

Simulation of Transition and Transfer

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U.S. Department of Energy

USDOE Office of Science (SC)

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Simulation of Transition and Transfer

(Mini-Workshop on RHIC RF Systems)

*July 11-15, 1988
Collider Center*

J. Wei
BNL

SIMULATION OF TRANSITION & TRANSFER

I. Transition Energy Crossing

* $\hat{V} = 100$ kV crossing transition

* Scheme of $\dot{\gamma}$ increase or $\dot{\gamma}_t$ jump

S. Y. Lee, A. G. Ruggiero

J. Claus

II. Transfer to High Frequency R.F. System

* Top energy bunch rotation

* Bunch rotation with 0.3 eV·s/a bunch

* Switch over near transition

J. M. Brennan, E. Raka

S. Y. Lee ...

I. Transition Energy Crossing

- $\hat{V} = 1.2 \text{ MV}$?

momentum spread too large, nonlinear

- $\hat{V} = 100 \text{ kV}$ crossing ?

much better,

space charge + nonlinear effect. $\gamma_t \left(\frac{e\phi}{p} \right)$

\Rightarrow phase space area blow up

- γ_t jump, or $\dot{\gamma}$ increase ?

Good, "clean" crossing

$$\begin{cases} \dot{\omega} = \frac{8e\dot{V}}{2\pi} (\sin\phi - \sin\phi_s) + \Delta_{s.c.} + \Delta_z \\ \dot{\phi} = \frac{h\nu_0}{P_0 R_0} \cdot \omega \cdot \eta(\omega) \end{cases} \quad (\omega = \frac{\Delta E}{\lambda_s})$$

* kinematic mismatching

$$\eta\left(\frac{\Delta p}{p}\right) = \eta_0 + \eta_1 \cdot \frac{\Delta p}{p} + \dots$$

* Low freq. impedance

space charge, inductive, capacitive

⇒ bunch mismatching

* High freq. impedance

resistive, inductive, capacitive (s.c.)

⇒ microwave instability

$$\underline{\eta = \eta \left(\frac{\Delta P}{P_0} \right)}$$

$$\Omega R = \beta c, \quad \frac{\Delta \Omega}{\Omega_0} = \frac{\beta}{\beta_0} \frac{R_0}{R} - 1$$

$$\frac{R}{R_0} = 1 + \alpha_1 \frac{\Delta P}{P_0} + \alpha_2 \left(\frac{\Delta P}{P_0} \right)^2 + \dots$$

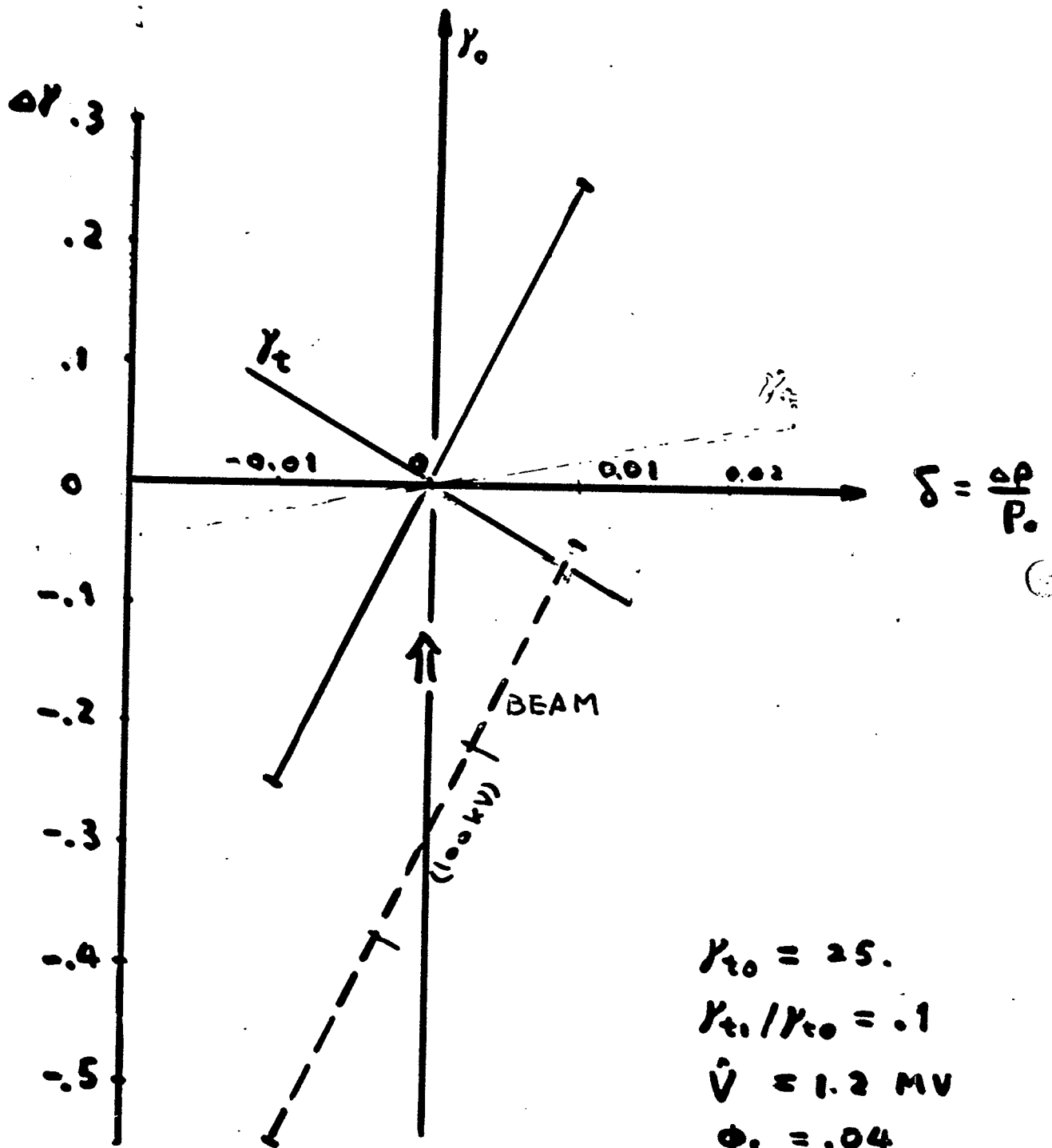
$$\frac{\beta}{\beta_0} = 1 + \frac{1}{\gamma_0^2} \frac{\Delta P}{P_0} - \frac{3\beta_0^2}{2\gamma_0^2} \left(\frac{\Delta P}{P_0} \right)^2 + \dots$$

$$\underline{\eta = \frac{\Delta \Omega}{\Omega} / \frac{\Delta P}{P_0} \approx \eta_0 + \left[\frac{3}{2} \frac{\beta_0^2}{\gamma_0^2} - \frac{\gamma_{E1}}{\gamma_{E0}^2} \right] \cdot \frac{\Delta P}{P_0}}$$

$$\delta = \frac{\Delta \gamma}{\gamma} = \left(1 + \frac{1}{\beta_0^2 \gamma_0^2} \right) \cdot \frac{\Delta P}{P_0}$$

$$\left(\frac{1}{\gamma_0^2 \gamma_0^2} - \frac{1}{\gamma_{E0}^2} \right)$$

Mismatch at transition (RHIC)



$$\gamma_{t0} = 25.$$

$$\gamma_{t1} / \gamma_{t0} = .1$$

$$\dot{V} = 1.2 \text{ MV}$$

$$\phi_s = .04$$

$$\Delta\gamma = 2.07 \cdot 10^{-5} / \text{turn}$$

$$\dot{\gamma} = 1.62 / \text{sec.}$$

$$\tau_0 = 12.79 \mu\text{s}$$

Estimate of δ growth.

$$\ddot{\delta} = \frac{qe\hat{v}}{2\pi p_0 R_0} \cos\varphi_0 k\Omega_0 \left(\frac{2\dot{\gamma}t}{\gamma_T^{0.3}} + \eta, \delta \right) \delta$$

$$i \int_{t_c}^0 \Omega_s^{t_c} dt = \left(\frac{k\Omega_0 qe\hat{v} \cos\varphi_0}{2\pi p_0 R_0} \right)^{1/2} \int_{t_c}^0 \left(\frac{2\dot{\gamma}t}{\gamma_T^{0.3}} + \eta, \delta \right)^{1/2} dt$$

$$\approx \frac{\Omega}{3\dot{\gamma}} \left(\frac{kqe\hat{v} \cos\varphi_0}{2\pi p_0 \beta c} \right)^{1/2} \left(\frac{3}{2} - \lambda \right)^{3/2} \hat{\delta}^{3/2}$$

AGS

$$295 \hat{\delta}^{3/2}$$

Growth factor: $e \approx 1.1$

RHIC

$$5200 \hat{\delta}^{3/2}$$

$e \approx 6.3$

$$\delta \propto \delta_0 e$$

$$\left(k\hat{v} \cos\varphi_0 \right)^{1/2} \hat{\delta}^{3/2}$$

Relevant time scales:

- τ_0 : revolution period
- $\tau_{\text{syn.}}$: synchrotron osc. period
- T_c : characteristic non-adiabatic time

$$T_c = \left[\frac{A m_0 c^2}{R_0^2 h} \cdot \frac{\gamma_t^4}{2 \dot{\gamma}} \cdot \frac{2 \pi}{z e \dot{V} |\cos \phi_s|} \right]^{1/3}$$

$\frac{2}{R_s} \left| \frac{dR_s}{dt} \right| > 1$

- $T_{\text{n.l.}}$: non-linear time

$$T_{\text{n.l.}} = \frac{\Delta P_{\text{n.l.}}}{\dot{\gamma}} = \frac{3 \beta_s^2 - \alpha_s}{4 \dot{\gamma}} \cdot \left(\frac{\Delta p}{p} \right)_{2.5\sigma}$$

$$\eta(\delta) = \eta_0 + \eta_1 \delta + \dots$$

time when $|\eta_1 \delta| > |\eta_0|$

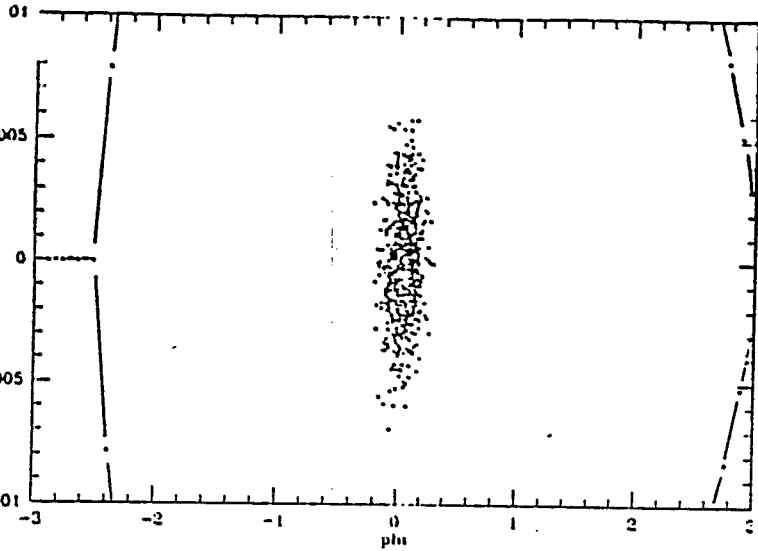
& of different sign

- $T_{\text{m.w.}}$: microwave instability growth time

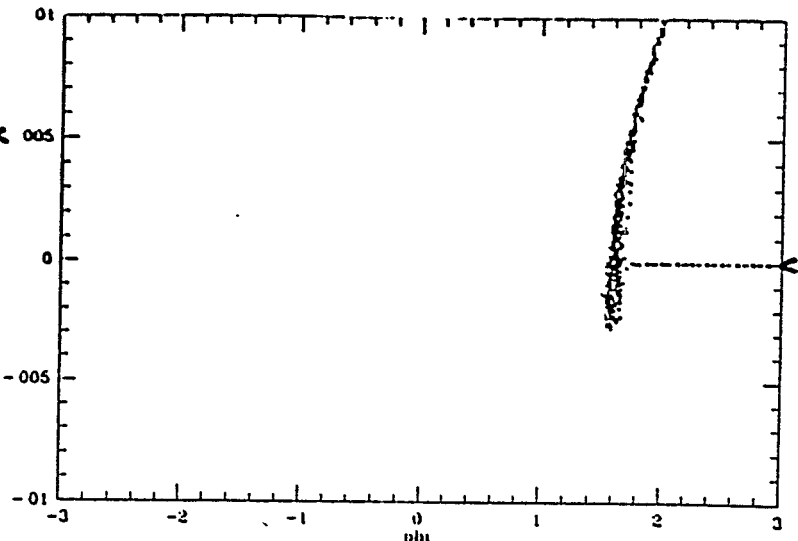
$\hat{V} = 1.2 \text{ MV}$
 $\sin \phi_s = 0.04$

γ_+ CROSSING

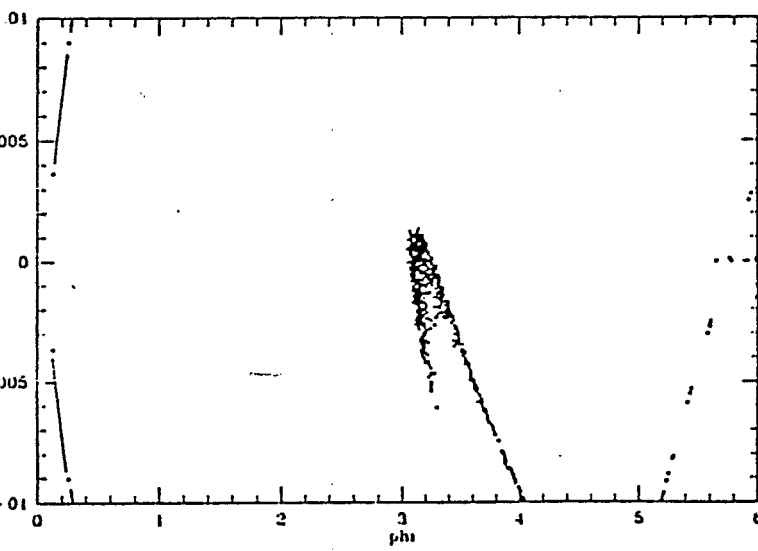
Au^{+79}



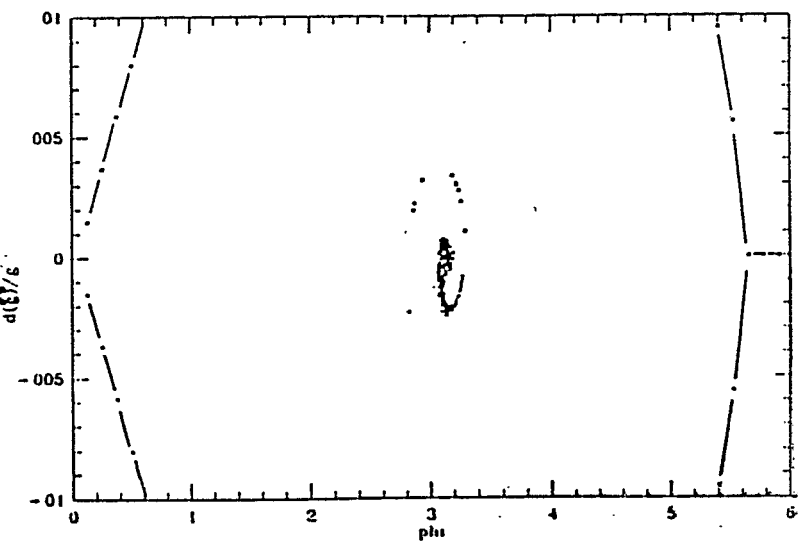
- 80 ms



0 ms



+ 25 ms



+ 180 ms

$\eta(\frac{\Delta P}{P})$, mismatching \Rightarrow 70% Loss

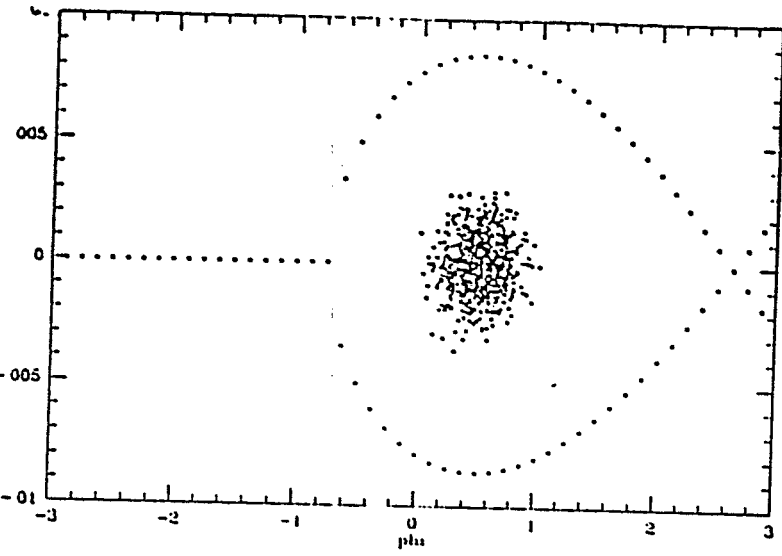
$\tau_0 \ll T_c < T_{n.l.} < \tau_{syn.}$

$$\hat{V} = 100 \text{ kV}$$

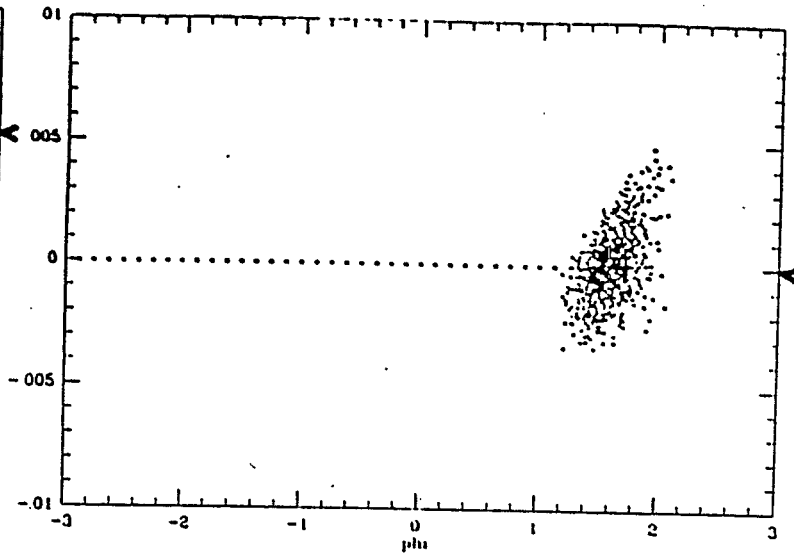
$$\sin \phi_s = 0.48$$

γ_t CROSSING

A_u^{+79}

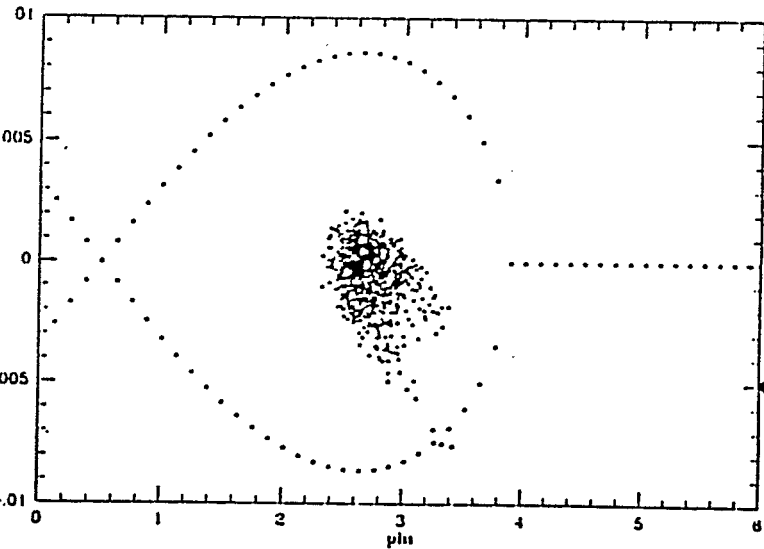


-80 ms

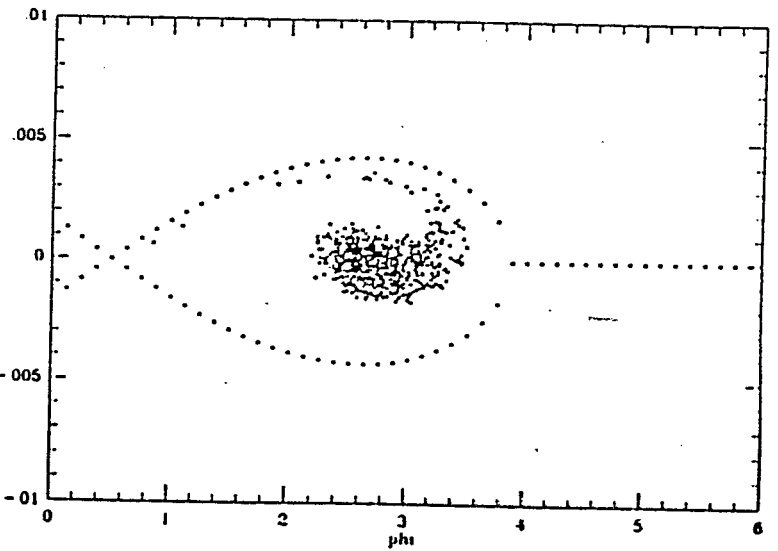


0 ms

0.3 eV·s/amu, w/o s.c.



+80ms



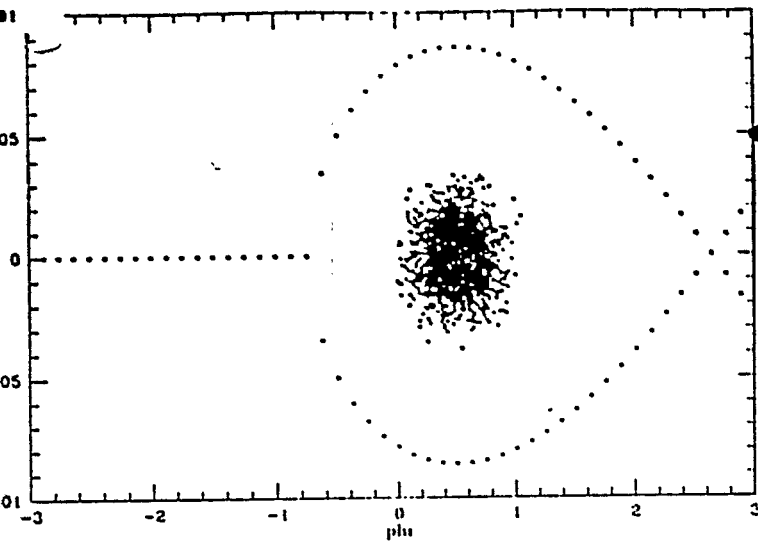
+230 ms

$\eta \left(\frac{\Delta P}{P} \right)$ mismatching \Rightarrow 0.9% Loss

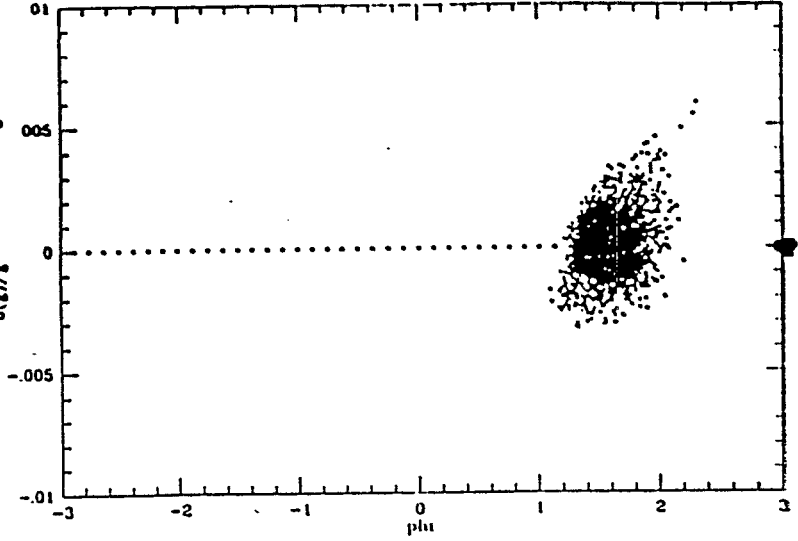
$$\tau_0 \ll T_{n.l.} < T_c < \tau_{syn.}$$

$\hat{V} = 100 \text{ kV}$
 $\sin \phi_s = 0.48$

γ_t CROSSING. space charge Au



-80ms

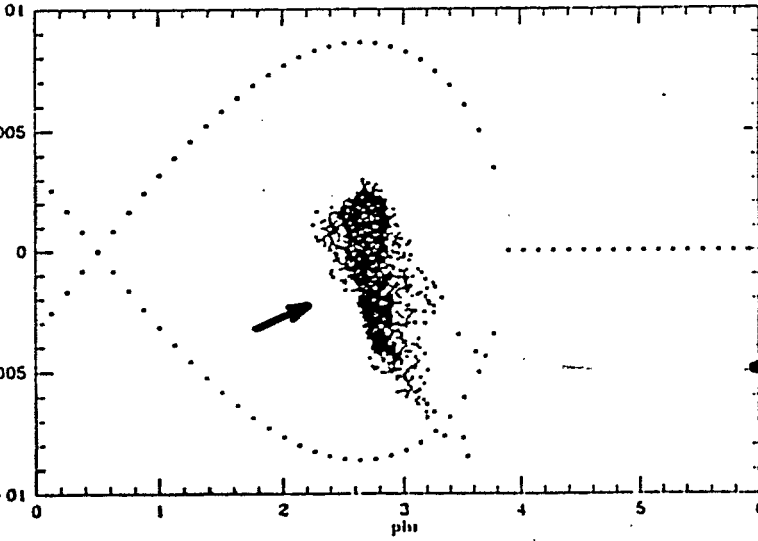


0ms

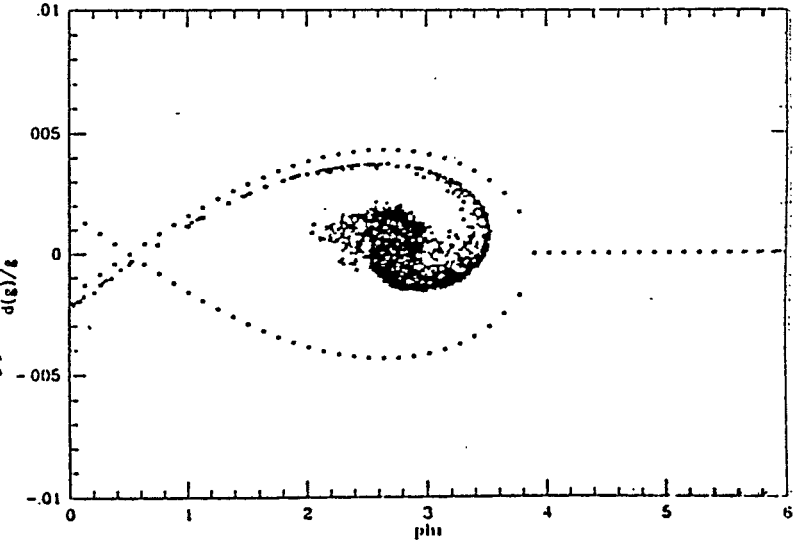
0.3 eV·s / amu, with s.c.

$\frac{Z}{N} = 1.2 \Omega, 1.1 \times 10^9 / \text{bunch}$

$l_{\text{bin}} = 2b, \Rightarrow \text{microwave freq.}$



+80ms



+230ms

mismatching + spacecharge \Rightarrow 2.1% Loss

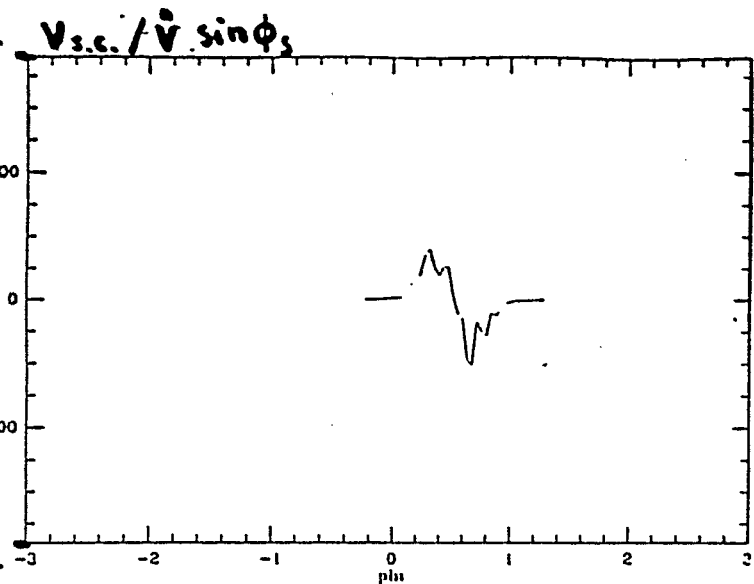
$\tau_0 \ll T_{n.l.} < T_c < \tau_{\text{syn.}}$

$\sim 1 \text{ eV}\cdot\text{sec}/\text{amu}$

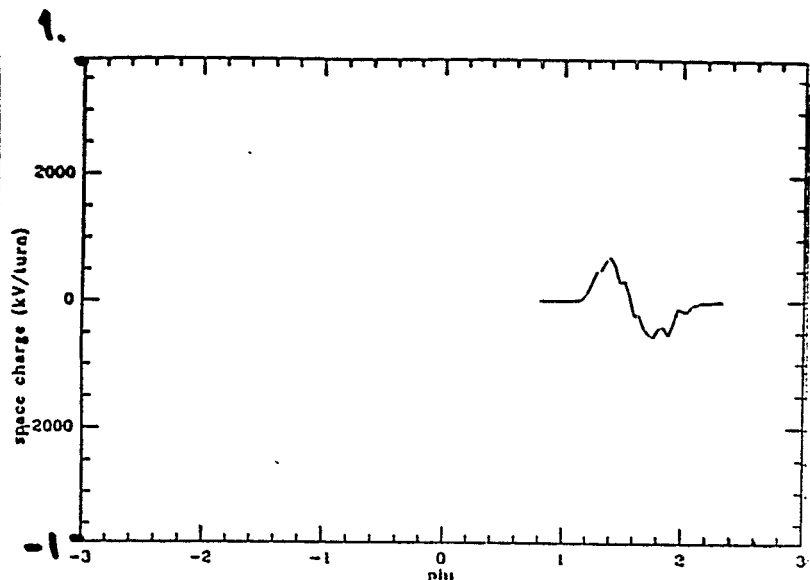
$$\hat{V} = 100 \text{ kV}$$

$$\sin \phi_s = 0.48$$

Space charge voltage

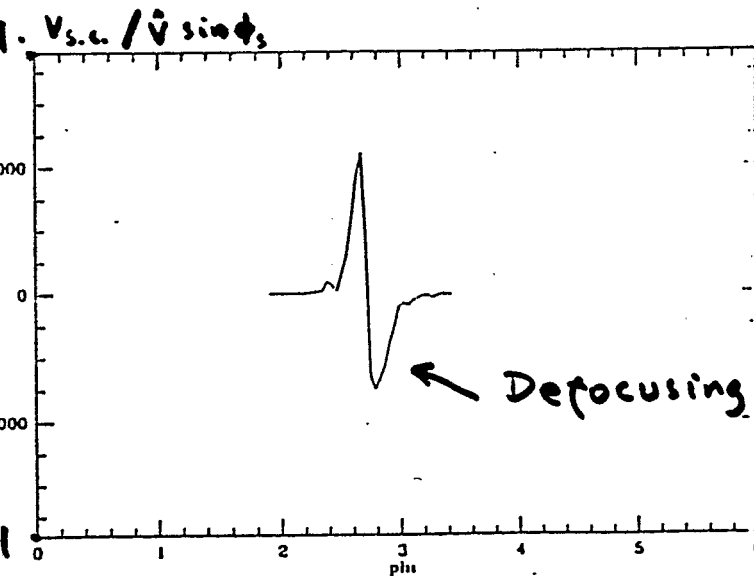


-80 ms

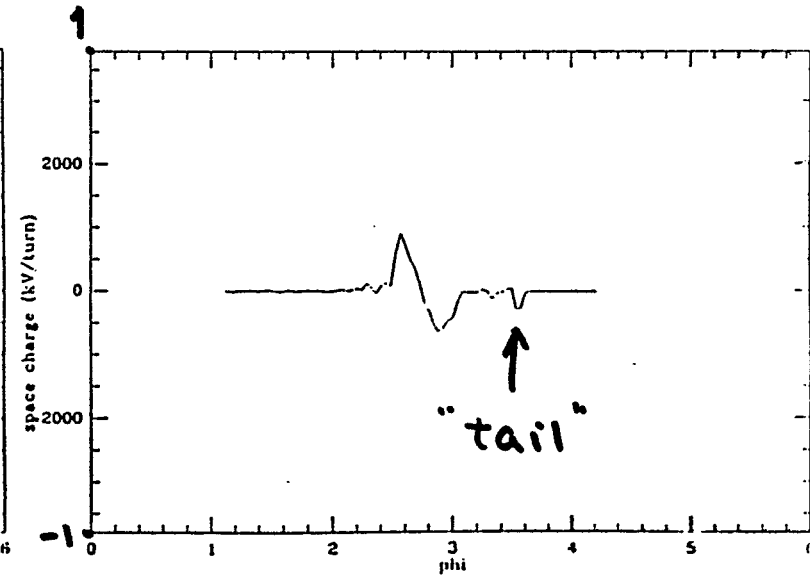


0 ms

$$\frac{Z}{h} = 1.2 \Omega, \text{ Capacitive}$$



$+80 \text{ ms}$

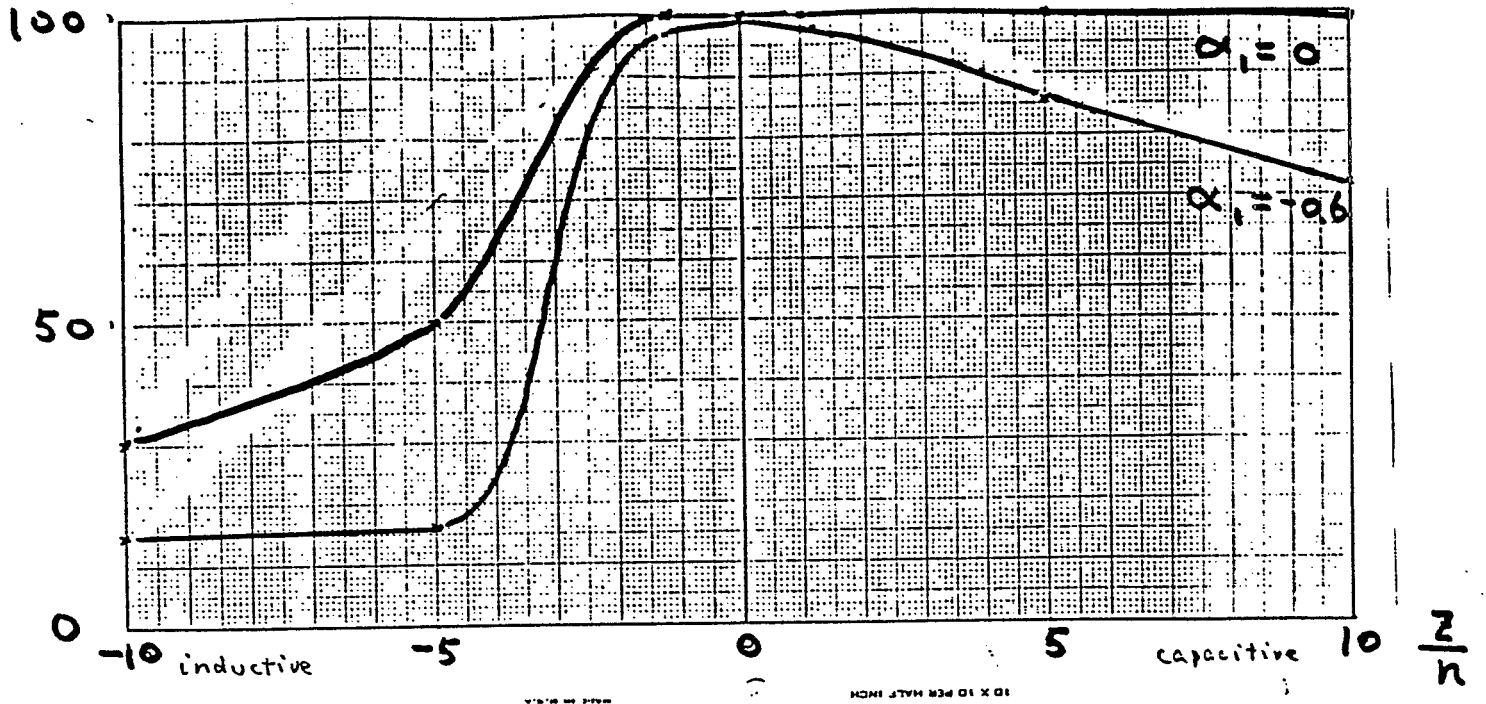


$+230 \text{ ms}$

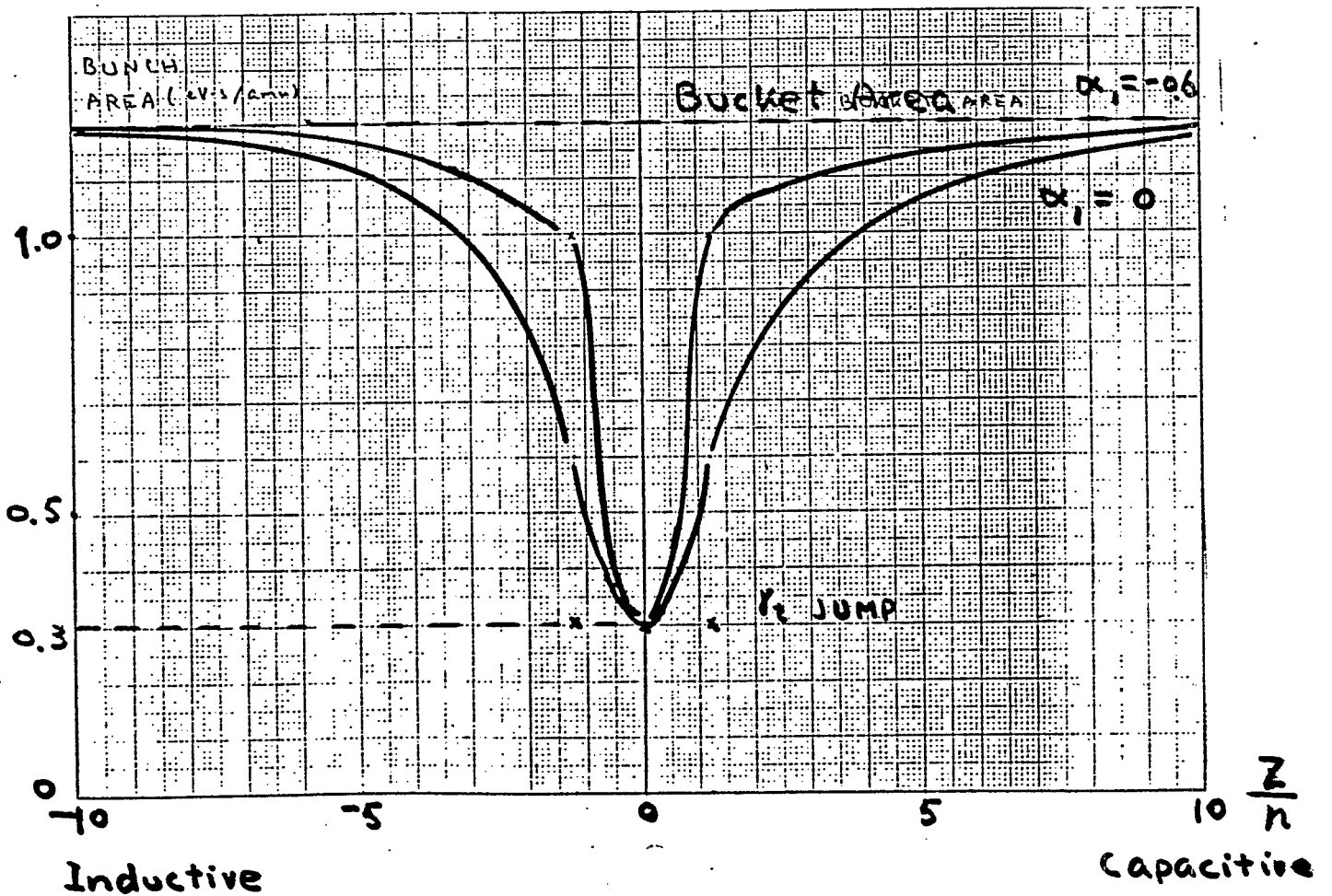
Defocusing space charge force

Crossing Efficiency (%)

A_u^{199} . RHIC
 $\hat{V}_1 = 100 \text{ kV}$
 $\sin \phi_s = 0.48$



Bunch Area After Transition



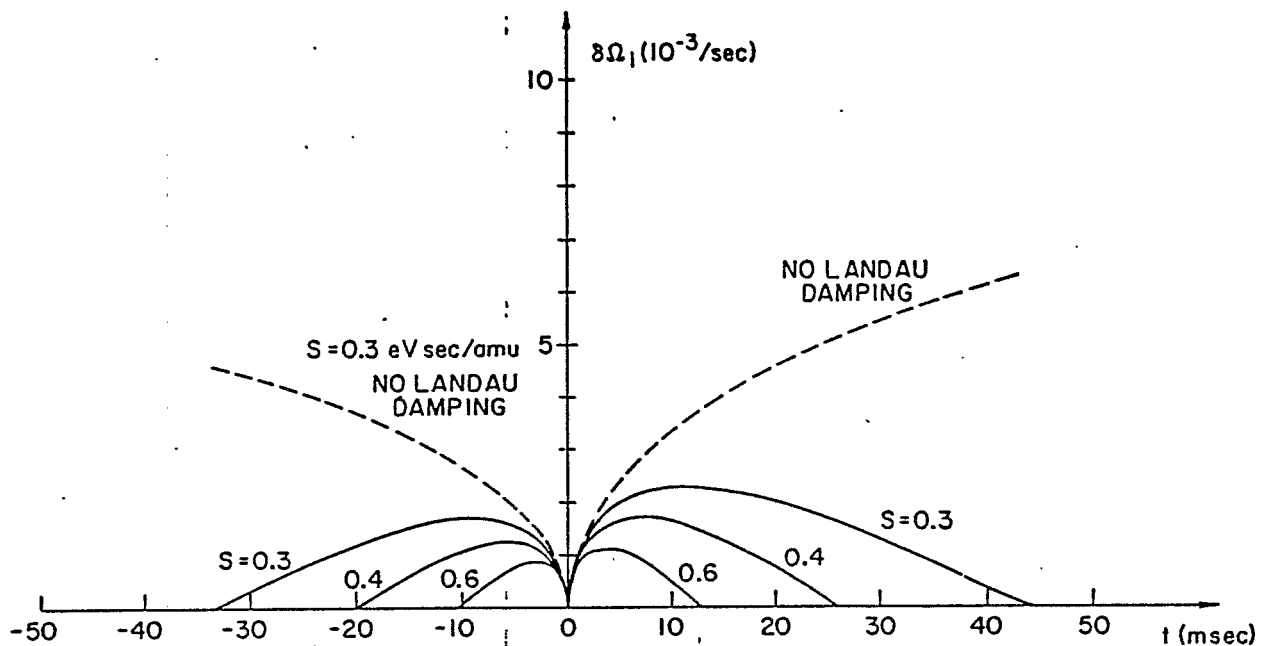


Fig. IV-19. The imaginary part of the microwave frequency, $\delta\Omega_i$, representing the growth rate of the instability, is plotted as function of time during crossing of the transition energy. The curves are calculated with the initial phase space as parameter. The dashed curve represents the growth rate without Landau damping.

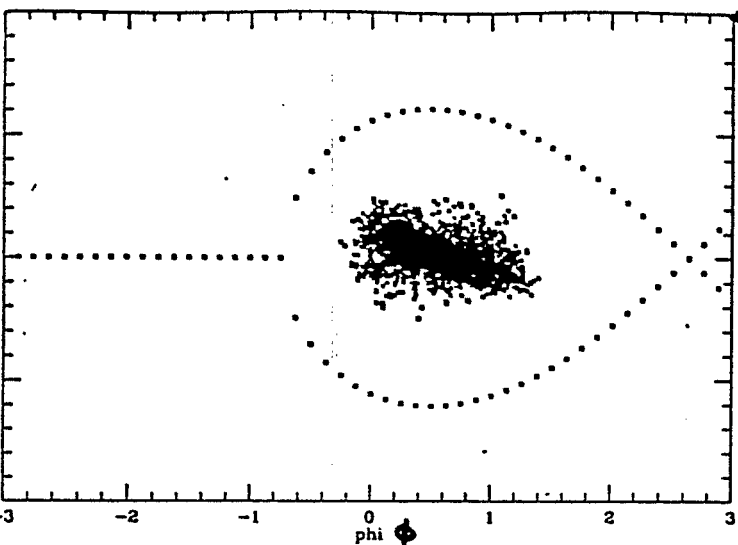
* Analytical solution based on
perturbation theory

$V = 100 \text{ kV}$
 $\sin \phi_s = 0.48$

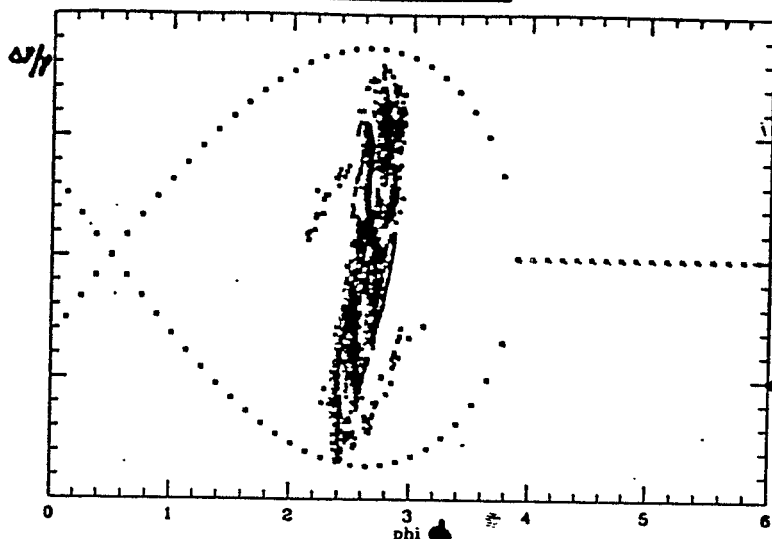
λ_c CROSSING

$Z/n = 10 \Omega$
CAPACITIVE

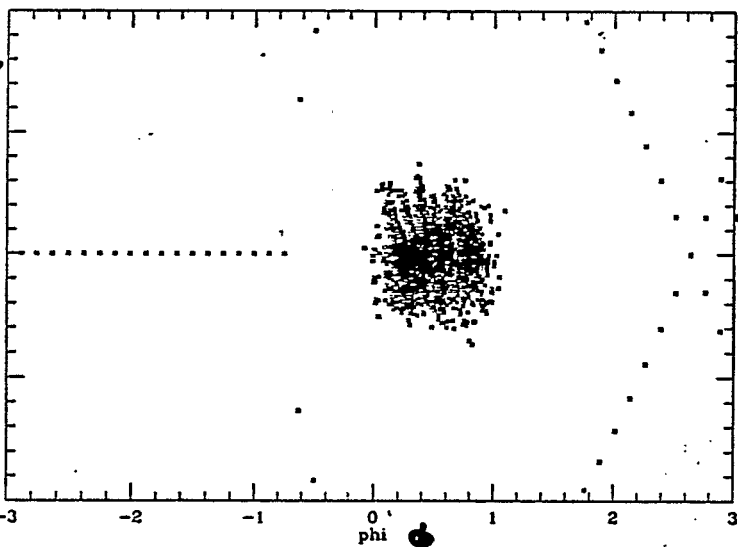
A_u^{+79}
 $\alpha_1 = 0$



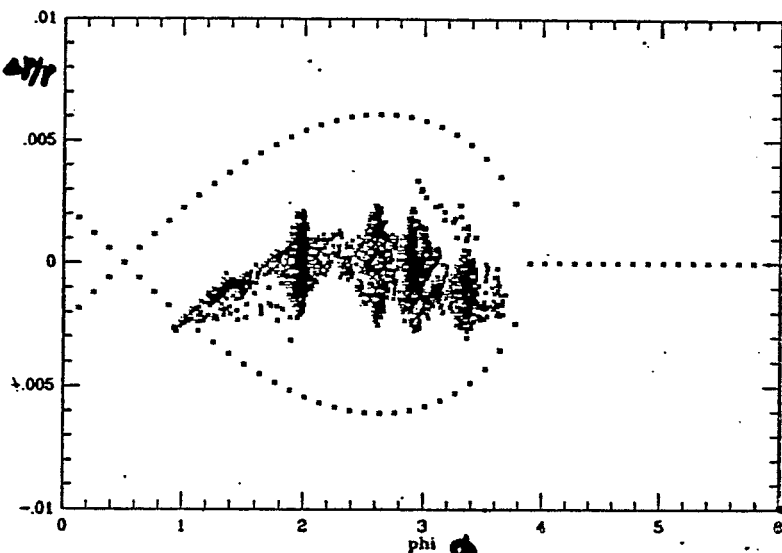
-160 ms



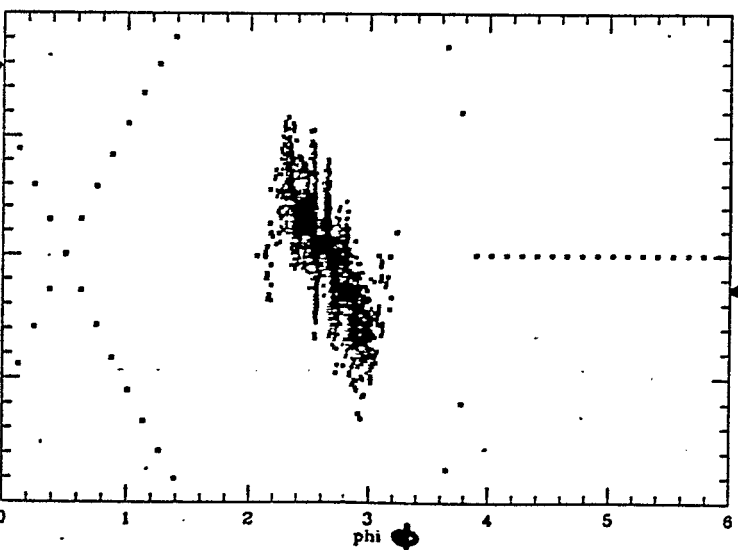
+80 ms



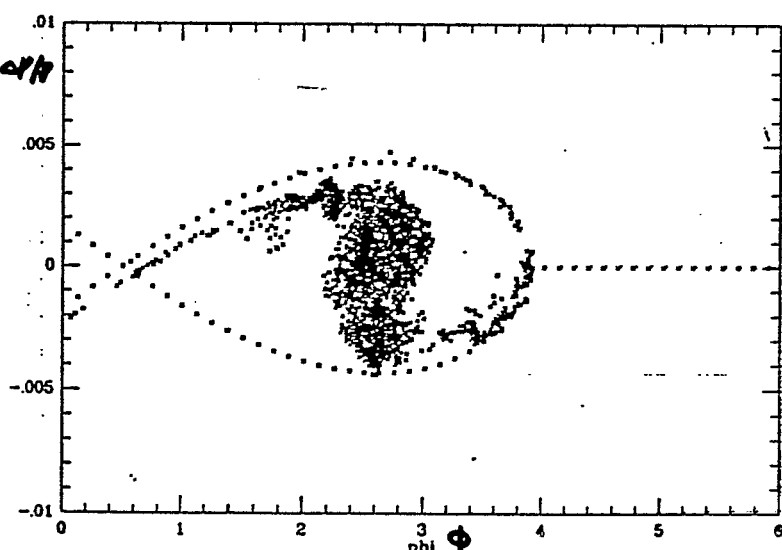
-25 ms



+160 ms



+25 ms

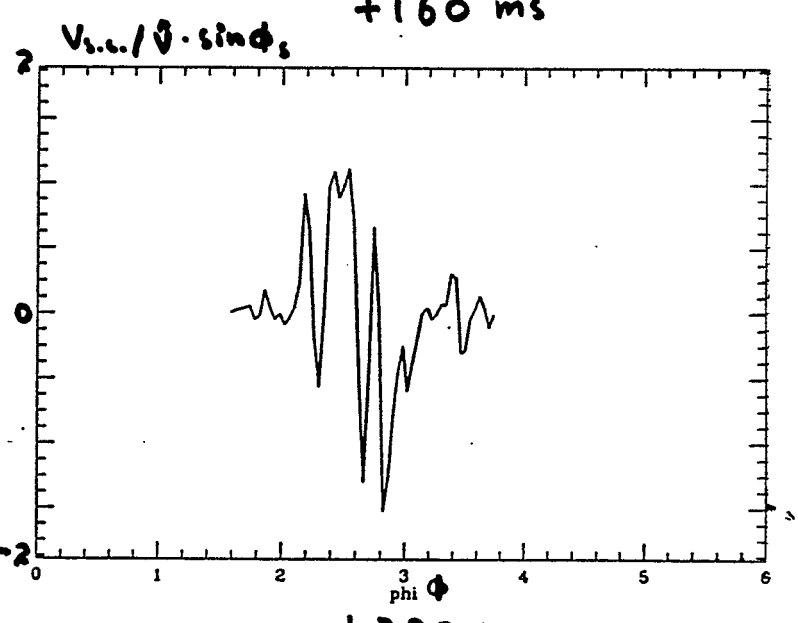
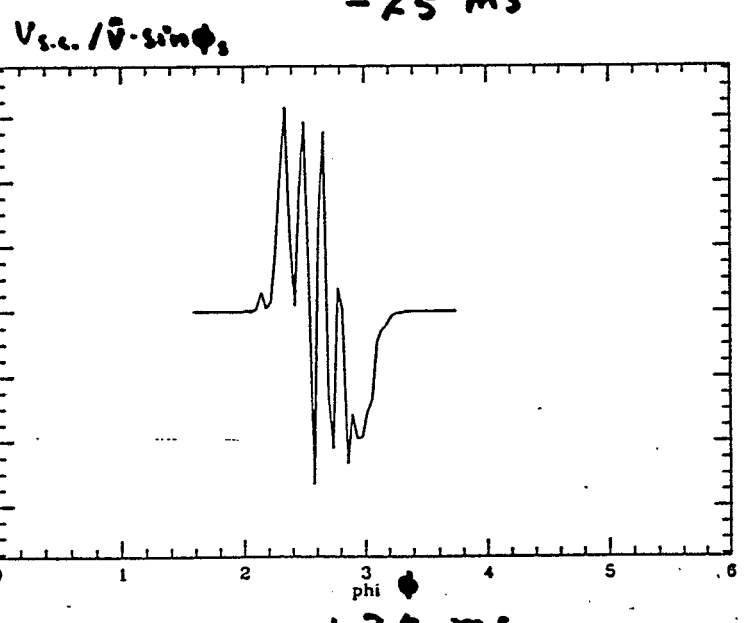
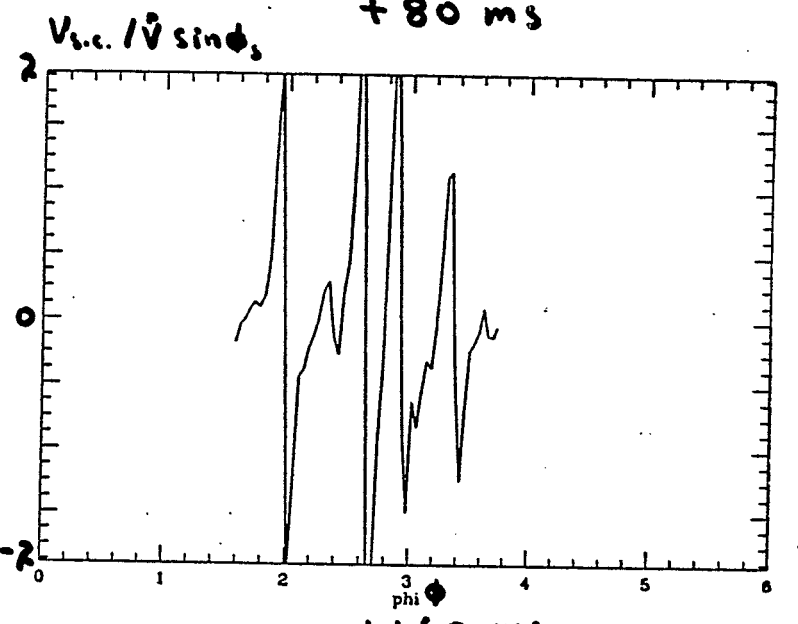
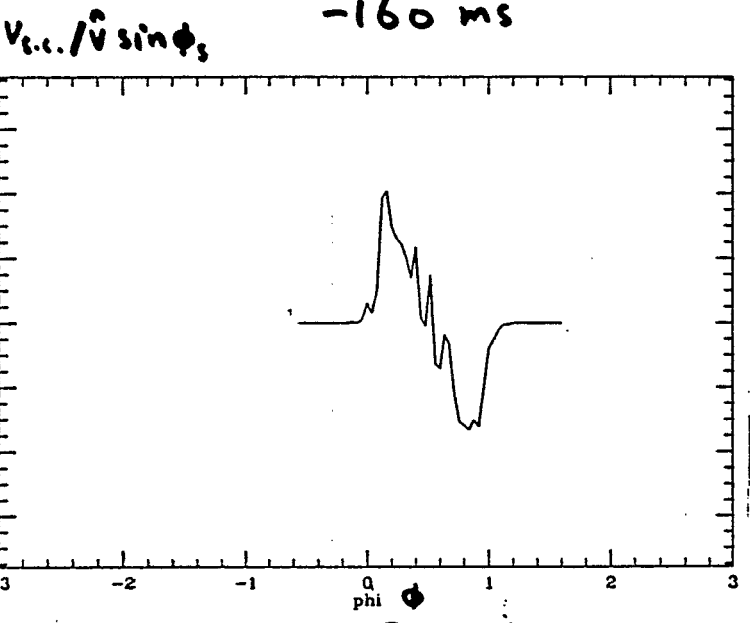
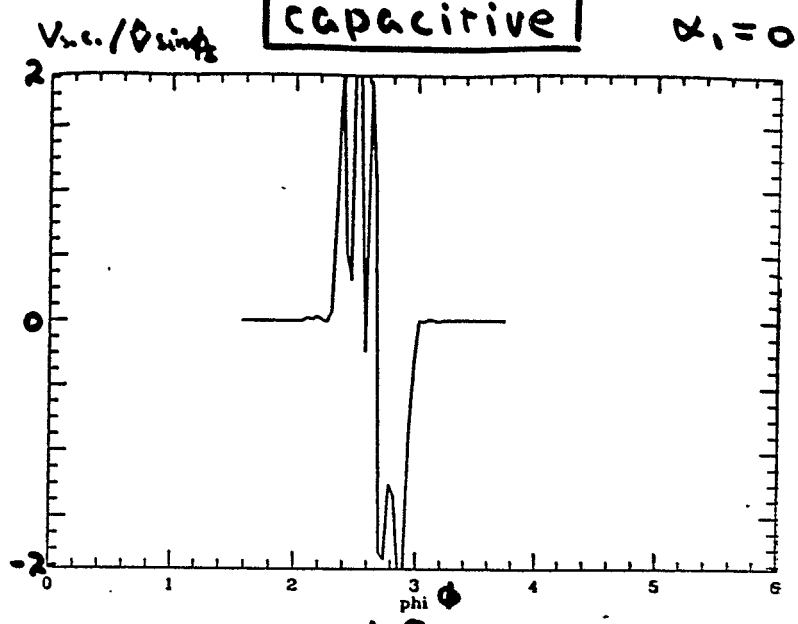
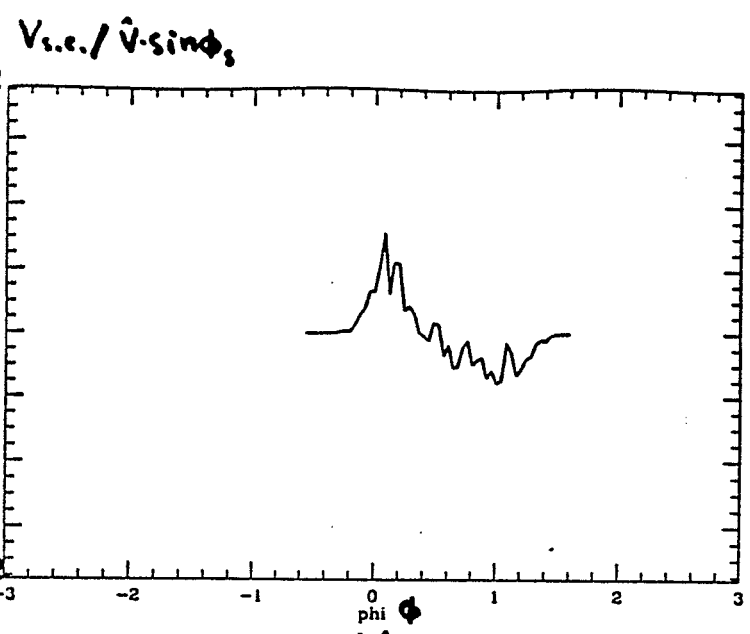


+320 ms

Efficiency 99%

$Z/n = 10 \Omega$
capacitive

Au^{+79}
 $\alpha_1 = 0$



-160 ms

+80 ms

-25 ms

+160 ms

+25 ms

+320 ms

To preserve the original $0.3 \text{ eV} \cdot \text{s}/\text{amu}$
& to minimize loss

- * γ_t jump, or
- * increase $\dot{\gamma}$ near transition
(without changing \dot{B}), or
- * make total $\frac{Z}{n} \sim 0$

• Resistive wall impedance
very very small effect

$10^2 * \left| \frac{Z}{n} \right|_{\text{stainless steel}} \Rightarrow \text{microwave instability}$

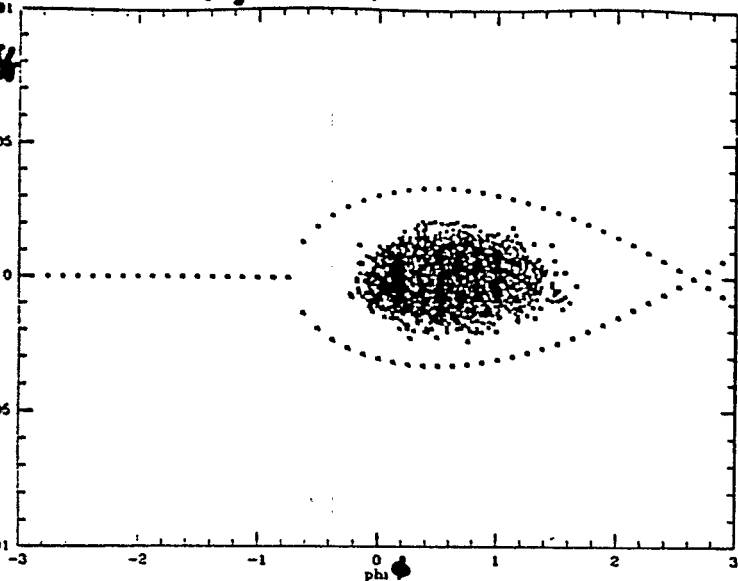
@ $\frac{Z}{n_c} \sim 5 \Omega$, $n_c = \frac{R_0}{b}$

$V = 100 \text{ kV}$
 $\sin \phi_s = 0.48$

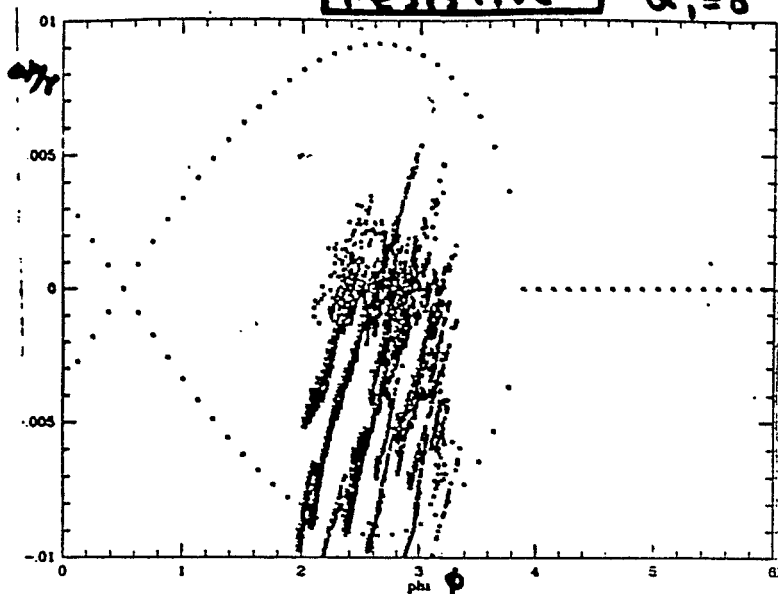
γ_s crossing

$R = 10^{-8} \Omega$
resistive

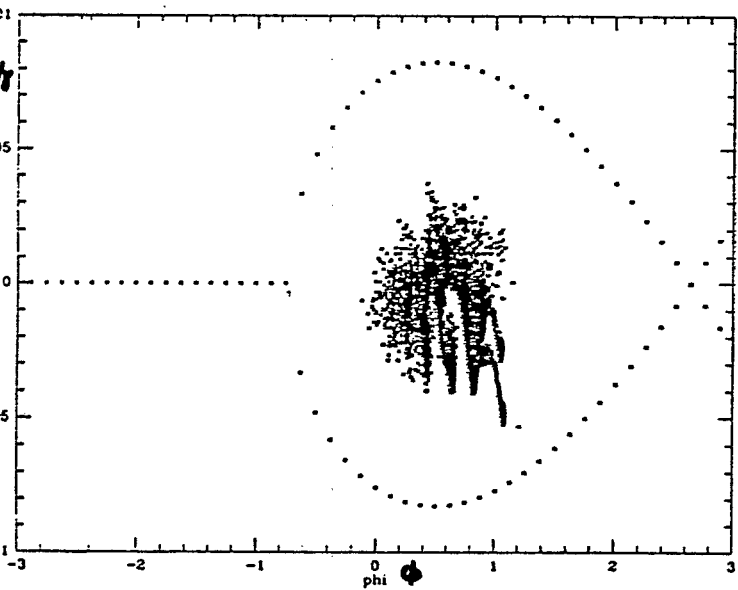
A_{in}''
 $\alpha_1 = 0$



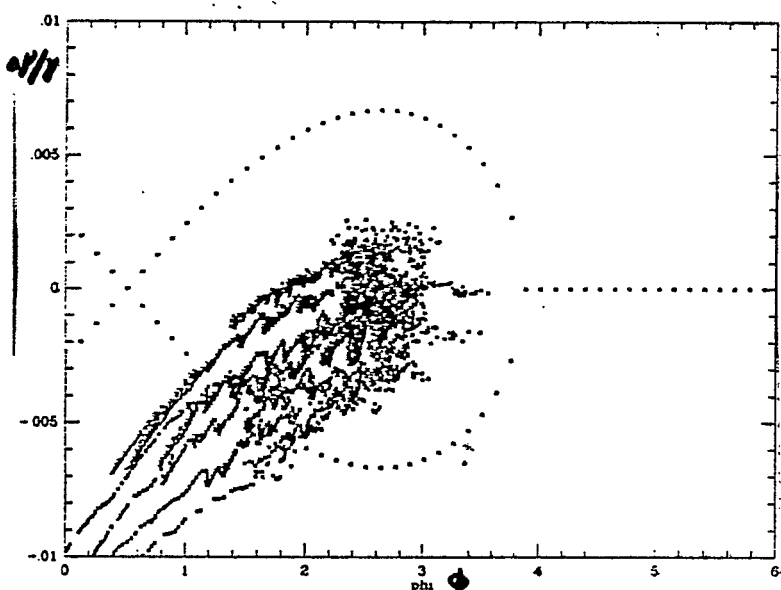
-520 ms



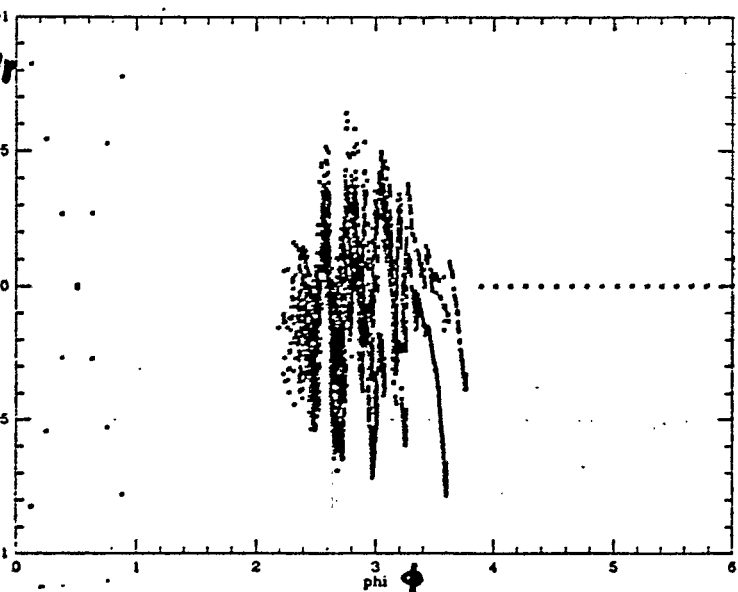
+100 ms



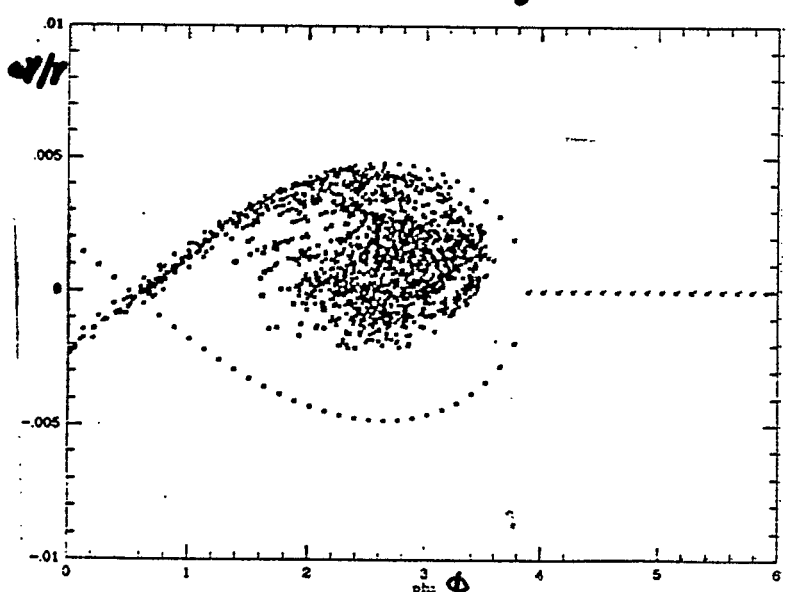
-85 ms



+165 ms



+40 ms

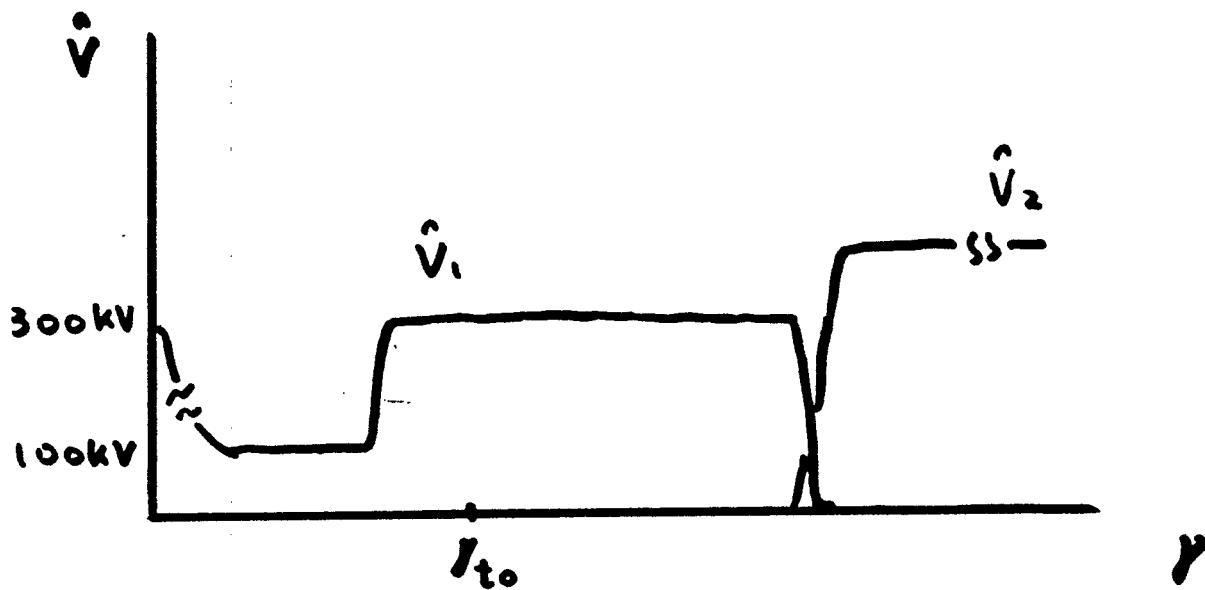
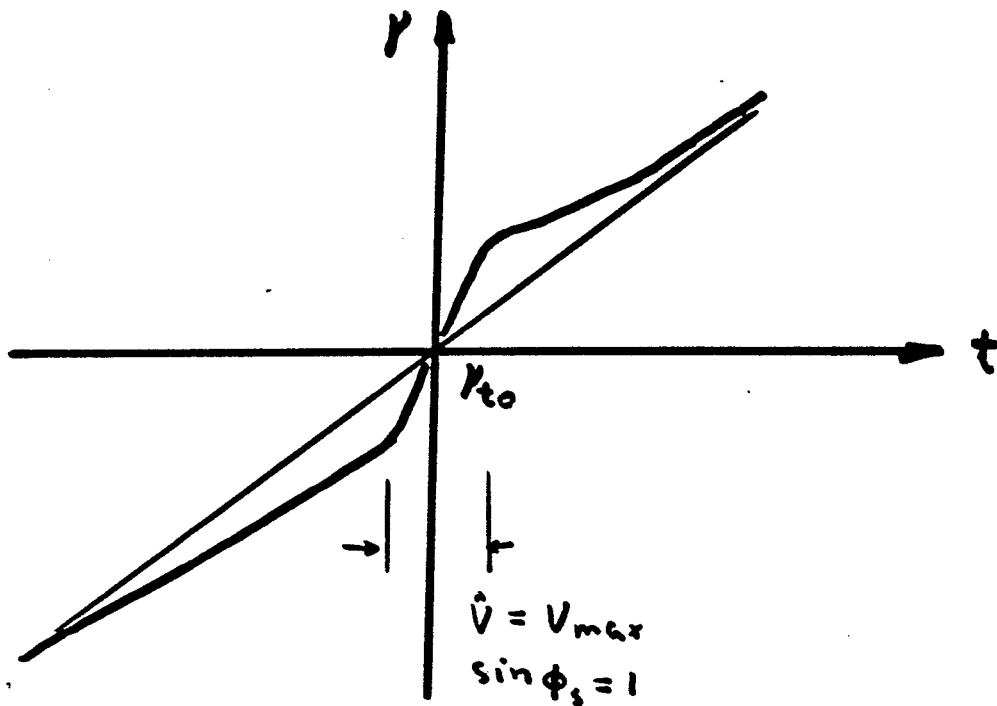


+290 ms

Efficiency 70%

7200 particles used.

$\dot{\gamma}$ increase crossing transition

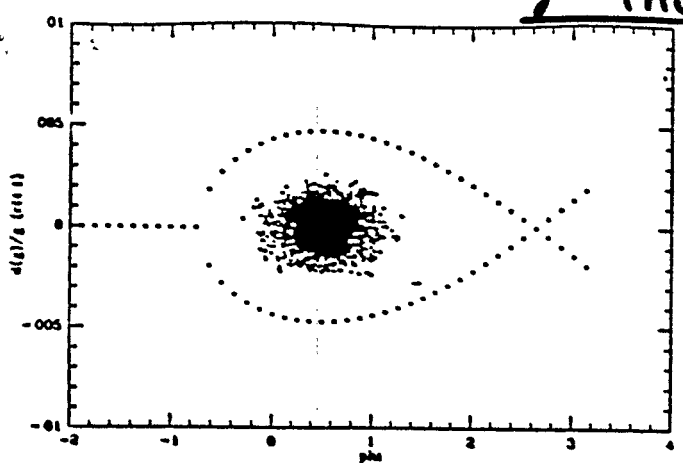


Total momentum aperture needed $\pm 0.8\%$

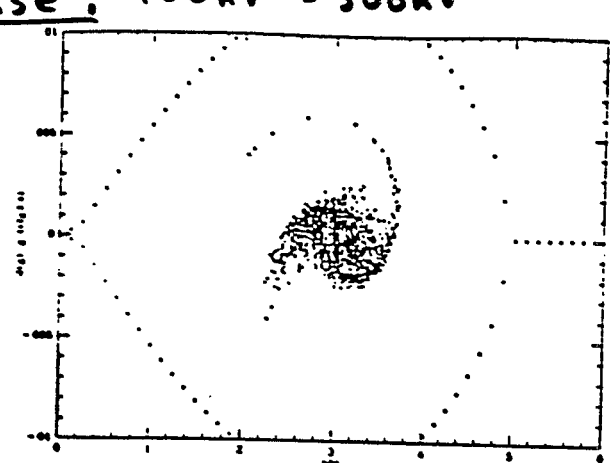
$\Delta\gamma = 0.38$ in 40 ms, used in simul.

INCREASE, 100KV - 300KV

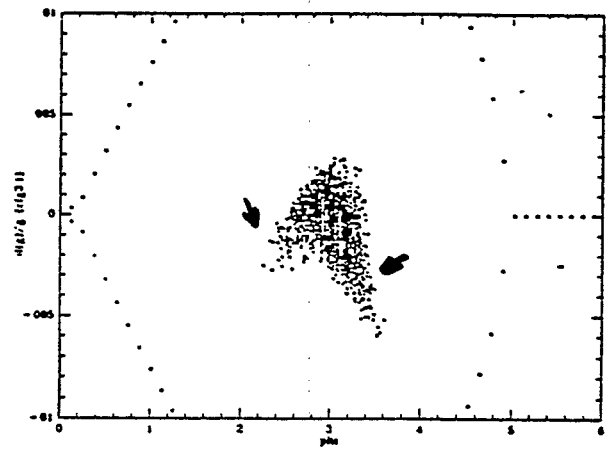
A_u^{+79}
0.3 eV-s/a
 $\dot{\gamma}$ increase
 $\alpha_1 = -0.6$
 $\frac{v}{c} = 1.2 \alpha$
Capac.



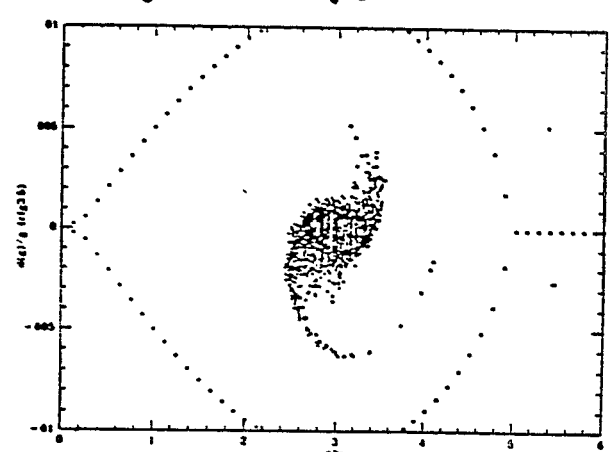
↓ -150 ms



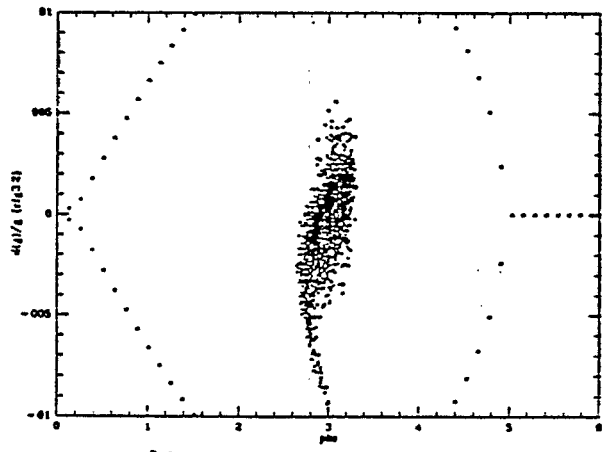
↓ +175 ms



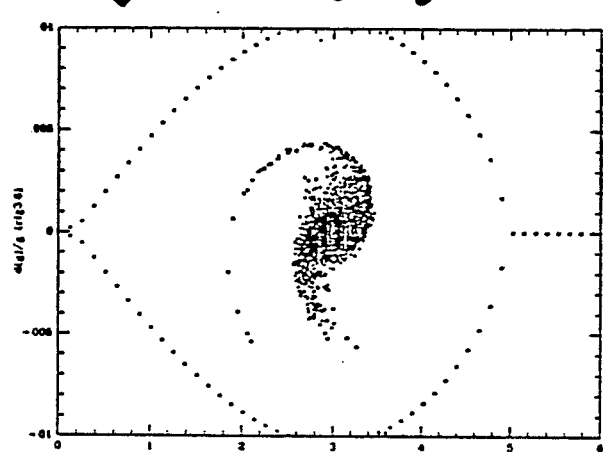
↓ +25 ms



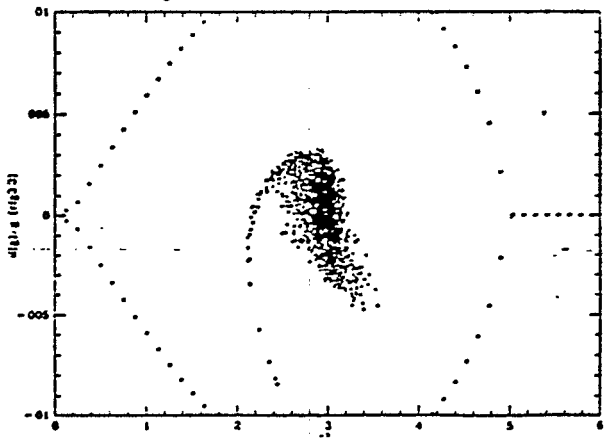
↓ +225 ms



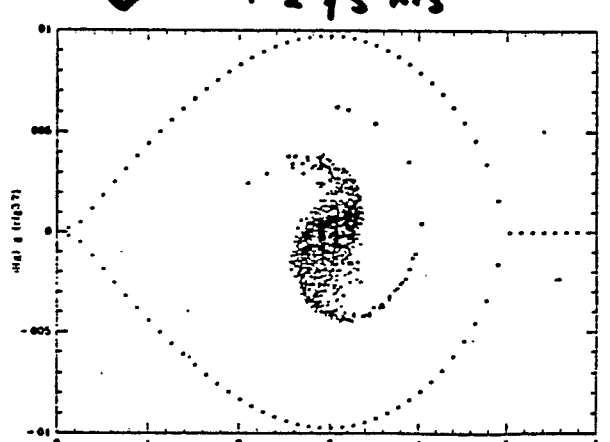
↓ +75 ms



↓ +275 ms



↓ +125 ms



+325 ms

Eff. 100%
0.3 ~ 0.4
eV-s/a

$$\hat{V} = 100 \text{ kV}$$

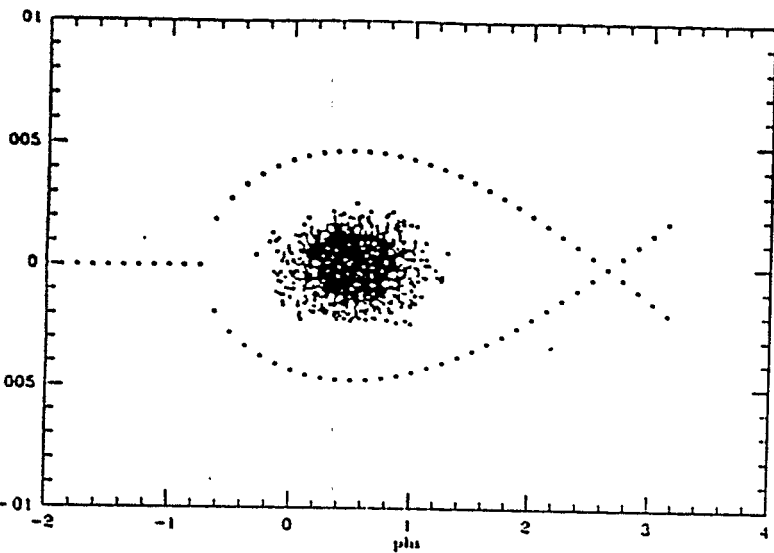
$$\sin \phi_s = 0.48$$

γ_t JUMP

$$\Delta \gamma_t = 0.6$$

in 60 ms

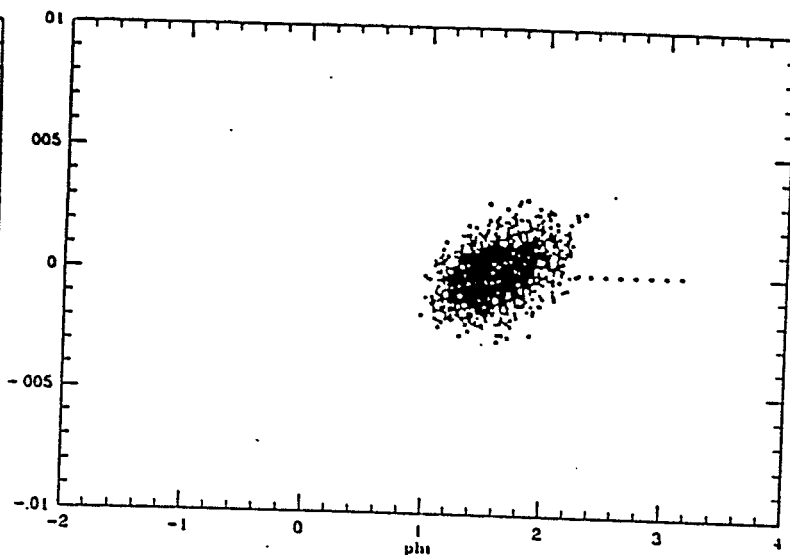
Au^{+79}



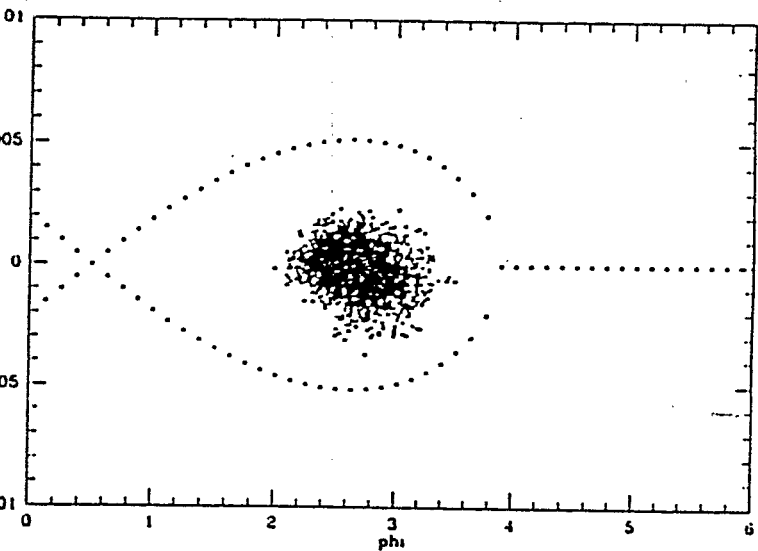
- 80 ms

$$0.3 \text{ eV} \cdot \text{s} / \text{amu}$$

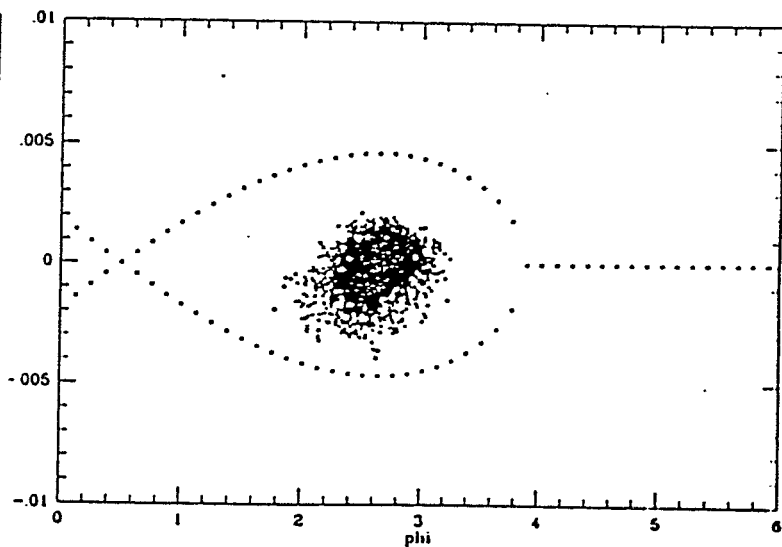
$$\frac{Z}{n} = 1.2 \Omega$$



0 ms



+ 25 ms



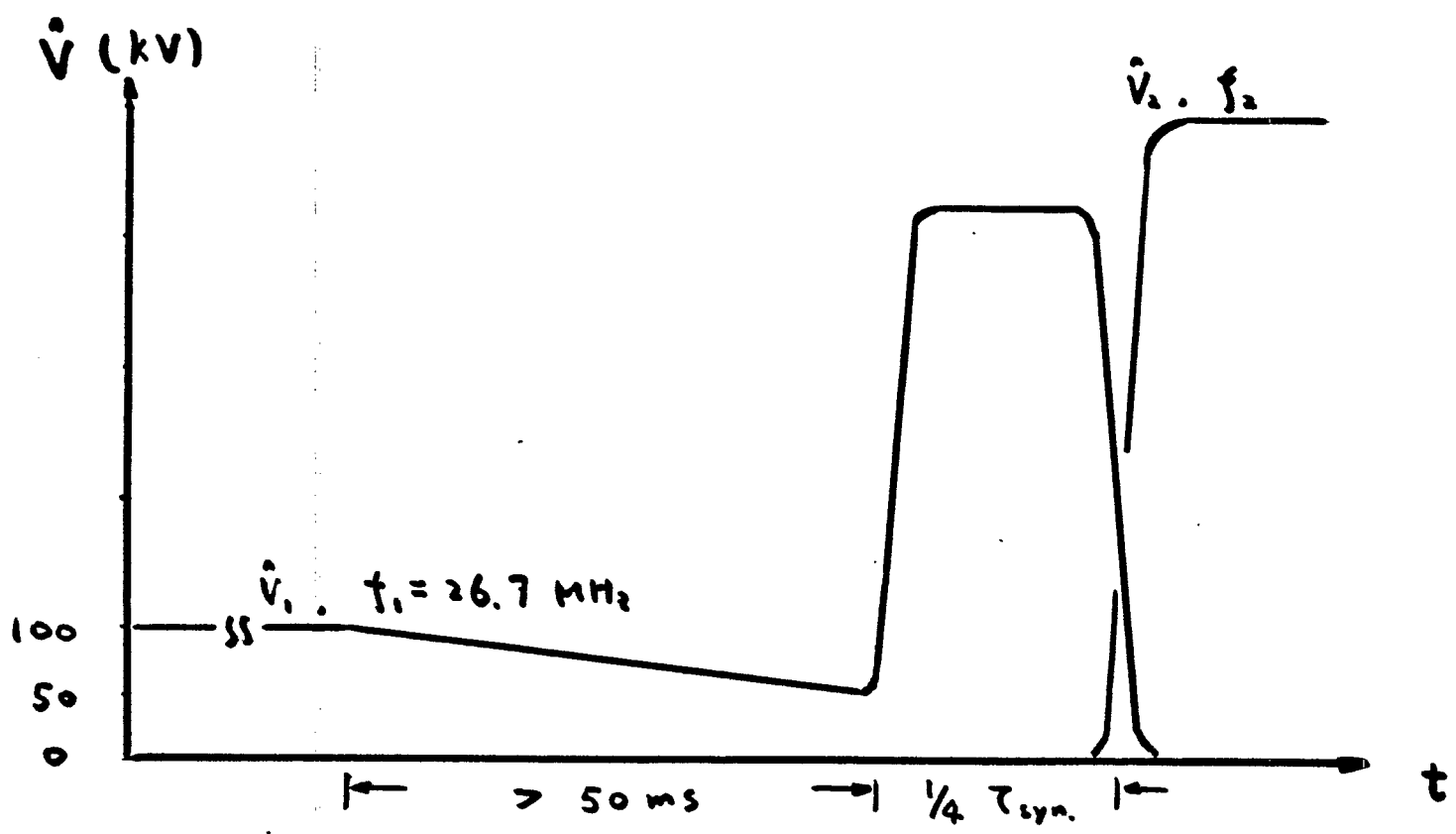
+ 80 ms

$\eta(\frac{\Delta p}{p})$ & s.c. included. No Loss

$\tau_0 \ll \tau_{n.l.} \ll \tau_c < \tau_{syn}$ Negligible Blow up

II. Transfer to High Freq. RF System

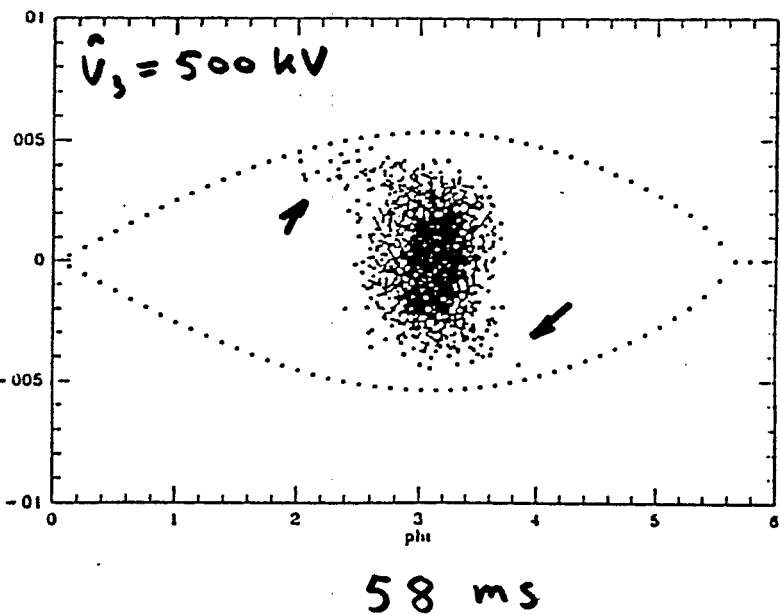
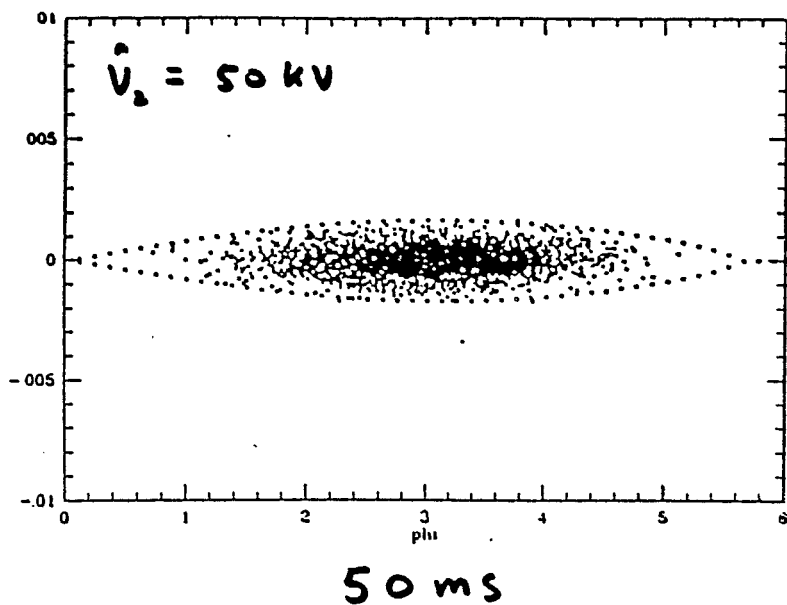
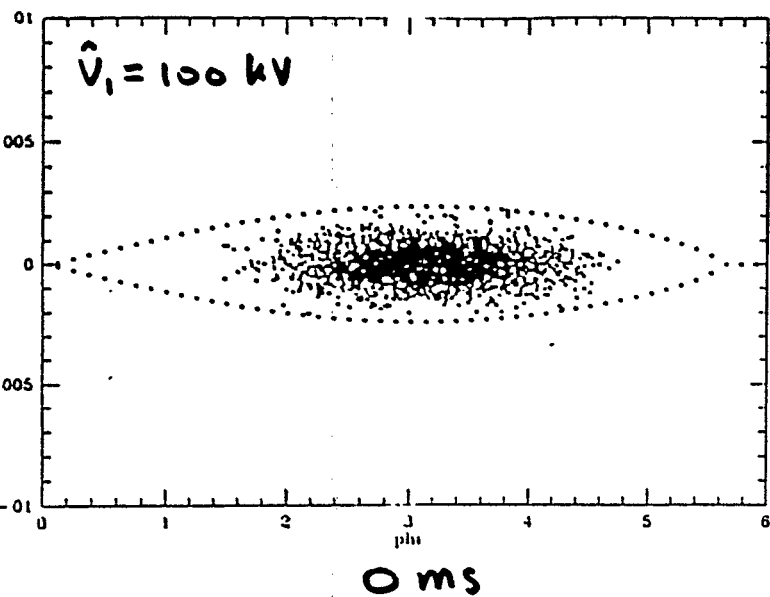
- 1 eV·s/amu bunch area. (general case)
Bunch rotation (squeeze $\frac{\Delta p}{p}$, rotate, recapture)
- 0.3 eV·s/amu bunch area
 - * Adiabatic compression ✓ A. G. Ruggiero
 - * "Simple" rotation ✓ E. Raka
 - * Unstable fixed point rotation ✓ S. T. Lee
J. Wei
- Switch over near transition
proposed by J. M. Brennan
 - ✓ , when combined with γ_t jump
or $\dot{\gamma}$ increase
- 160 MHz, tight bucket length



Bunch rotation

Bunch Rotation, 1 eV.s/a

A_u^{+79}



1 eV.s/amu

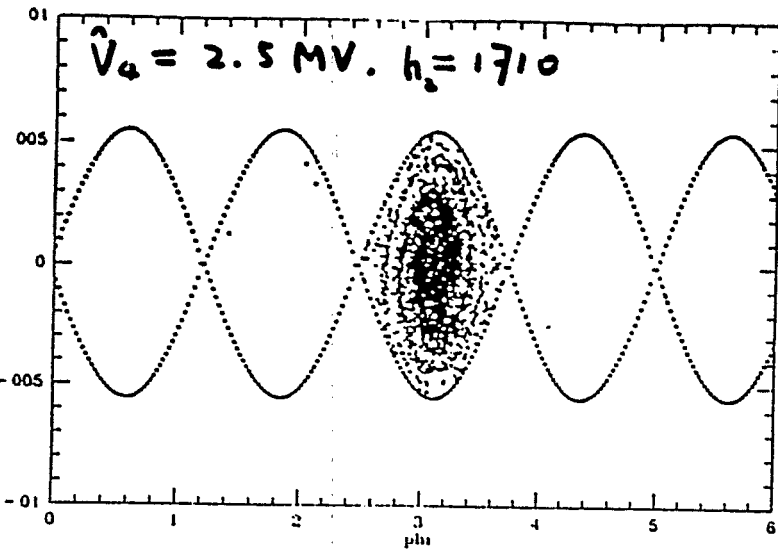
$\gamma_{\text{top}} = 30$

with s.c.

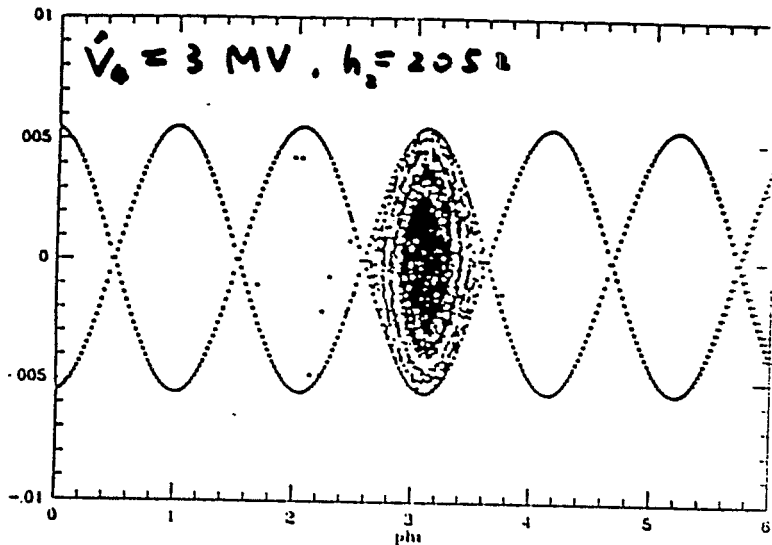
Bunch Rotation Recapture

A_u^{+79}

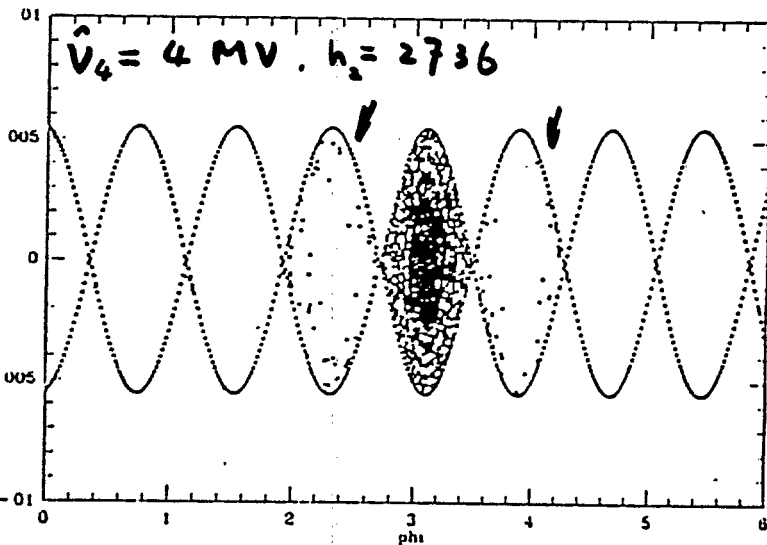
RHIC



134 MHz 98%
survival



160 MHz 95%
surv.



214 MHz 87%
surv.

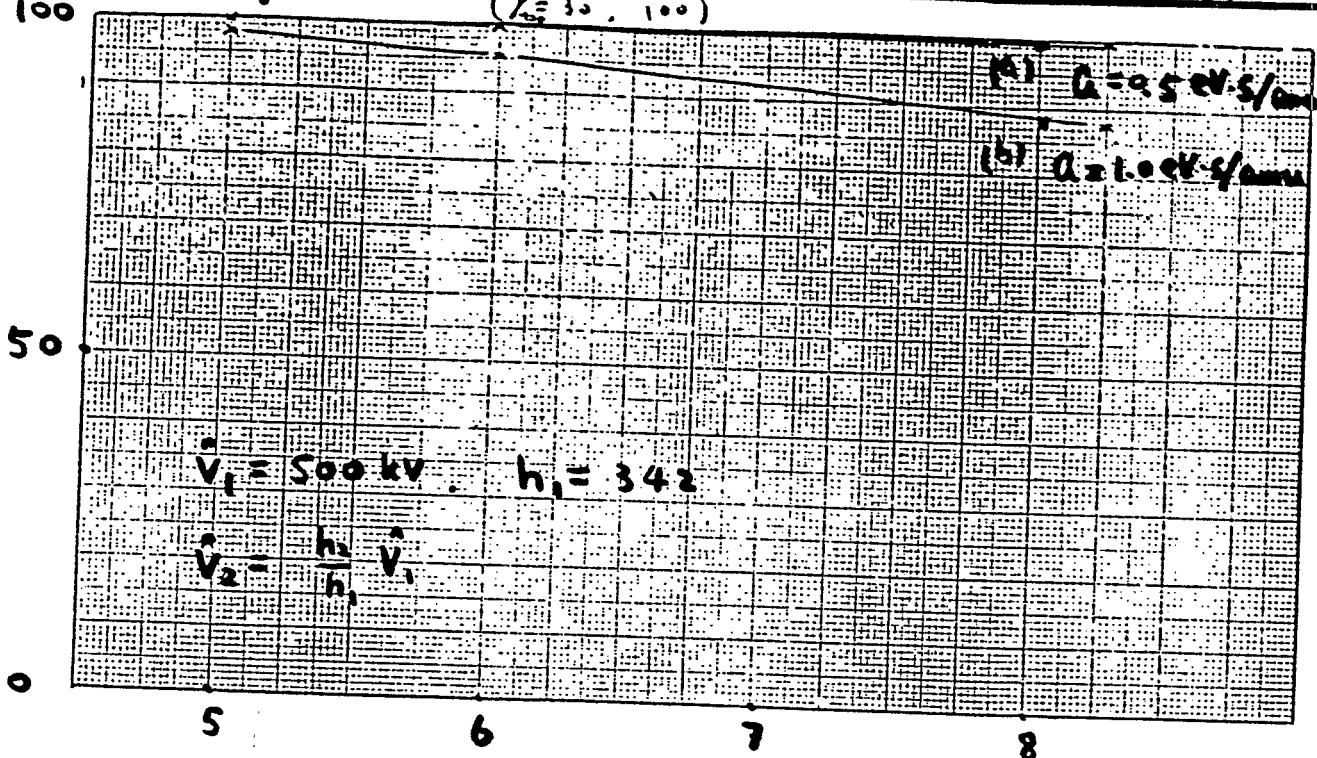
$$\hat{V}_3 = 500 \text{ kV}$$

$$\hat{V}_4 = \frac{h_2}{h_1} \hat{V}_3$$

$$h_1 = 342$$

$$(\text{At } \gamma_{\text{top}} = 30)$$

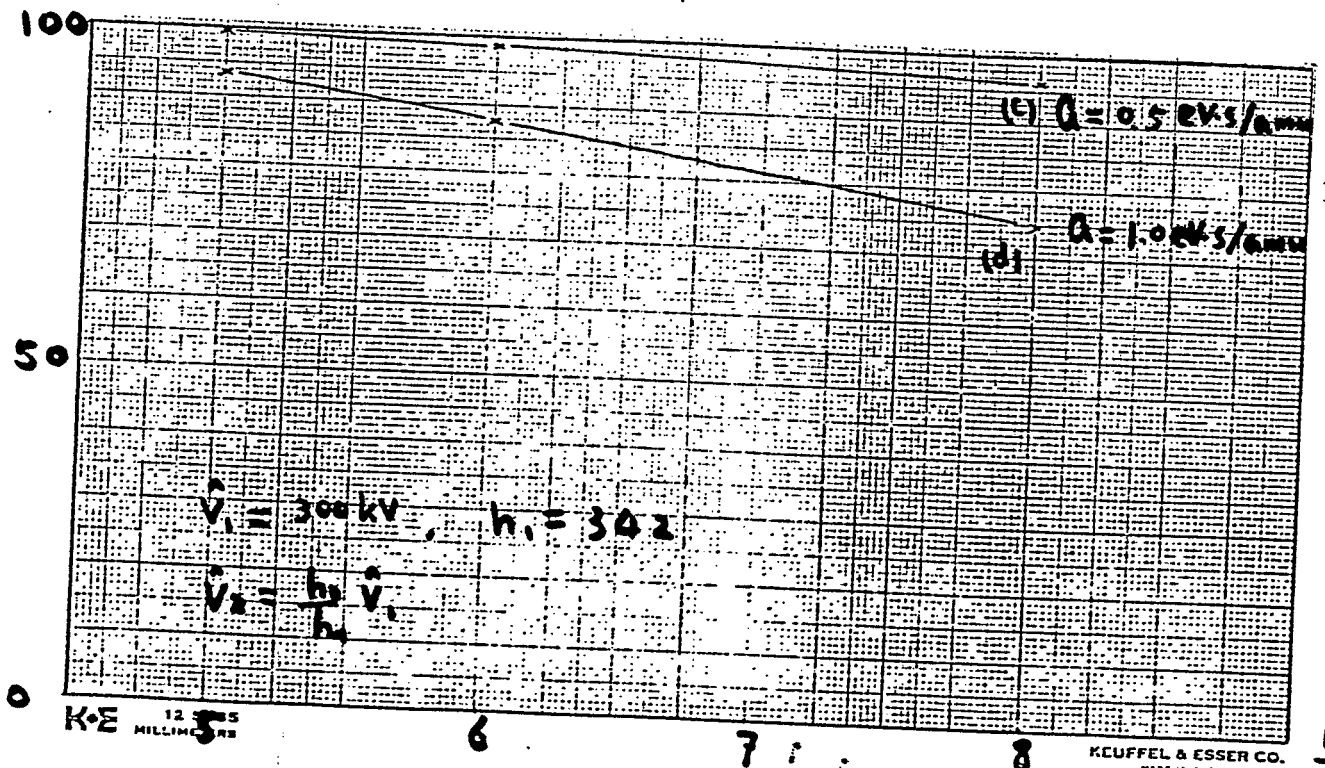
Efficiency (%) Top Energy Bunch Rotation Efficiency
(V₀ = 30, 100)



- a) $V_1: 100 \text{ kV} \rightarrow 15 \text{ kV} \rightarrow 500 \text{ kV}$
- b) $V_1: 100 \text{ kV} \rightarrow 50 \text{ kV} \rightarrow 500 \text{ kV}$

$\frac{h_2}{h_1}$

Efficiency (%)



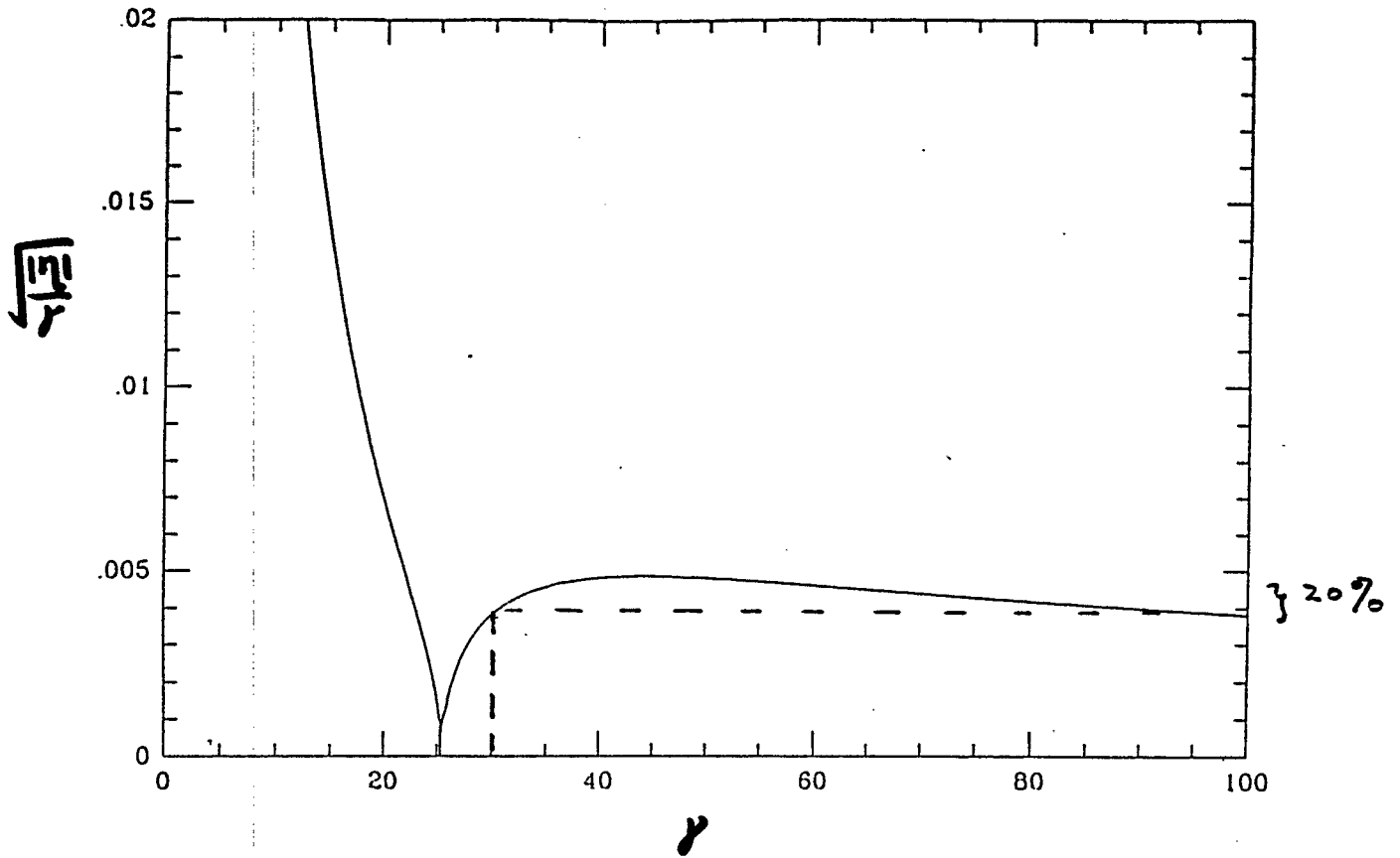
- c) $V_1: 100 \text{ kV} \rightarrow 15 \text{ kV} \rightarrow 300 \text{ kV}$
- d) $V_1: 100 \text{ kV} \rightarrow 50 \text{ kV} \rightarrow 300 \text{ kV}$

$\frac{h_2}{h_1}$

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Energy dependence of Rotation



$$\Omega_s \propto \sqrt{\frac{|n|}{\gamma}}$$

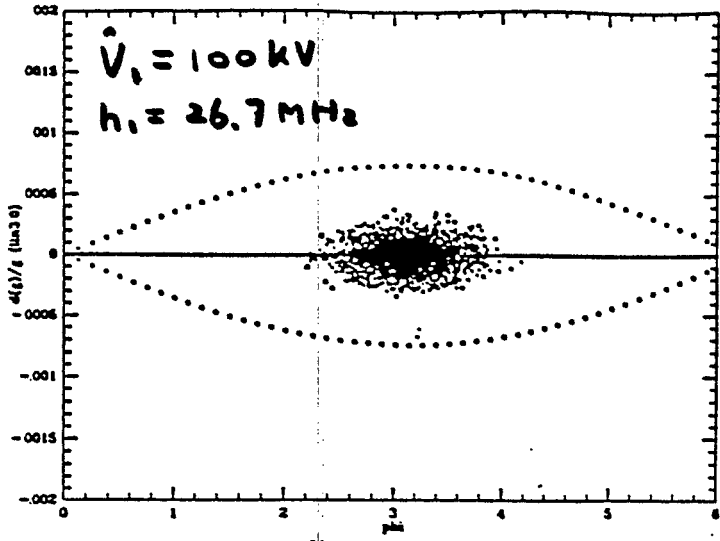
timing

$$\hat{V}_3 \propto \sqrt{\frac{|n|}{\gamma}}$$

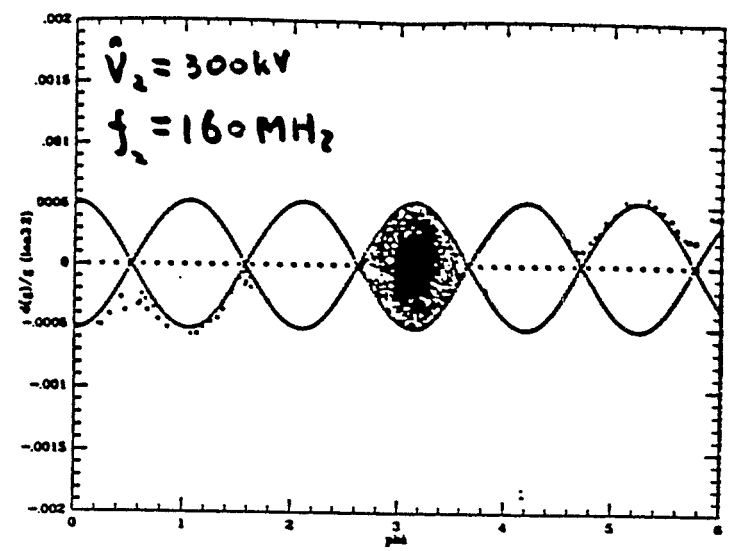
rotation voltage

0.3 eV·s/amu "Simple" rotation

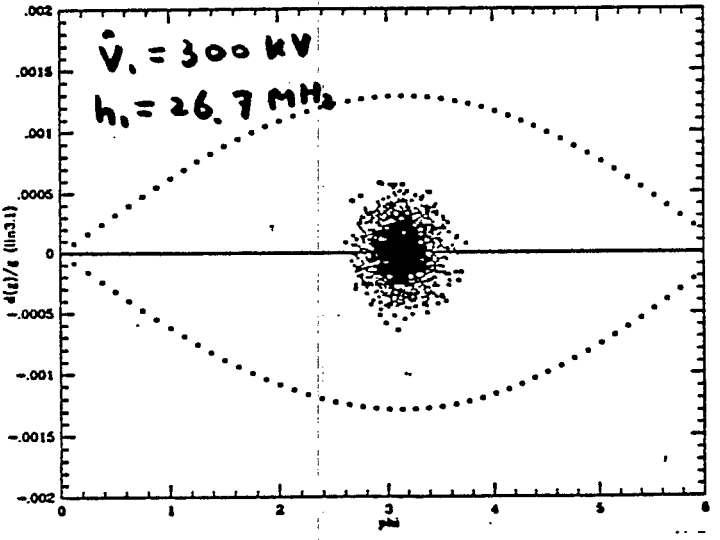
Au⁷⁹
RHIC



0 ms



36 ms

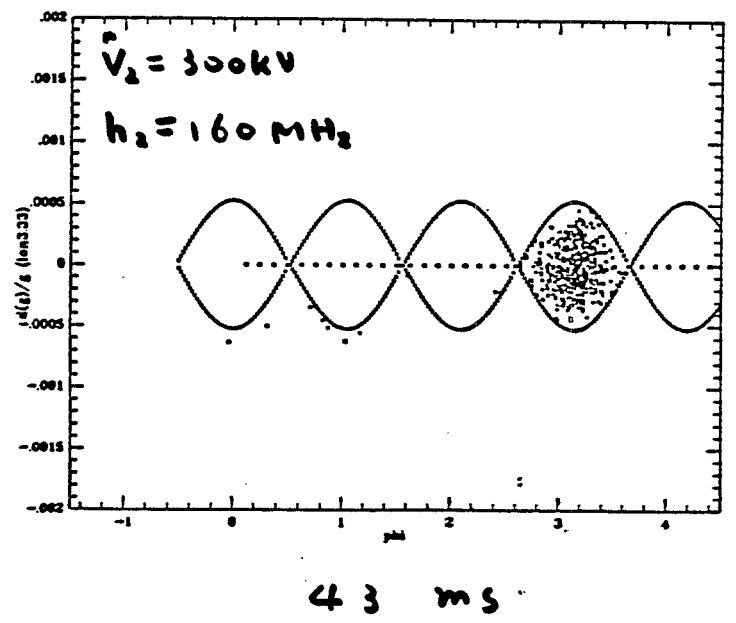
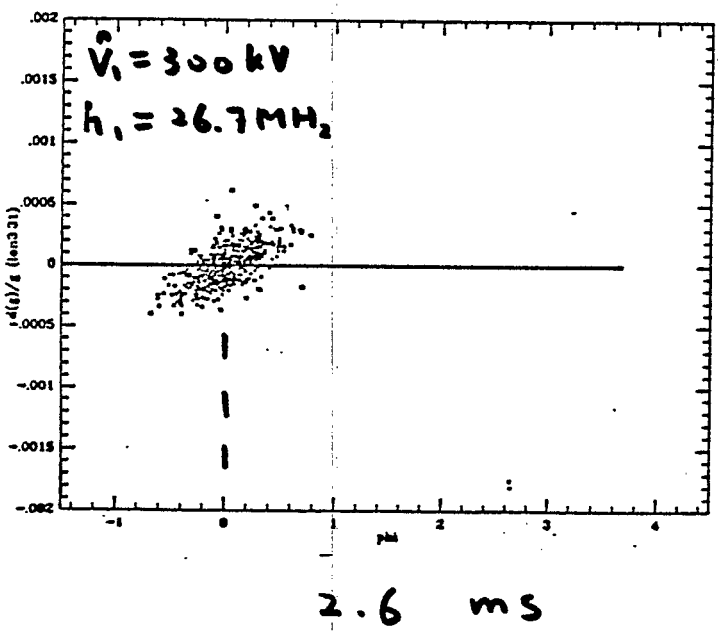
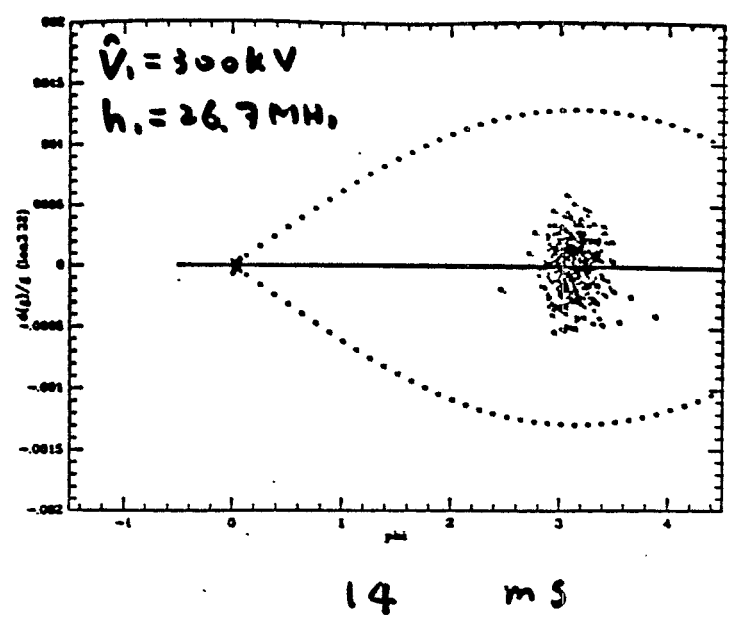
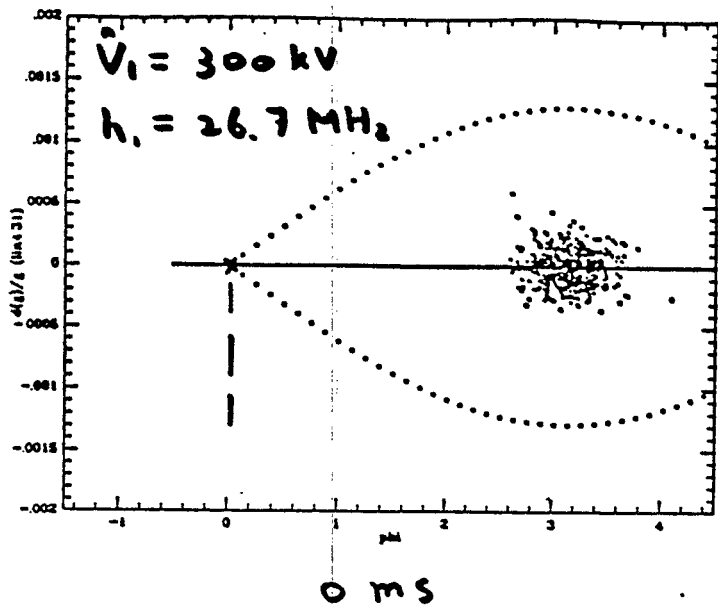


10 ms

98% in 0.39 eV·s/amu
bucket

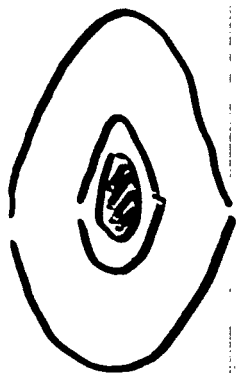
(At $\gamma_{\text{top}} = 100$)

Rotation by using unstable fixed point Au^{+79} RHIC



98% in $0.39 \text{ eV} \cdot \text{s/amu}$
bucket
(At $\gamma_{\text{top}} = 100$)

When matched



$$\frac{\Delta P}{P} / \frac{\Delta \phi}{h} \sim \hat{v}^{1/2} \cdot h^{1/2} \cdot \eta^{-1/2}$$

||
const

$$\hat{v}_1 = 300 \text{ kV}$$

$$h_1 = 342$$

$$\hat{v}_2 = 300 \text{ kV}$$

$$h_2 = 6 * 342$$

A gain of $\frac{\hat{v}_2'}{\hat{v}_1} = 6$

AGS

$$\hat{v}_1 = 12 \text{ kV}$$

$$\Rightarrow \hat{v}_1' = 100 \text{ kV}$$

$$\frac{\hat{v}_1'}{\hat{v}_1} \sim 8$$

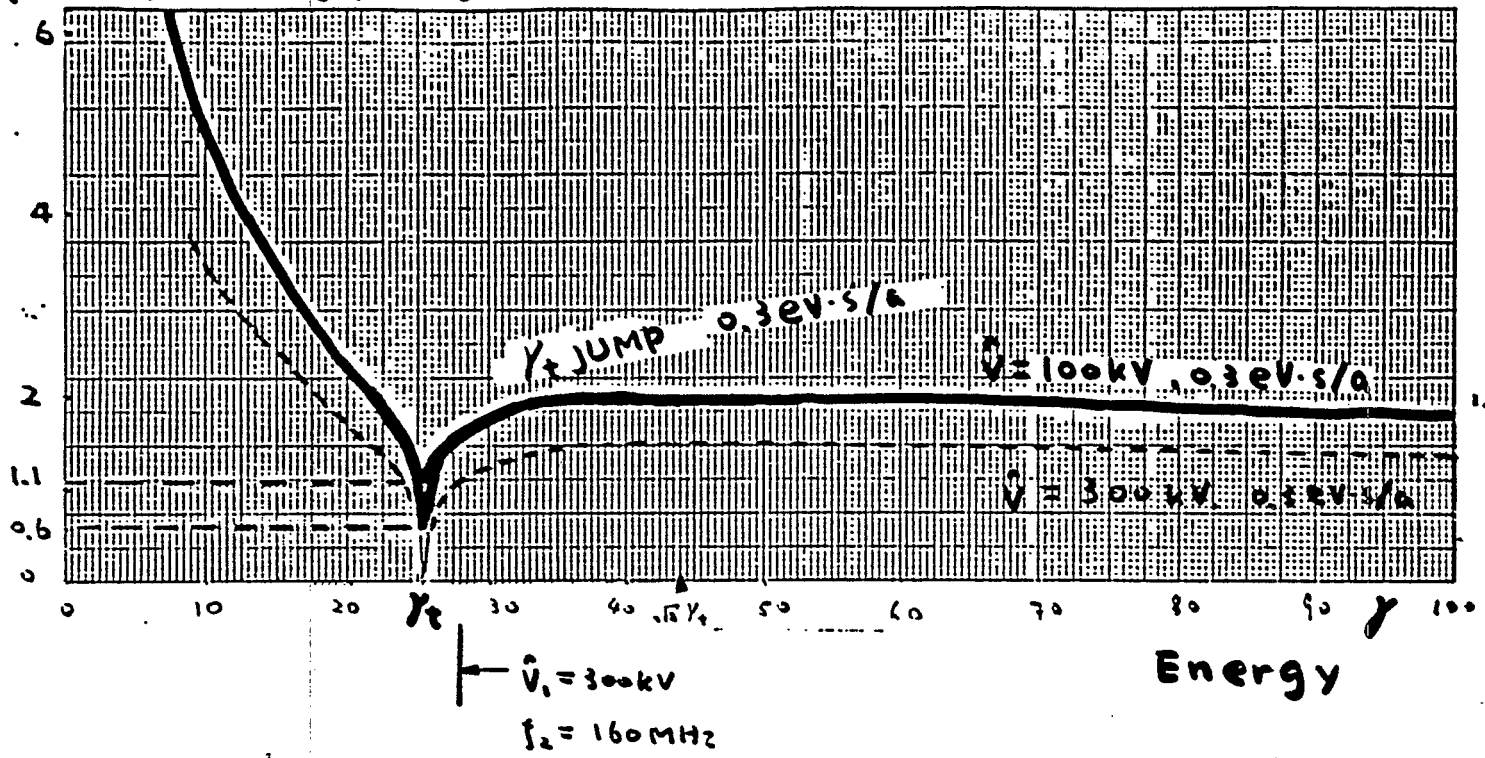
But

$$\Omega_s = \sqrt{\frac{2 e V h \eta c^3}{2 \pi A m_0 c^2 \cdot R_s^2 \cos \phi_s}}$$

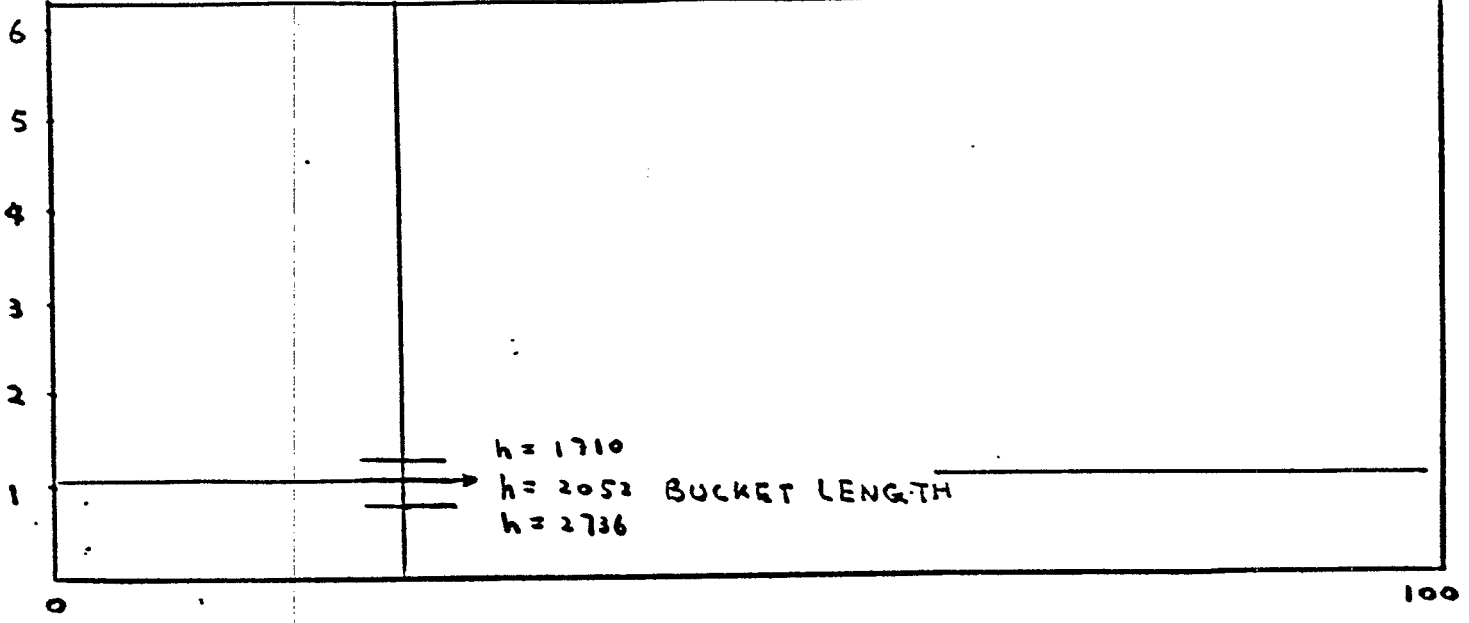
$$\Rightarrow \tau_{\text{syn}} = 5 \text{ ms} \quad ?$$

RHIC, $\tau_{\text{syn}} = 40 \text{ ms}$ (300 kV)

Bunch Length (rad. .26.7 MHz phase)

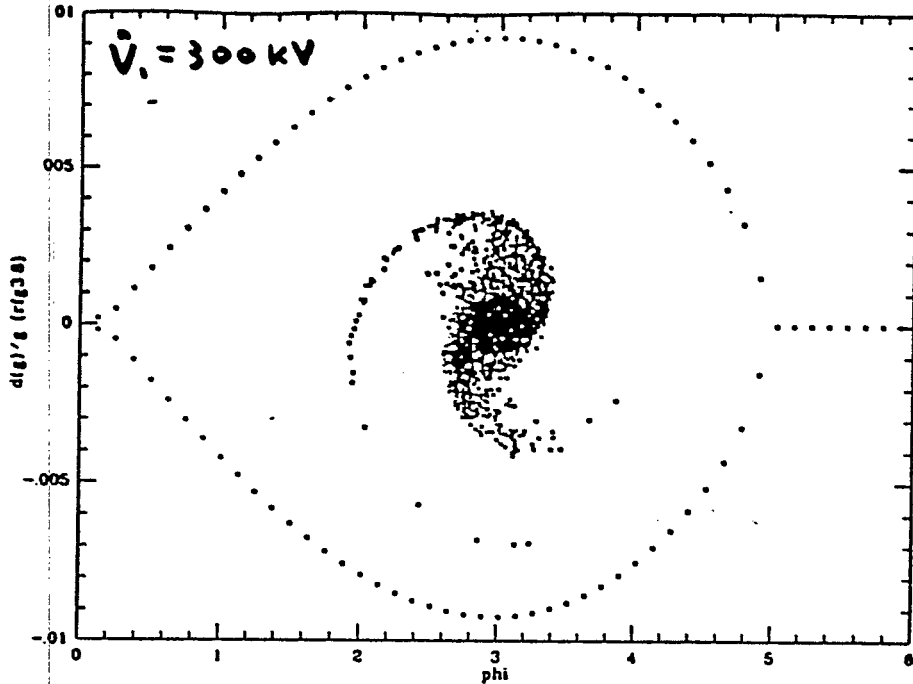


- Switch over near transition γ_t .

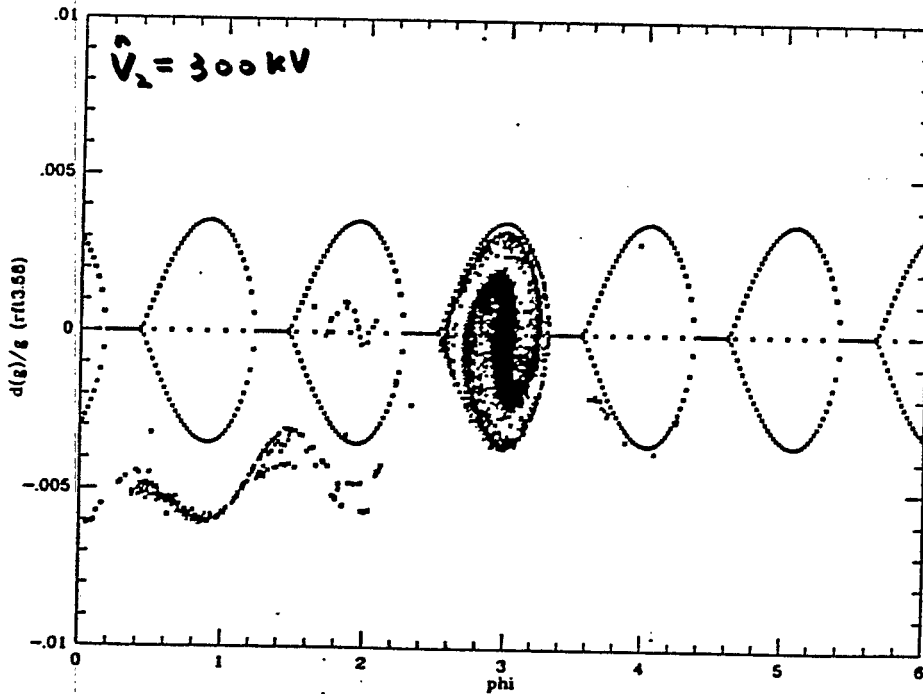


Switch over near transition

(after a β increase)



+ 375 ms



+ 440 ms

A_u^{+19}

$\gamma_{tr.} = 26.14$

$\gamma_{t_0} = 25.4376$

$\hat{V}_1 = 100 \sim 300 \text{ kV}$

$\hat{V}_2 = 300 \text{ kV}$

$h_1 = 342$

$h_2 = 2052$

$\alpha_1 = -0.6$

$\frac{2}{\pi} = 1.2 \Omega$

capac.

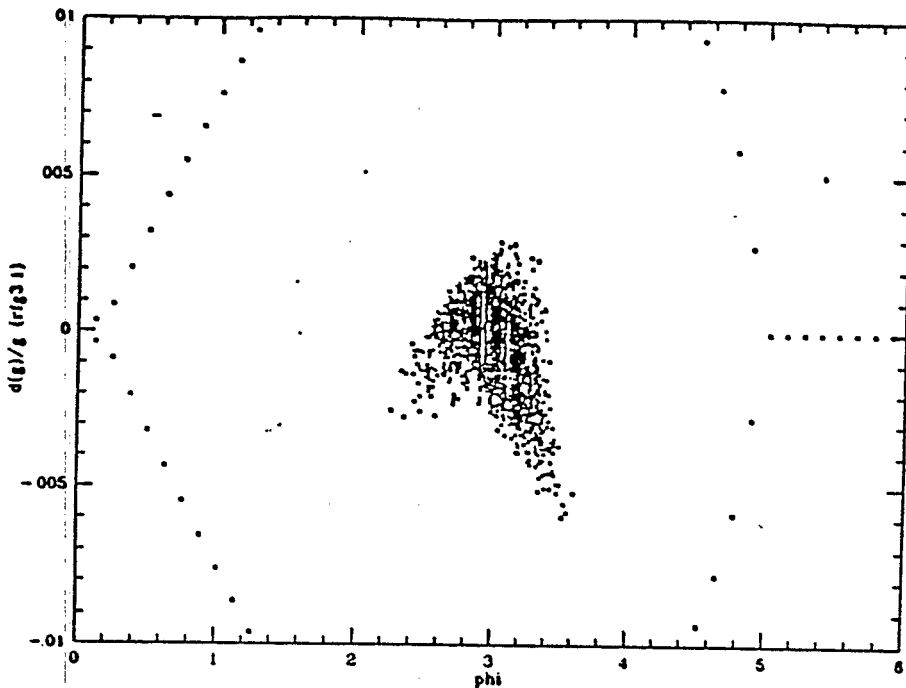
Transfer Efficiency

90%

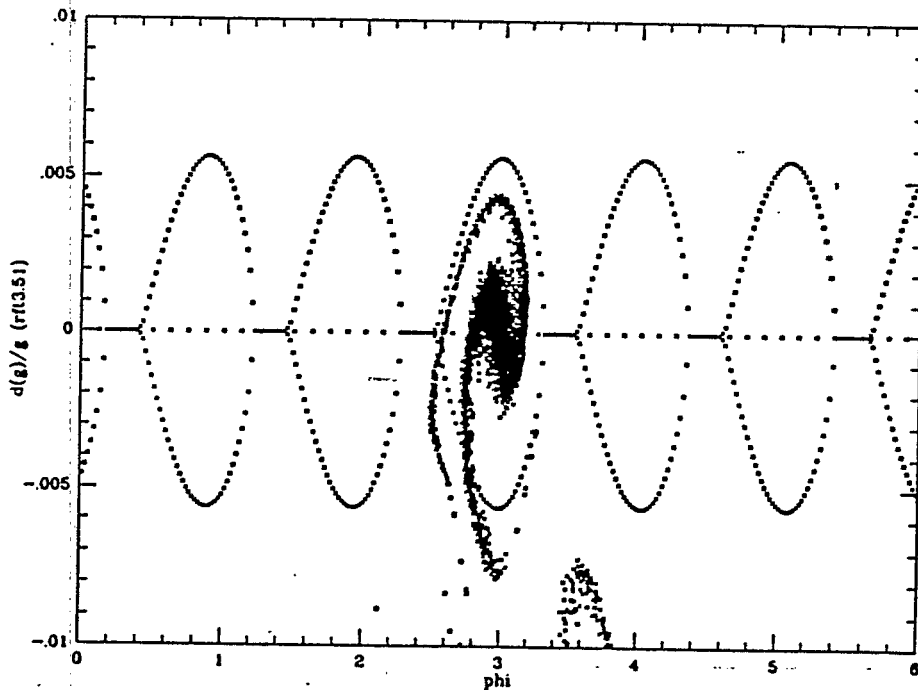
in $0.6 \text{ eV} \cdot \text{s}/\text{am}$

Switch over near transition
(after a γ increase)

A_u^{179}
 $\gamma_{tr.} = 25.65$
 $\gamma_{t_0} = 25.4376$
 $\hat{V}_1 = 100 \sim 300 \text{ kV}$
 $\hat{V}_2 = 300 \text{ kV}$
 $h_1 = 342$
 $h_2 = 2052$
 $\alpha_1 = -0.6$
 $\frac{Z}{n} = 1.2 \Omega$
 capac.



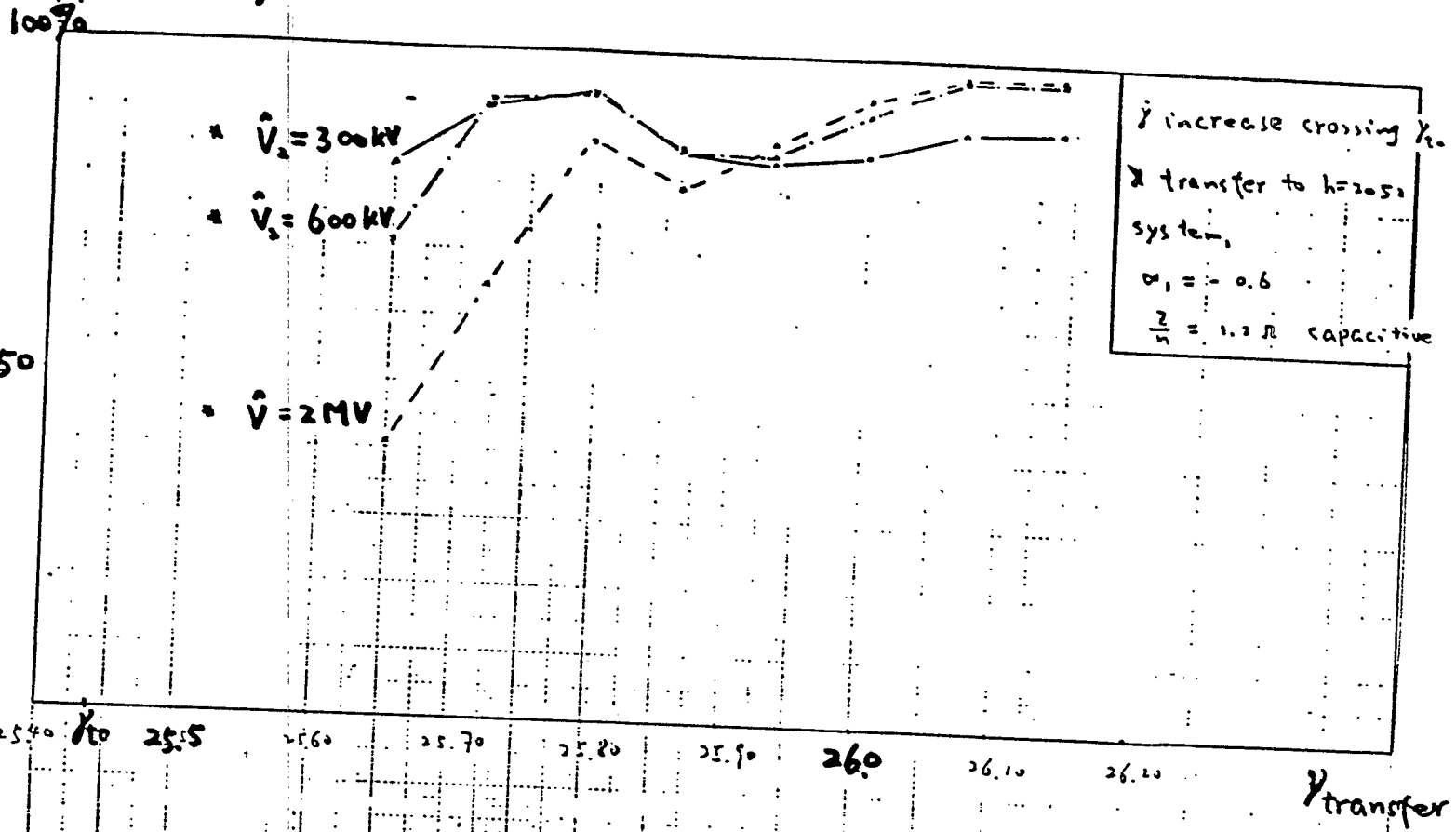
+ 25 ms



+ 90 ms

Transfer
Efficiency
83 %
in 0.6 eV·s/a

Transfer Efficiency



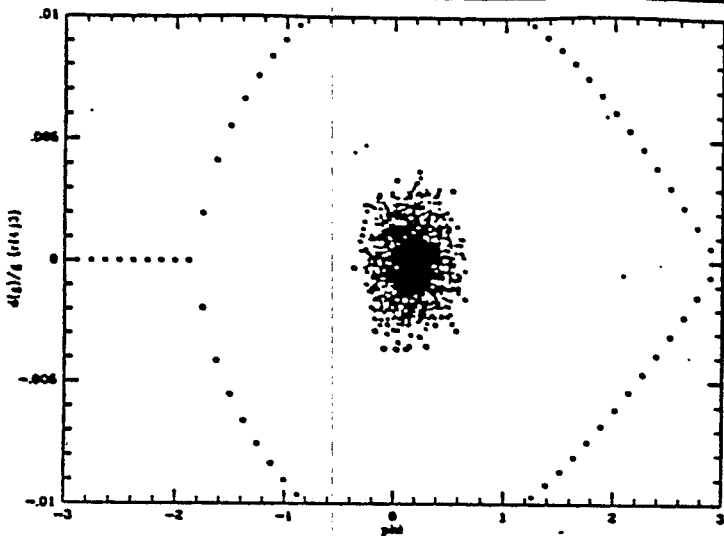
* γ increase crossing X_0 $V_1 = 100 \text{ kV} \rightarrow 300 \text{ kV}$	-----	$V_2 = 2 \text{ MV}$	$\sin \phi_2 = 0.024$	$A_2 = 2.3 \text{ ev/s/e}$
	-----	$V_2 = 600 \text{ kV}$	$\sin \phi_1 = 0.08$	$A_0 = 1.2 \text{ ev/s/e}$
	-----	$V_2 = 300 \text{ kV}$	$\sin \phi_3 = 0.16$	$A_0 = 0.6 \text{ ev/s/e}$

- γ increase of $\Delta\gamma = 0.38$ in 40 ms
- then transfer to 160 MHz RF system
on $\gamma = \gamma_{\text{transfer}}$

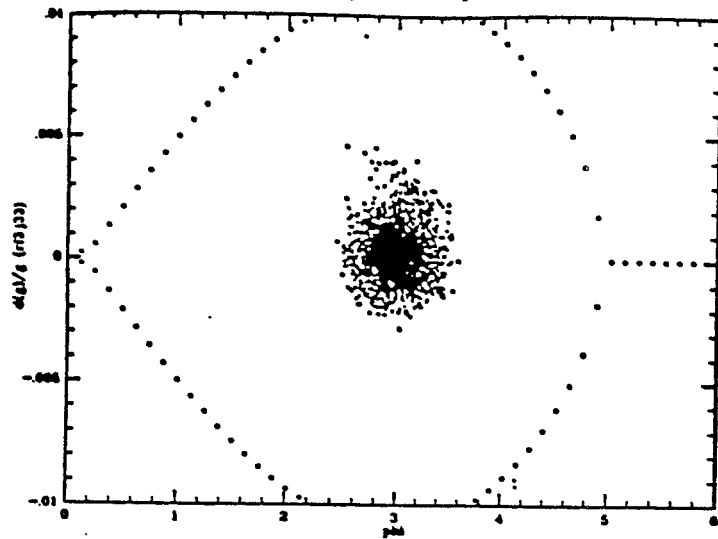
γ_t jump, $\hat{V}_i = 300$ kV

$\Delta\gamma_i = 0.6$
in 60 ms

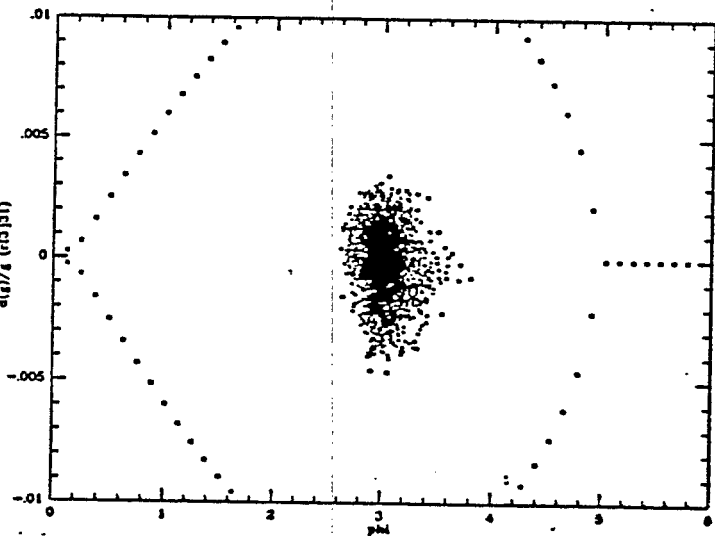
Au¹⁹⁷
RHIC



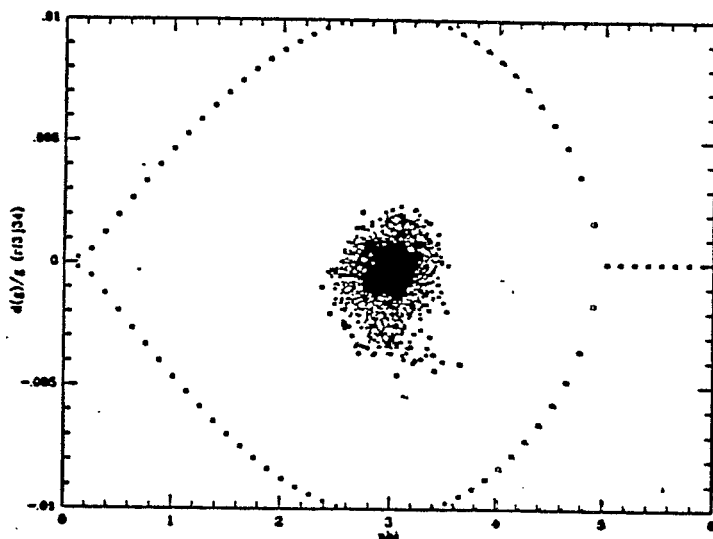
-80 ms



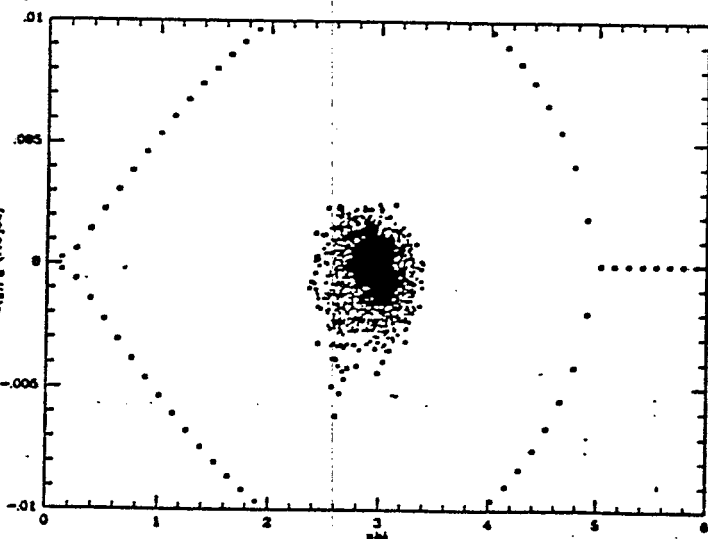
+125 ms



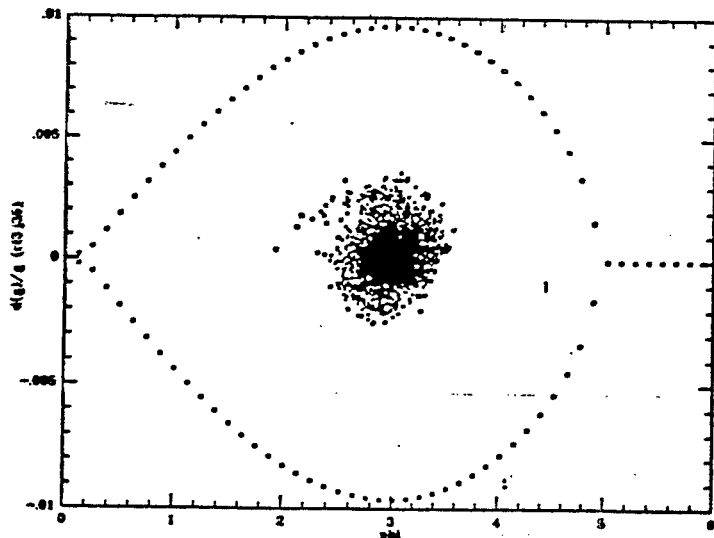
+25 ms



+175 ms



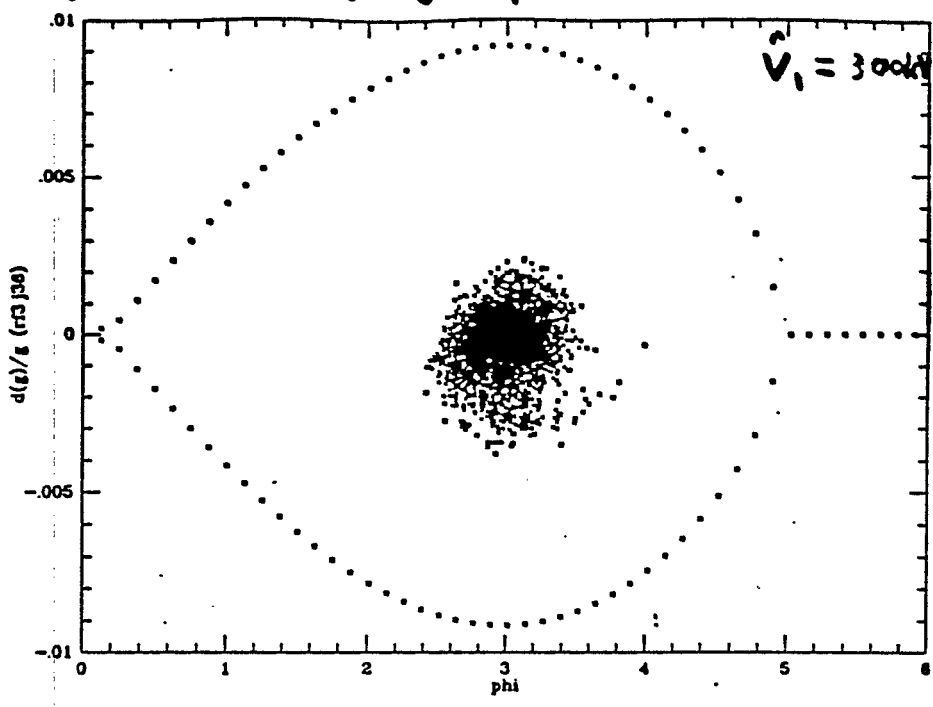
+75 ms



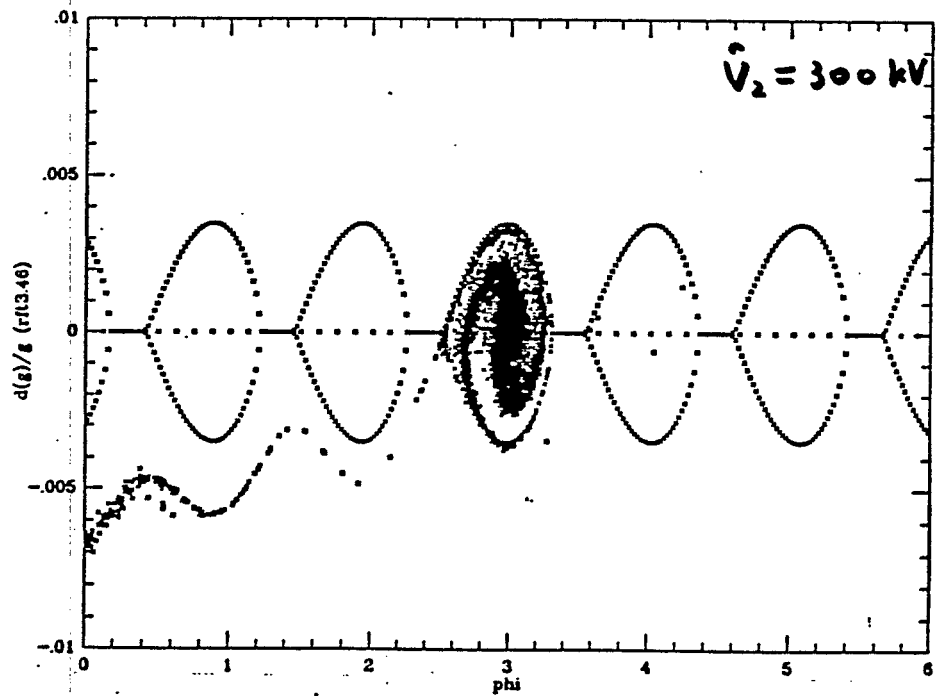
+225 ms No Loss

Switch over near transition
(after a V_t jump)

A_u^{+29}
RHIC



+ 275 ms

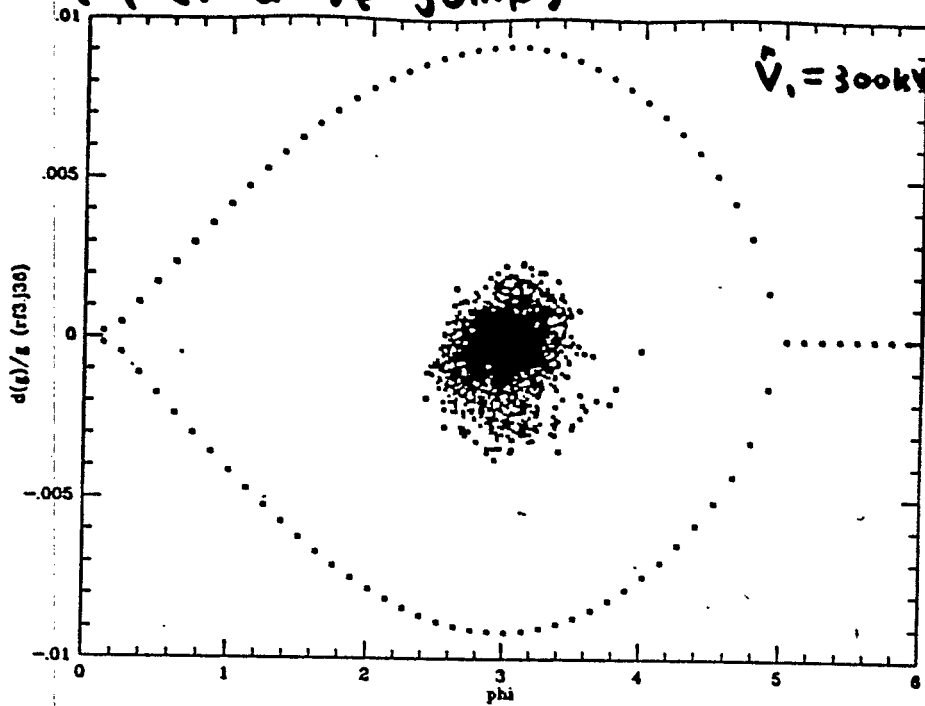


+ 340 ms

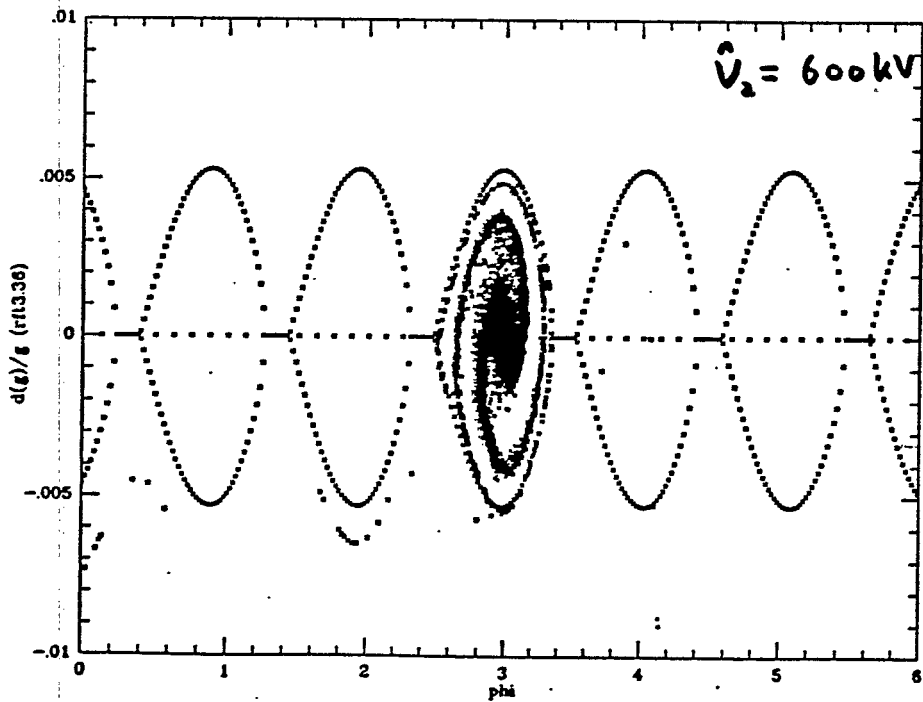
92% surv.
in 0.6 ev. s/c
bucket

Switch over near transition
(after a γ_t jump)

A_{γ}^{+39}
RHIC



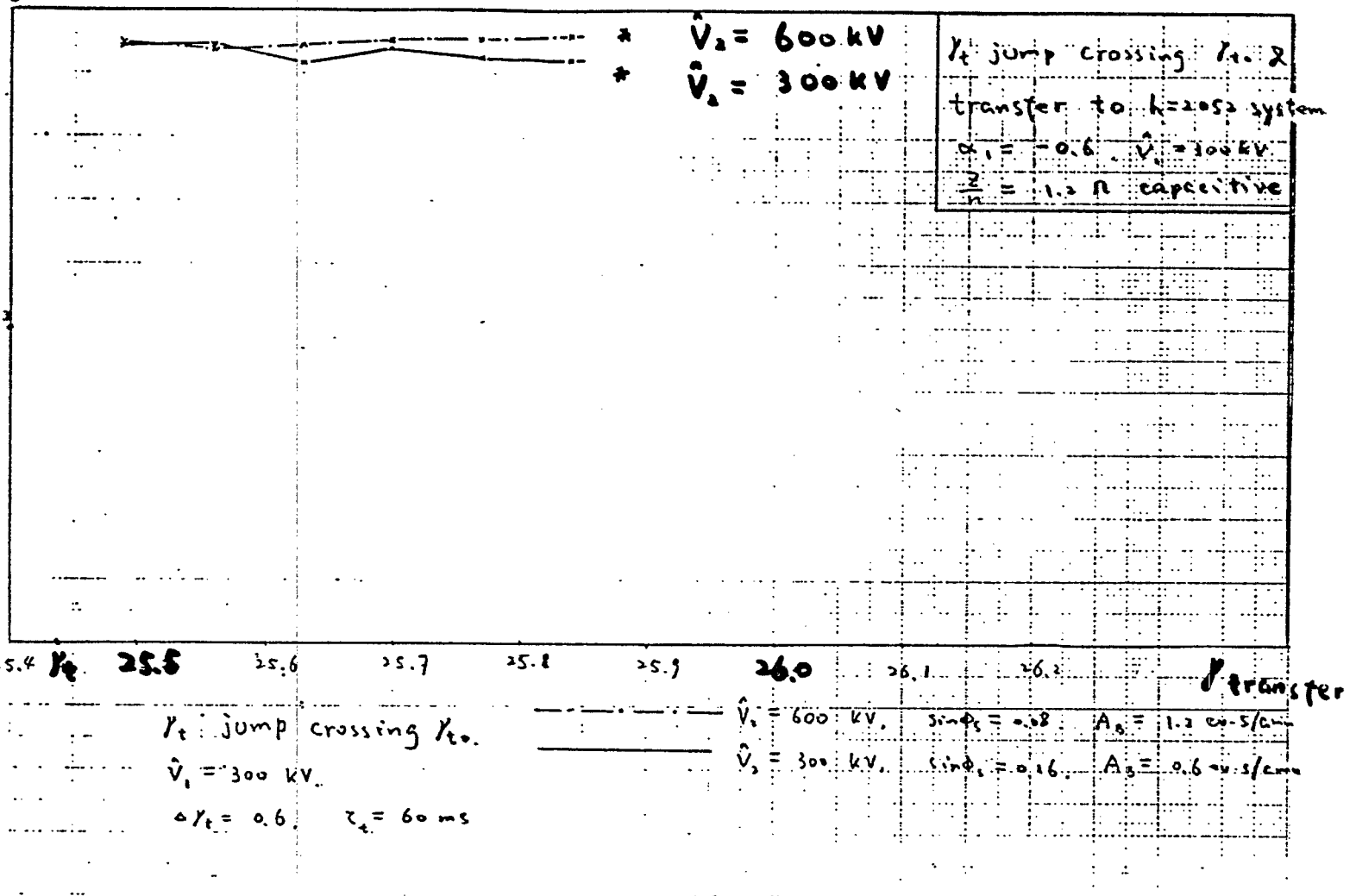
+ 275 ms



+ 340 ms

96% surv.
in 1.2 eV.s/a
bucket

Transfer efficiency



- γ_t jump of $\Delta \gamma_t = 0.6$ in 60 ms, $\hat{V}_1 = 300 \text{ kV}$
 - then transfer to 160 MHz RF System
- on $\gamma = \gamma_{\text{transfer}}$

Summary on the previous study:

- * A γ_t jump, or a $\dot{\gamma}$ increase near transition is very helpful
- * A r.f. voltage of 300 kV for 26.7 MHz system helps in achieving $\dot{\gamma}$ increase, rotation and providing sufficient bucket area at low energy
- * A second system of 160 MHz is comfortable for r.f. system transfer.