot{\delta} = \frac{qe\hat{V}}{2\pi p_0 R_0}\cos\varphi_0 \, h\Omega_0 \left(\frac{2\dot{\gamma} t}{\gamma_T^{o3}} + \eta_1 \delta \right) \delta$$

$$i\int_{t_c}^{0} \Omega_s(t)\, dt = \left(\frac{h\Omega\, qe\hat{V}\cos\varphi_0}{2\pi p_0 R_0} \right)^{1/2} \int_{t_c}^{0} \left(\frac{2\dot{\gamma} t}{\gamma_T^{o3}} + \eta_1 \delta \right)^{1/2} dt$$

$$\simeq \frac{\Omega}{3\dot{\gamma}} \left(\frac{h\, qe\hat{V}\cos\varphi_0}{2\pi p_0 \beta c} \right)^{1/2} \left(\frac{3}{2} - \lambda \right)^{3/2} \hat{\delta}^{3/2}$$

	AGS	RHIC
Growth factor:	$e^{295\,\hat{\delta}^{3/2}} \simeq 1.1$	$e^{5200\,\hat{\delta}^{3/2}} \simeq 6.3$

$$\delta \propto \delta_0 \boxed{e^{(h\hat{V}\cos\varphi_0)^{1/2}\,\hat{\delta}^{3/2}}}$$

Relevant time scales:

- τ_0 : revolution period
- $\tau_{syn.}$: synchrotron osc. period
- T_c : characteristic non-adiabatic time $\quad \frac{2}{\Omega_s^2} \cdot \left| \frac{d\Omega_s}{dt} \right| > 1$

  $$T_c = \left[\frac{A m_0 c^2}{R_0^2 h} \cdot \frac{\gamma_t^4}{2\dot{\gamma}} \cdot \frac{2\pi}{Z e \hat{V} |\cos\phi_s|} \right]^{1/3}$$

- $T_{n.l.}$: non-linear time

  $$T_{n.l.} = \frac{\Delta \gamma_{n.l.}}{\dot{\gamma}} = \frac{3\beta_s^2 - \alpha_1}{4\dot{\gamma}} \cdot \left(\frac{\Delta p}{p}\right)_{2.5\sigma}$$

  $$\eta(\delta) = \eta_0 + \eta_1 \delta + \cdots$$

  time when $|\eta_1 \delta| > |\eta_0|$ & of different sign

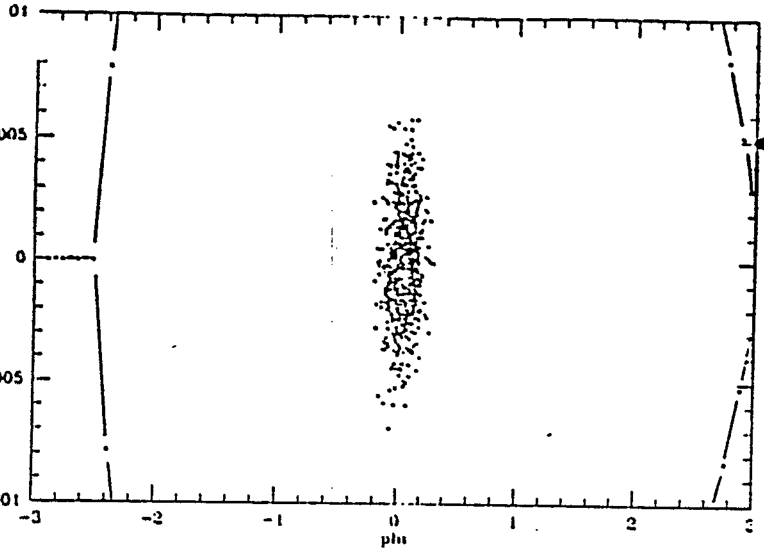
- $T_{m.w.}$: microwave instability growth time.

$\hat{V} = 1.2$ MV

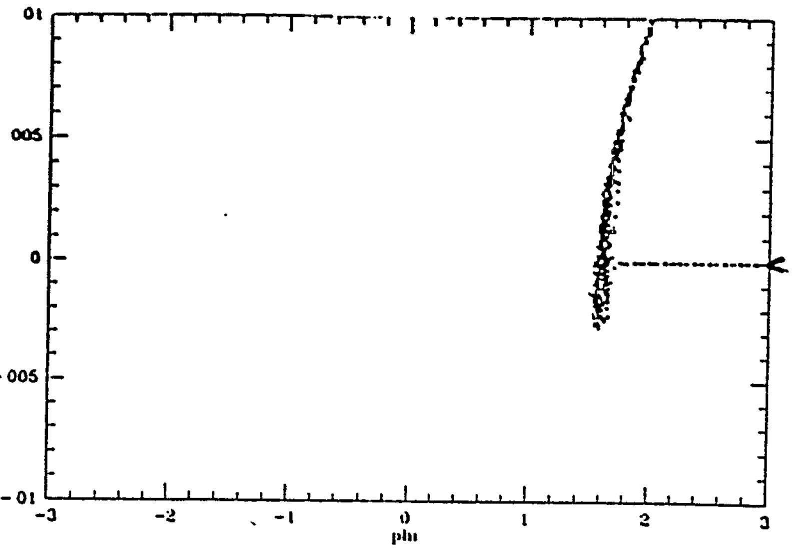
$\sin\phi_s = 0.04$

γ_t CROSSING

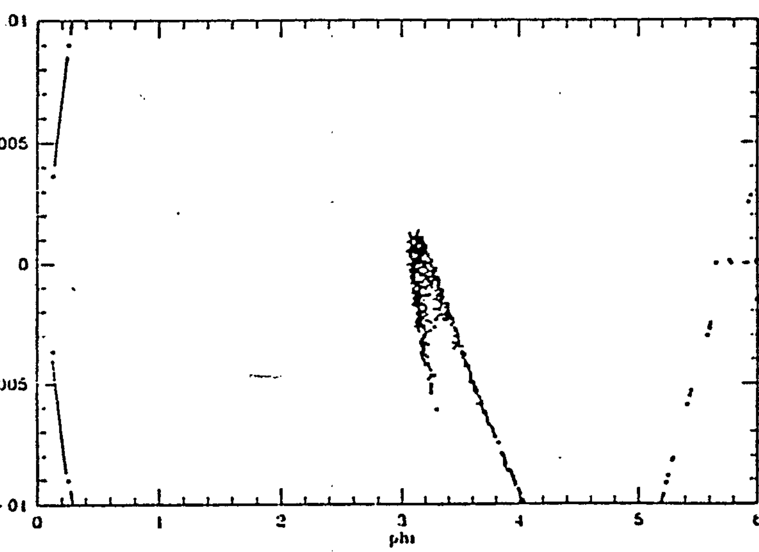
Au^{+79}



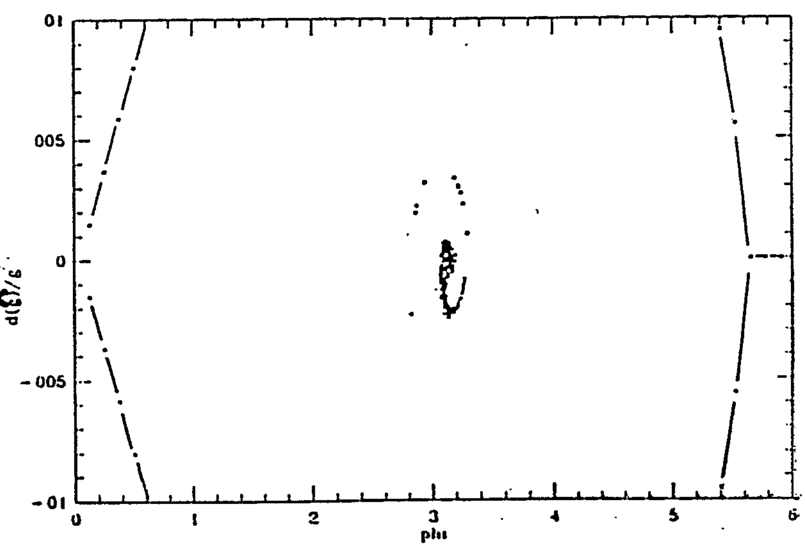
−80 ms



0 ms



+25 ms



+180 ms

$\eta(\frac{\Delta p}{p})$, mismatching ⇒ 70% Loss

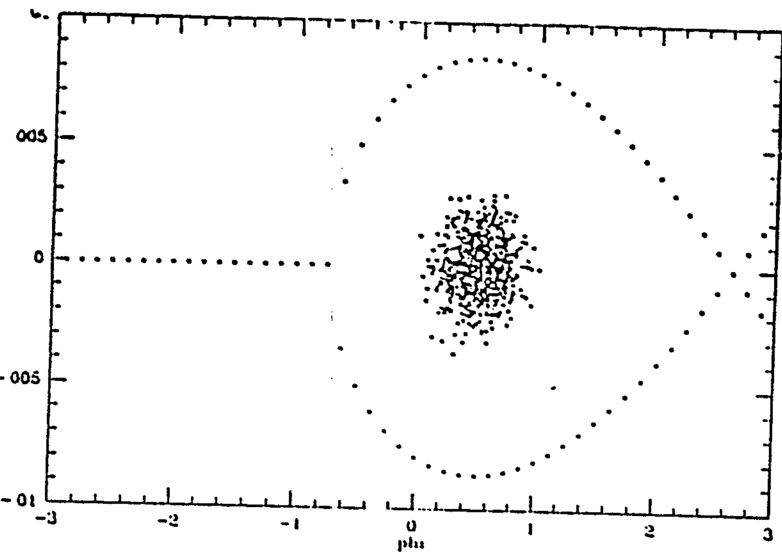
$\tau_0 \ll T_c < T_{n.l.} < \tau_{syn.}$

$\hat{V} = 100$ kV

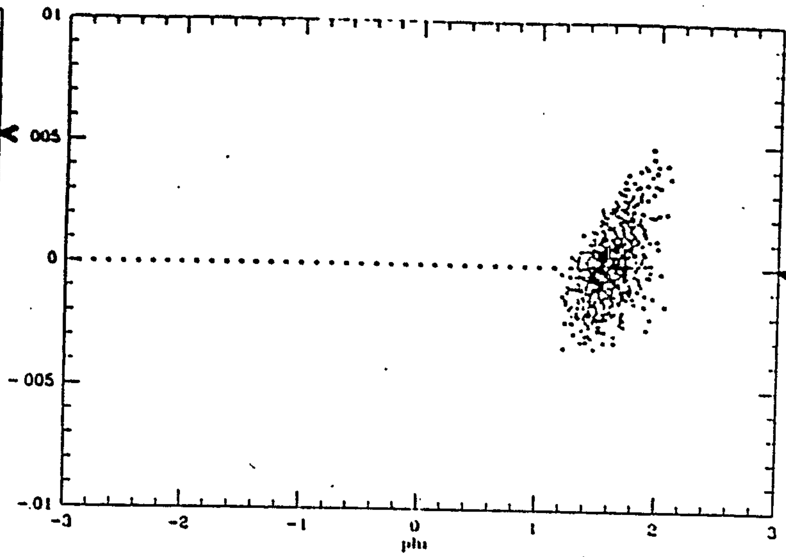
$\sin\phi_s = 0.48$

γ_t CROSSING

Au^{+79}

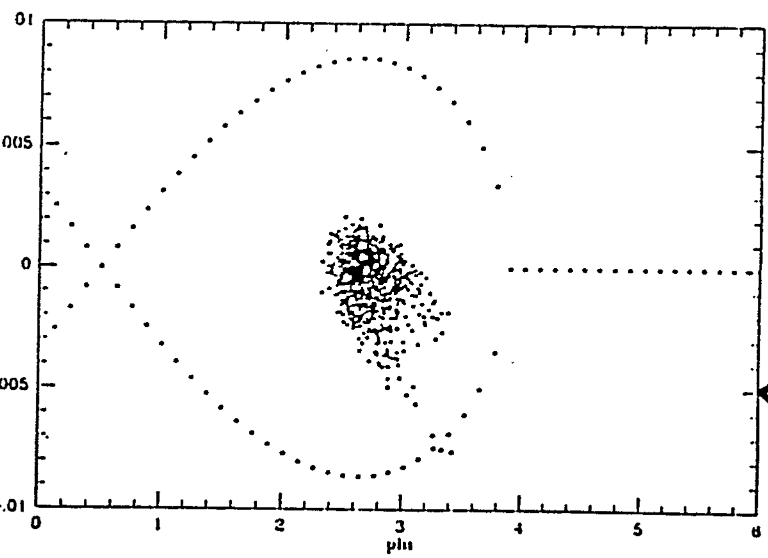


−80 ms

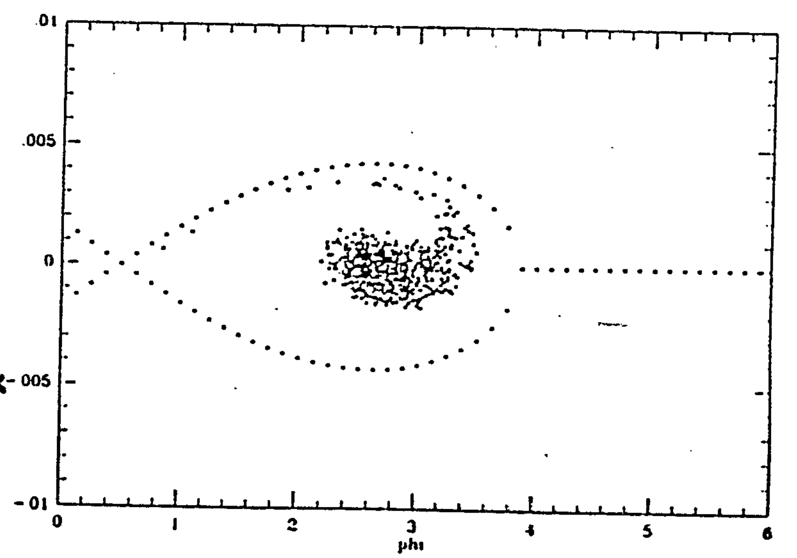


0 ms

0.3 eV·s/amu, w/o s.c.



+80 ms



+230 ms

$\eta\left(\frac{\Delta p}{p}\right)$ mismatching $\Rightarrow$ 0.9% Loss

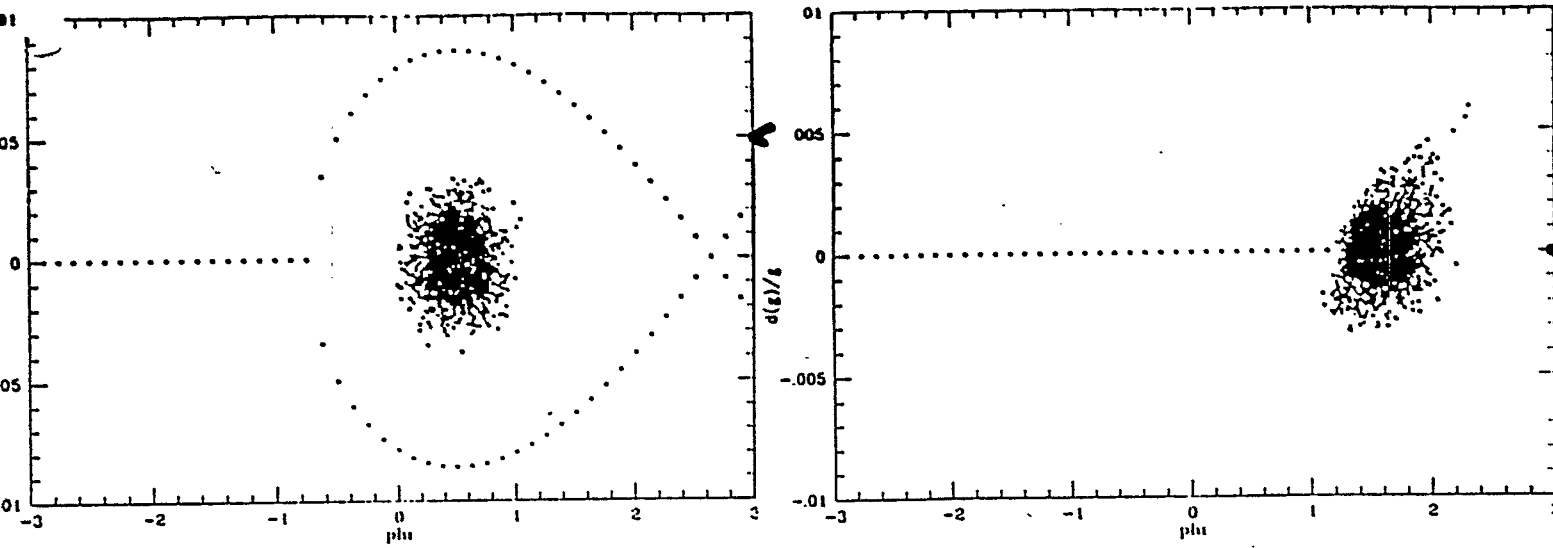
$\tau_0 \ll T_{n.l.} < T_c < \tau_{syn.}$

$\hat{V}$ = 100 kV

$\sin \phi_s = 0.48$

Au

γ_t CROSSING. space charge

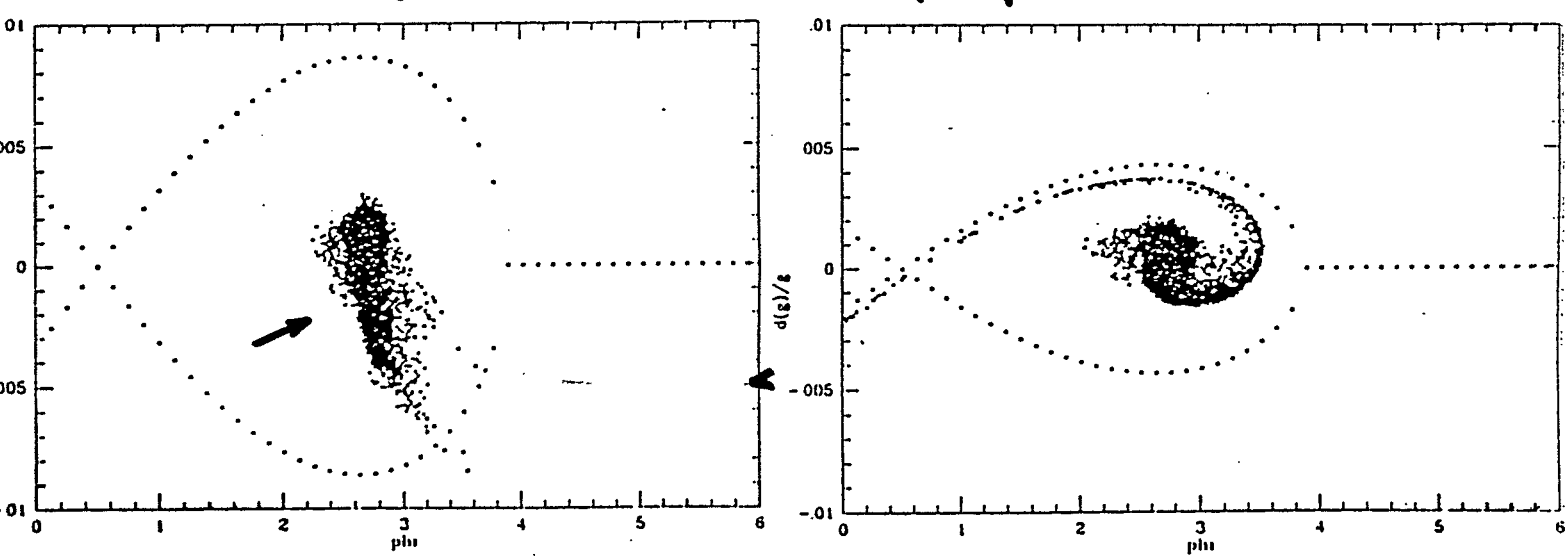


−80 ms   0 ms

0.3 eV·s/amu, with s.c.

$\frac{Z}{n}$ = 1.2 Ω, 1.1×10^9 /bunch

l_{bin} = 2b. ⇒ microwave freq.



+80 ms   +230 ms

mismatching + spacecharge ⇒ 2.1% Loss

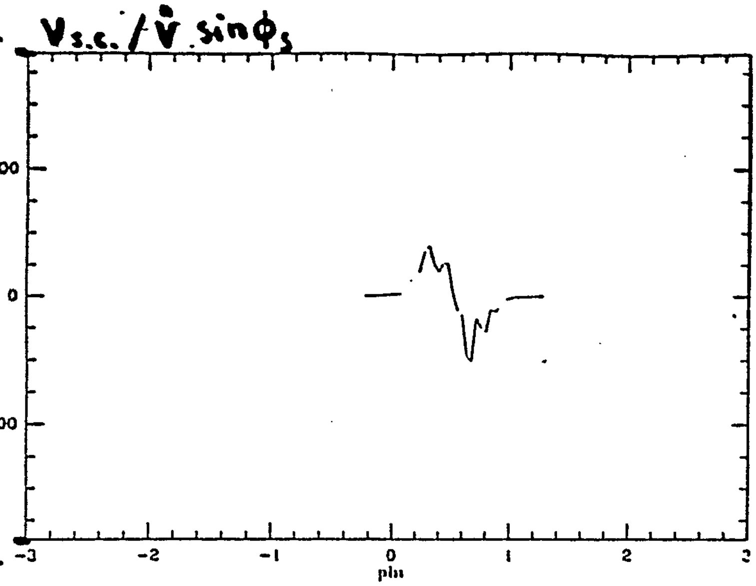
$\tau_0 \ll T_{n.l.} < T_c < \tau_{syn.}$

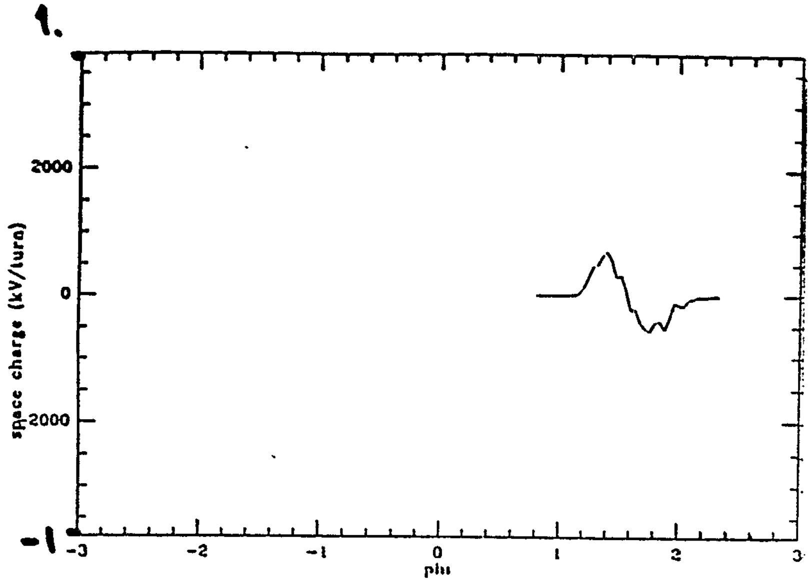
~1 eV·sec/amu

$\hat{V} = 100\,kV$

$\sin\phi_s = 0.48$

Space charge voltage

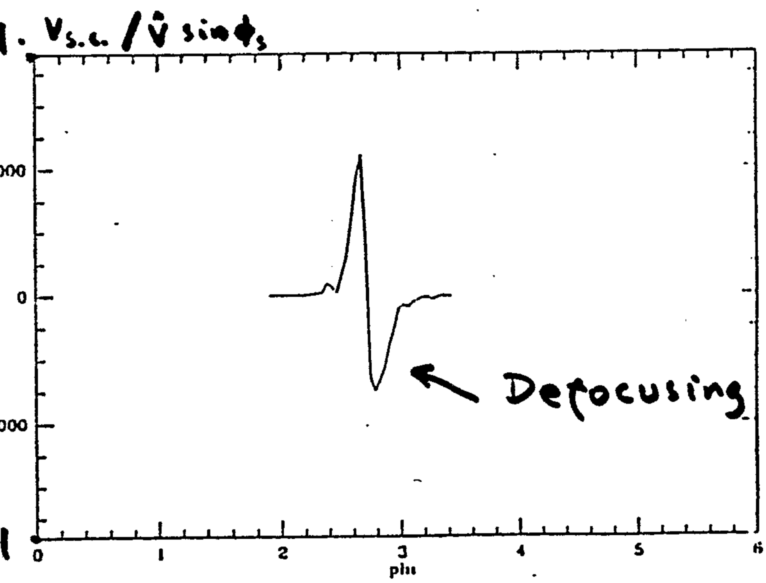


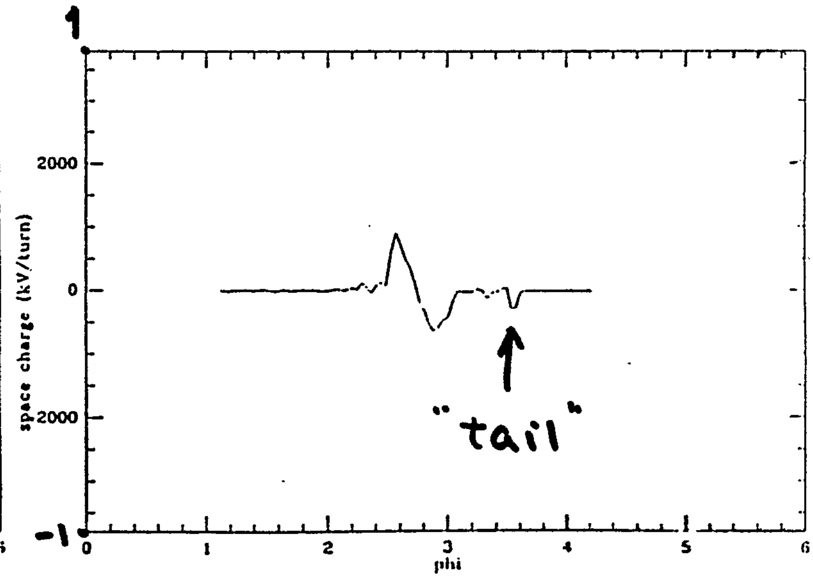


−80 ms

0 ms

$\frac{Z}{n} = 1.2\ \Omega$, Capacitive

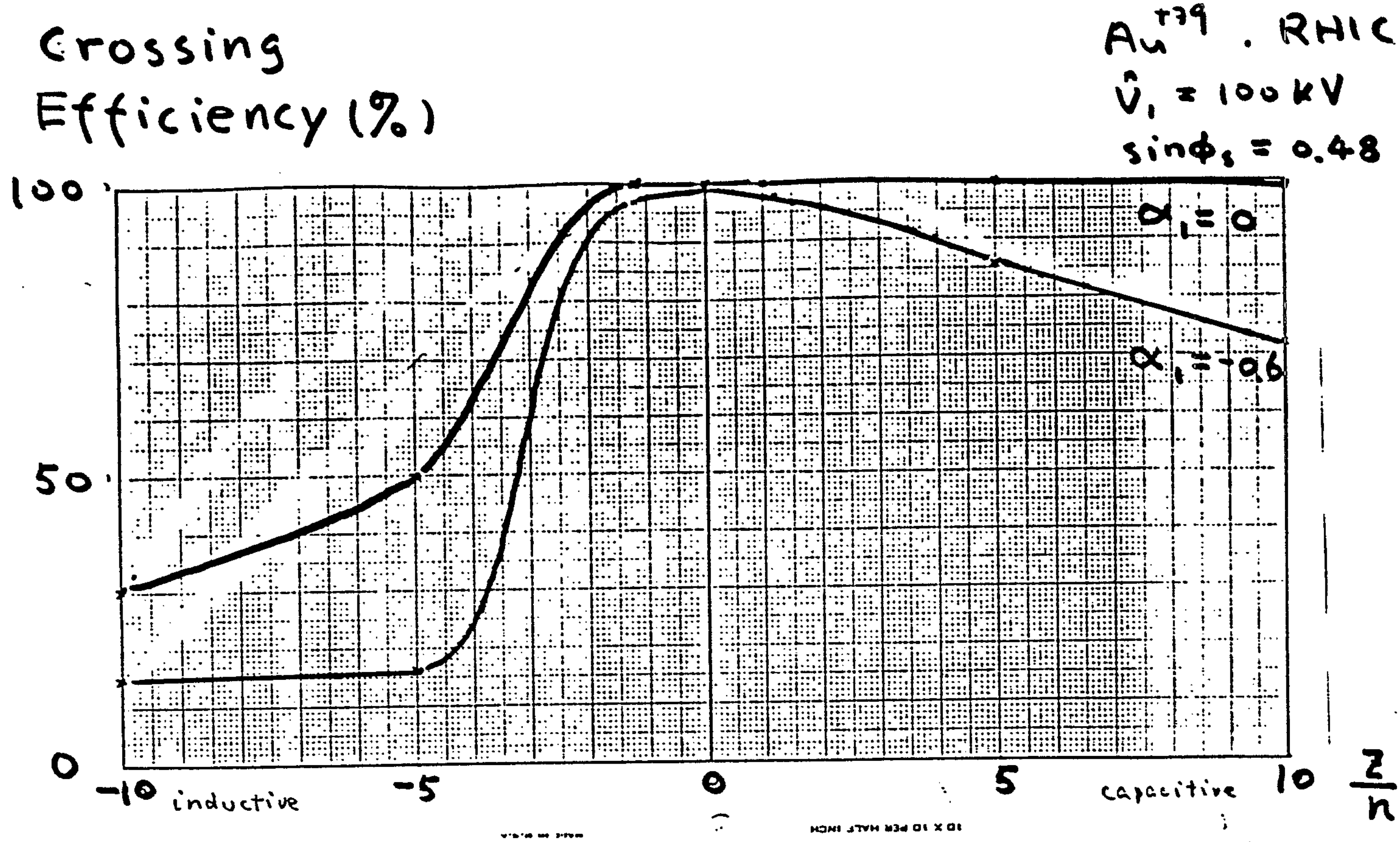




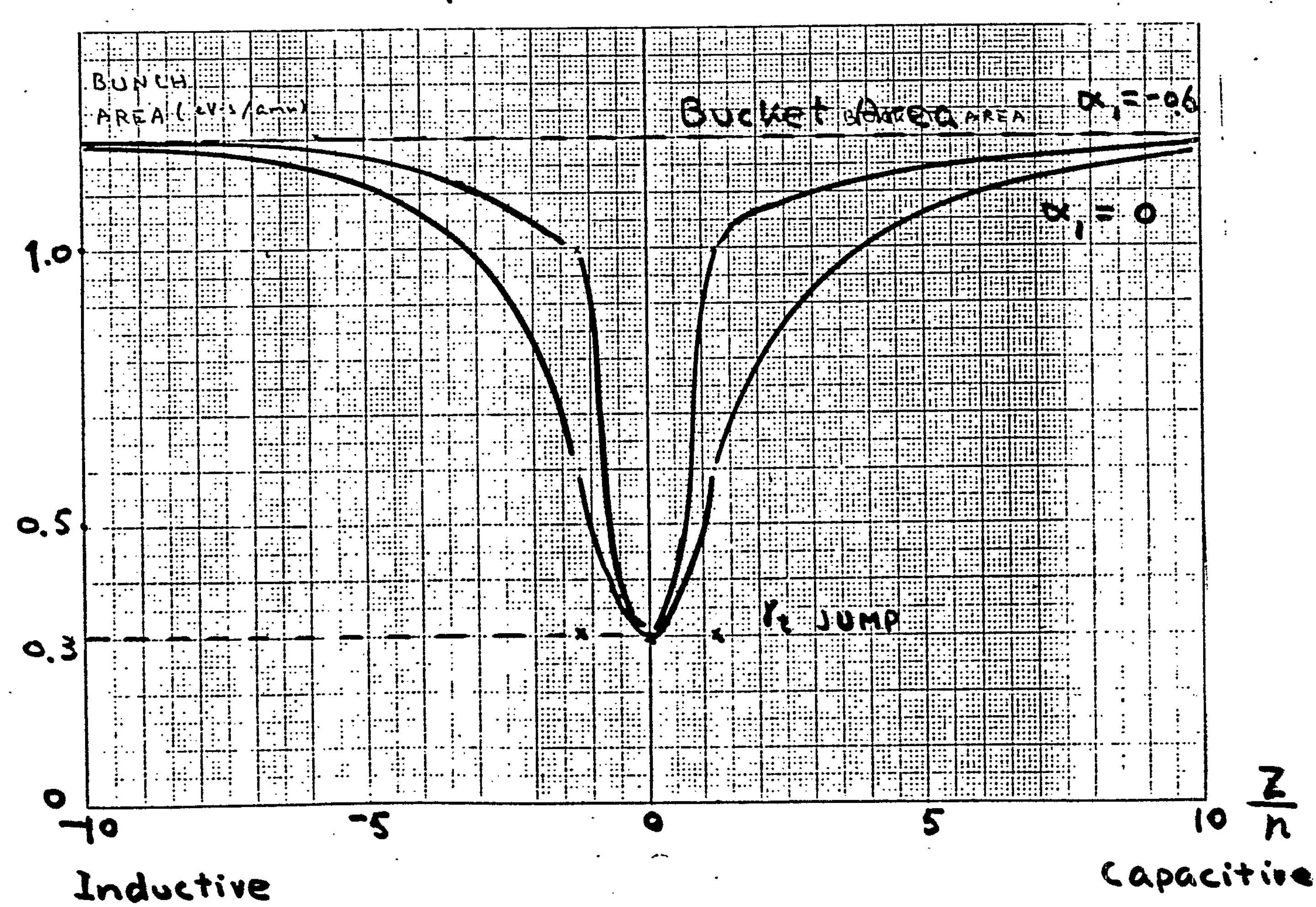
+80 ms

+230 ms

Defocusing space charge force



Bunch Area After Transition



S.Y. Lee & J.M. Wang

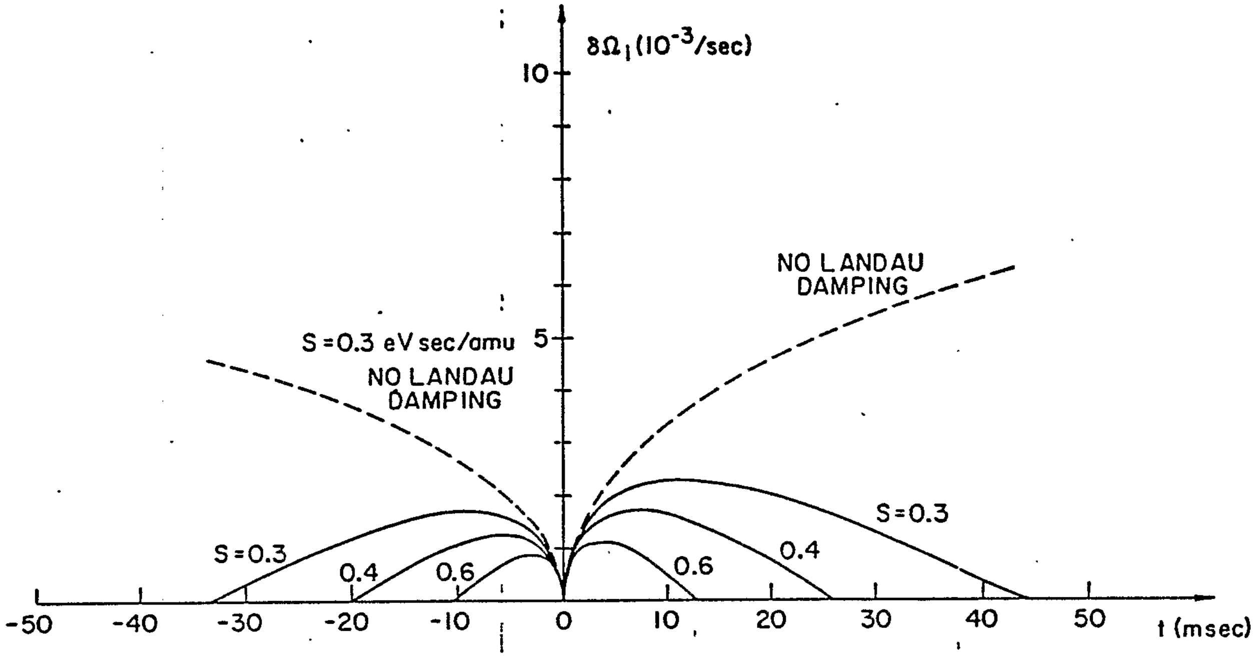


Fig. IV-19. The imaginary part of the microwave frequency, $\delta\Omega_i$, representing the growth rate of the instability, is plotted as function of time during crossing of the transition energy. The curves are calculated with the initial phase space as parameter. The dashed curve represents the growth rate without Landau dumping.

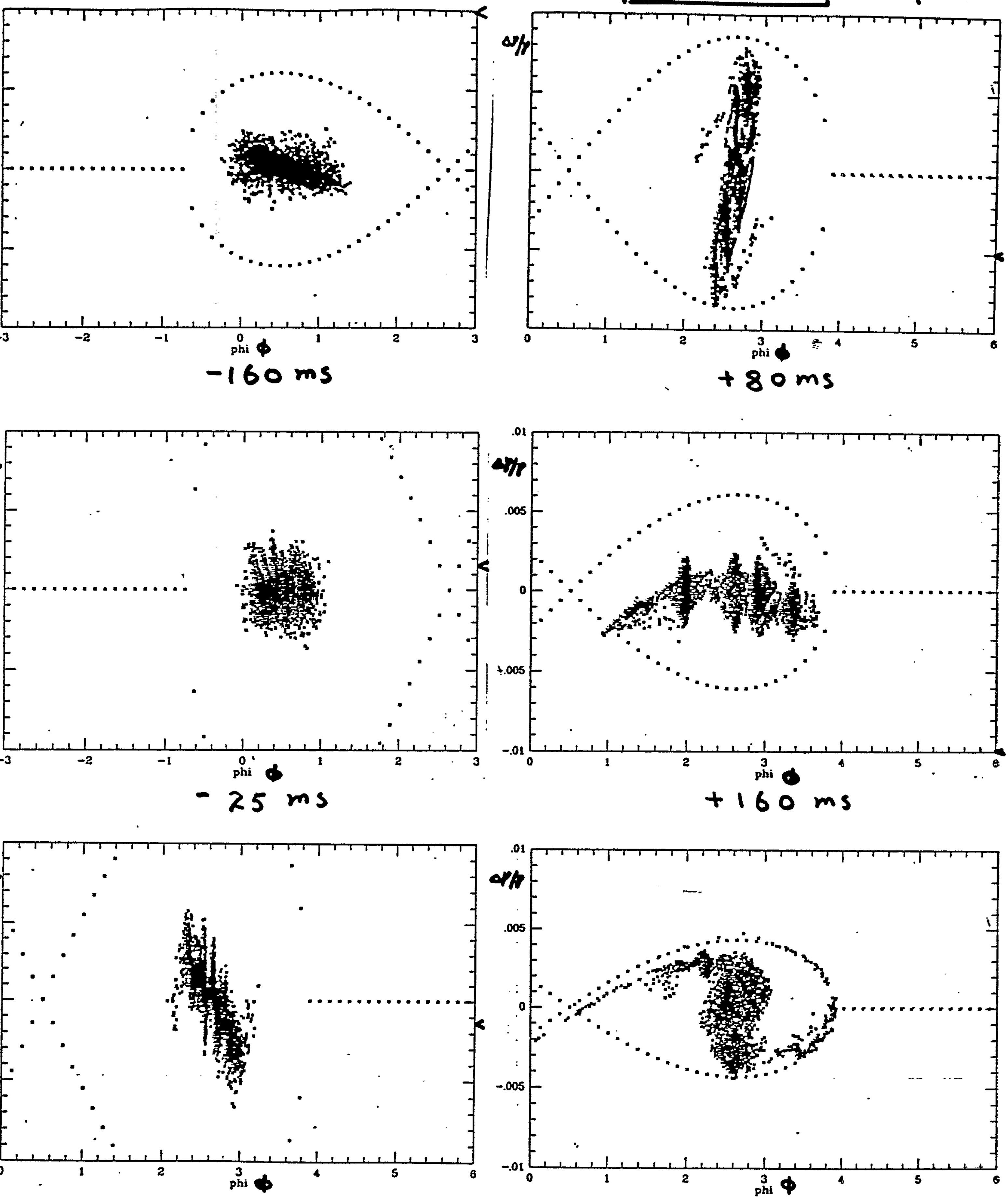
* Analytical solution based on perturbation theory

$\hat{V} = 100$ kV $\sin\phi_s = 0.48$

γ_t crossing

$Z/n = 10\ \Omega$ capacitive

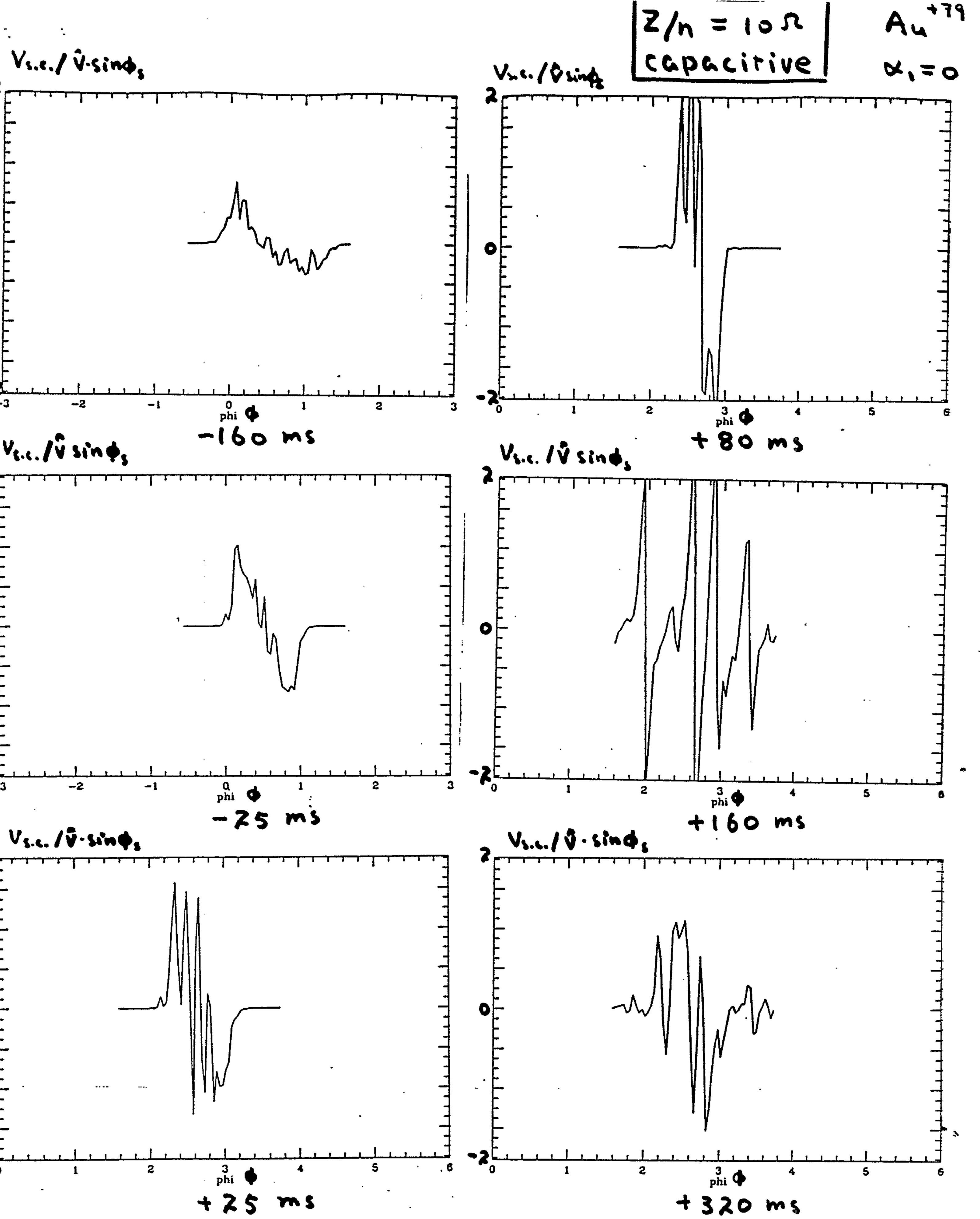
Au^{+79} $\alpha_1 = 0$



−160 ms +80 ms

−25 ms +160 ms

+25 ms Efficiency 99% +320 ms



Z/n = 10 Ω capacitive
Au+79
α1 = 0
Vs.c./V̂·sinφs
-3
-2
-1
0
1
2
3
phi φ
-160 ms
Vs.c./V̂ sinφs
2
0
-2
0
1
2
3
4
5
6
phi φ
+80 ms
Vs.c./V̂ sinφs
-3
-2
-1
0
1
2
3
phi φ
-25 ms
Vs.c./V̂ sinφs
2
0
-2
0
1
2
3
4
5
6
phi φ
+160 ms
Vs.c./V̂·sinφs
0
1
2
3
4
5
6
phi φ
+25 ms
Vs.c./V̂·sinφs
2
0
-2
0
1
2
3
4
5
6
phi φ
+320 ms

To preserve the original 0.3 eV·s/amu
& to minimize loss

* γ_t jump, or
* increase $\dot{\gamma}$ near transition (without changing $\dot{B}$), or
* make total $\frac{Z}{n} \sim 0$

- Resistive wall impedance

very very small effect

$10^2 * \left|\frac{Z}{n}\right|_{\text{stainless steel}} \Rightarrow$ microwave instability

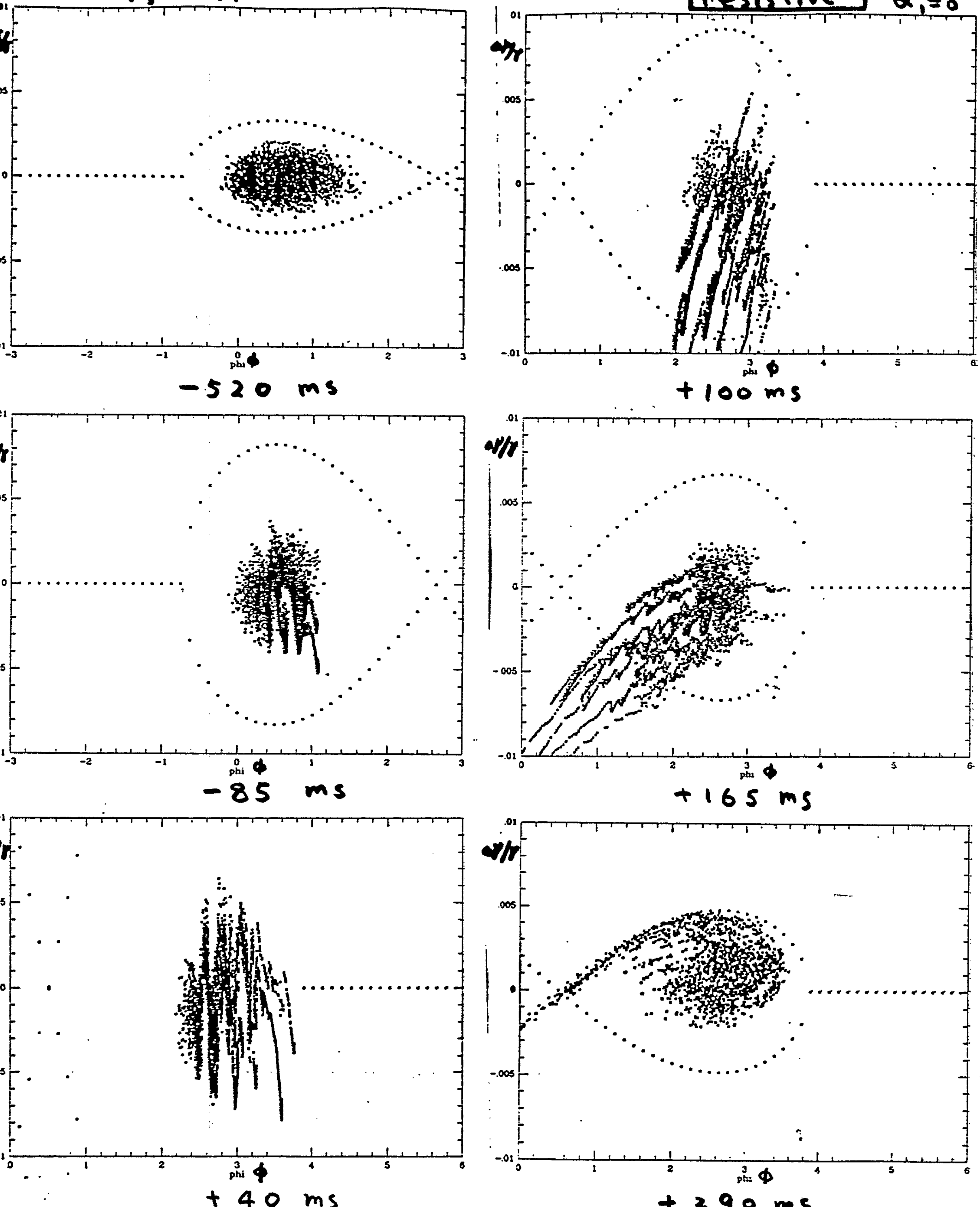
@ $\frac{Z}{n_c} \sim 5\ \Omega$, $n_c = \frac{R_0}{b}$

V = 100 kV
$\sin\phi_s = 0.48$

γ_t crossing

R = 10° Ω resistive

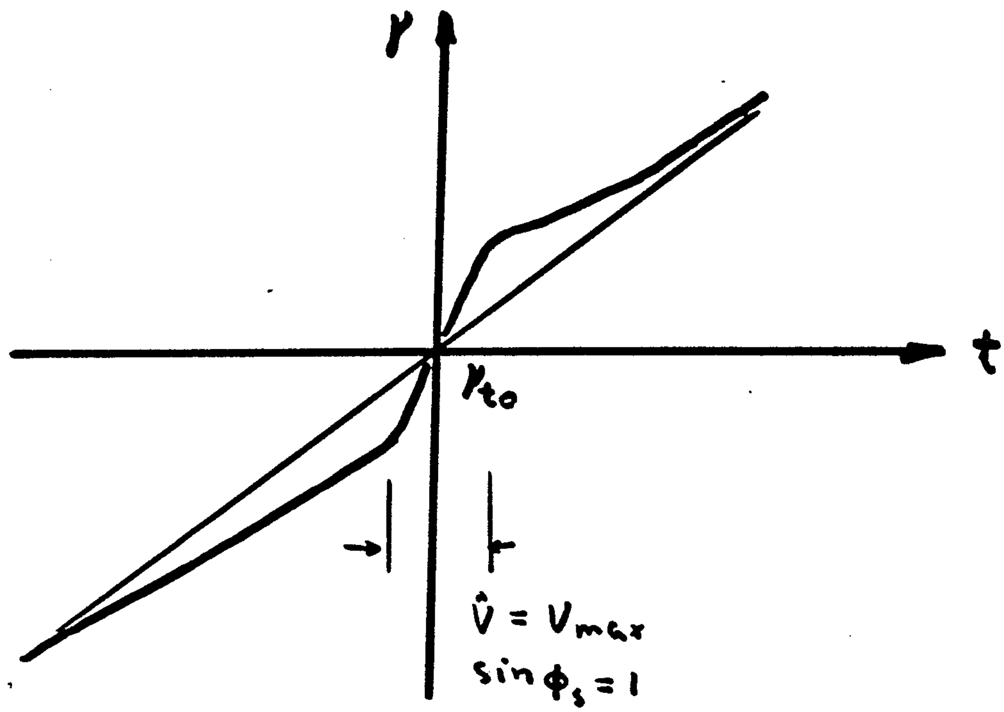
Au, $\alpha_1 = 0$

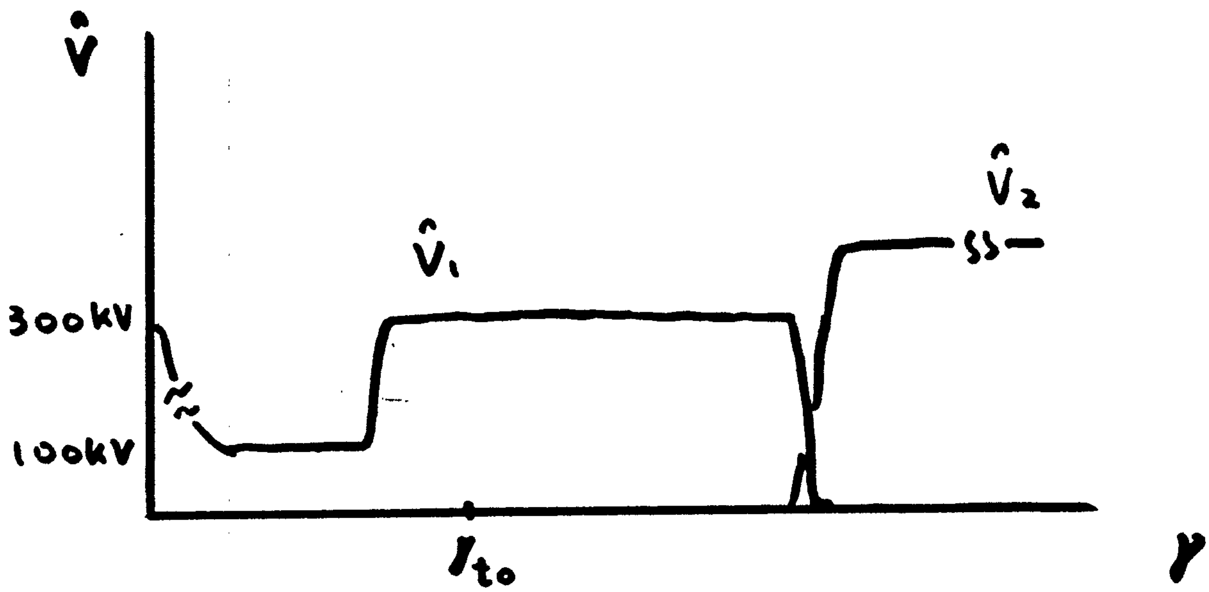


Efficiency 70%

7200 particles used.

$\dot{\gamma}$ increase crossing transition

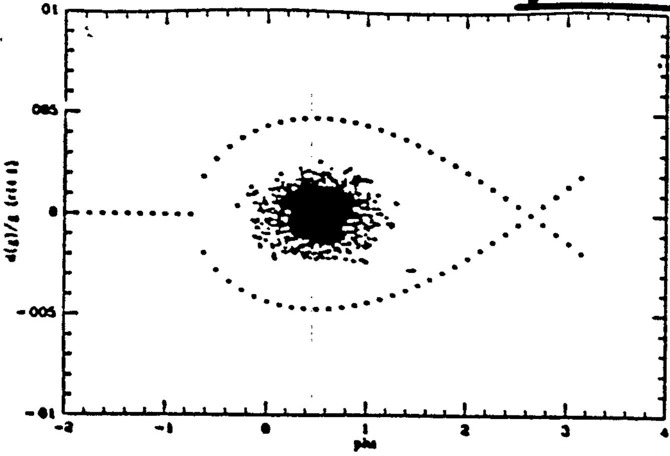




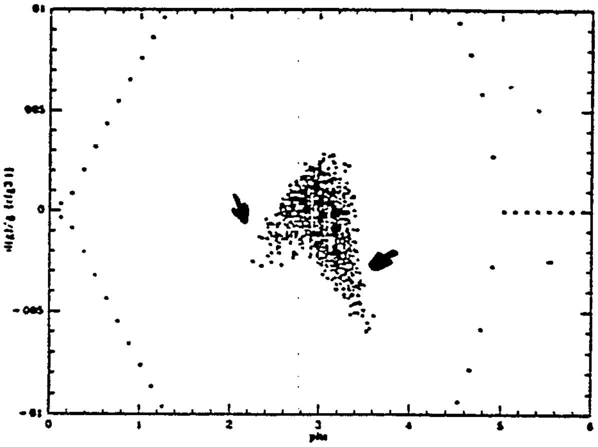
Total momentum aperture needed ±0.8%

$\Delta\gamma = 0.38$ in 40 ms. used in simul.

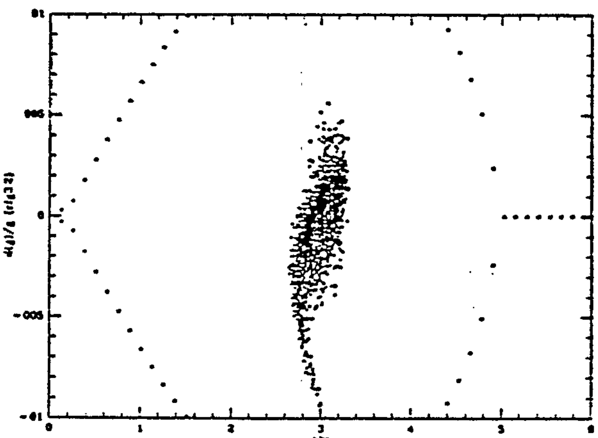
γ increase, 100kV → 300kV



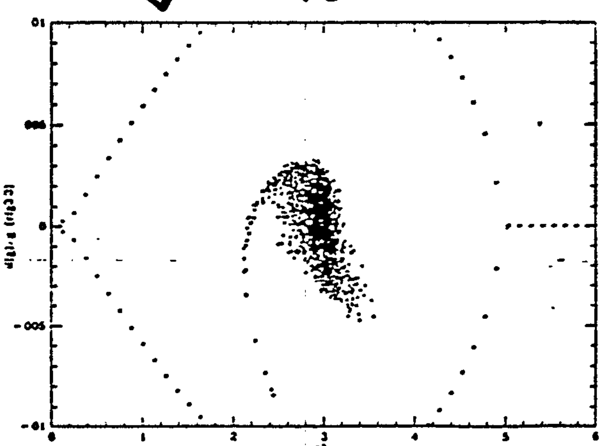
⇓ −150 ms



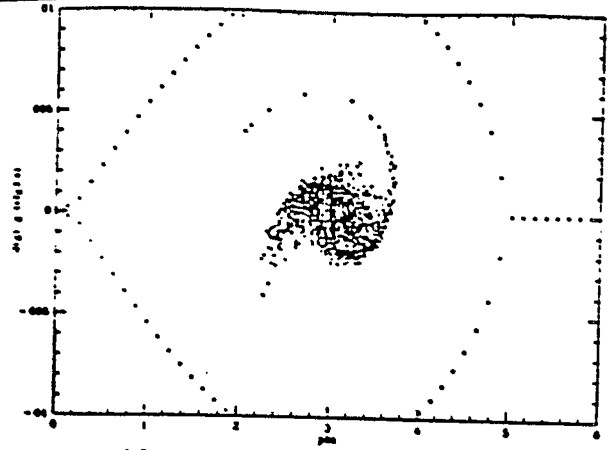
⇓ + 25 ms



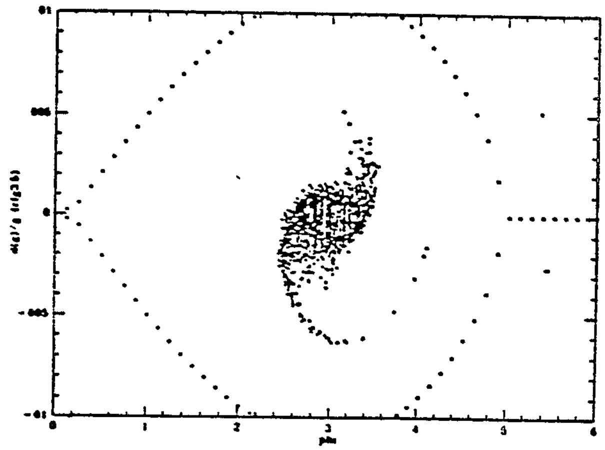
⇓ + 75 ms



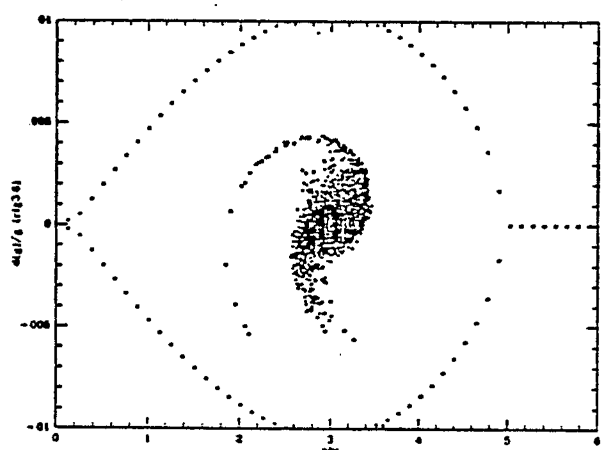
⇓ +125 ms



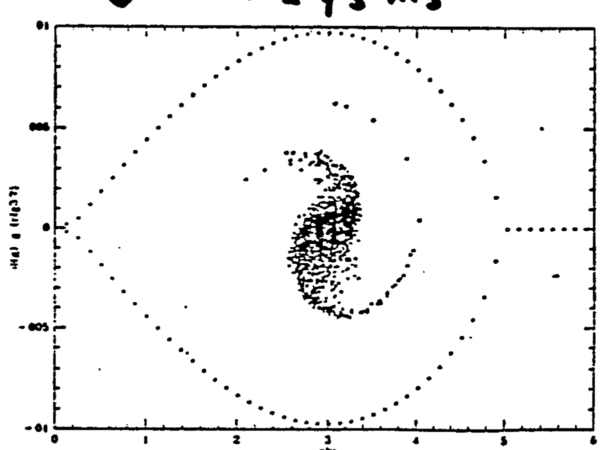
⇓ +175 ms



⇓ +225 ms



⇓ +275 ms



+325 ms

Au^{+79}

0.3 eV·s/a

$\dot{\gamma}$ increase

$\alpha_1 = -0.6$

$\frac{Z}{n} = 1.2\,\Omega$

capac.

Eff. 100%

0.3 ~ 0.4 eV·s/a

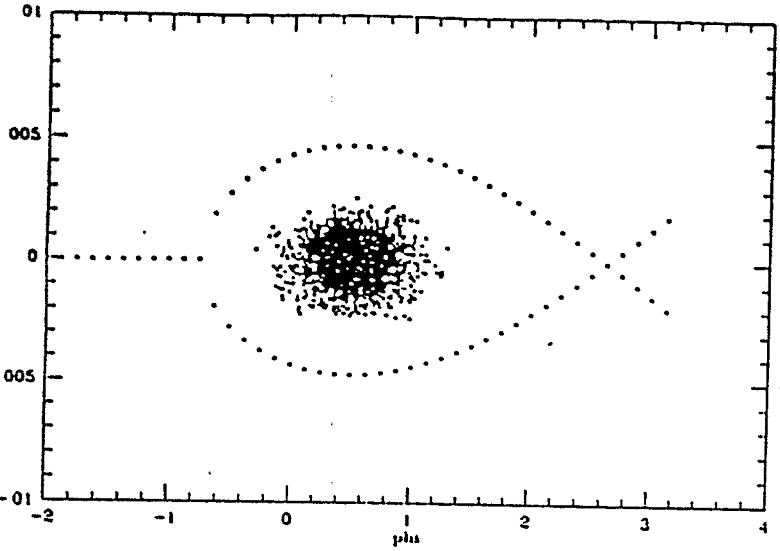
$\hat{V} = 100\,kV$

$\sin\phi_s = 0.48$

γ_t JUMP

$\Delta\gamma_t = 0.6$ in 60 ms

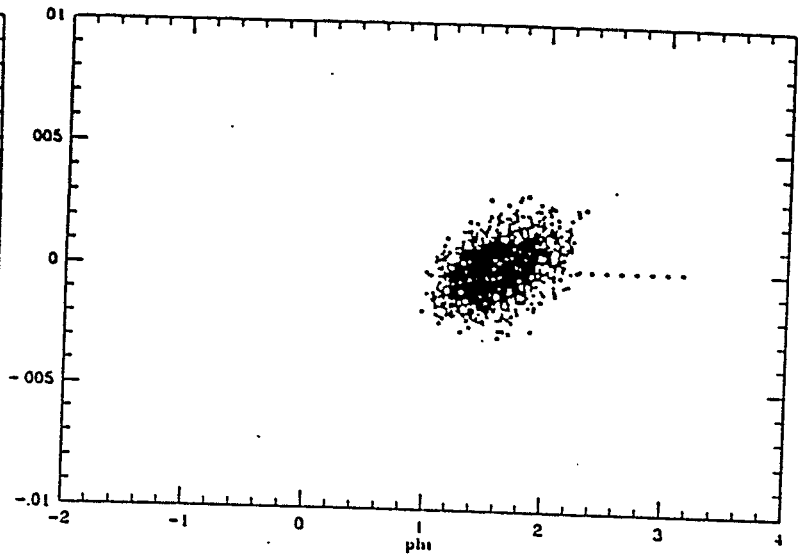
Au^{+79}



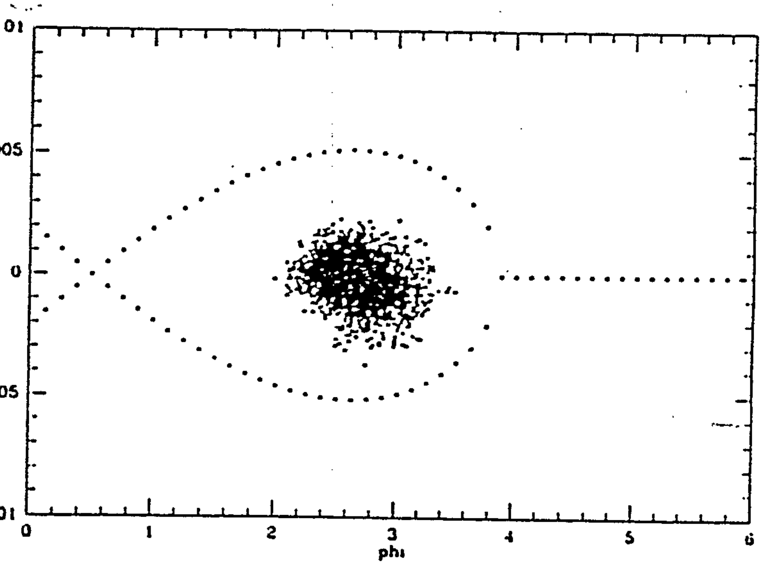
−80 ms

0.3 eV·s/amu

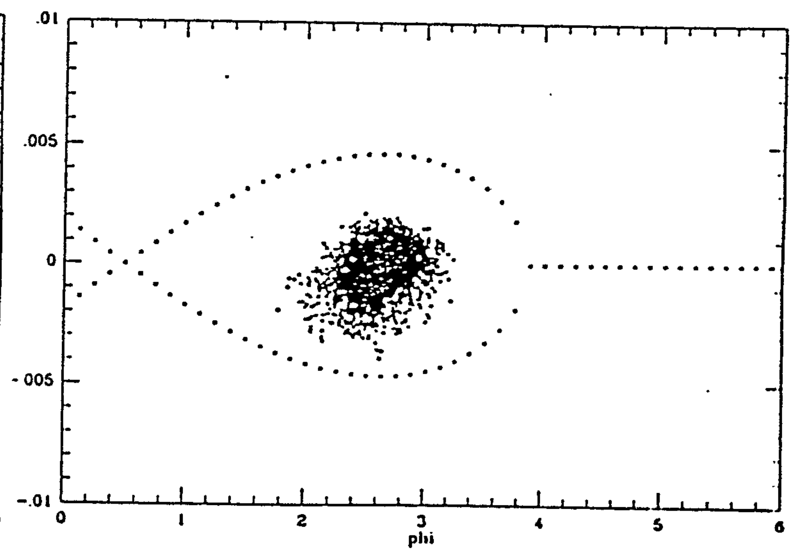
$\frac{Z}{n} = 1.2\ \Omega$



0 ms



+25 ms



+80 ms

$\eta(\frac{\Delta p}{p})$ & S.C. included. No Loss

$\tau_0 \ll T_{n.l.} \ll T_c < \tau_{syn.}$ Negligible Blow up

II. Transfer to High Freq. RF System

- 1 eV·s/amu bunch area. (general case)

  Bunch rotation (squeeze $\frac{\Delta p}{p}$, rotate, recapture)

- 0.3 eV·s/amu bunch area

  * Adiabatic compression ✓ A. G. Ruggiero

  * "Simple" rotation ✓ E. Raka

  * unstable fixed point rotation ✓ S. Y. Lee, J. Wei

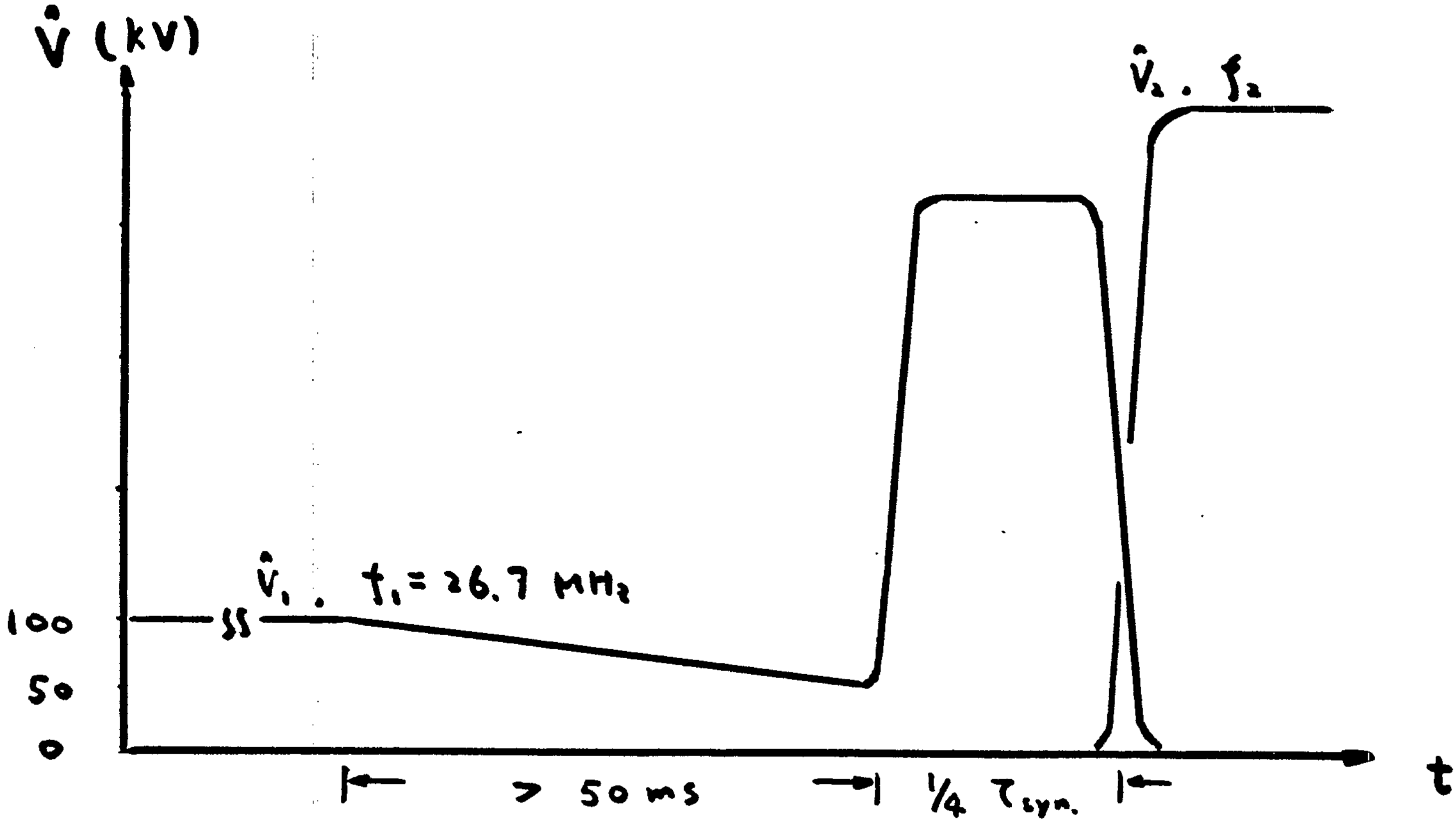
- Switch over near transition

  proposed by J. M. Brennan

  ✓, when combined with γ_t jump

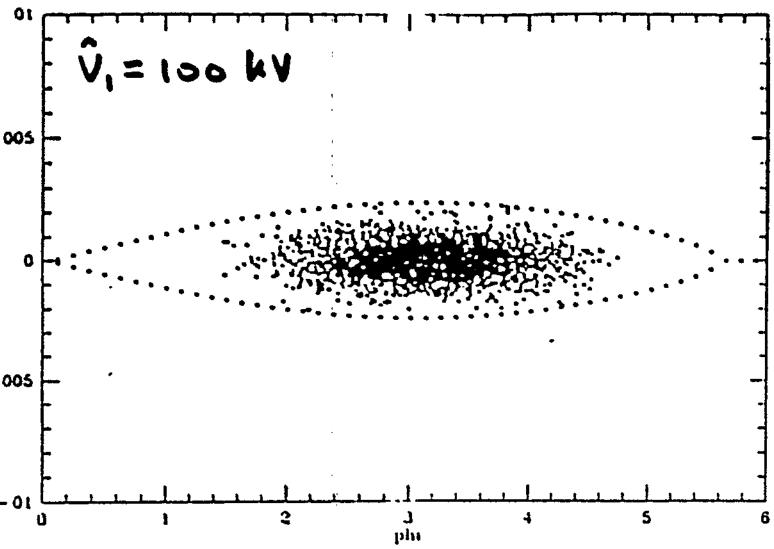
  or $\dot{\gamma}$ increase

- 160 MHz, right bucket length

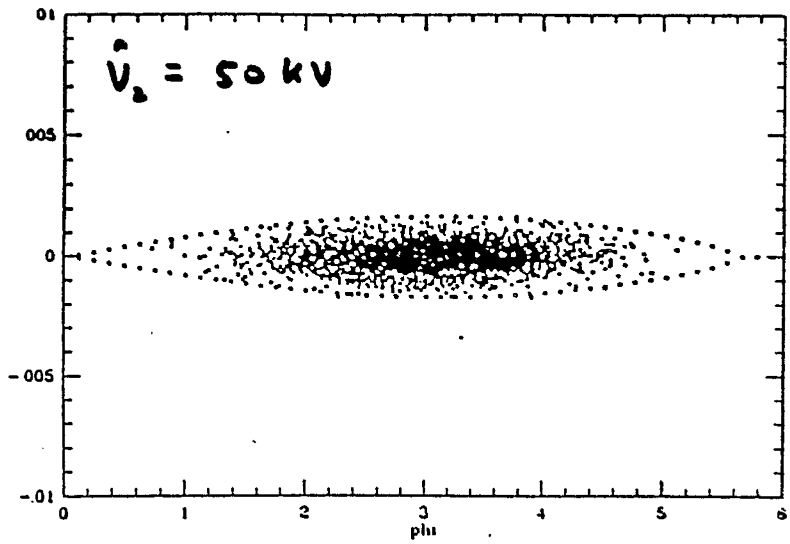


Bunch rotation

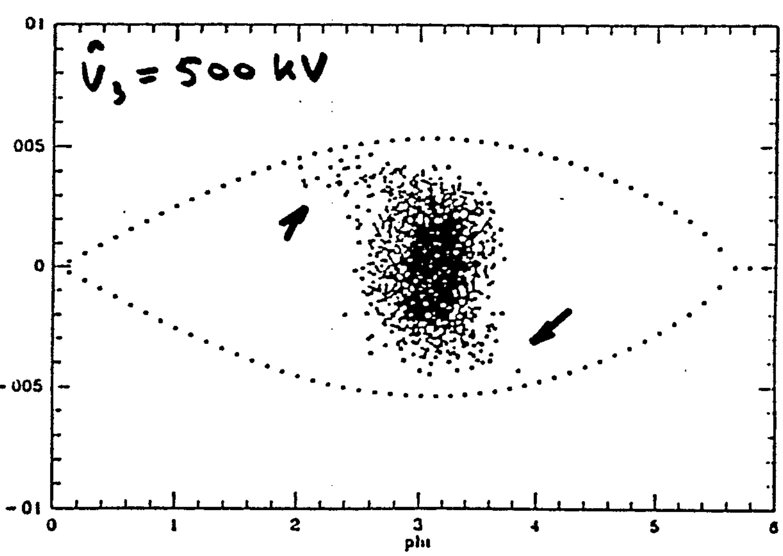
Bunch Rotation, 1 eV·s/a Au^{+79}



0 ms



50 ms



58 ms

1 eV·s/amu

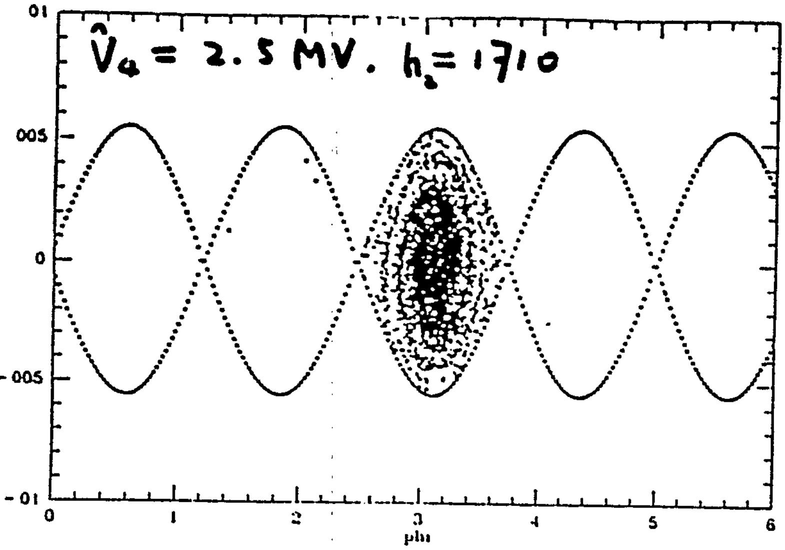
γ_{top} = 30

with s.c.

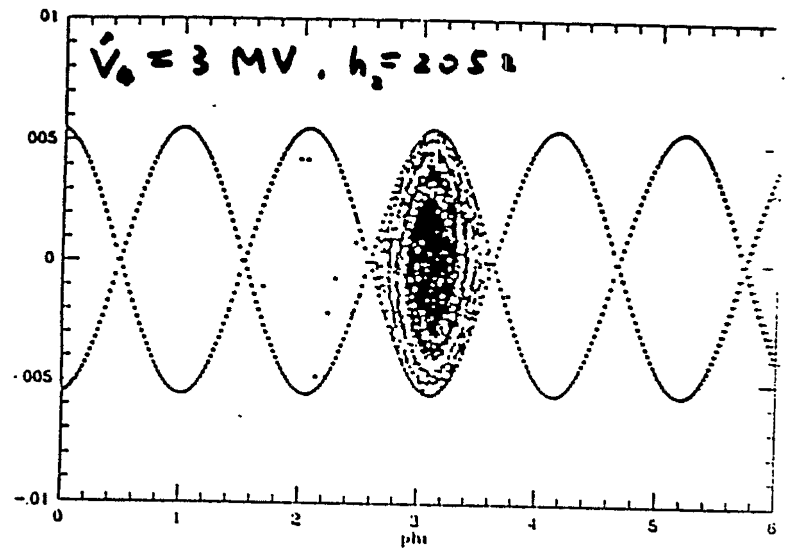
Bunch Rotation Recapture

Au^{+79}

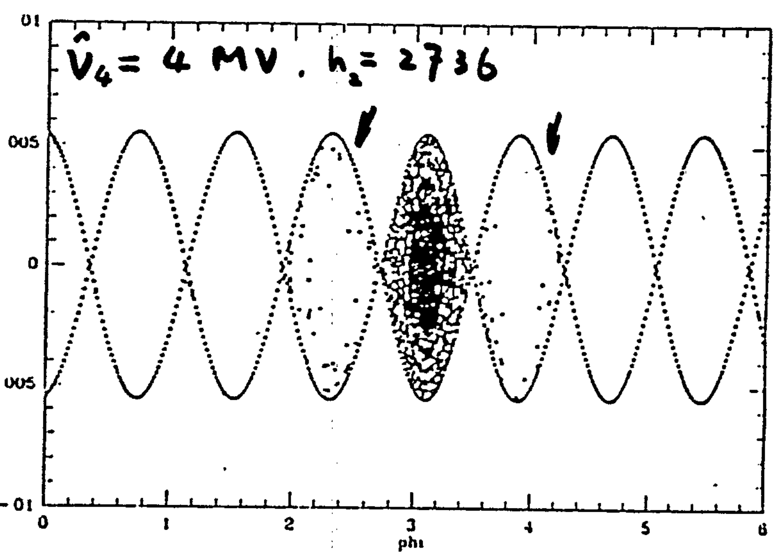
RHIC



134 MHz 98% survival



160 MHz 95% surv.



214 MHz 87% surv.

$\hat{V}_3 = 500$ kV

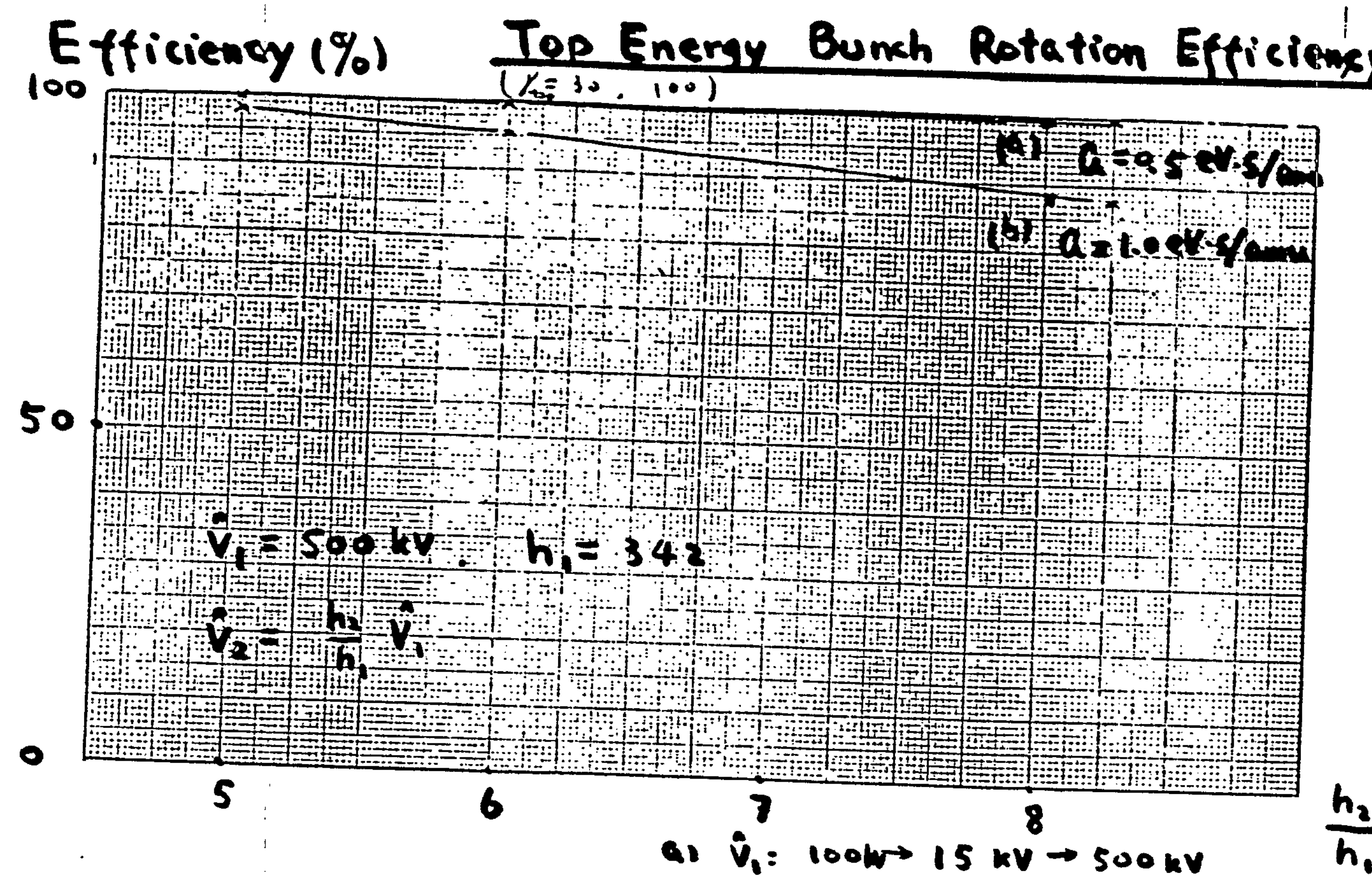
$\hat{V}_4 = \frac{h_2}{h_1} \hat{V}_3$

$h_1 = 342$

(At $\gamma_{top} = 30$)

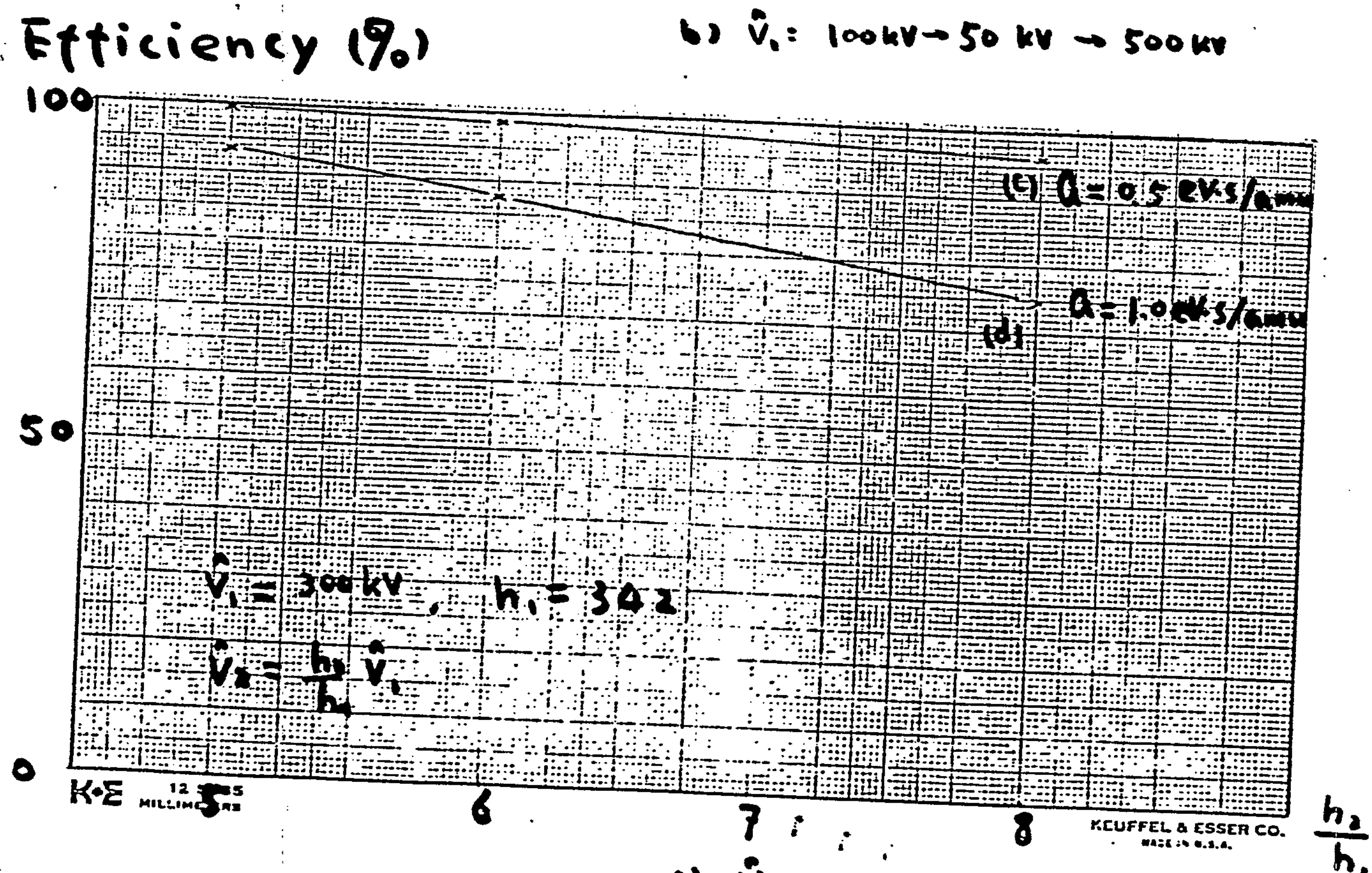
Top Energy Bunch Rotation Efficiency

(γ_T = 30, 100)



a) $\hat{V}_1$: 100 kV → 15 kV → 500 kV

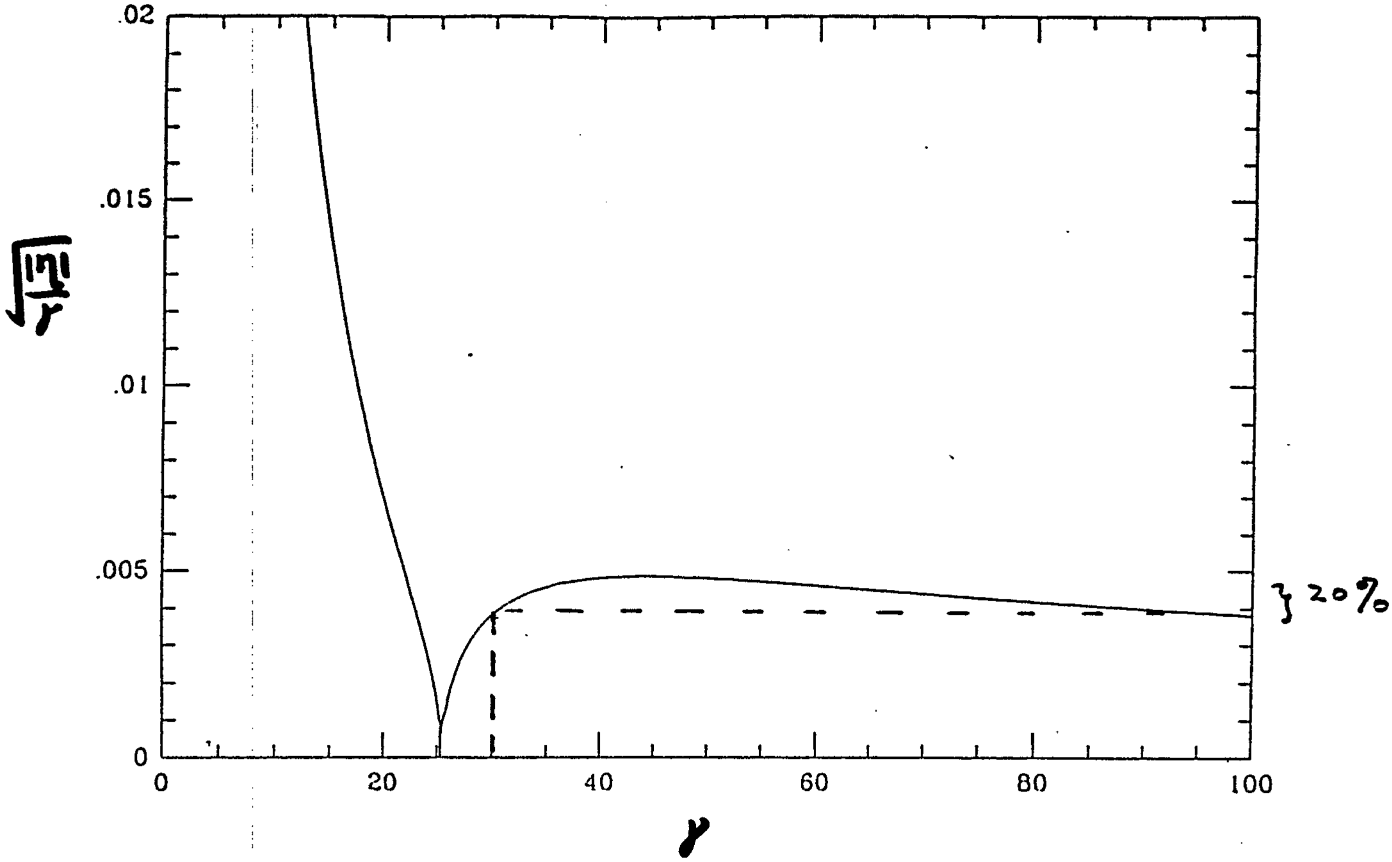
b) $\hat{V}_1$: 100 kV → 50 kV → 500 kV



c) $\hat{V}_1$: 100 kV → 15 kV → 300 kV

d) $\hat{V}_1$: 100 kV → 50 kV → 300 kV

Energy dependence of Rotation

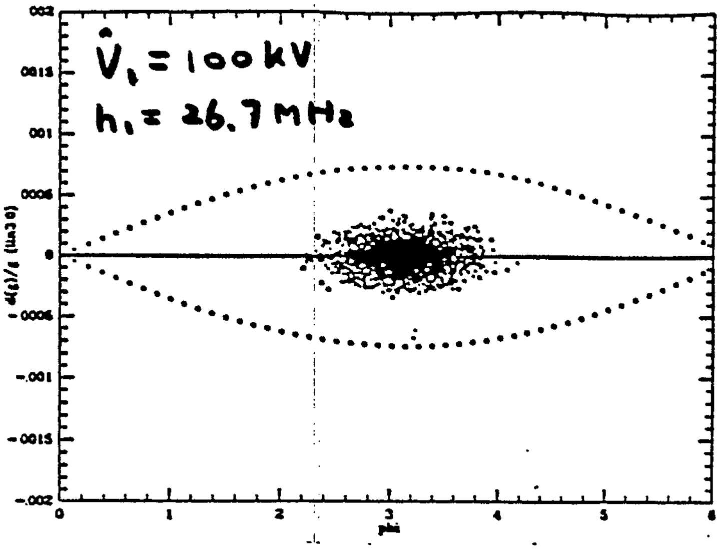


$$\Omega_s \propto \sqrt{\frac{|\eta|}{\gamma}} \qquad \text{timing}$$

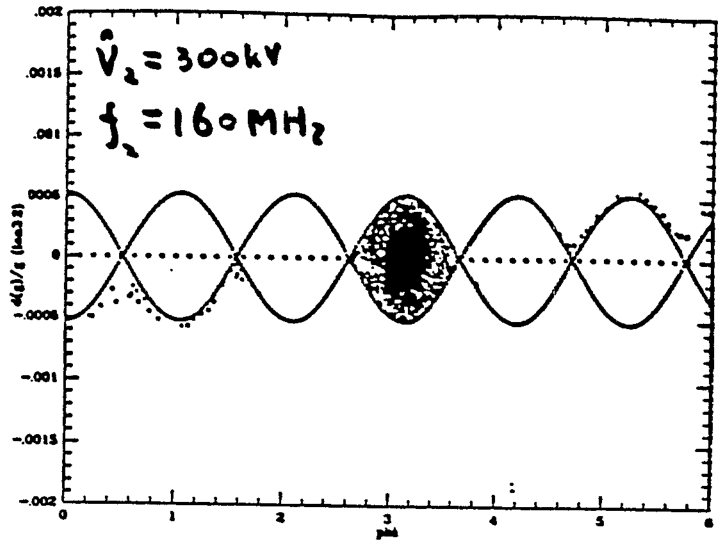
$$\hat{V}_3 \propto \sqrt{\frac{|\eta|}{\gamma}} \qquad \text{rotation voltage}$$

0.3 eV·s/amu "Simple" rotation

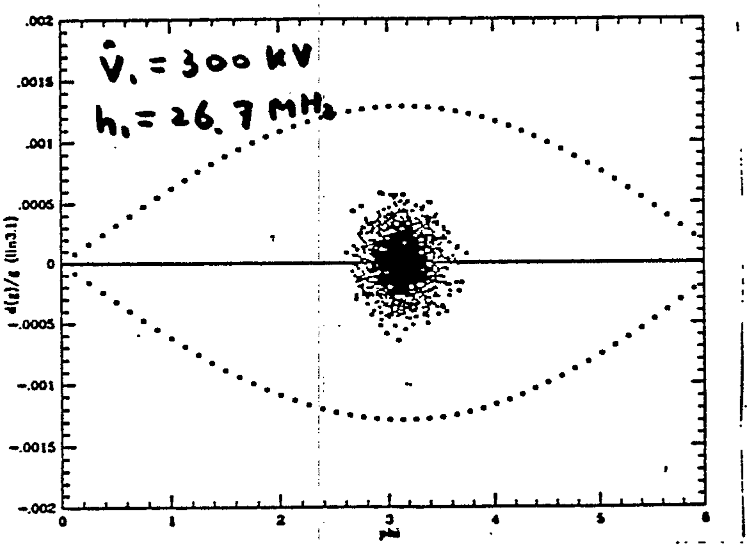
Au^{+79}
RHIC



0 ms



36 ms



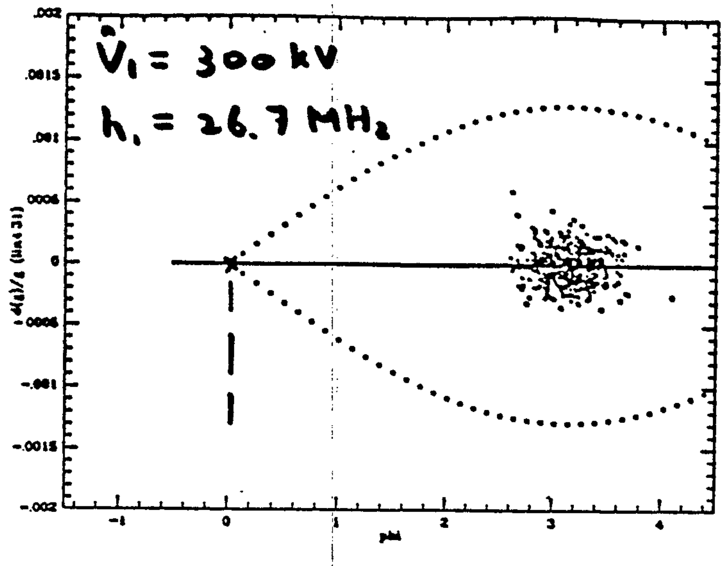
10 ms

98% in 0.39 eV·s/amu bucket

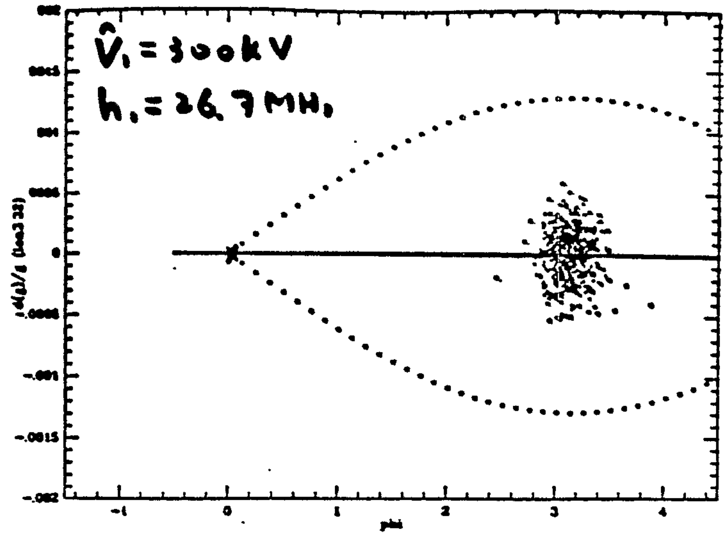
(At γ_{top} = 100)

Rotation by using unstable fixed point

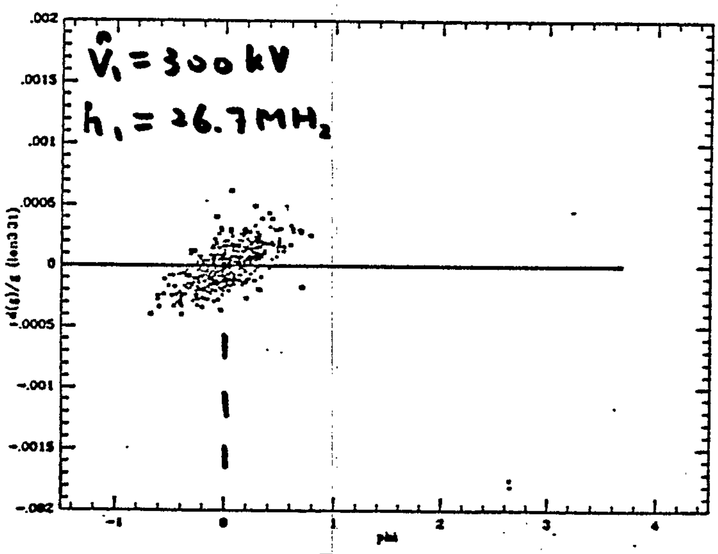
Au^{+79} RHIC



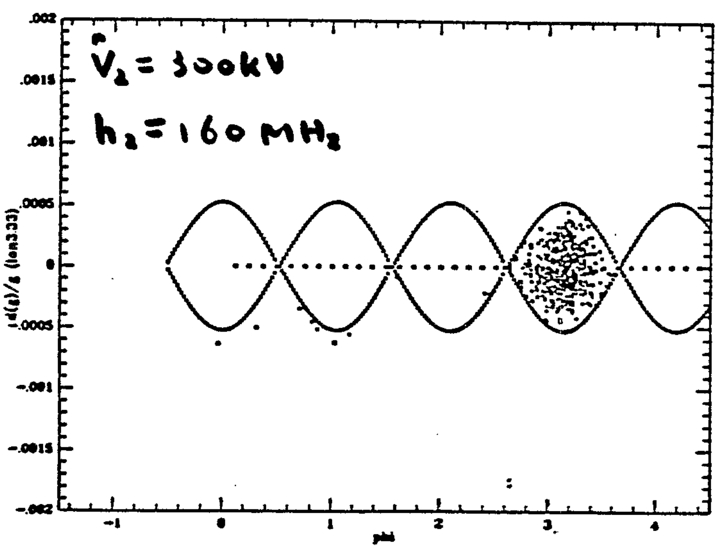
0 ms



14 ms



2.6 ms

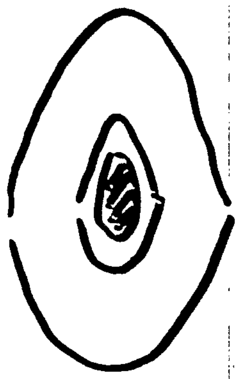


43 ms

98% in 0.39 eV·s/amu bucket

(At $\gamma_{top} = 100$)

When matched



$$\frac{\Delta p}{p} \Big/ \frac{\Delta\phi}{h} \sim \hat{V}^{1/2} \cdot h^{1/2} \cdot \eta^{-1/2}$$

$\shortparallel$ const

$\hat{V}_1 = 300$ kV. $\quad h_1 = 342$

$\hat{V}_2 = 300$ kV, $\quad h_2 = 6 * 342$

a gain of $\frac{\hat{V}_1'}{\hat{V}_1} = 6$

AGS

$\hat{V}_1 = 12$ kV $\Rightarrow$ $\hat{V}_1' = 100$ kV.

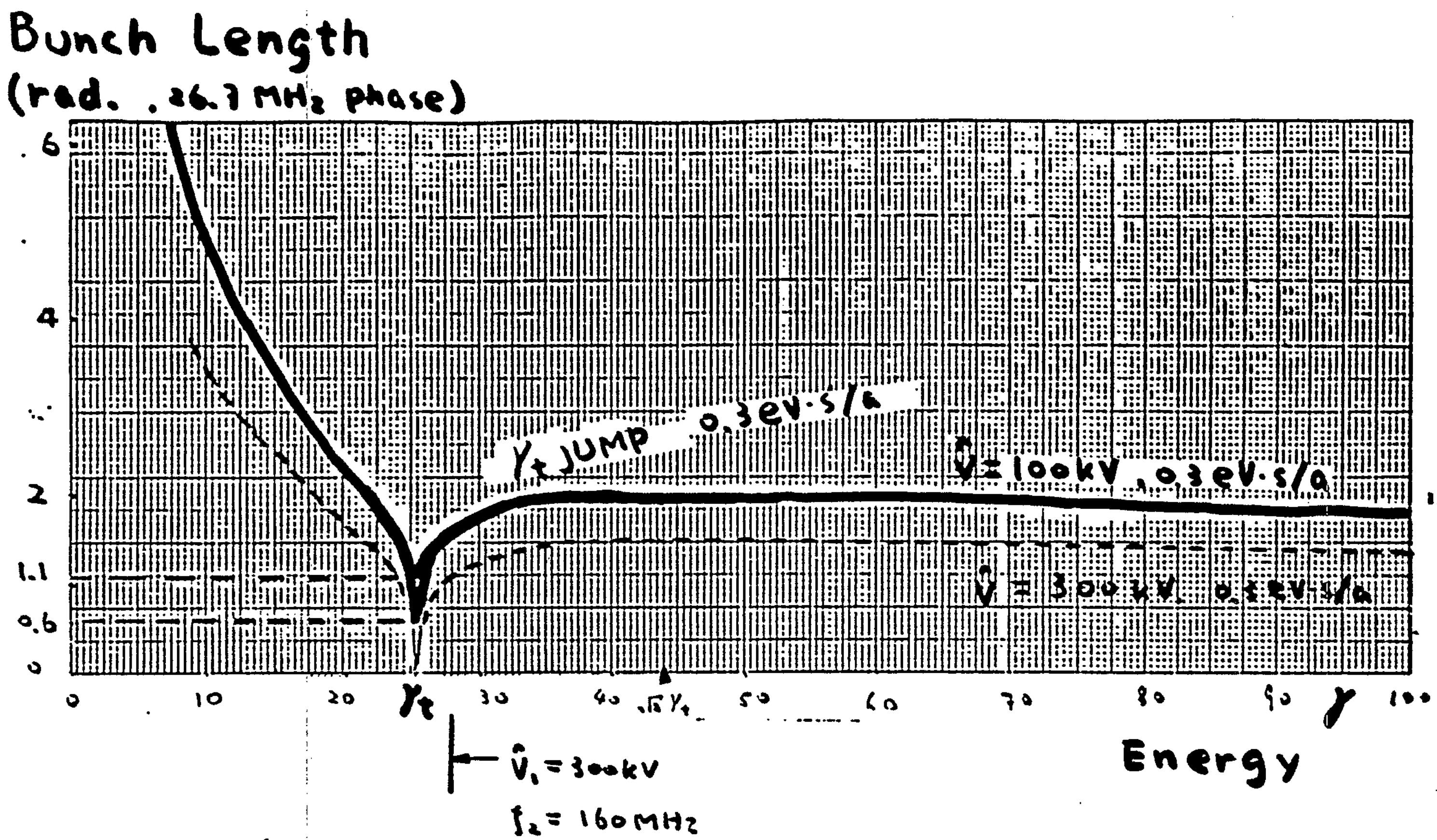
$\frac{\hat{V}_1'}{\hat{V}_1} \sim 8$.

But

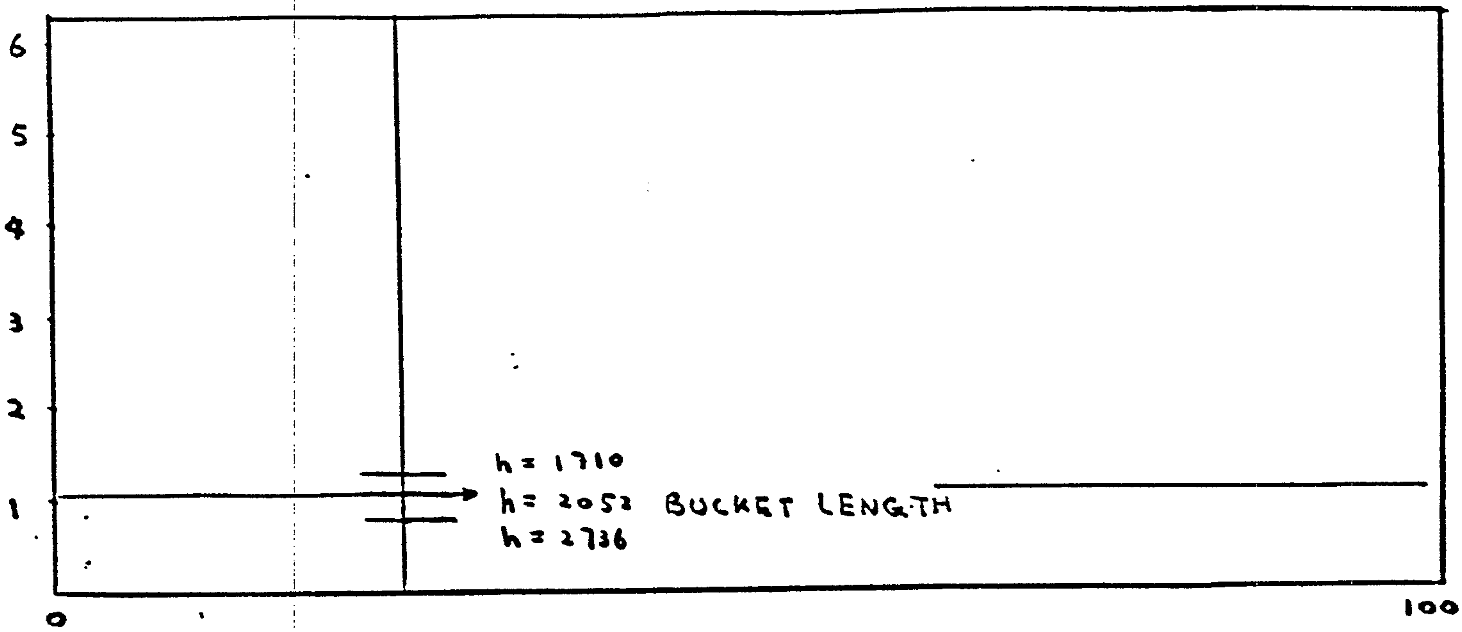
$$\Omega_s = \sqrt{\frac{2 e V h \eta c^2}{2\pi A m_0 c^2 \cdot R_0^2} \cos\phi_s}$$

$\Rightarrow \tau_{syn} = 5$ ms ?

RHIC, $\tau_{syn} = 40$ ms. (300kV)



- Switch over near transition γ_t.



6
5
4
3
2
1
0
100
h= 1710
h= 2052 BUCKET LENGTH
h= 2736

Switch over near transition
(after a $\dot{\gamma}$ increase)

Au^{+79}

$\gamma_{tr.} = 26.14$

$\gamma_{t0} = 25.4376$

$\hat{V}_1 = 100 \sim 300$ kV

$\hat{V}_2 = 300$ kV

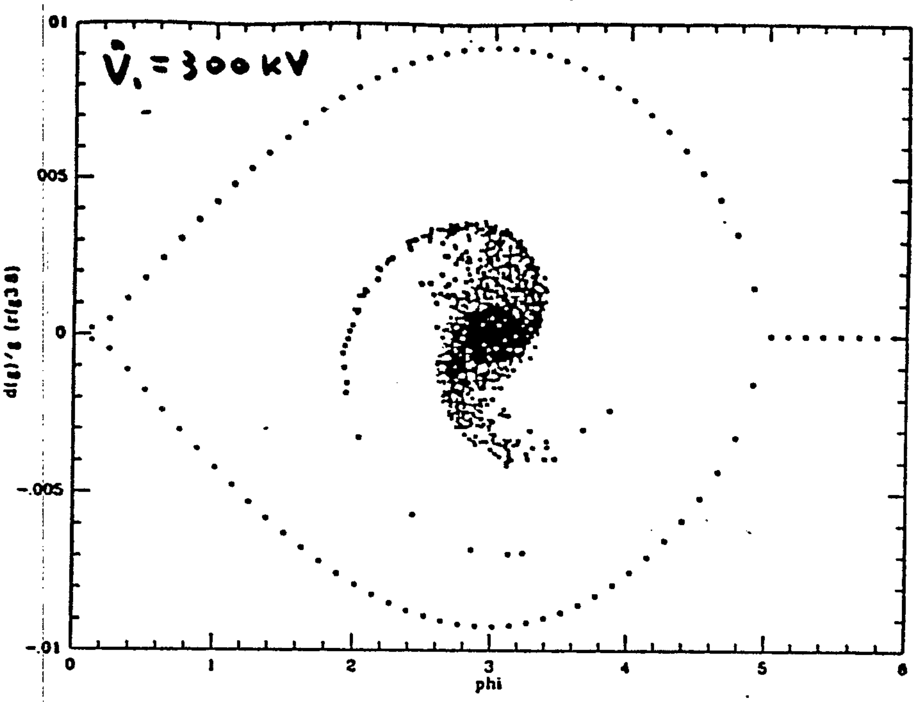
$h_1 = 342$

$h_2 = 2052$

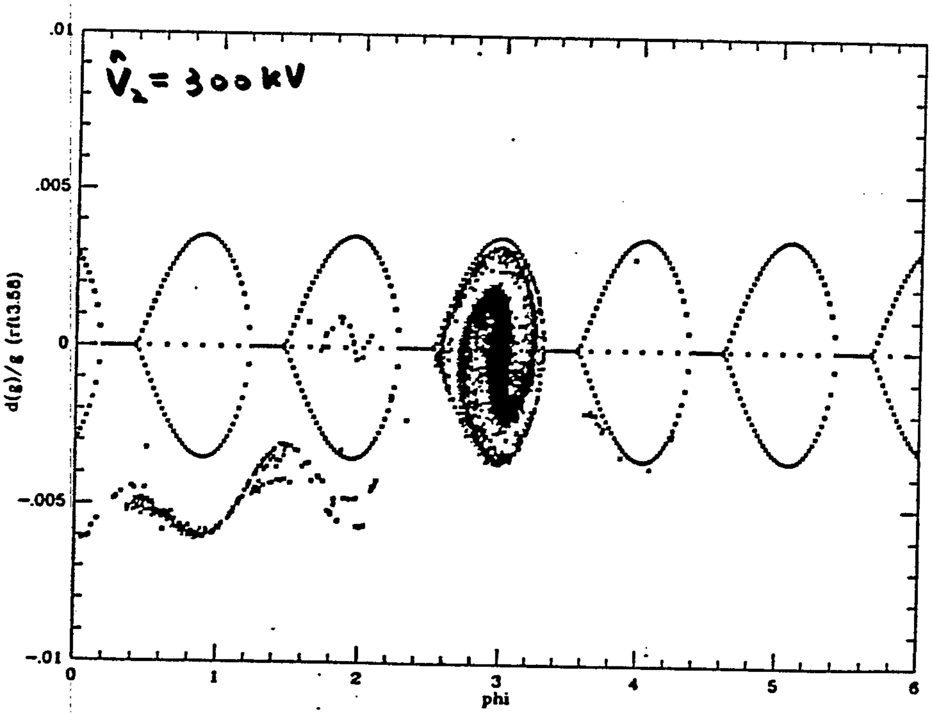
$\alpha_1 = -0.6$

$\frac{Z}{n} = 1.2\ \Omega$

capac.



+ 375 ms



+ 440 ms

Transfer Efficiency 90% in 0.6 eV·s/am

Switch over near transition
(after a $\dot{\gamma}$ increase)

Au^{+79}

$\gamma_{tr.} = 25.65$

$\gamma_{t_0} = 25.4376$

$\hat{V}_1 = 100 \sim 300\,kV$

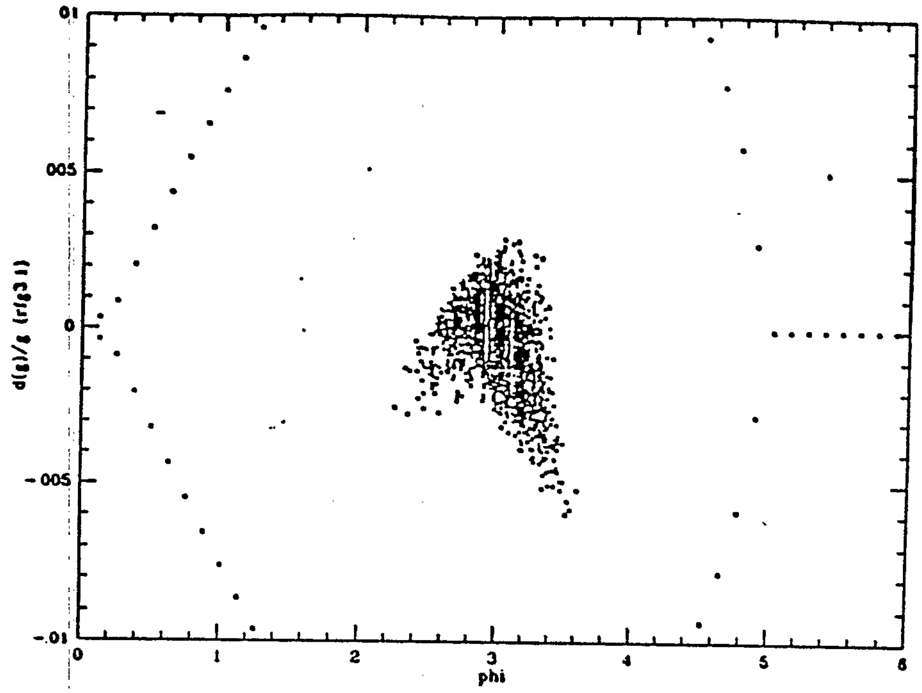
$\hat{V}_2 = 300\,kV$

$h_1 = 342$

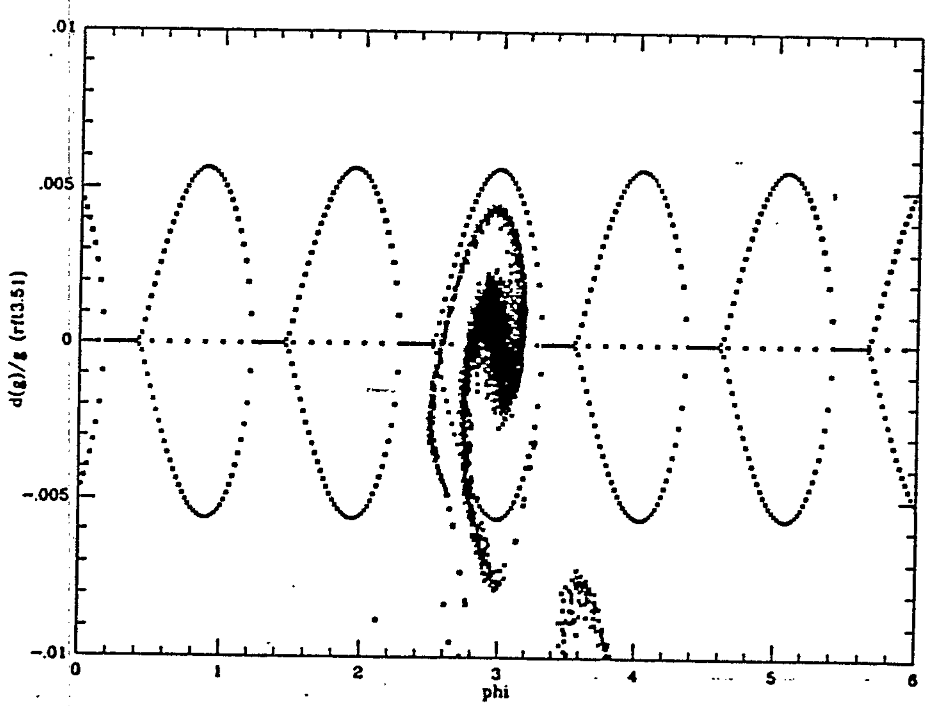
$h_2 = 2052$

$\alpha_1 = -0.6$

$\frac{Z}{n} = 1.2\ \Omega$ capac.

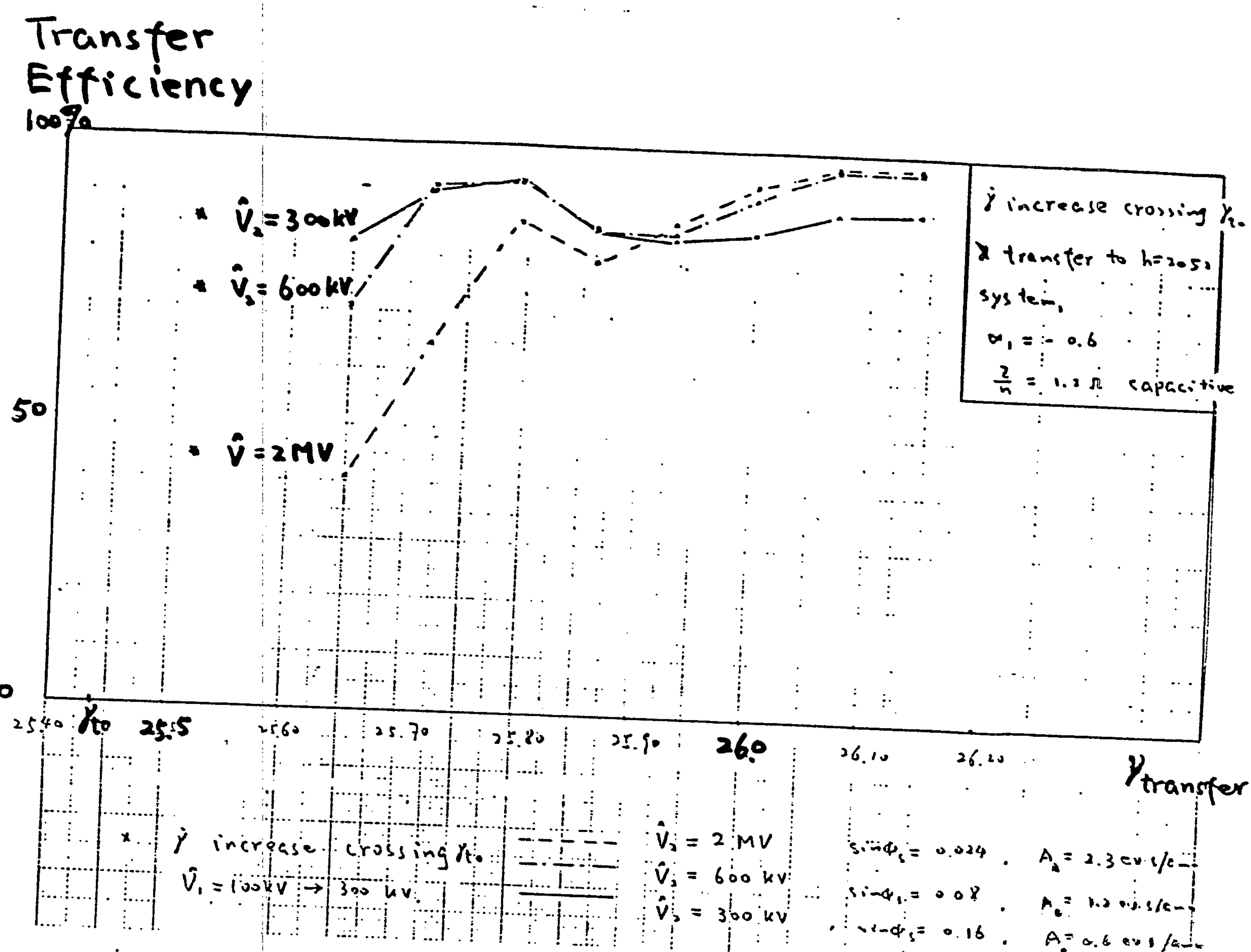


+ 25 ms



+ 90 ms

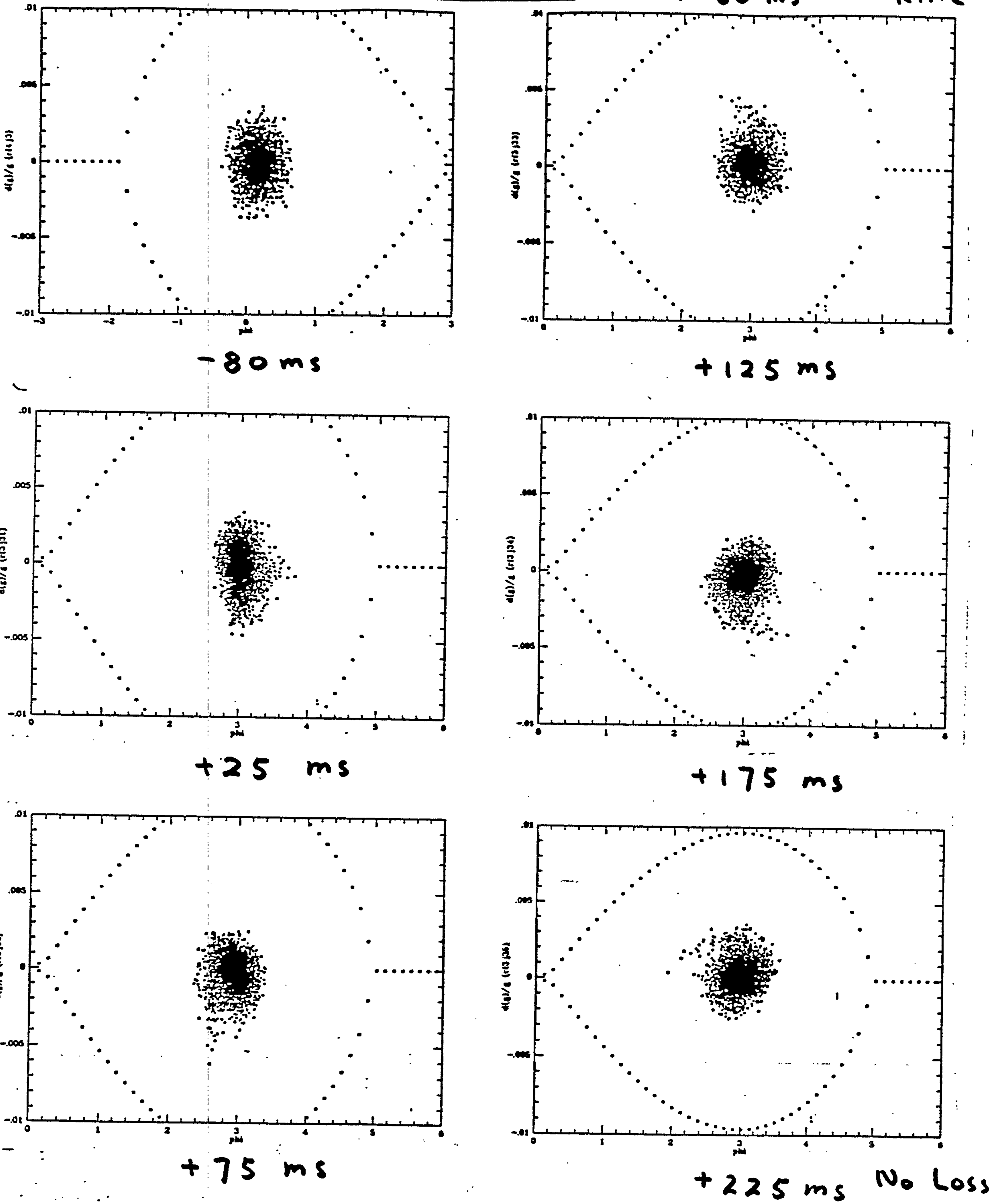
Transfer Efficiency 83 % in 0.6 eV·s/a



- $\dot{\gamma}$ increase of $\Delta\gamma = 0.38$. in 40 ms
- then transfer to 160 MHz RF System

on $\gamma = \gamma_{transfer}$

γ_t jump, $\hat{V}_1 = 300$ kV $\Delta\gamma_t = 0.6$ in 60 ms   Au RHIC

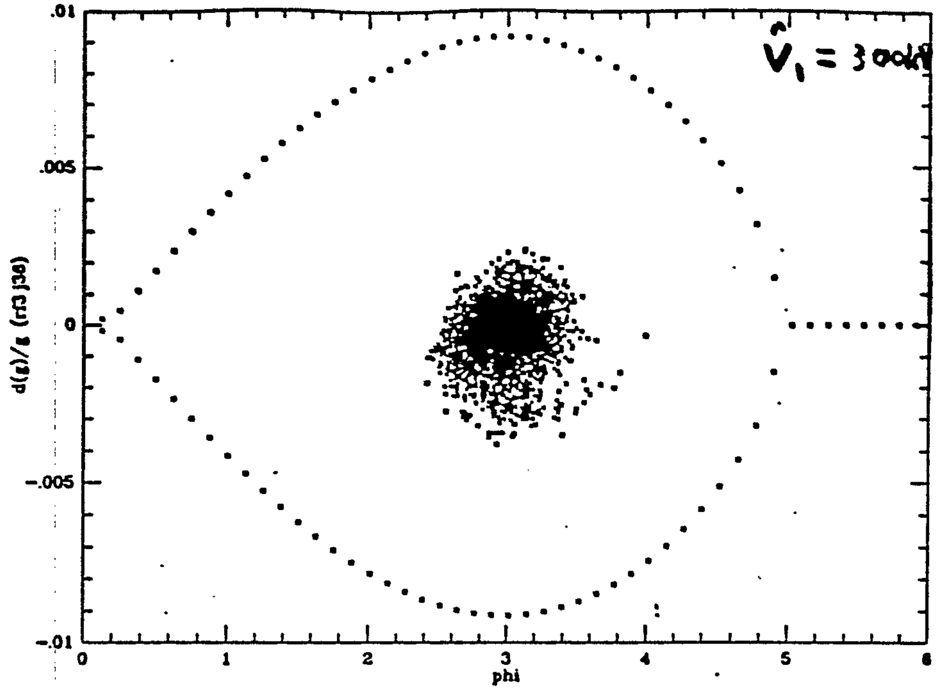


Switch over near transition

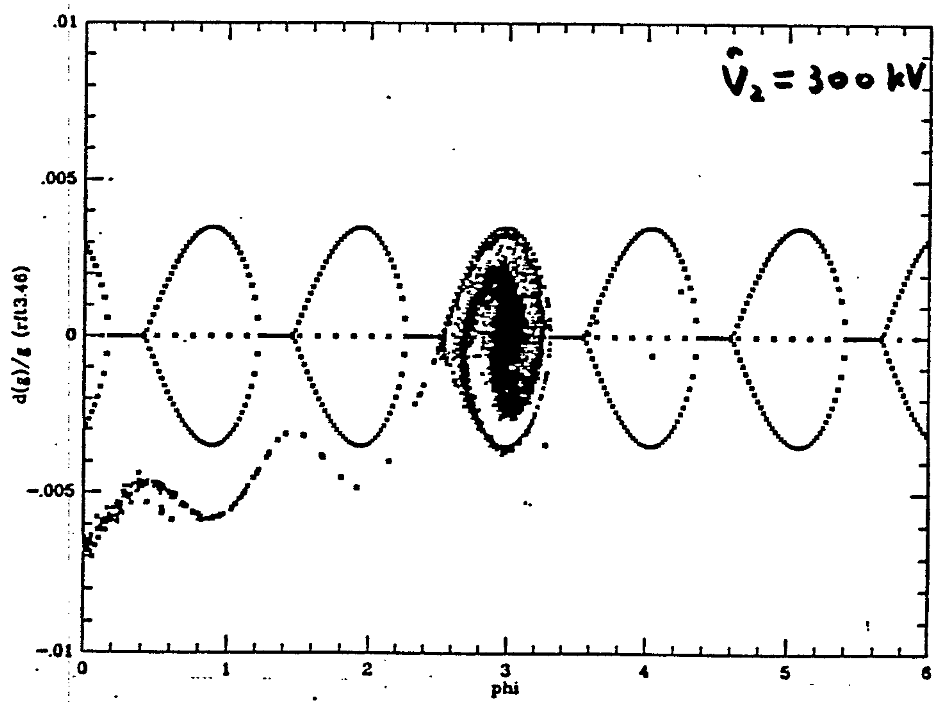
(after a γ_t jump)

Au^{+79}

RHIC



+ 275 ms

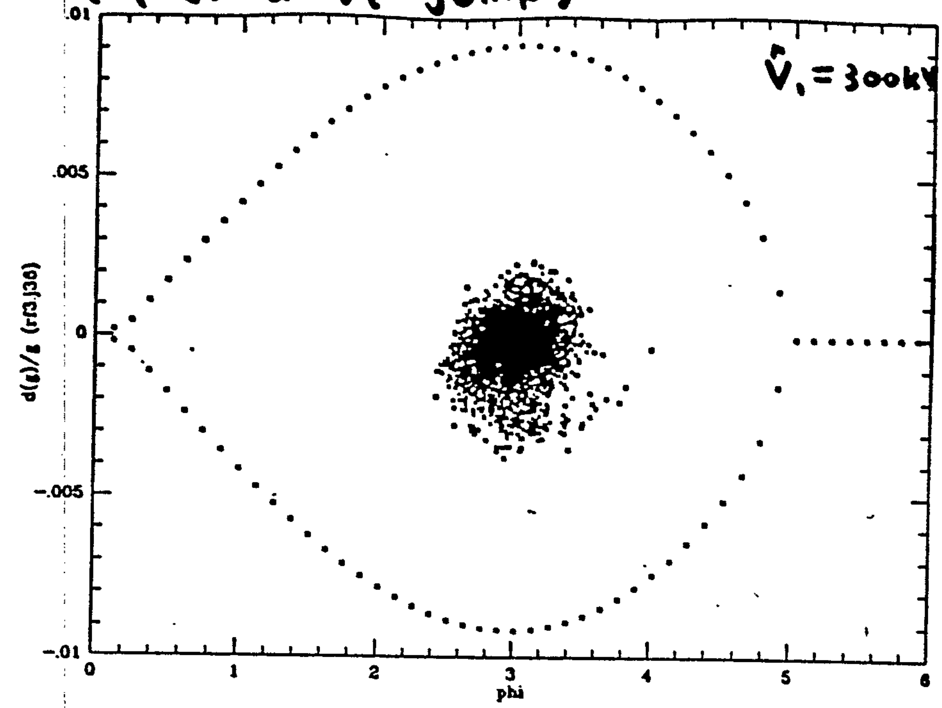


+ 340 ms

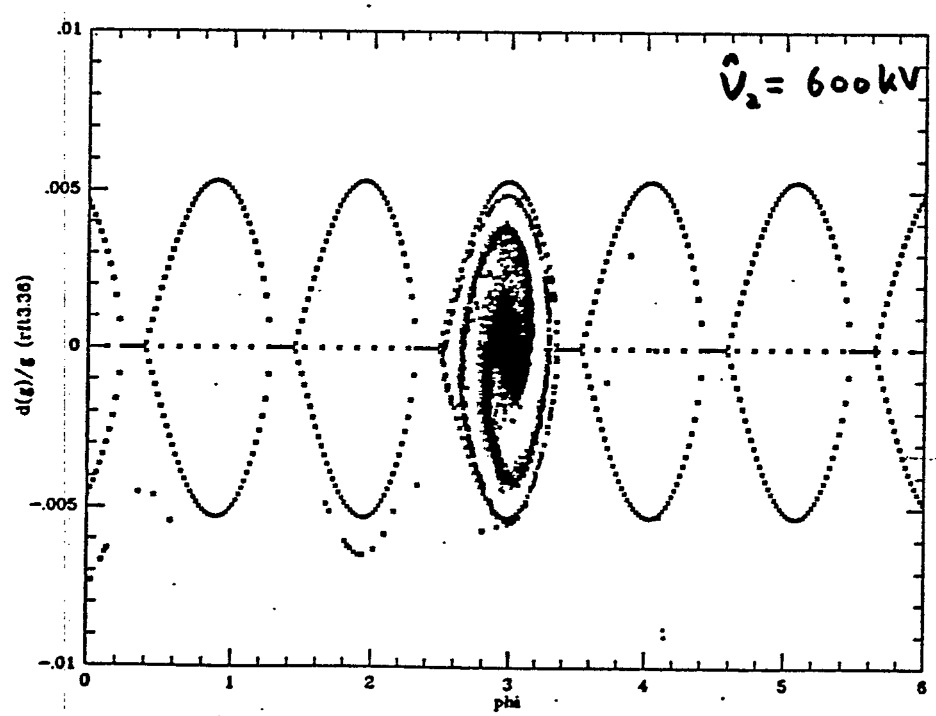
92% surv. in 0.6 eV·s/u bucket

Switch over near transition (after a γ_t jump)

Au^{+79} RHIC



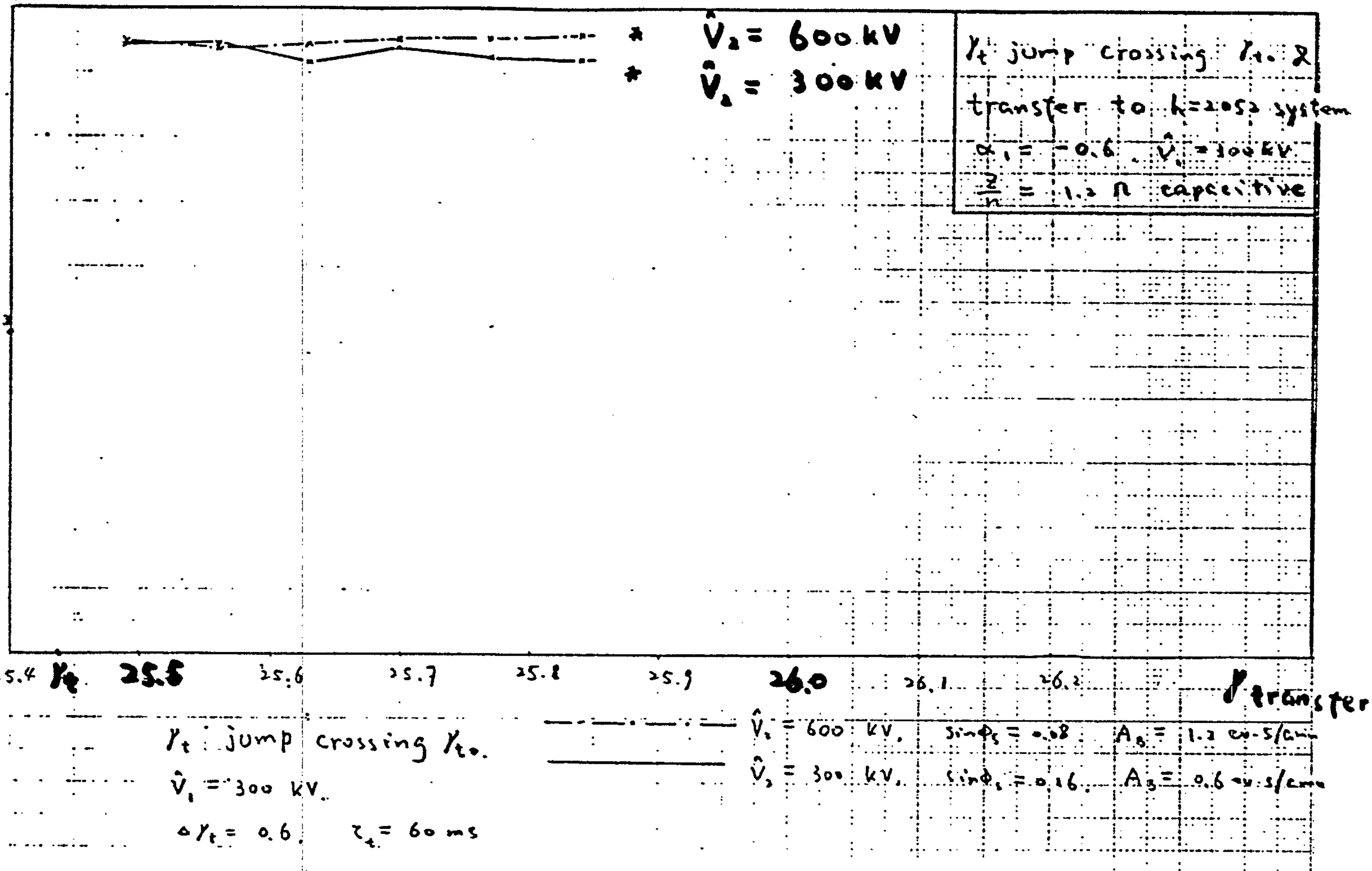
+275 ms



+340 ms

96% surv. in 1.2 eV·s/a bucket

Transfer efficiency



- γ_t jump of $\Delta\gamma_t = 0.6$ in 60 ms, $\hat{V}_1 = 300$ kV
- then transfer to 160 MHz RF System on $\gamma = \gamma_{transfer}$

Summary on the previous study:

* A γ_t jump, or a $\dot{\gamma}$ increase near transition is very helpful

* A r.f. voltage of 300 kV for 26.7 MHz system helps in achieving $\dot{\gamma}$ increase, rotation and providing sufficient bucket area at low energy

* A second system of 160 MHz is comfortable for r.f. system transfer.