

## Simulation of Transition and Transfer

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USDOE Office of Science (SC)

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Simulation of Transition and Transfer

*(Mini-Workshop on RHIC RF Systems)*

*July 11-15, 1988  
Collider Center*

J. Wei  
BNL

# SIMULATION OF TRANSITION & TRANSFER

## I. Transition Energy Crossing

- \*  $\hat{V} = 100 \text{ kV}$  crossing transition
- \* Scheme of  $\gamma$  increase or  $\gamma_t$  jump

S. Y. Lee, A. G. Ruggiero

J. Claus

## II. Transfer to High Frequency R.F. System

- \* Top energy bunch rotation
- \* Bunch rotation with  $0.3 \text{ eV}\cdot\text{s}/\text{a bunch}$
- \* Switch over near transition

J. M. Brennan, E. Raka

S. Y. Lee ...

# I. Transition Energy Crossing

- $\hat{V} = 1.2 \text{ MV}$  ?

momentum spread too large, nonlinear

- $\hat{V} = 100 \text{ kV}$  crossing ?

much better,

space charge + nonlinear effect.  $\gamma_t(\frac{\omega}{p})$

$\Rightarrow$  phase space area blow up

- $\gamma_t$  jump, or  $\dot{\gamma}$  increase ?

Good, "clean" crossing

$$\begin{cases} \dot{\omega} = \frac{q e \tilde{V}}{2\pi} (\sin \phi - \sin \phi_s) + \Delta_{s.c.} + \Delta_z \\ \dot{\phi} = \frac{h \Omega_0}{p_0 R_0} \cdot \omega \cdot \eta(\omega) \end{cases} \quad (\omega = \frac{\Delta E}{\hbar})$$

\* kinematic mismatching

$$\eta\left(\frac{\Delta p}{p}\right) = \eta_0 + \eta_1 \cdot \frac{\Delta p}{p} + \dots$$

\* Low freq. impedance

space charge, inductive, capacitive

$\Rightarrow$  bunch mismatching

\* High freq. impedance

resistive, inductive, capacitive (s.c.)

$\Rightarrow$  microwave instability

$$\underline{\eta = \eta \left( \frac{\Delta P}{P_0} \right)}$$

$$\Omega R = \beta c, \quad \frac{\Delta \Omega}{\Omega_0} = \frac{\beta}{\beta_0} \frac{R_0}{R} - 1$$

$$\frac{R}{R_0} = 1 + \alpha_1 \frac{\Delta P}{P_0} + \alpha_2 \left( \frac{\Delta P}{P_0} \right)^2 + \dots$$

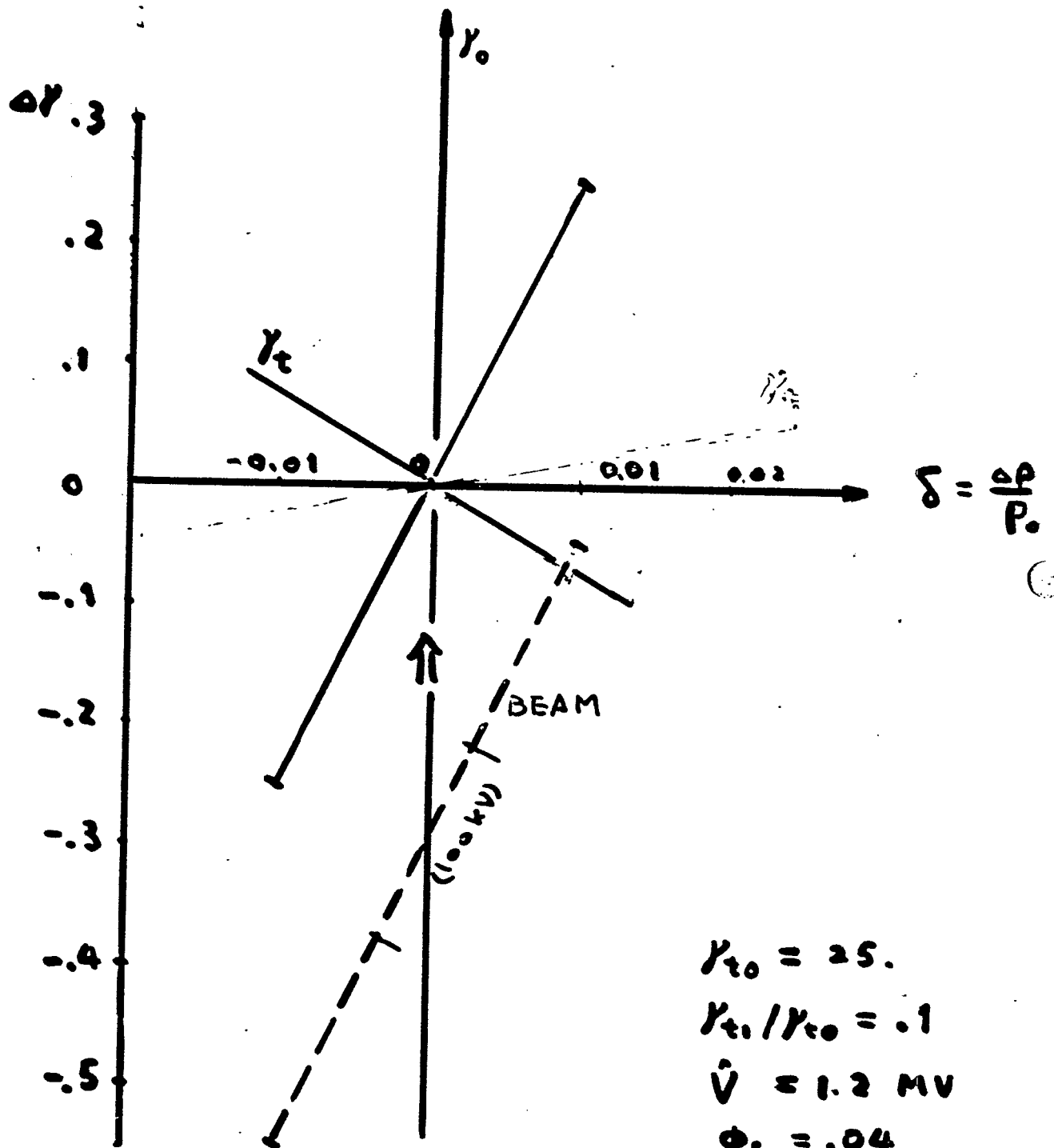
$$\frac{\beta}{\beta_0} = 1 + \frac{1}{\gamma_0^2} \cdot \frac{\Delta P}{P_0} - \frac{3\beta_0^2}{2\gamma_0^2} \cdot \left( \frac{\Delta P}{P_0} \right)^2 + \dots$$

$$\underline{\eta = \frac{\Delta \Omega}{\Omega} / \frac{\Delta P}{P_0} \approx \eta_0 + \left[ \frac{3}{2} \frac{\beta_0^2}{\gamma_0^2} - \frac{\gamma_{E1}}{\gamma_{T0}^2} \right] \cdot \frac{\Delta P}{P_0}}$$

$$\delta = \frac{\Delta \gamma}{\gamma} = \left( 1 + \frac{1}{\beta_0^2 \gamma_0^2} \right) \cdot \frac{\Delta P}{P_0}$$

$$\sim \left( \frac{1}{\gamma_0^2 \gamma_k^2} - \frac{1}{\gamma_k^4} \right)$$

# Mismatch at transition (RHIC)



$$\gamma_{t0} = 25.$$

$$\gamma_{t1}/\gamma_{t0} = .1$$

$$\hat{V} = 1.2 \text{ MV}$$

$$\phi_s = .04$$

$$\Delta\gamma = 2.07 \cdot 10^{-5} / \text{turn}$$

$$\dot{\gamma} = 1.62 / \text{sec.}$$

$$\tau_0 = 12.79 \mu\text{s}$$



Estimate of  $\delta$  growth.

$$\ddot{\delta} = \frac{qe\hat{V}}{2\pi p_0 R_0} \cos \varphi_0 h \Omega_0 \left( \frac{2\dot{\gamma}t}{\gamma_T^{0.3}} + \eta, \delta \right) \delta$$

$$i \int_{t_c}^0 \Omega_s \exp(i\omega t) dt = \left( \frac{h \Omega q e \hat{V} \cos \varphi_0}{2\pi p_0 R_0} \right)^{1/2} \int_{t_c}^0 \left( \frac{2\dot{\gamma}t}{\gamma_T^{0.3}} + \eta, \delta \right)^{1/2} dt$$

$$\approx \frac{\Omega}{3\dot{\gamma}} \left( \frac{h q e \hat{V} \cos \varphi_0}{2\pi p_0 \beta c} \right)^{1/2} \left( \frac{3}{2} - \lambda \right)^{3/2} \hat{\delta}^{3/2}$$

AGS  
295  $\hat{\delta}^{3/2}$   
Growth factor:  $e \approx 1.1$

RHIC  
5200  $\hat{\delta}^{3/2}$   
 $e \approx 6.3$

$$\delta \propto \delta_0 e^{\left( \frac{h \hat{V} \cos \varphi_0}{2\pi p_0 \beta c} \right)^{1/2} \hat{\delta}^{3/2}}$$

## Relevant time scales:

- $\tau_0$  : revolution period
- $\tau_{syn.}$  : synchrotron osc. period
- $T_c$  : characteristic non-adiabatic time

$$\frac{2}{\Omega_s} \left| \frac{d\Omega_s}{dt} \right| > 1$$

$$T_c = \left[ \frac{A m_0 c^2}{R_0^2 h} \cdot \frac{\gamma_t^4}{2 \dot{\gamma}} \cdot \frac{2 \pi}{Z e \hat{V} |\cos \phi_s|} \right]^{1/3}$$

- $T_{n.l.}$  : non-linear time

$$T_{n.l.} = \frac{\Delta p_{n.l.}}{\dot{\gamma}} = \frac{3 \beta_s^2 - \alpha_s}{4 \dot{\gamma}} \cdot \left( \frac{\Delta p}{p} \right)_{2.5\sigma}$$

$$\eta(\delta) = \eta_0 + \eta_1 \delta + \dots$$

$$\text{time when } |\eta_1 \delta| > |\eta_0|$$

& of different sign

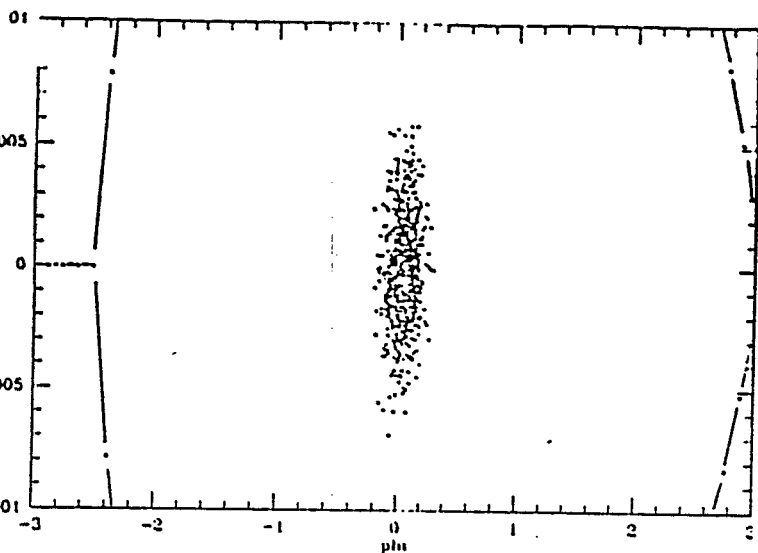
- $T_{m.w.}$  : microwave instability growth time

$$\hat{V} = 1.2 \text{ MV}$$

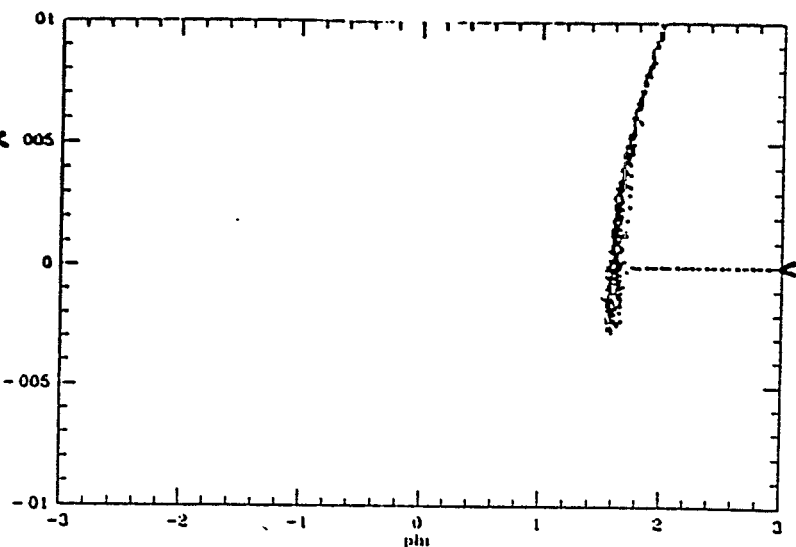
$$\sin \phi_s = 0.04$$

$\gamma_+$  CROSSING

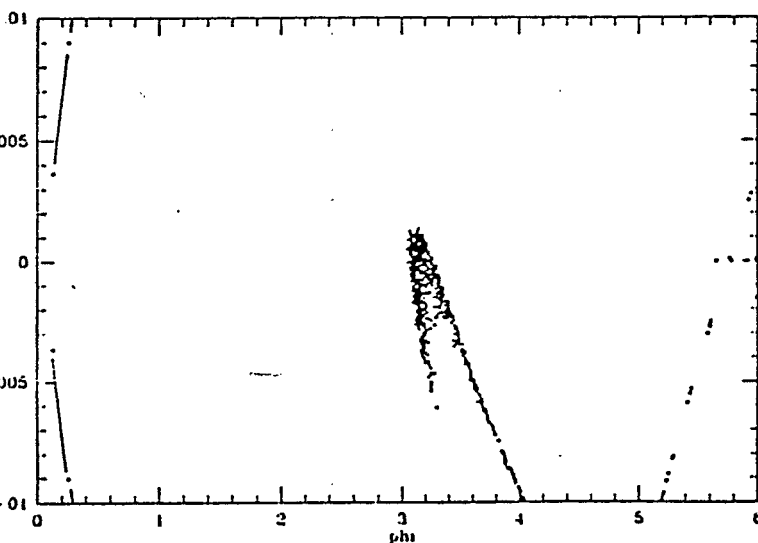
$Au^{+79}$



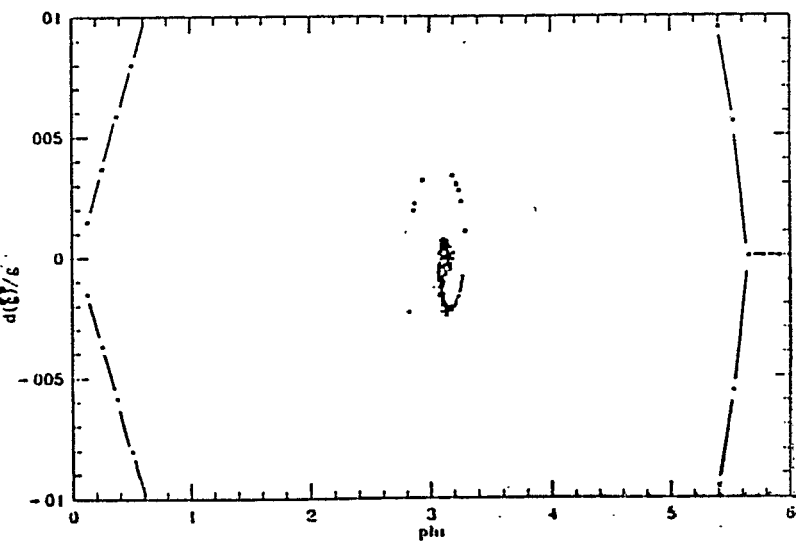
$-80 \text{ ms}$



$0 \text{ ms}$



$+25 \text{ ms}$



$+180 \text{ ms}$

$\eta(\frac{\phi}{P})$ . mismatching  $\Rightarrow 70\%$  Loss

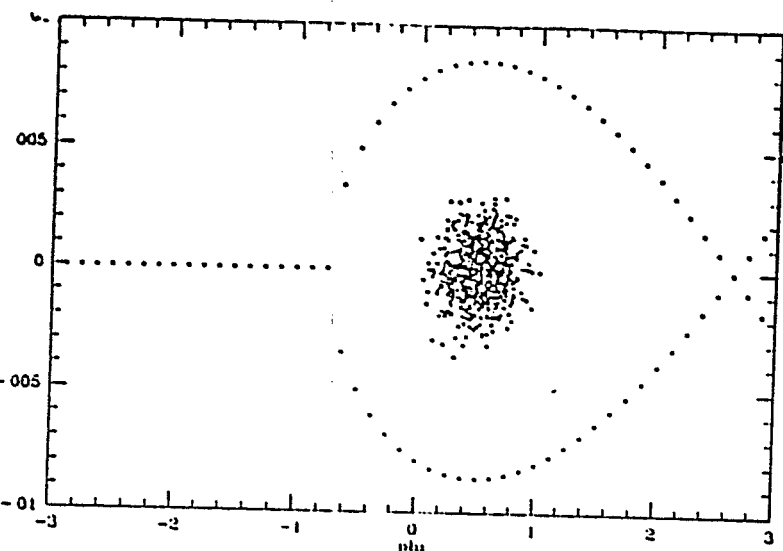
$$\tau_0 \ll T_c < T_{n.l.} < \tau_{syn.}$$

$$\hat{V} = 100 \text{ kV}$$

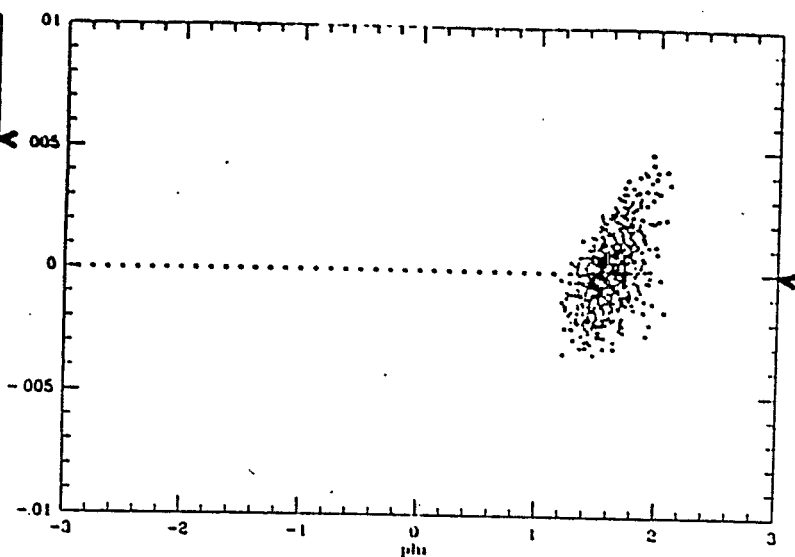
$$\sin \phi_s = 0.48$$

$\gamma_t$  CROSSING

$A_u^{+79}$

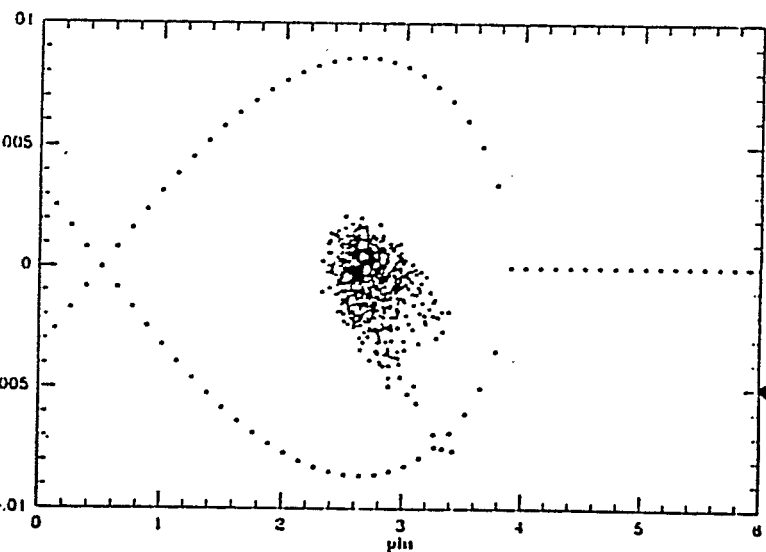


$-80 \text{ ms}$

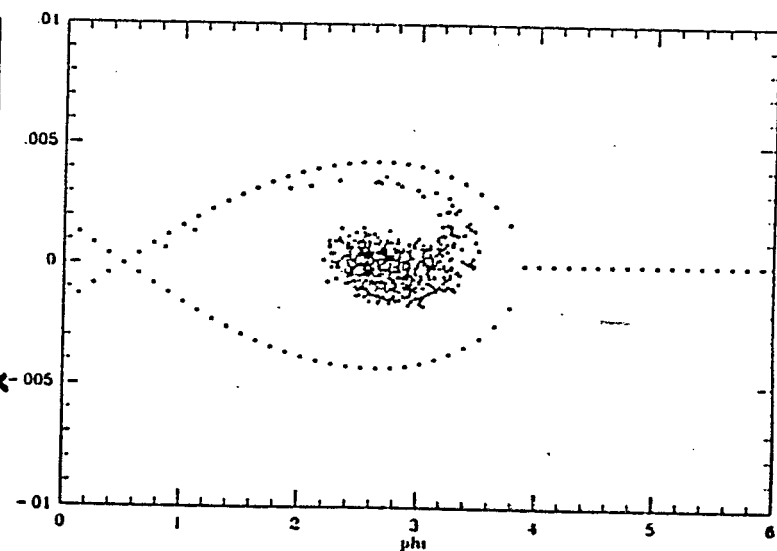


$0 \text{ ms}$

$0.3 \text{ eV} \cdot \text{s} / \text{amu}$ , w/o s.c.



$+80 \text{ ms}$



$+230 \text{ ms}$

$\eta(\frac{\Delta p}{p})$  mismatching  $\Rightarrow$  0.9% Loss

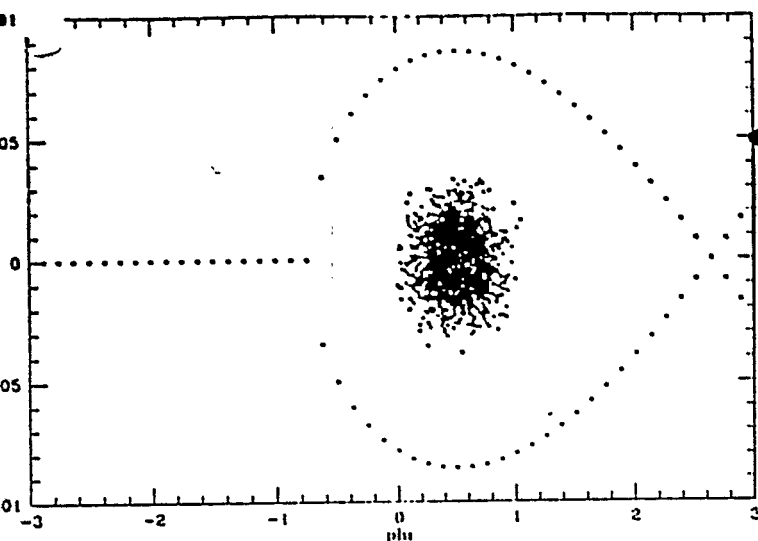
$$\tau_0 \ll T_{n.l.} < T_c < \tau_{syn.}$$

$$\hat{V} = 100 \text{ kV}$$

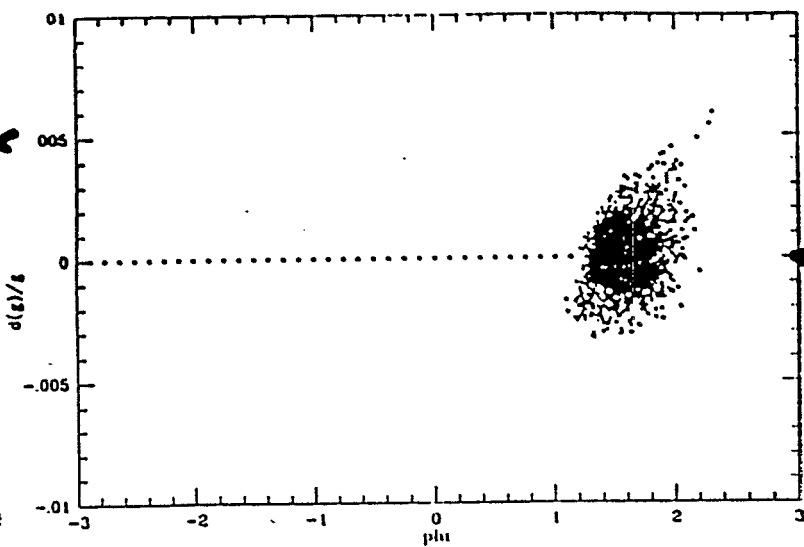
$$\sin \phi_s = 0.48$$

$\gamma_t$  CROSSING. space charge

$A_u$



-80 ms

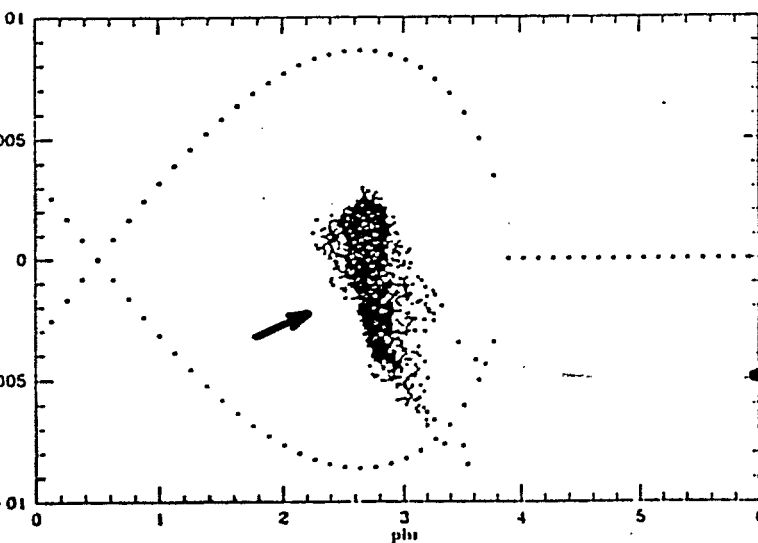


0 ms

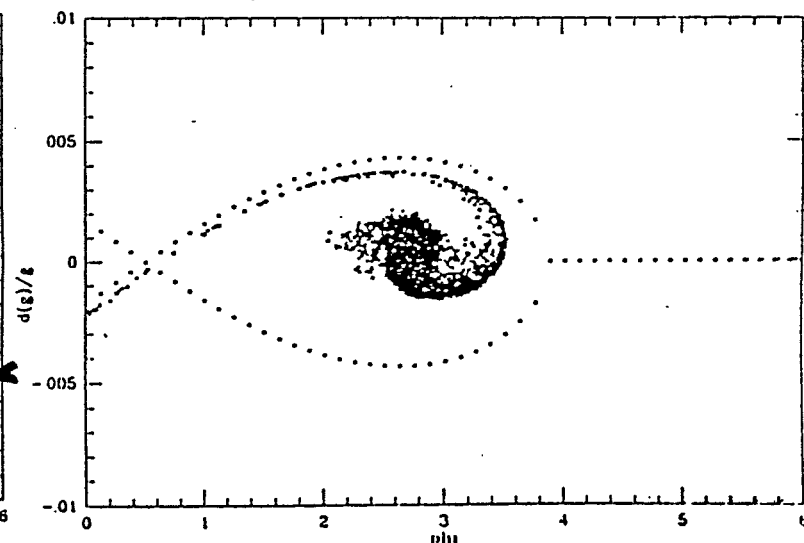
0.3 eV·s / amu, with s.c.

$$\frac{Z}{h} = 1.2 \Omega, \quad 1.1 \times 10^9 / \text{bunch}$$

$l_{\text{bin}} = 2b, \Rightarrow$  microwave freq.



+80 ms



+230 ms

mismatching + spacecharge  $\Rightarrow$  2.1% Loss

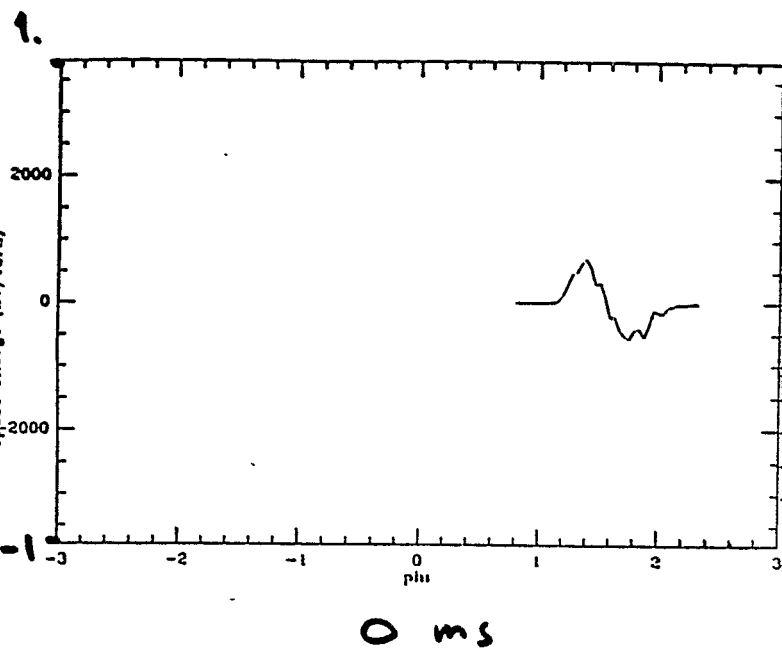
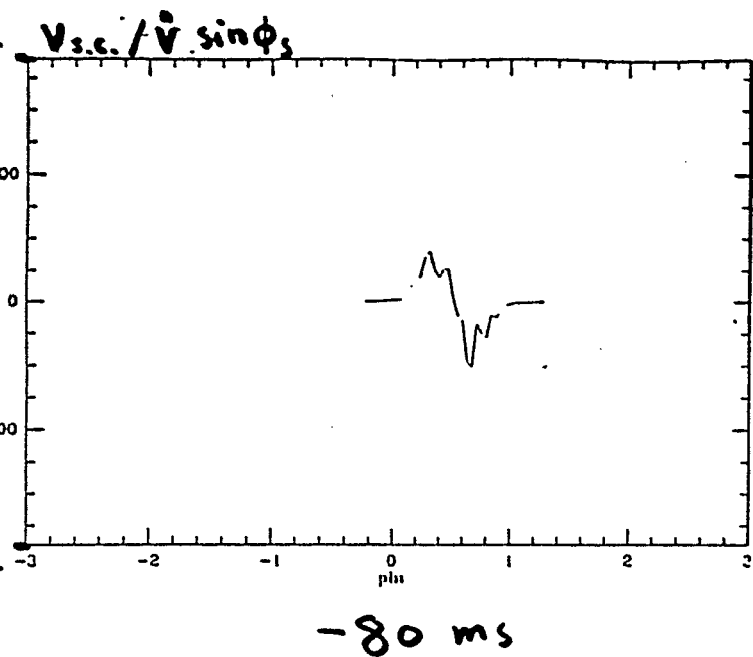
$$\tau_0 \ll T_{n.l.} < T_c < \tau_{\text{syn.}}$$

$$\sim 1 \text{ eV} \cdot \text{sec} / \text{amu}$$

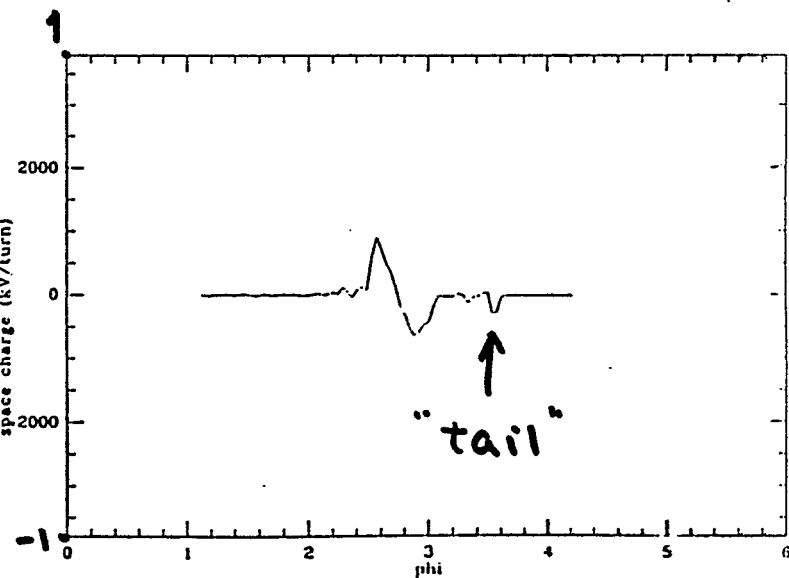
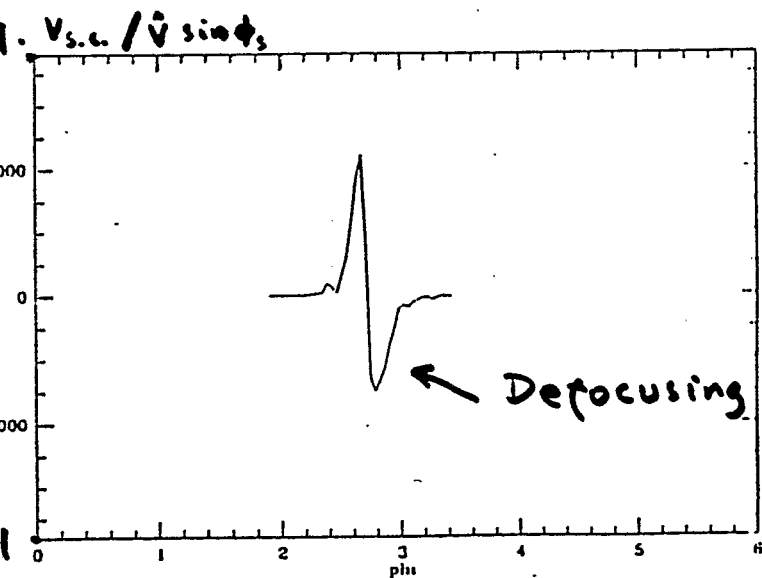
$$\hat{V} = 100 \text{ kV}$$

$$\sin \phi_s = 0.48$$

## Space charge voltage



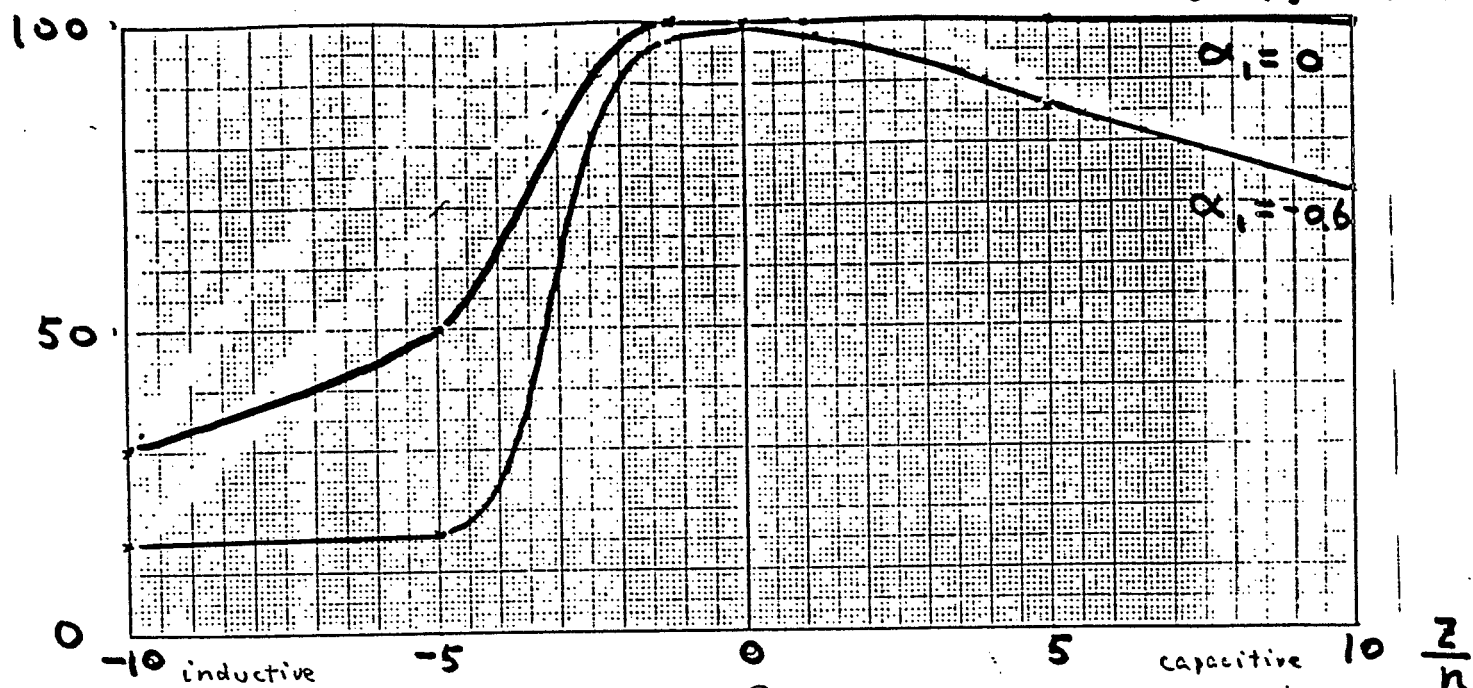
$$\frac{Z}{h} = 1.2 \Omega, \text{ Capacitive}$$



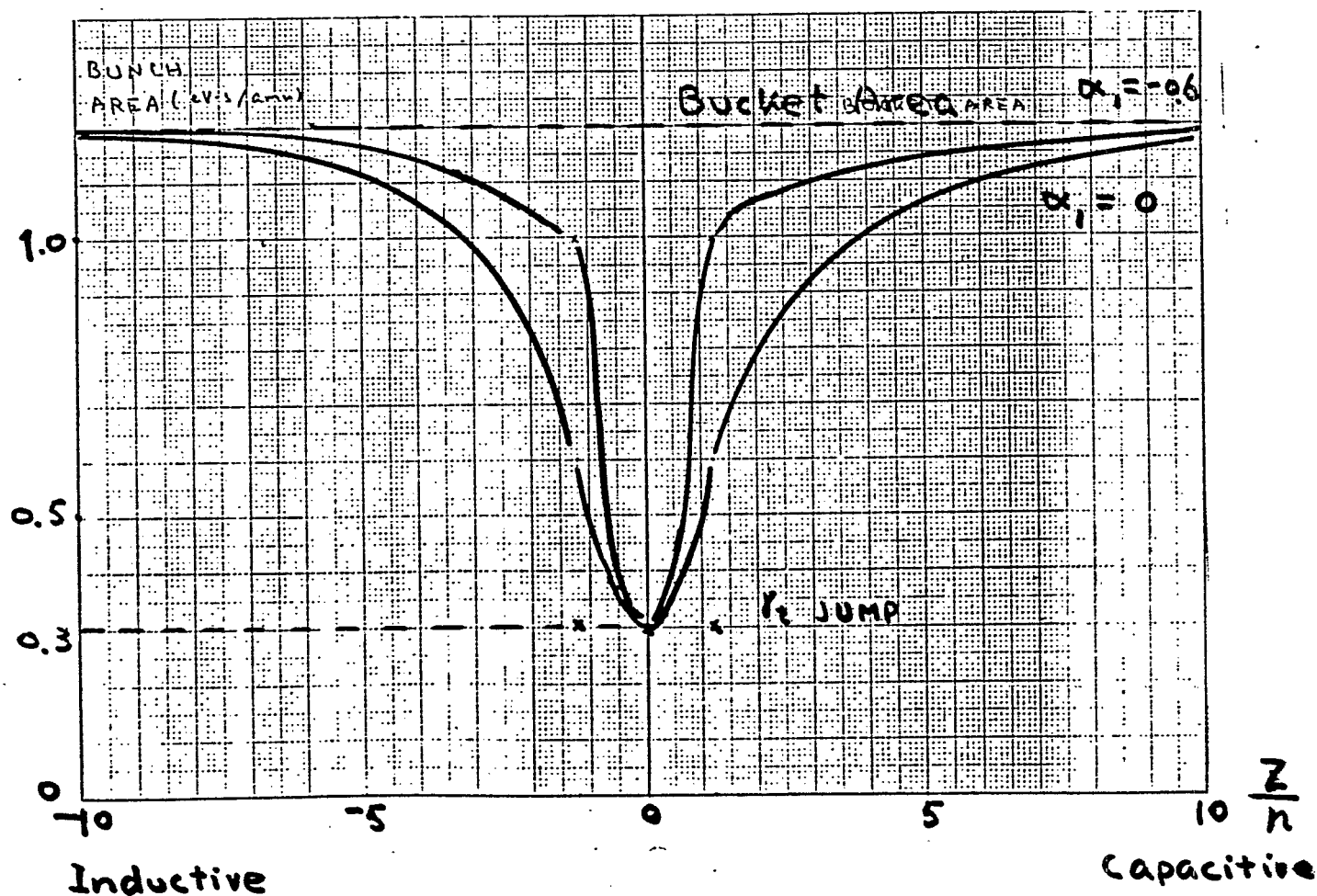
Defocusing space charge force

# Crossing Efficiency (%)

$A_u^{199}$  RHIC  
 $\hat{V}_1 = 100 \text{ kV}$   
 $\sin \phi_s = 0.48$



## Bunch Area After Transition



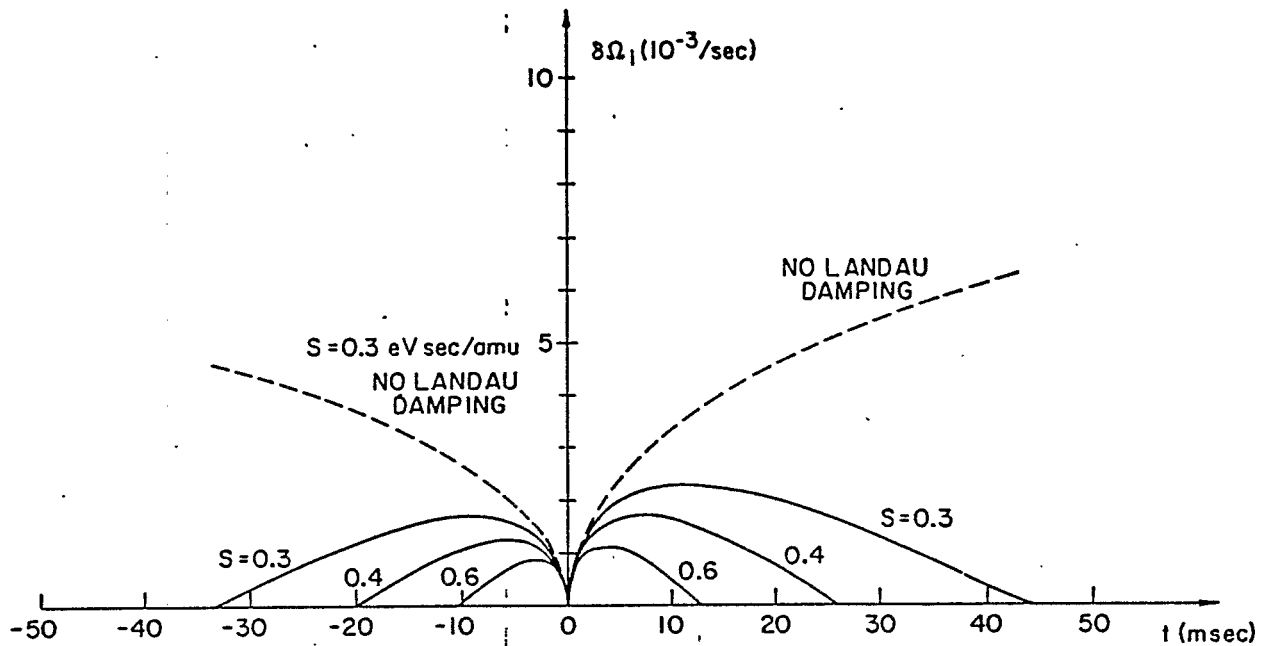


Fig. IV-19. The imaginary part of the microwave frequency,  $\delta\Omega_i$ , representing the growth rate of the instability, is plotted as function of time during crossing of the transition energy. The curves are calculated with the initial phase space as parameter. The dashed curve represents the growth rate without Landau damping.

\* Analytical solution based on  
perturbation theory

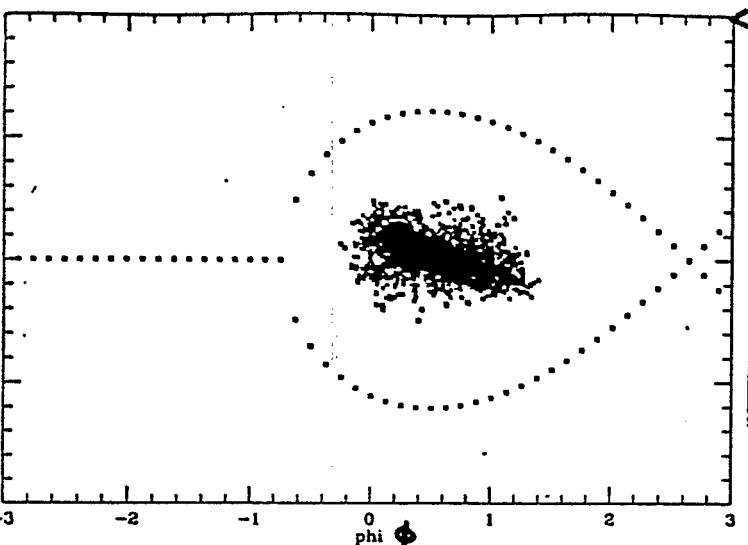


$V = 100 \text{ kV}$   
 $\sin \phi_s = 0.48$

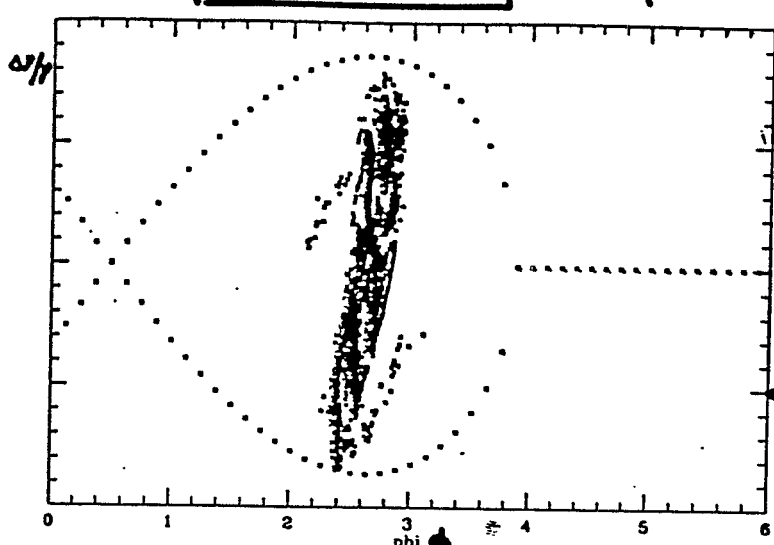
$\gamma$  CROSSING

$Z/n = 10 \Omega$   
 capacitive

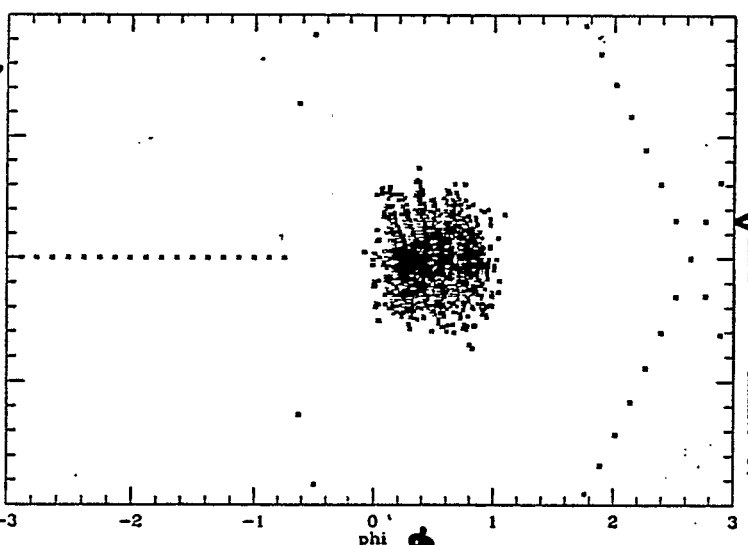
$A_n^{+79}$   
 $\alpha_1 = 0$



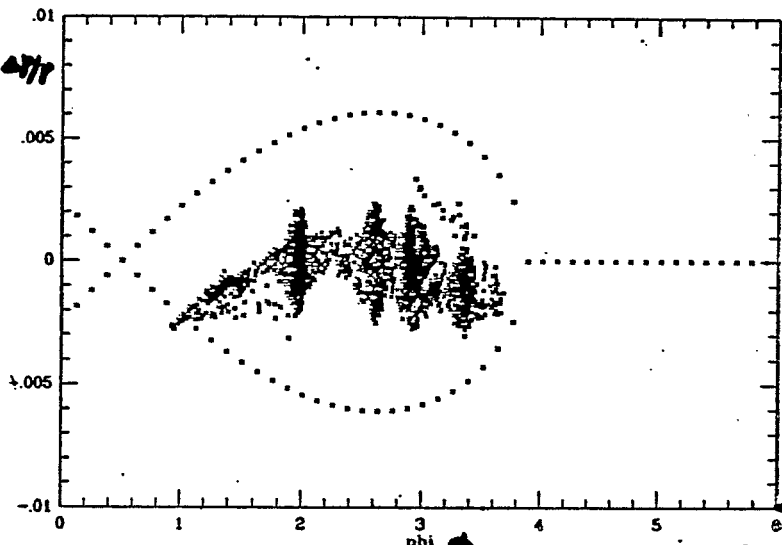
$-160 \text{ ms}$



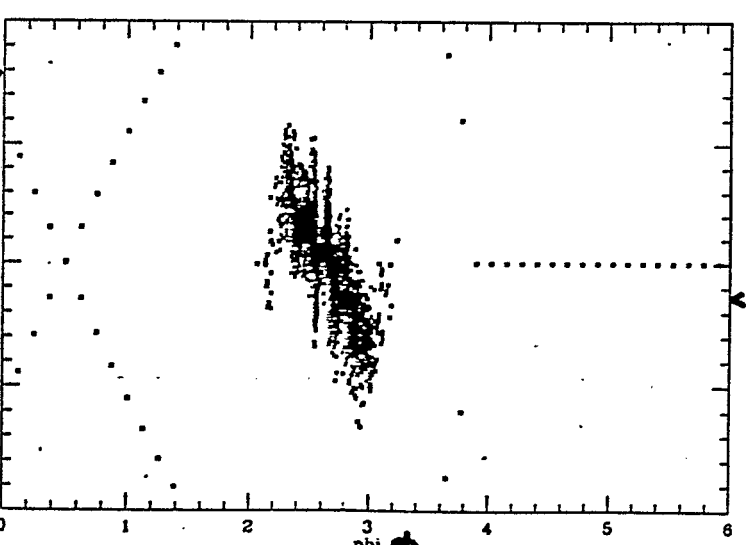
$+80 \text{ ms}$



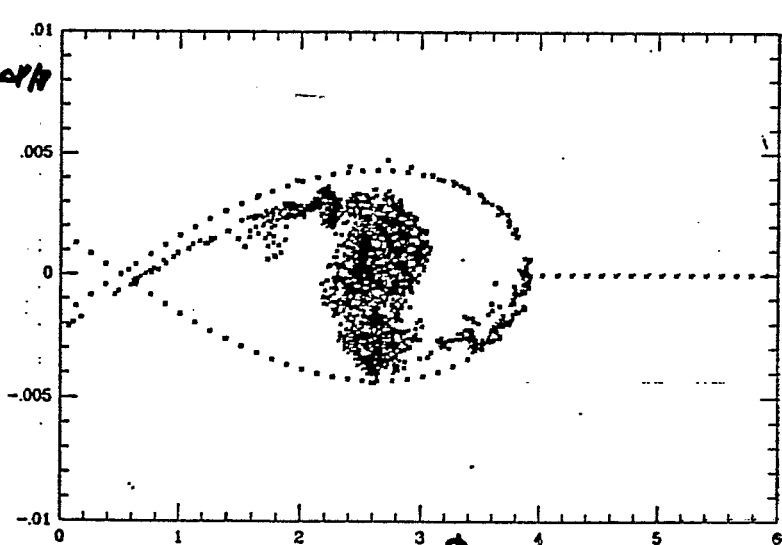
$-25 \text{ ms}$



$+160 \text{ ms}$



$+25 \text{ ms}$



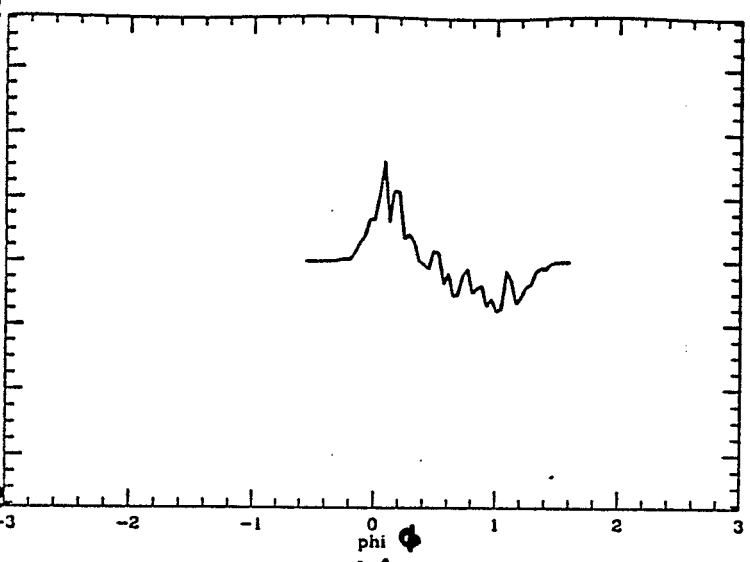
$+320 \text{ ms}$

Efficiency 99%

$Z/n = 10 \Omega$   
capacitive

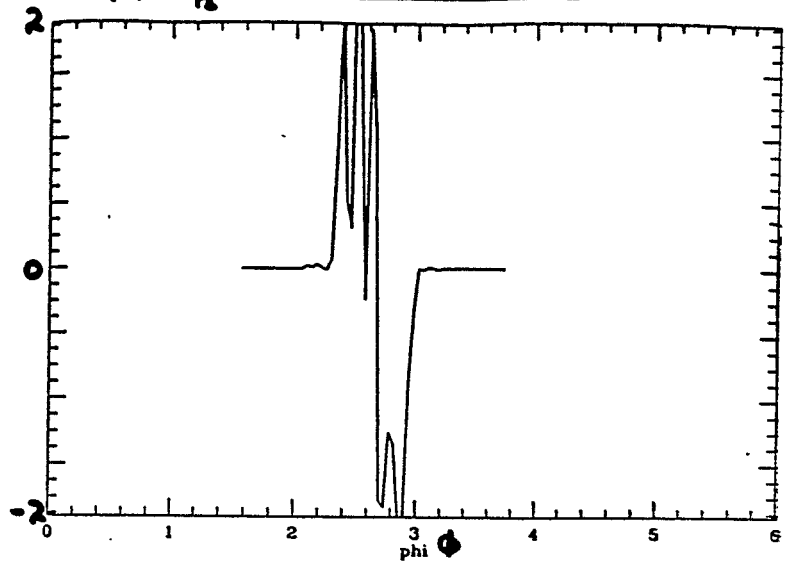
$A_u^{+79}$   
 $\alpha_1 = 0$

$V_{s.c.}/\hat{V} \sin \phi_s$



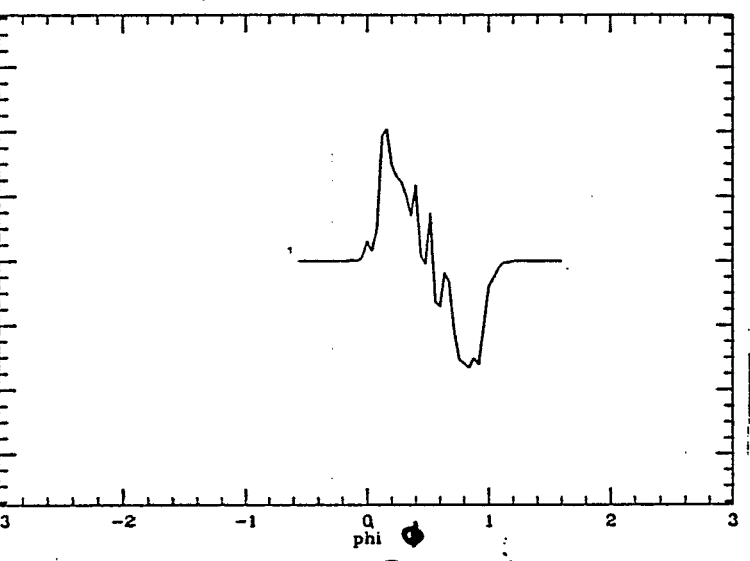
-160 ms

$V_{s.c.}/\hat{V} \sin \phi_s$



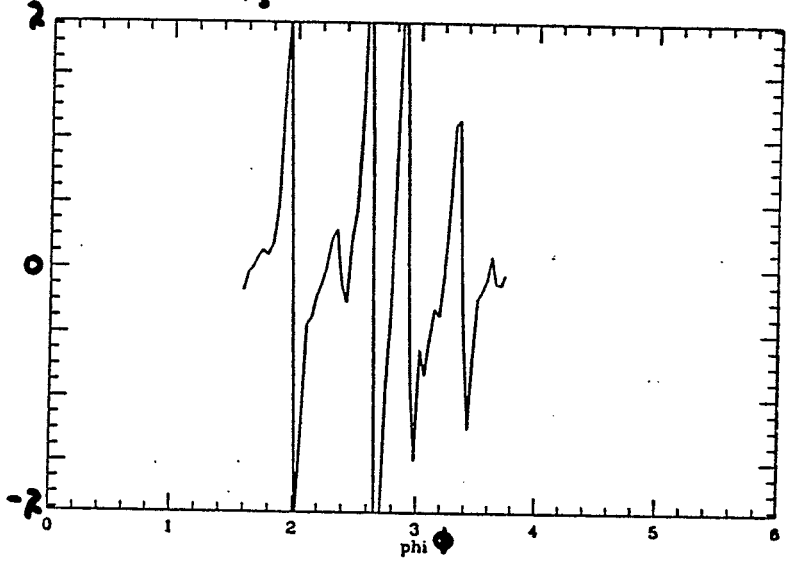
+80 ms

$V_{s.c.}/\hat{V} \sin \phi_s$



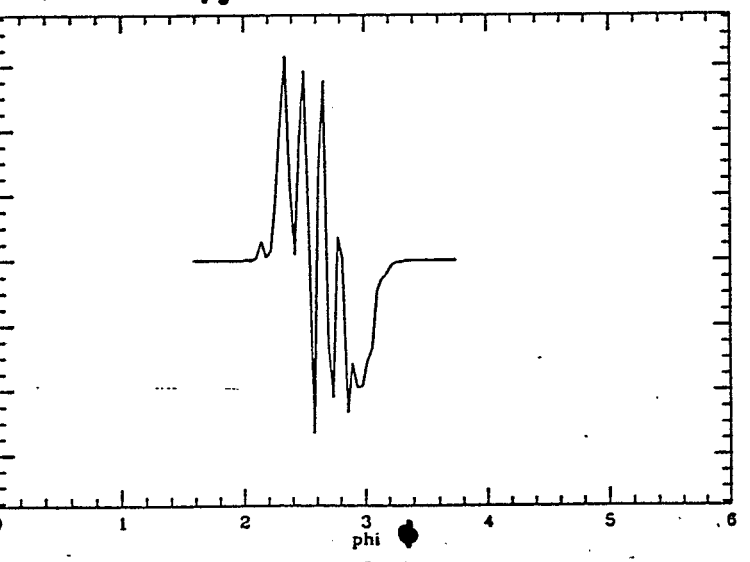
-25 ms

$V_{s.c.}/\hat{V} \sin \phi_s$



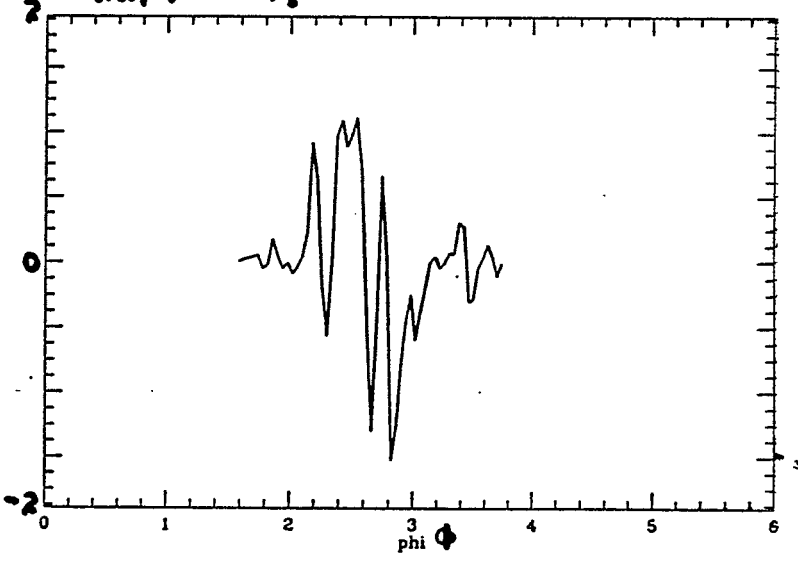
+160 ms

$V_{s.c.}/\hat{V} \sin \phi_s$



+25 ms

$V_{s.c.}/\hat{V} \sin \phi_s$



+320 ms

To preserve the original  $0.3 \text{ eV} \cdot \text{s/amu}$   
& to minimize loss

- \*  $\gamma_t$  jump, or
- \* increase  $\dot{\gamma}$  near transition  
(without changing  $\dot{B}$ ), or
- \* make total  $\frac{Z}{n} \sim 0$

---

• Resistive wall impedance  
very very small effect

$$10^2 * \left| \frac{Z}{n} \right|_{\substack{\text{stainless} \\ \text{steel}}} \Rightarrow \text{microwave instability}$$

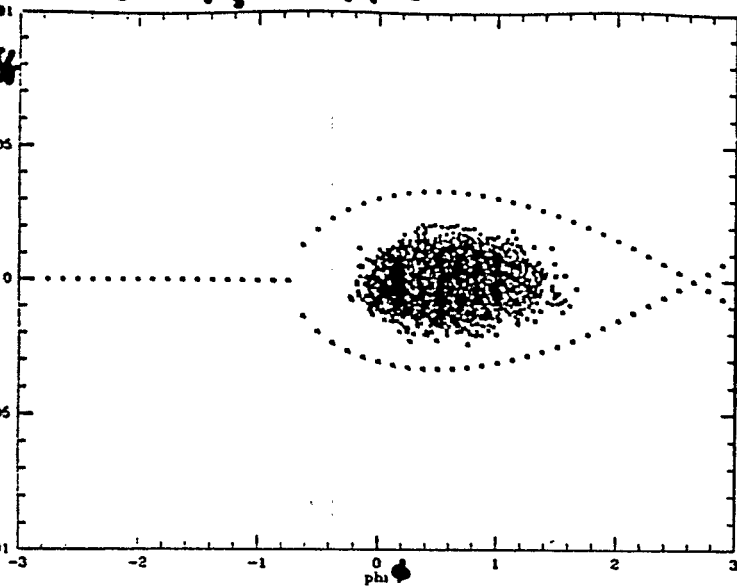
$$@ \frac{Z}{n_c} \sim 5 \, \Omega, \quad n_c = \frac{R_0}{b}$$

$V = 100 \text{ kV}$   
 $\sin \phi_s = 0.48$

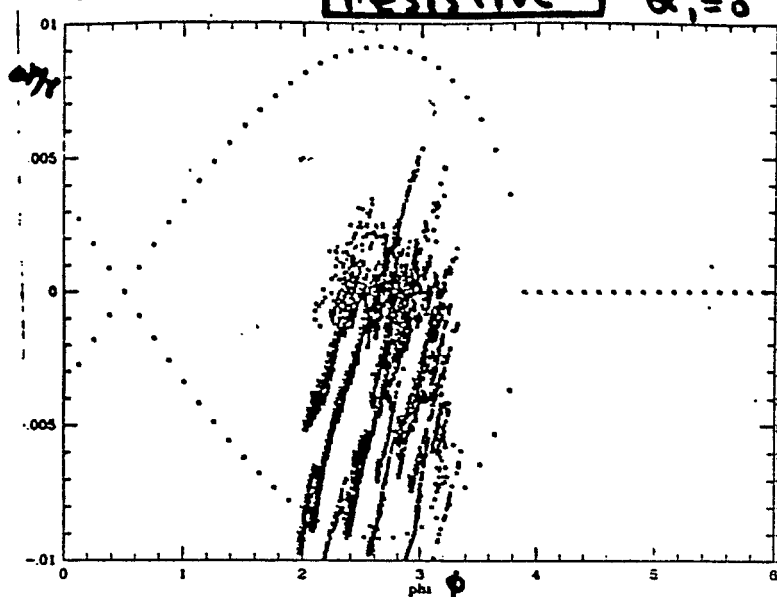
$\gamma_s$  crossing

$R = 10^{-8} \Omega$   
 resistive

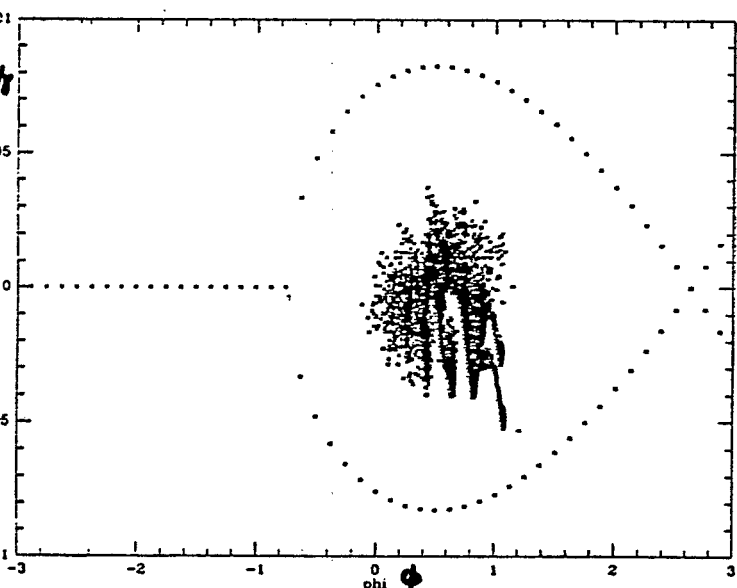
$Au^{19}$   
 $\alpha_s = 0$



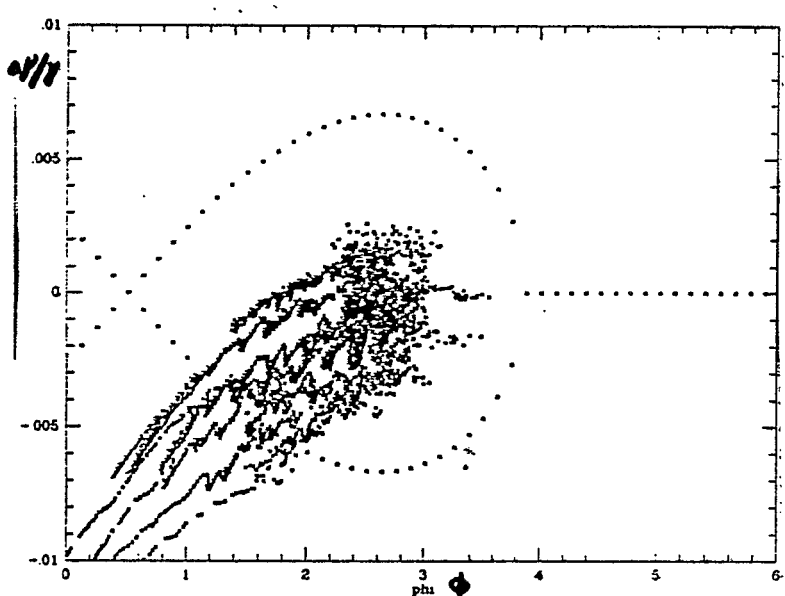
-520 ms



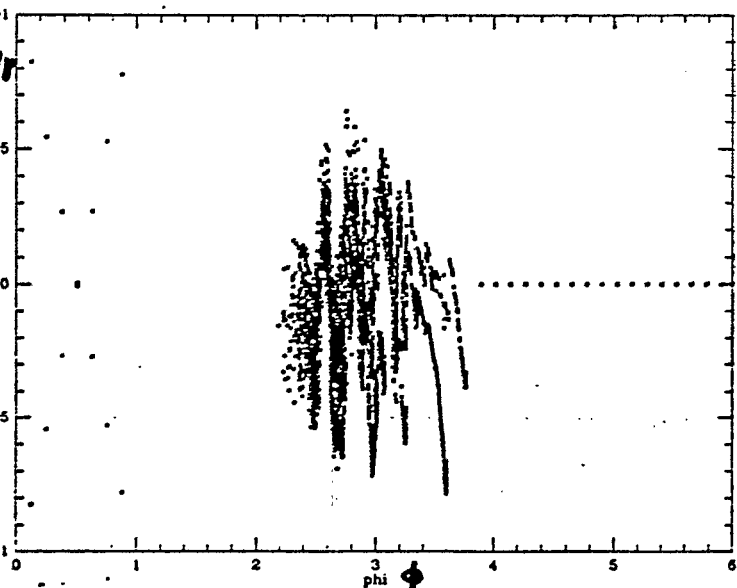
+100 ms



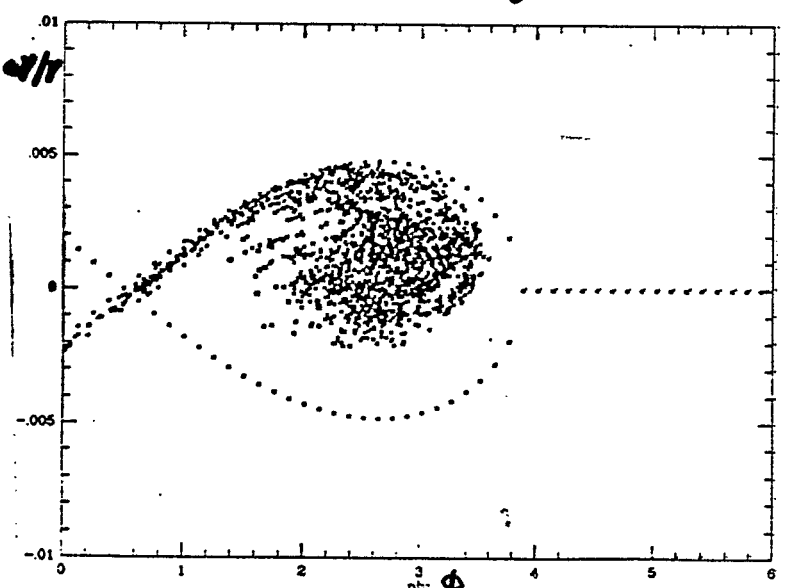
-85 ms



+165 ms



+40 ms

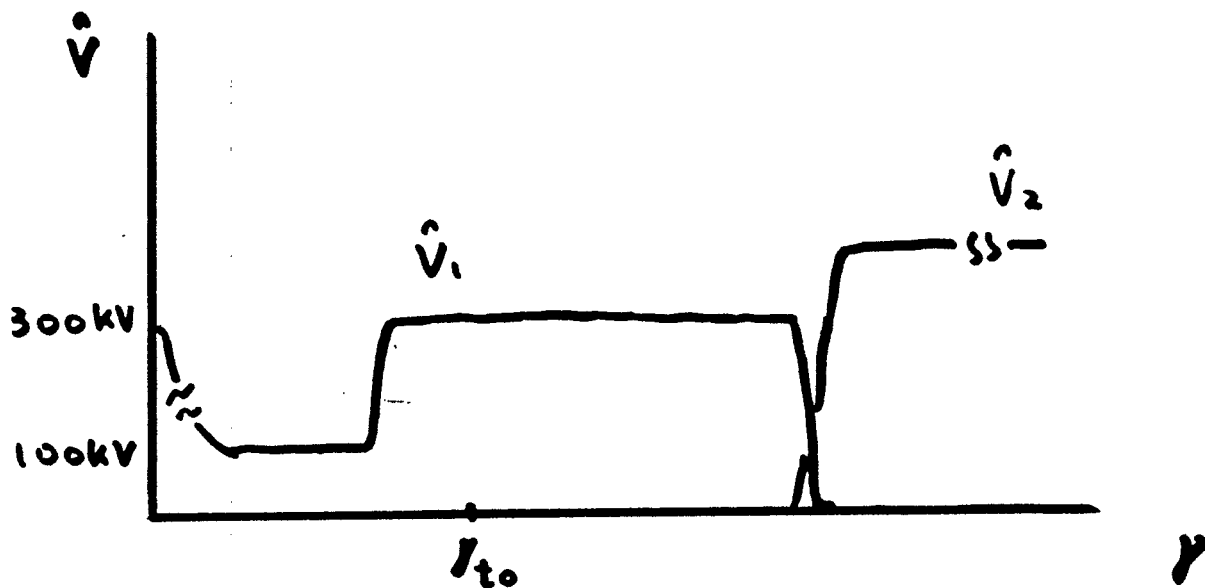
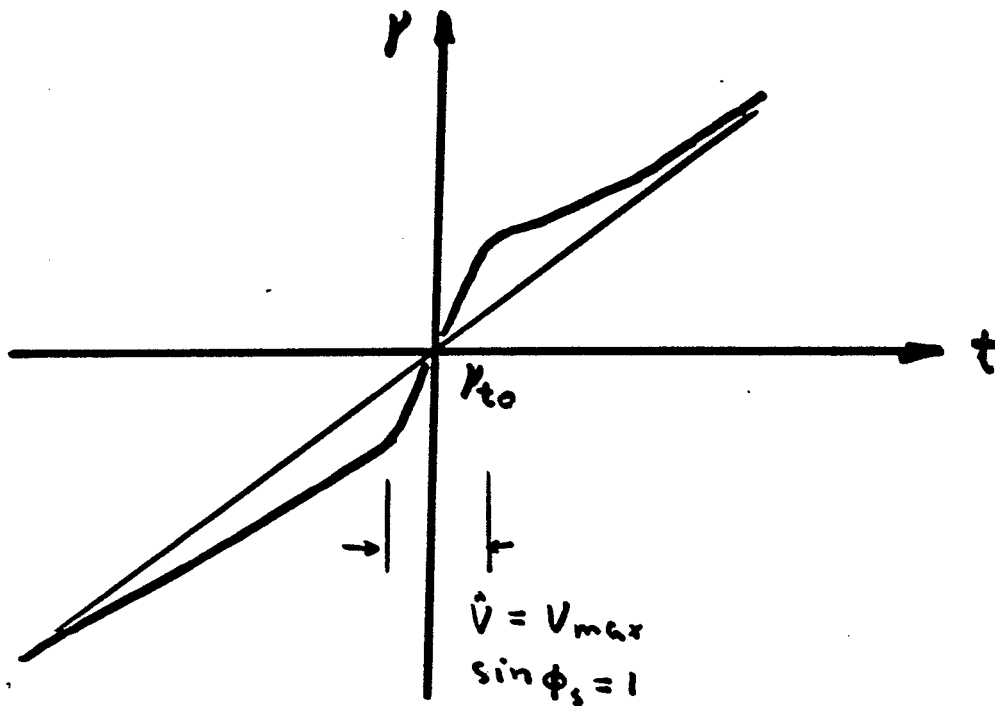


+290 ms

Efficiency 70%

7200 particles  
 used.

# $\dot{\gamma}$ increase crossing transition



Total momentum aperture needed  $\pm 0.8\%$

$\Delta\gamma = 0.38$  in 40 ms. used in simul.

INCREASE, 100V → 300V

$A_u^{+79}$

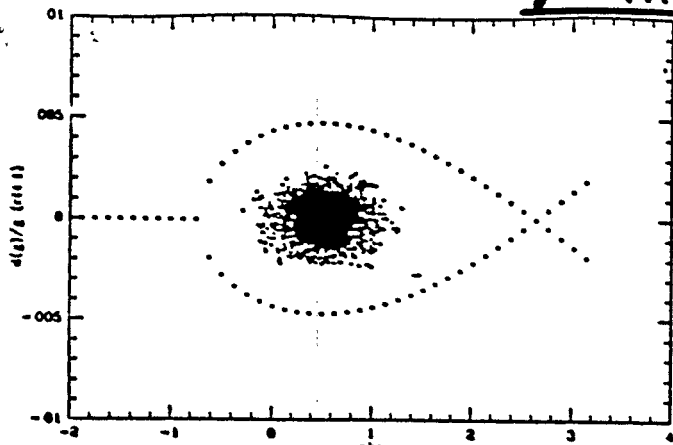
0.3 eV·s/a

$\dot{\gamma}$  increase

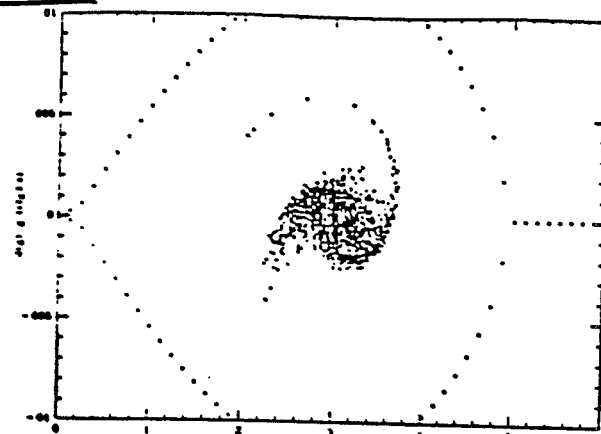
$\alpha_1 = -0.6$

$\frac{2}{\pi} = 1.2 \Omega$

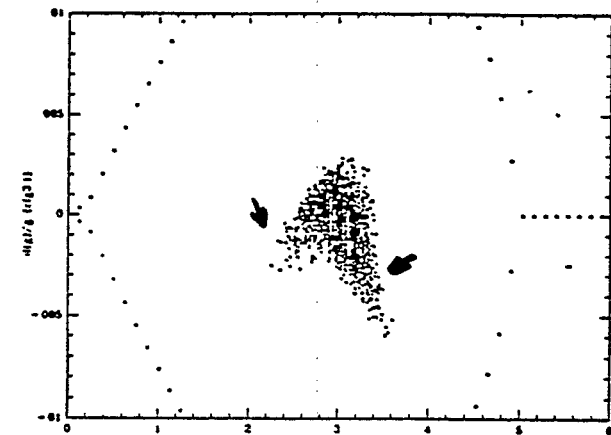
capac.



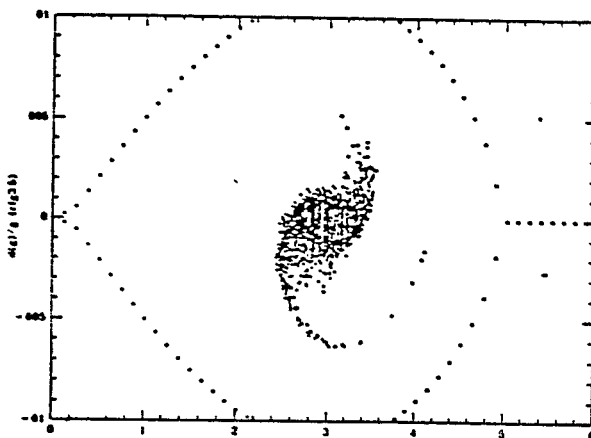
↓ -150 ms



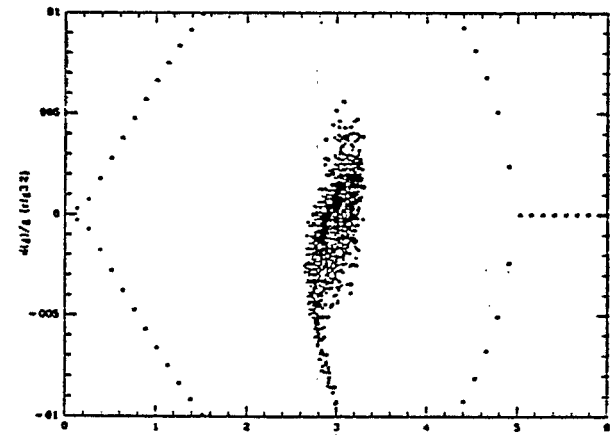
↓ +175 ms



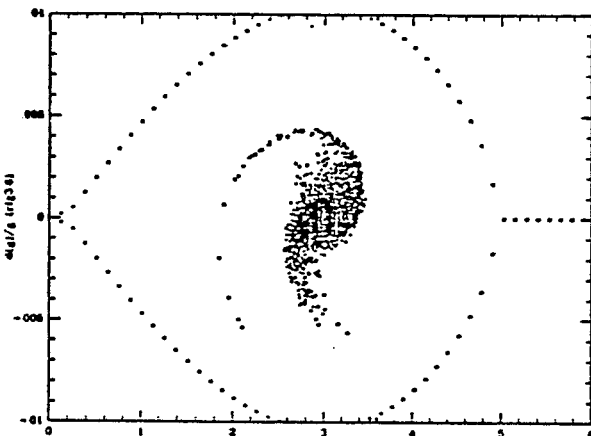
↓ +25 ms



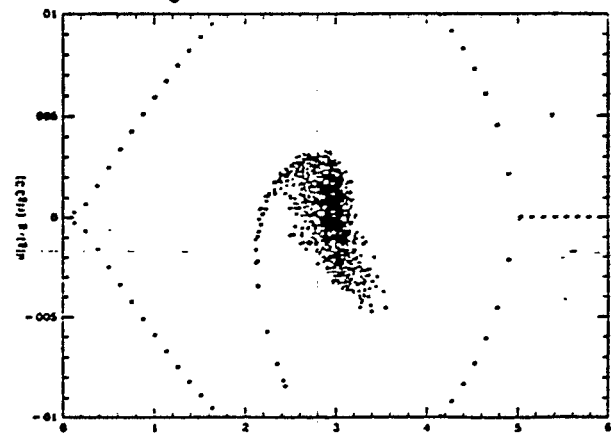
↓ +225 ms



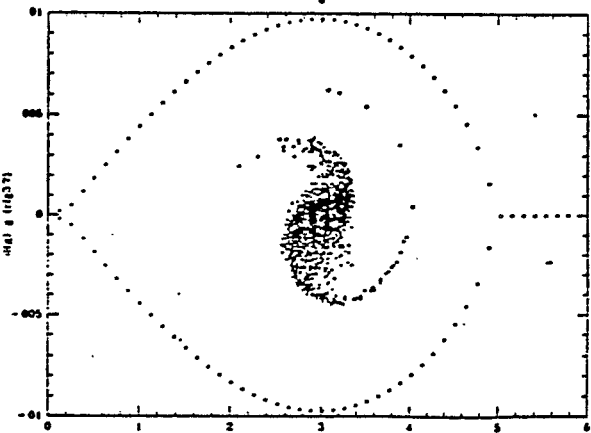
↓ +75 ms



↓ +275 ms



↓ +125 ms



+325 ms

Eff. 100%

0.3 ~ 0.4  
eV·s/a

$$\hat{V} = 100 \text{ kV}$$

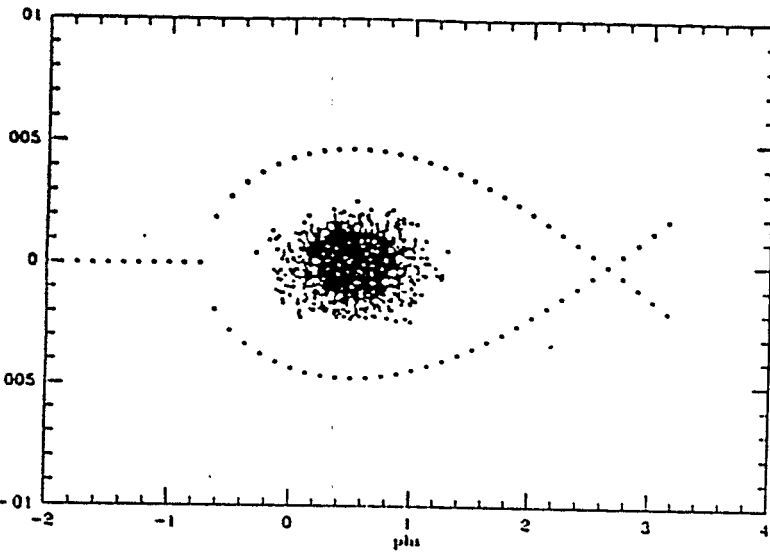
$$\sin \phi_s = 0.48$$

$\chi_t$  JUMP

$$\Delta \chi_t = 0.6$$

in 60 ms

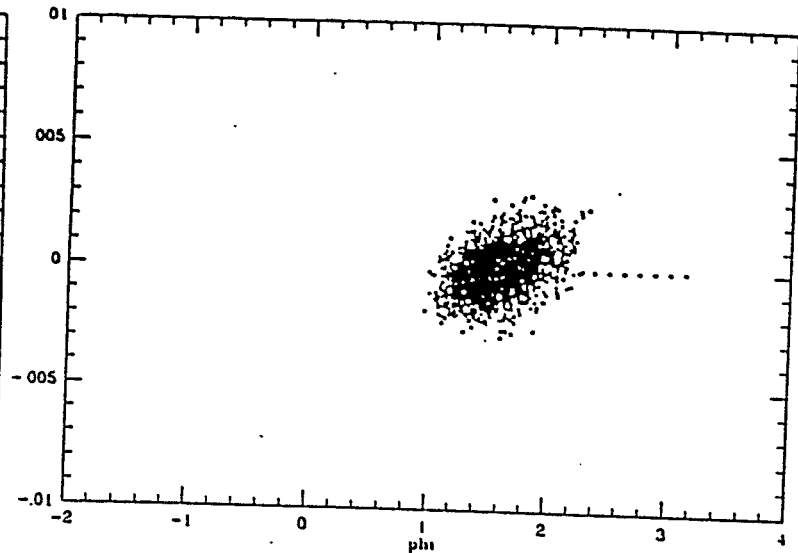
$A_u^{+79}$



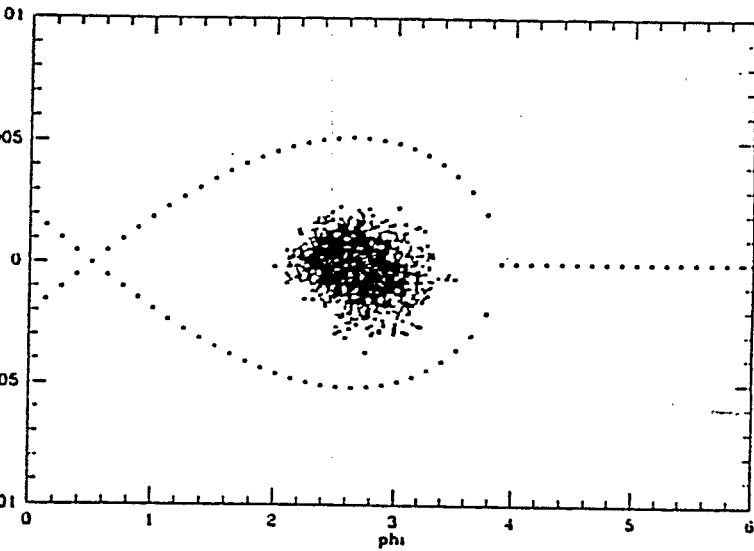
- 80 ms

$$0.3 \text{ eV} \cdot \text{s} / \text{amu}$$

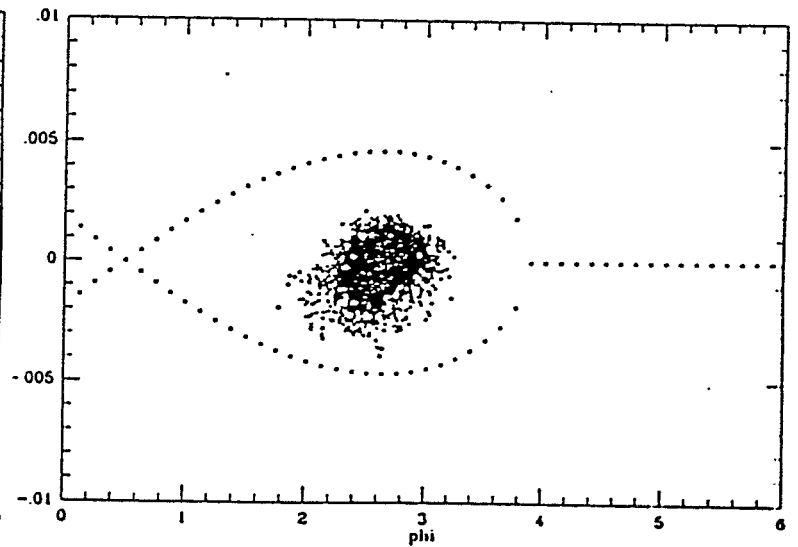
$$\frac{Z}{n} = 1.2 \Omega$$



0 ms



+ 25 ms



+ 80 ms

$\eta(\frac{\Delta p}{p})$  & s.c. included. No Loss

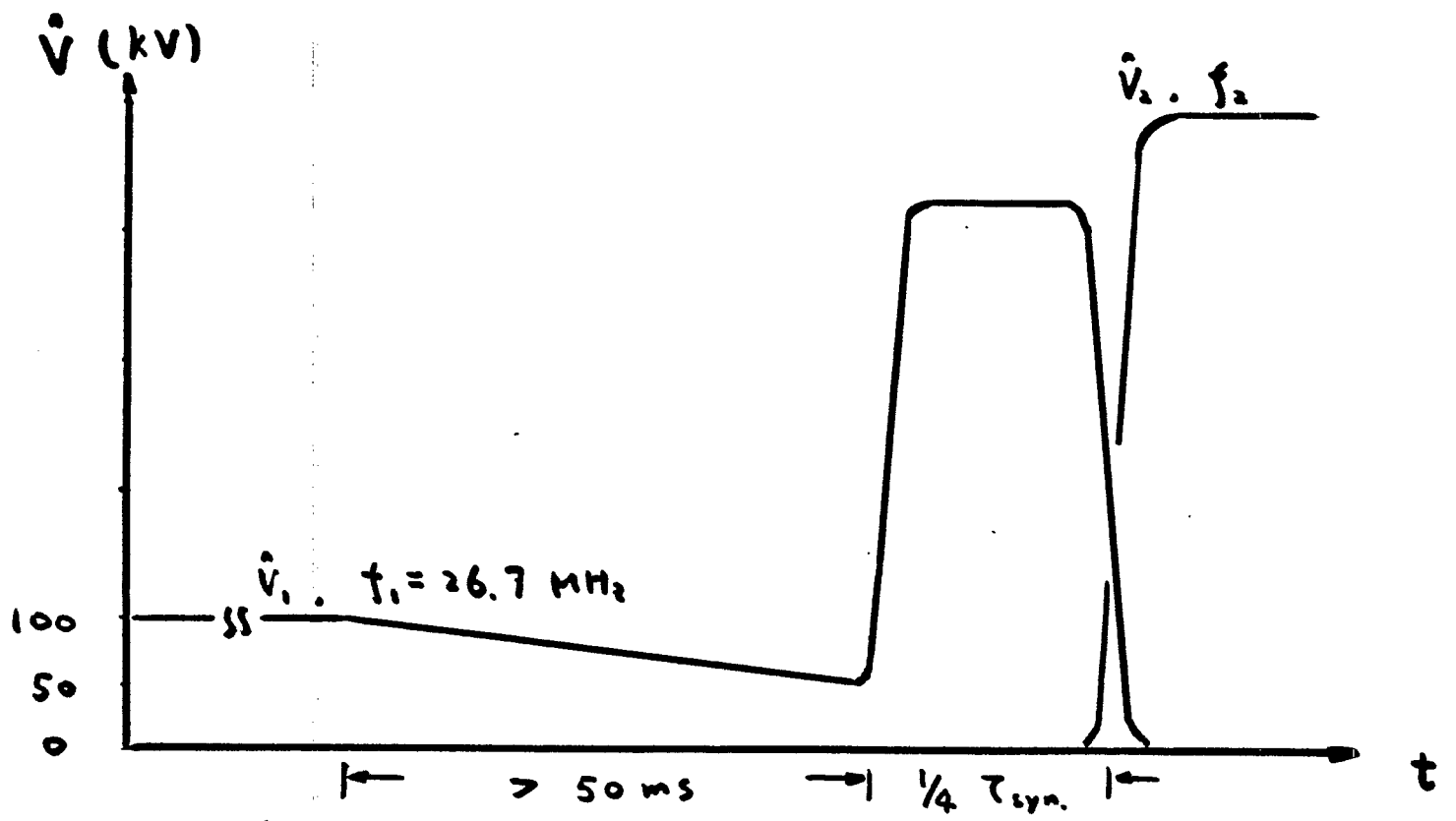
$$\tau_0 \ll T_{n.l.} \ll T_c < \tau_{syn.}$$

Negligible Blow up

## II. Transfer to High Freq. RF System

- 1 eV·s/amu bunch area. (general case)  
Bunch rotation (squeeze  $\frac{\Delta p}{p}$ , rotate, recapture)
- 0.3 eV·s/amu bunch area
  - \* Adiabatic compression ✓ A. G. Ruggiero
  - \* "Simple" rotation ✓ E. Raka
  - \* unstable fixed point rotation ✓ S. Y. Lee  
J. Wei
- Switch over near transition  
proposed by J. M. Brennan
  - ✓, when combined with  $\gamma_t$  jump  
or  $\dot{\gamma}$  increase
- 160 MHz, right bucket length

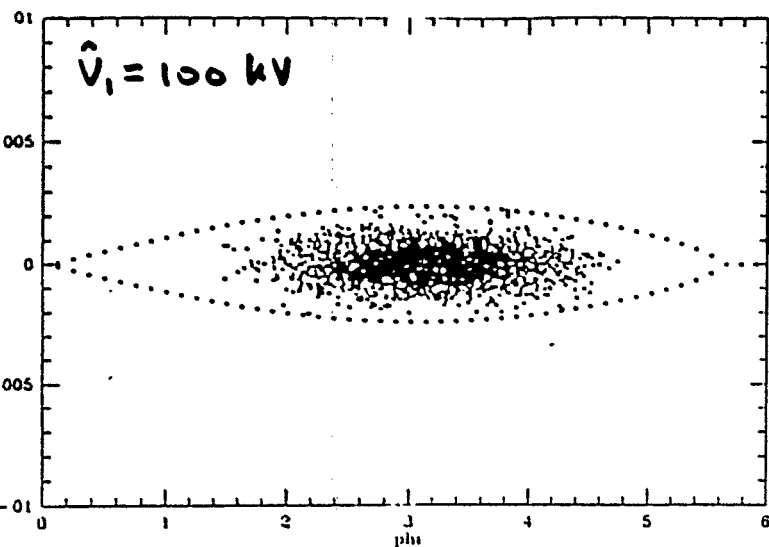




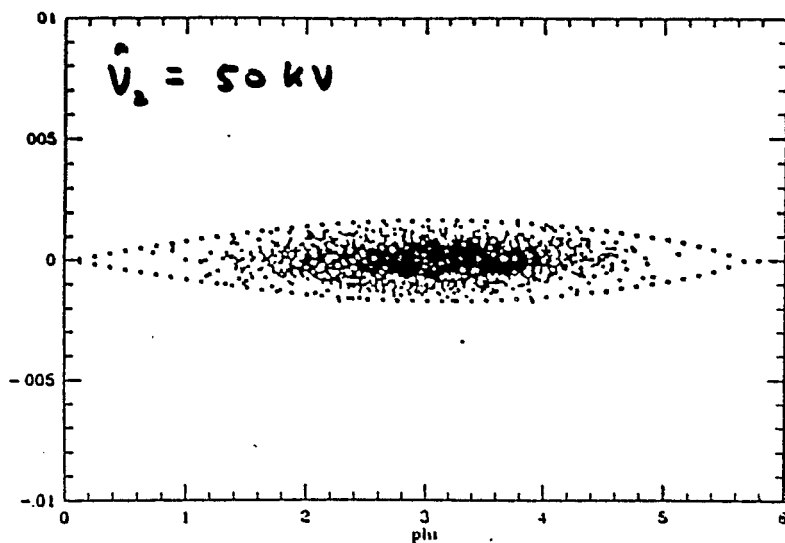
Bunch rotation

# Bunch Rotation, 1 eV.s/a

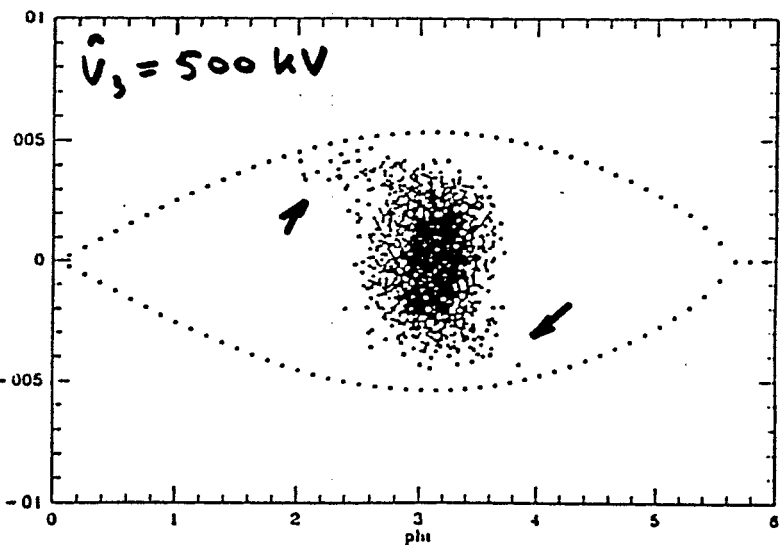
$A_u^{+79}$



0 ms



50 ms



58 ms

1 eV.s/amu

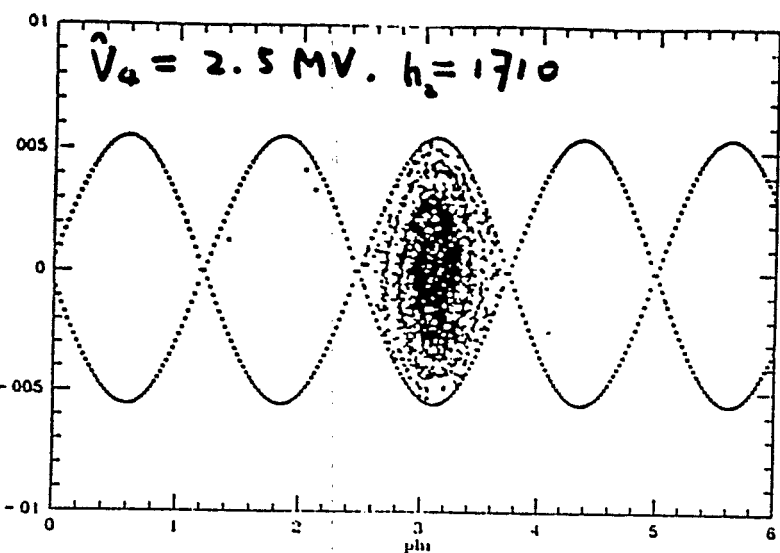
$\gamma_{\text{top}} = 30$

with s.c.

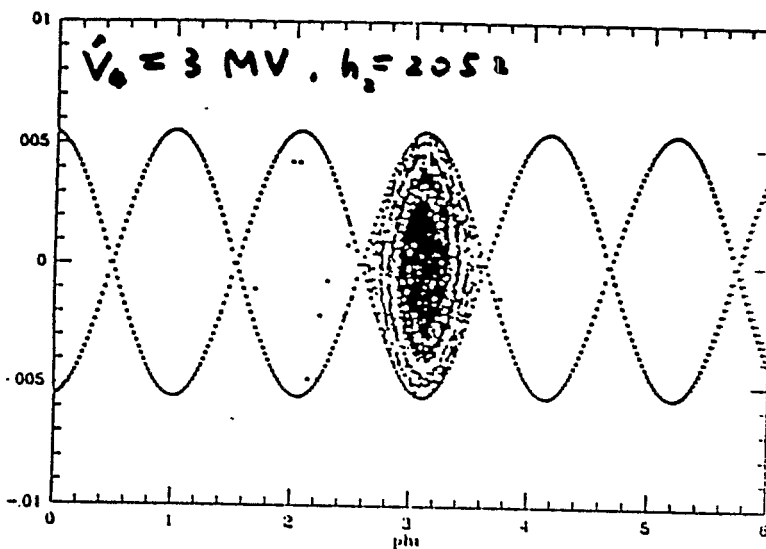
# Bunch Rotation Recapture

$A_n^{+79}$

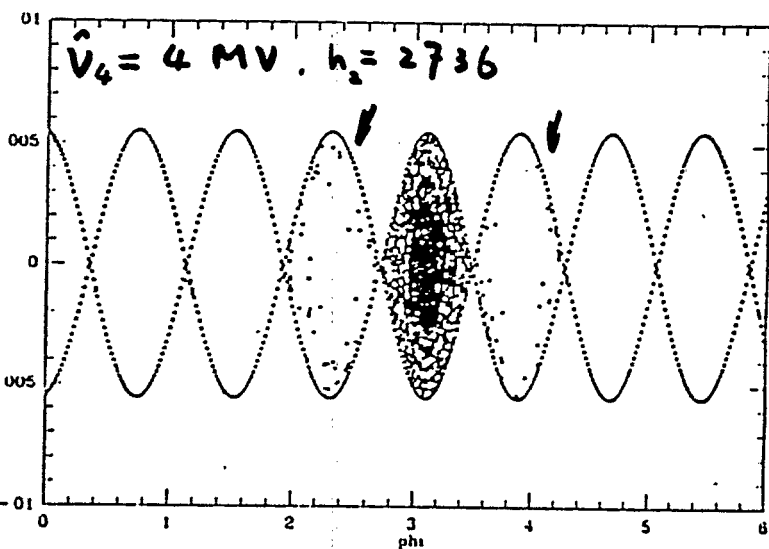
RHIC



134 MHz 98%  
survival



160 MHz 95%  
surv.



214 MHz 87%  
surv.

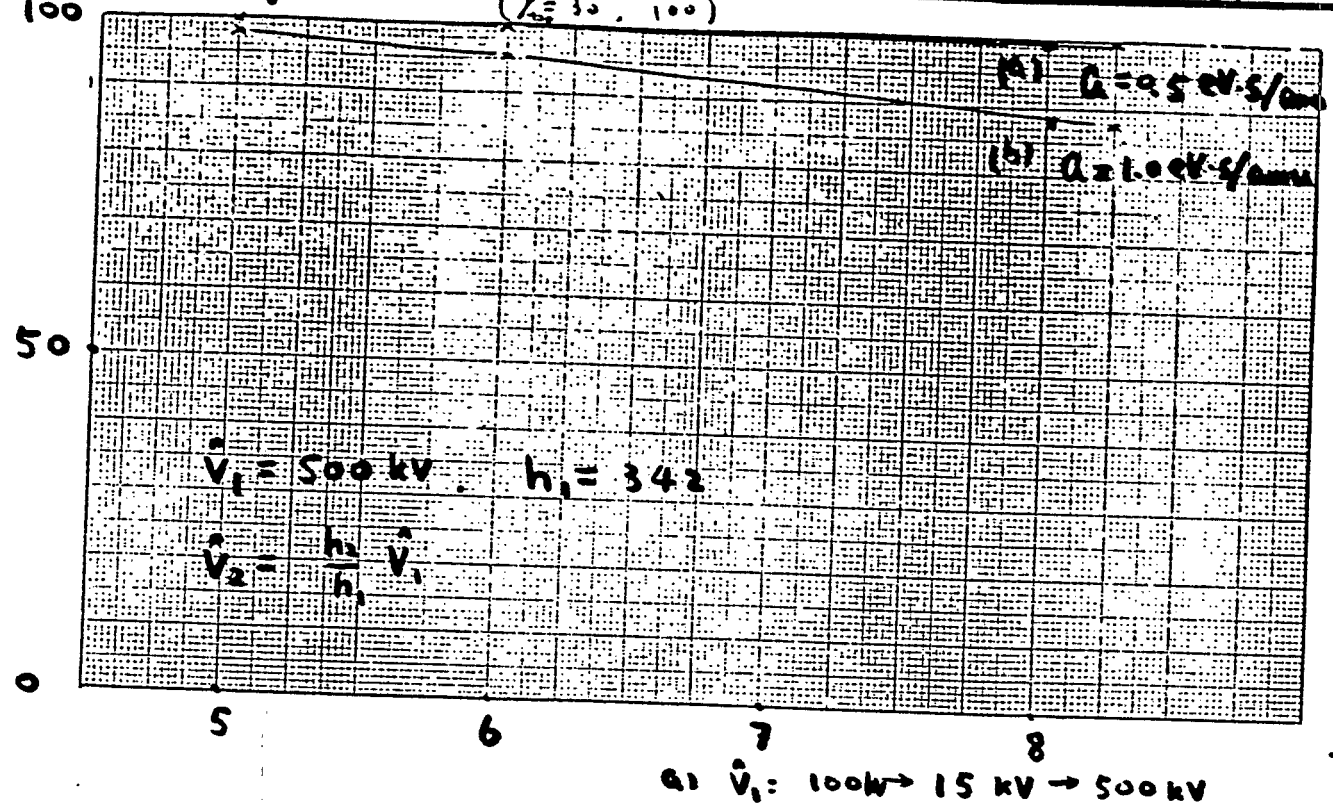
$$\hat{V}_3 = 500 \text{ kV}$$

$$\hat{V}_4 = \frac{h_2}{h_1} \hat{V}_3$$

$$h_1 = 342$$

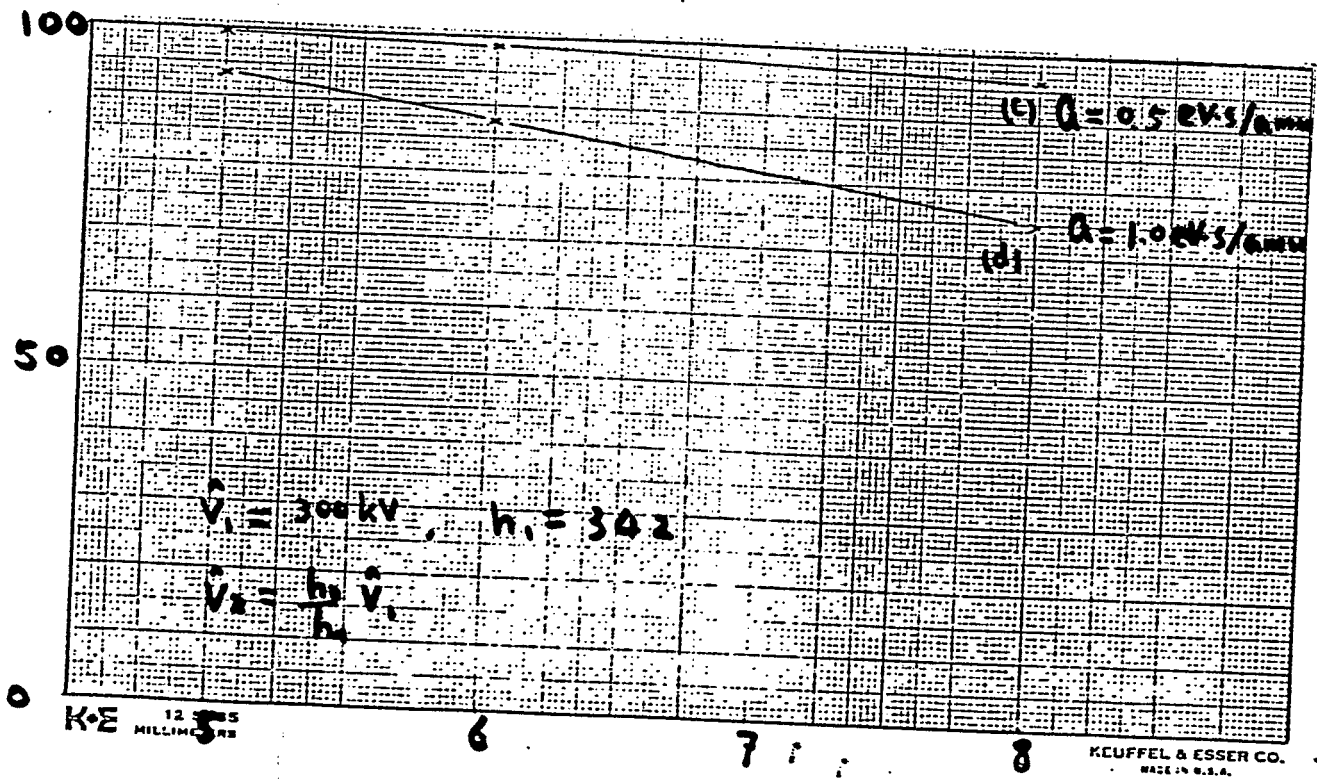
$$(\text{At } \gamma_{\text{top}} = 30)$$

# Efficiency (%)      Top Energy Bunch Rotation Efficiency ( $V_0 = 30, 100$ )



- a)  $V_1: 100 \text{ kV} \rightarrow 15 \text{ kV} \rightarrow 500 \text{ kV}$
- b)  $V_1: 100 \text{ kV} \rightarrow 50 \text{ kV} \rightarrow 500 \text{ kV}$

## Efficiency (%)

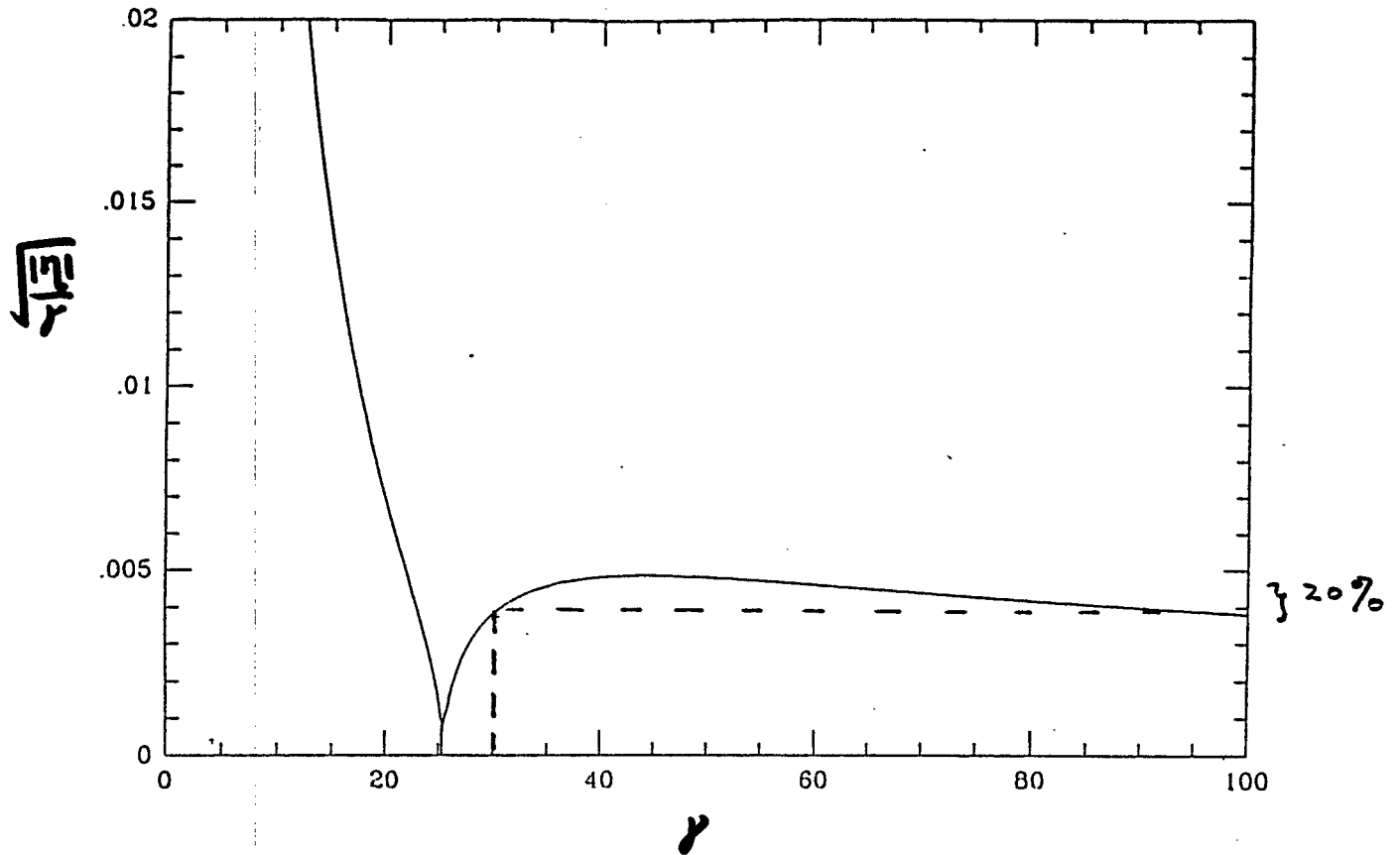


- c)  $V_1: 100 \text{ kV} \rightarrow 15 \text{ kV} \rightarrow 300 \text{ kV}$
- d)  $V_1: 100 \text{ kV} \rightarrow 50 \text{ kV} \rightarrow 300 \text{ kV}$

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# Energy dependence of Rotation



$$\Omega_s \propto \sqrt{\frac{|\eta|}{\gamma}}$$

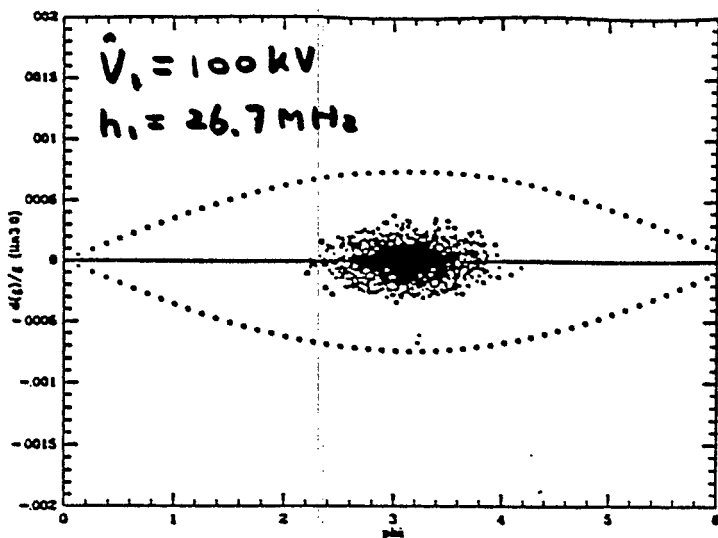
timing

$$\hat{V}_3 \propto \sqrt{\frac{|\eta|}{\gamma}}$$

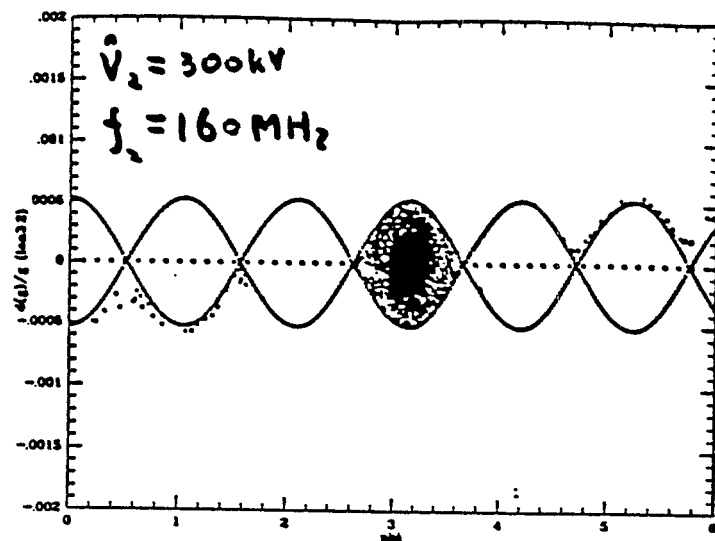
rotation voltage

# 0.3 eV·s/amu "Simple" rotation

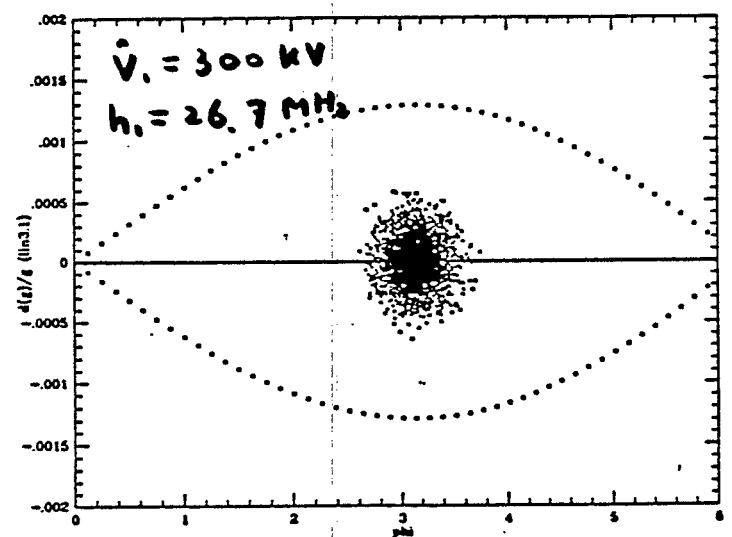
$Au^{+79}$   
RHIC



0 ms



36 ms



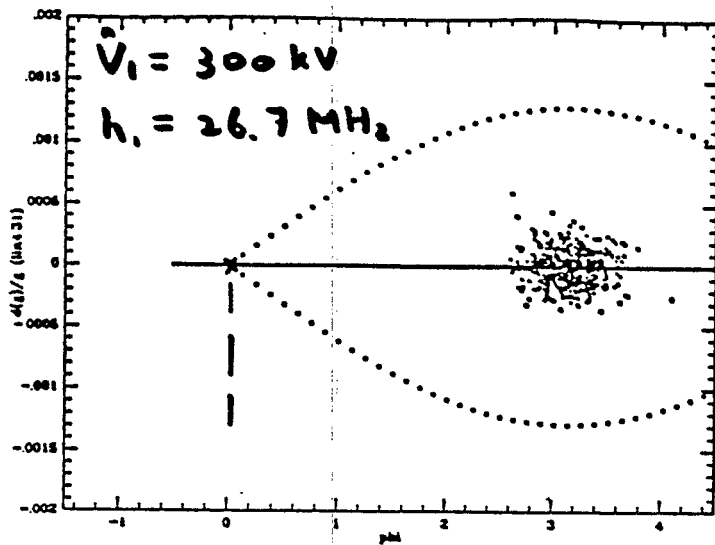
10 ms

98% in 0.39 eV·s/amu  
bucket

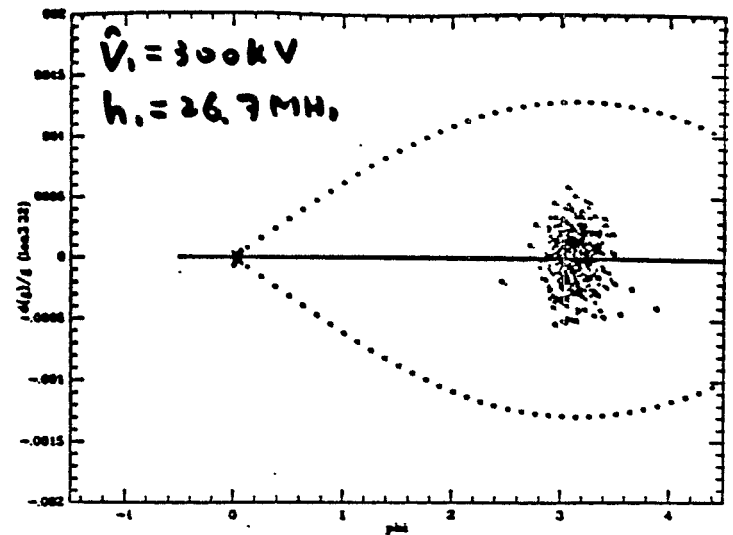
(At  $\gamma_{\text{top}} = 100$ )

# Rotation by using unstable fixed point $A_u^{+19}$

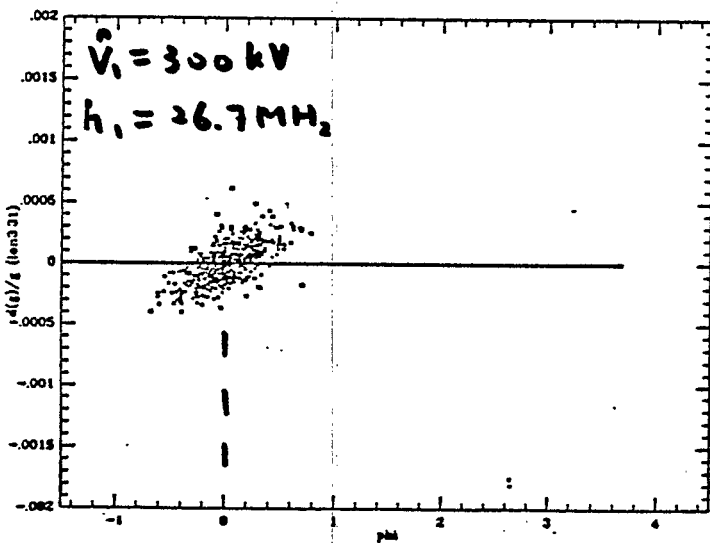
RHIC



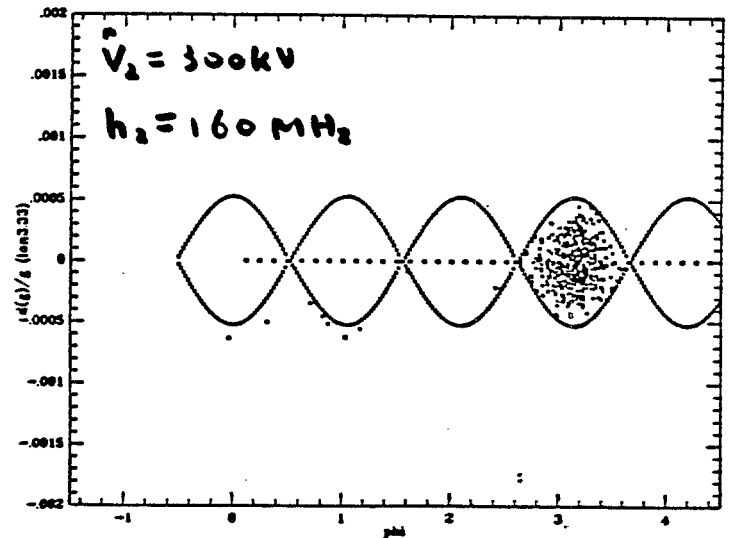
0 ms



14 ms



2.6 ms

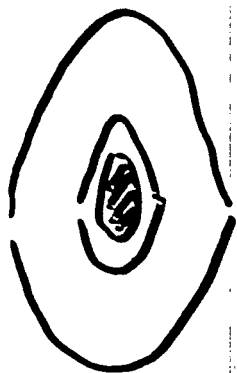


43 ms

98% in  $0.39 \text{ eV} \cdot \text{s/amu}$   
 bucket

(At  $\gamma_{\text{top}} = 100$ )

When matched



$$\frac{\Delta P}{P} / \frac{\Delta \phi}{h} \sim \hat{V}^{1/2} \cdot h^{1/2} \cdot \eta^{-1/2}$$

$\parallel$   
const

$$\hat{V}_1 = 300 \text{ kV}$$

$$h_1 = 342$$

$$\hat{V}_2 = 300 \text{ kV}$$

$$h_2 = 6 \times 342$$

a gain of  $\frac{\hat{V}_2'}{\hat{V}_1} = 6$

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AGS

$$\hat{V}_1 = 12 \text{ kV}$$

$$\Rightarrow \hat{V}_1' = 100 \text{ kV}$$

$$\frac{\hat{V}_1'}{\hat{V}_1} \sim 8$$

But

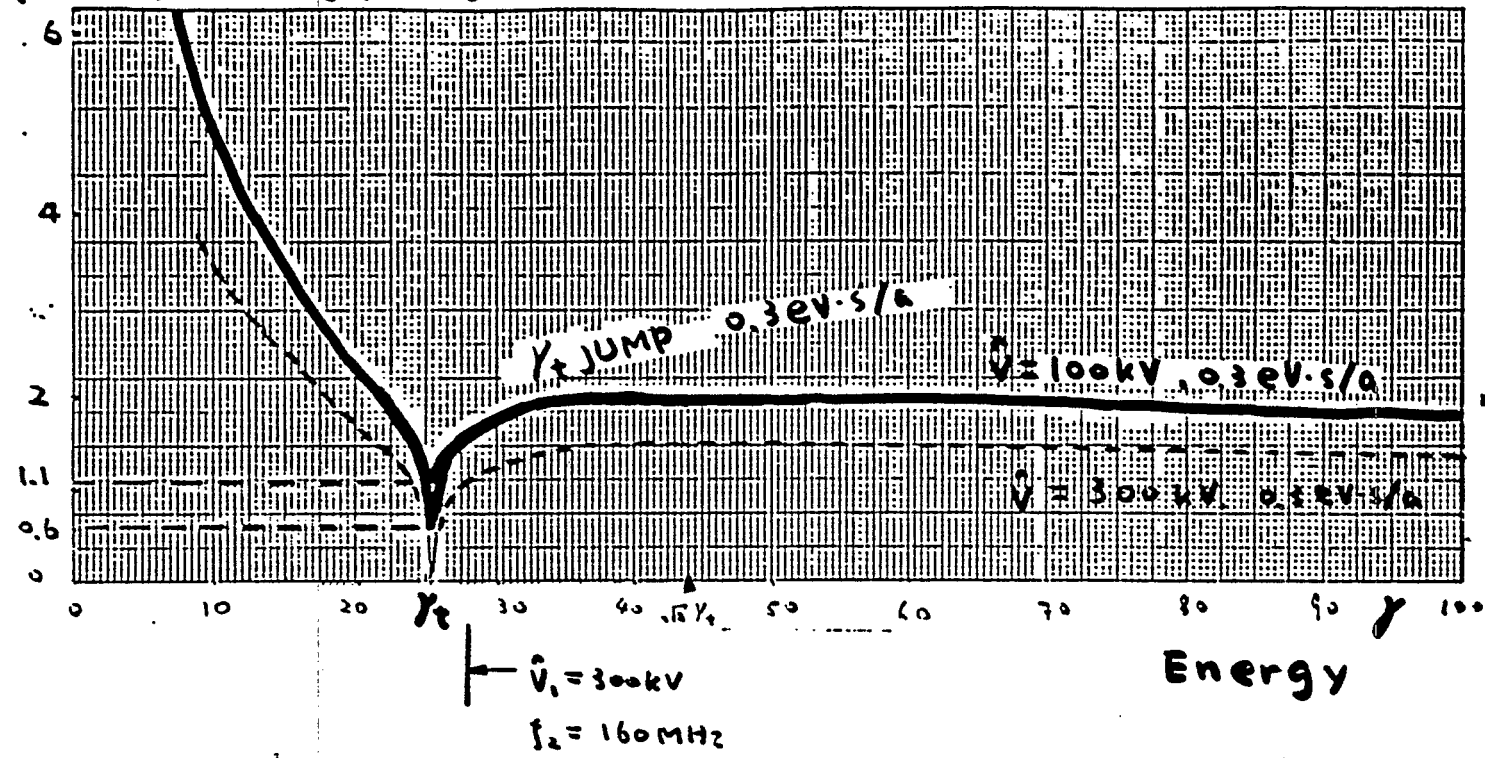
$$\Omega_s = \sqrt{\frac{2 e V h \eta c^2}{2 \pi A m_0 c^2 R_s^2 \cos \phi_s}}$$

$$\Rightarrow \tau_{\text{syn}} = 5 \text{ ms} \quad ?$$

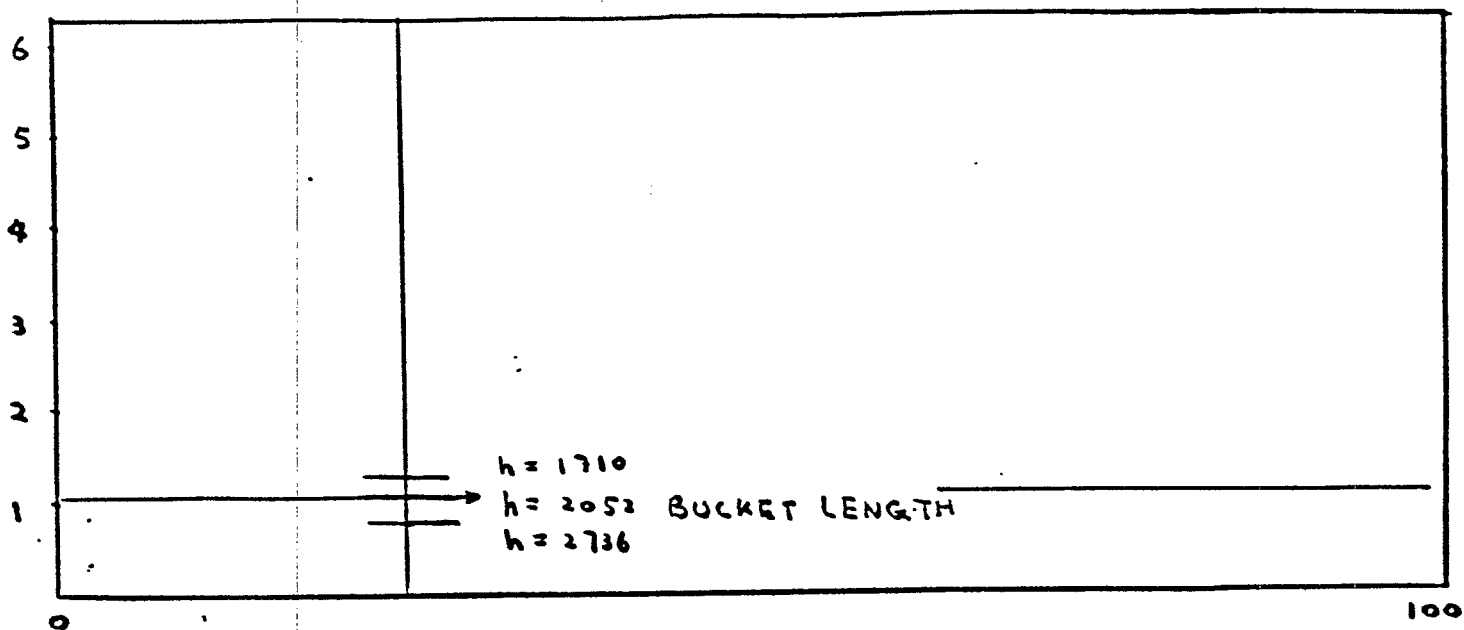
$$\text{RHIC, } \tau_{\text{syn}} = 40 \text{ ms} \quad (300 \text{ kV})$$



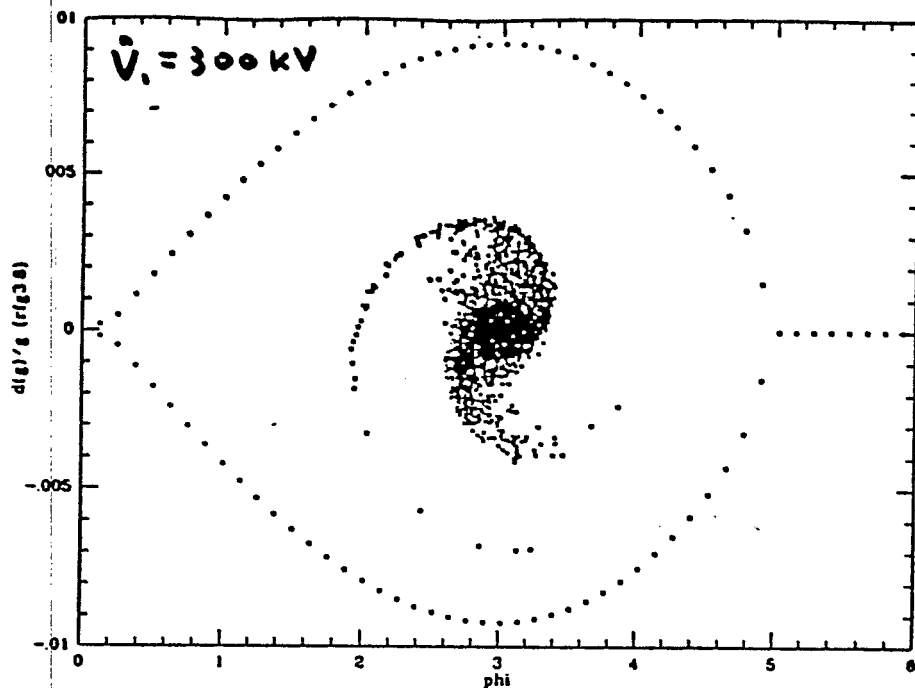
# Bunch Length (rad. .26.7 MHz phase)



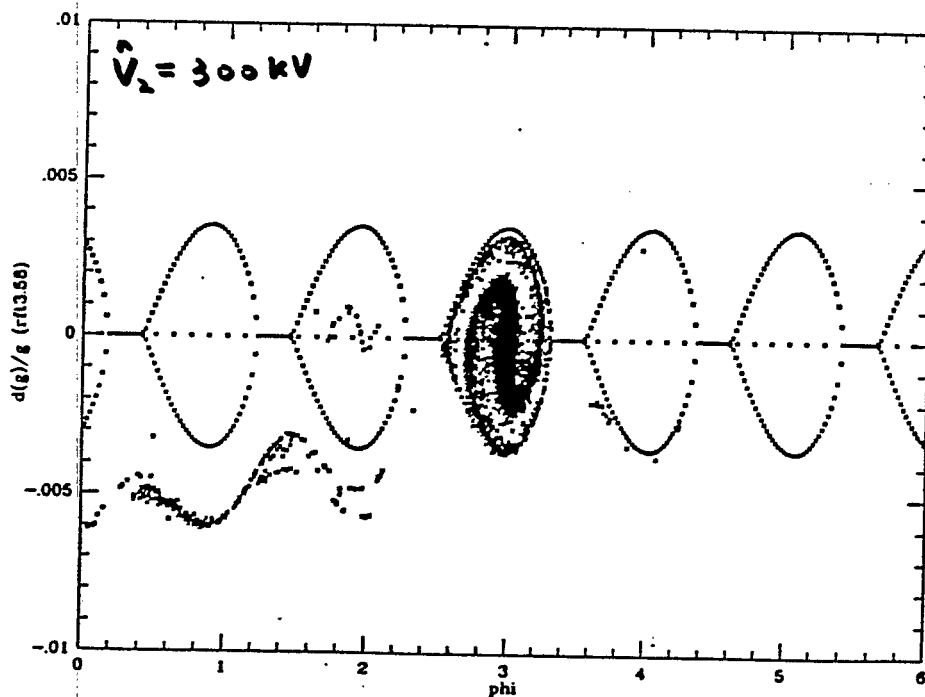
- Switch over near transition  $\gamma_t$ .



# Switch over near transition (after a $\beta$ increase)



+ 375 ms



+ 440 ms

$A_u^{+19}$

$\gamma_{tr.} = 26.14$

$\gamma_{t0} = 25.4376$

$\hat{V}_1 = 100 \sim 300 \text{ kV}$

$\hat{V}_2 = 300 \text{ kV}$

$h_1 = 342$

$h_2 = 2052$

$\alpha_1 = -0.6$

$\frac{2}{n} = 1.2 \text{ n}$

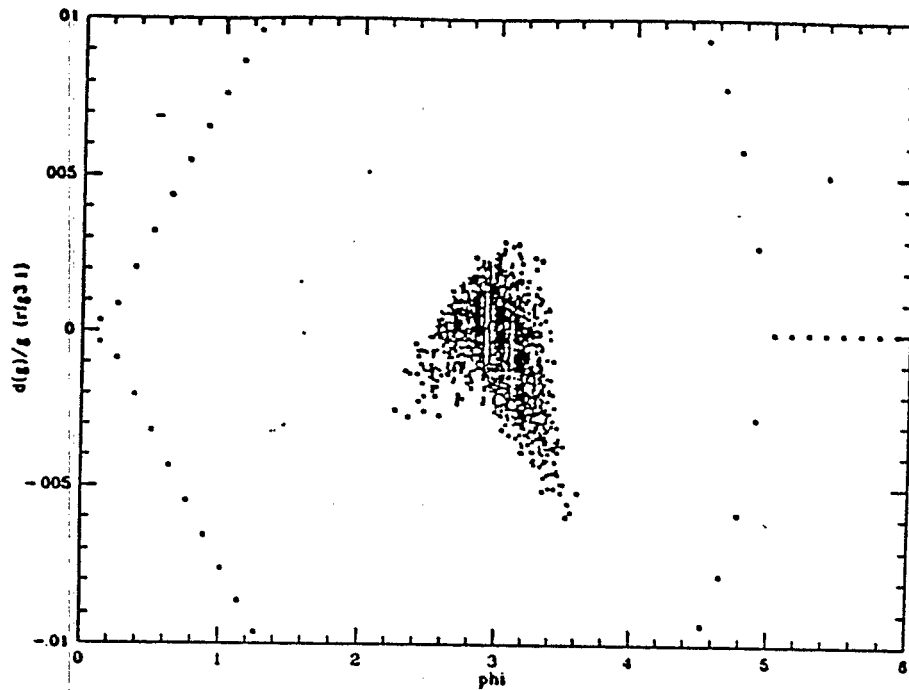
capac.

Transfer  
Efficiency

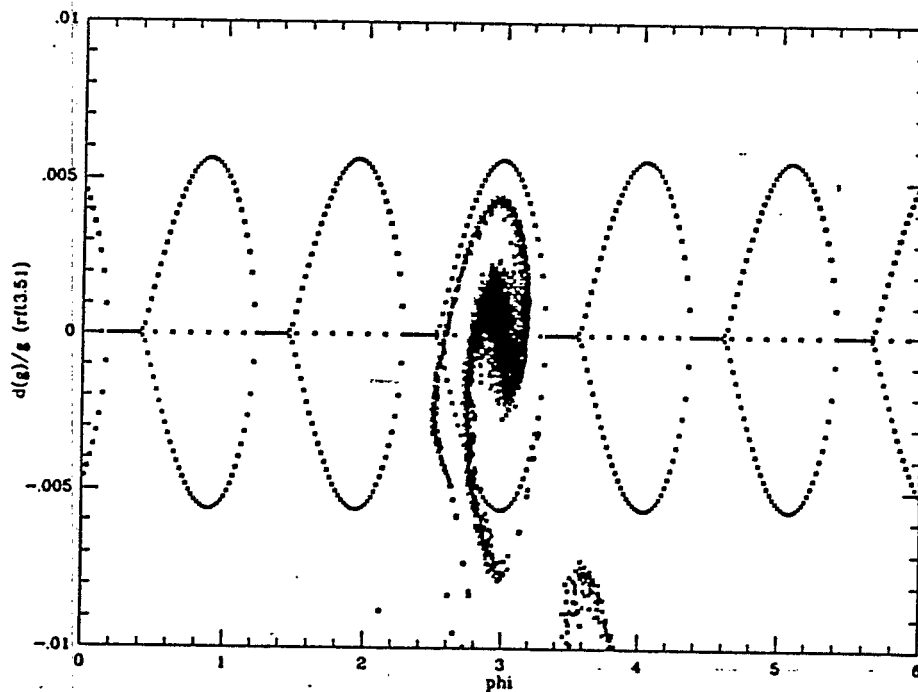
90%

in  $0.6 \text{ eV} \cdot \text{s}/\text{am}$

# Switch over near transition (after a $\dot{\gamma}$ increase)



+ 25 ms



+ 90 ms

$$A_u^{+79}$$

$$\gamma_{tr.} = 25.65$$

$$\gamma_{t0} = 25.4376$$

$$\hat{V}_1 = 100 \sim 300 \text{ kV}$$

$$\hat{V}_2 = 300 \text{ kV}$$

$$h_1 = 342$$

$$h_2 = 2052$$

$$\alpha_1 = -0.6$$

$$\frac{2}{n} = 1.2 \Omega$$

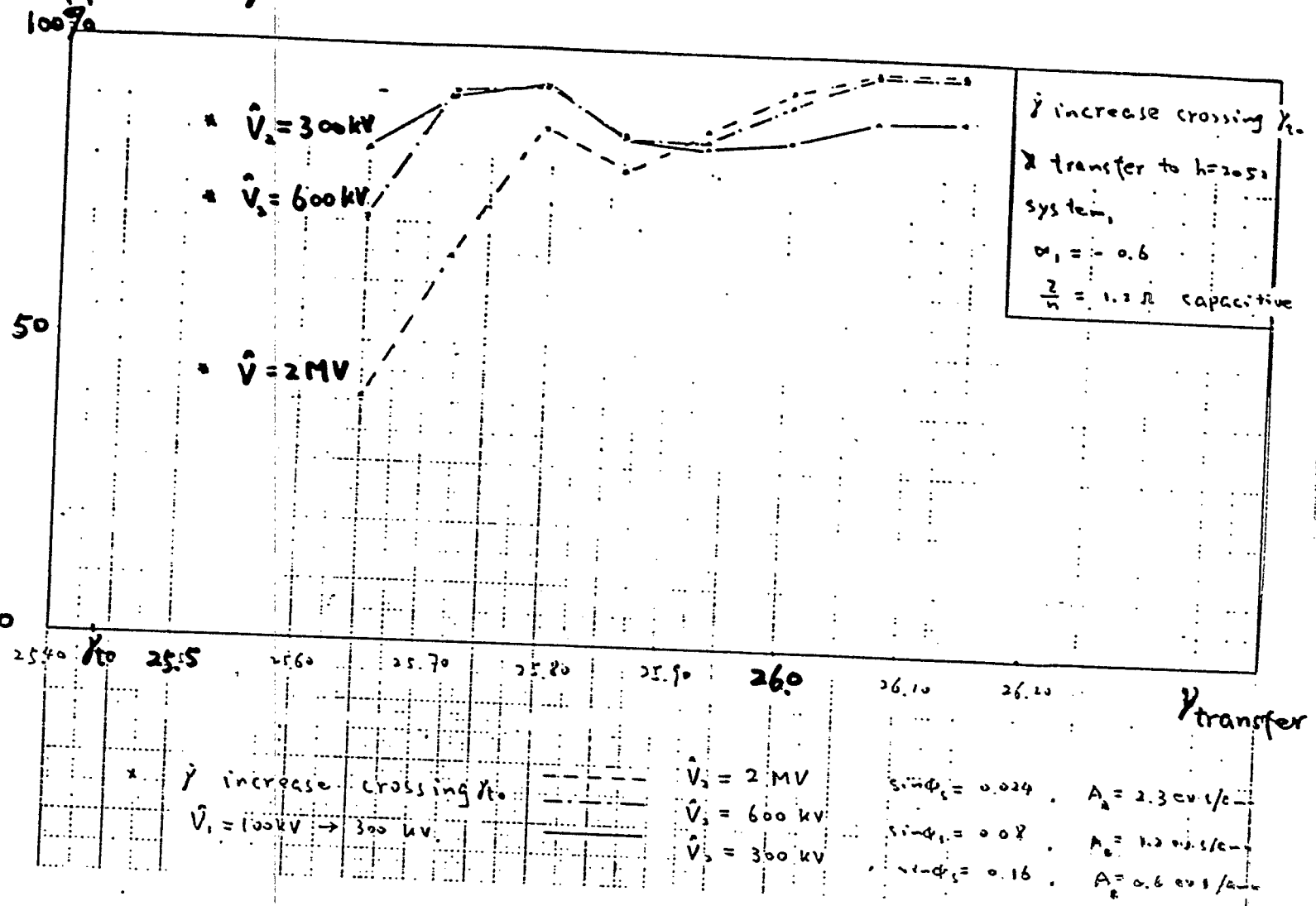
capac.

Transfer  
Efficiency

83 %

in 0.6 eV·s/a

# Transfer Efficiency

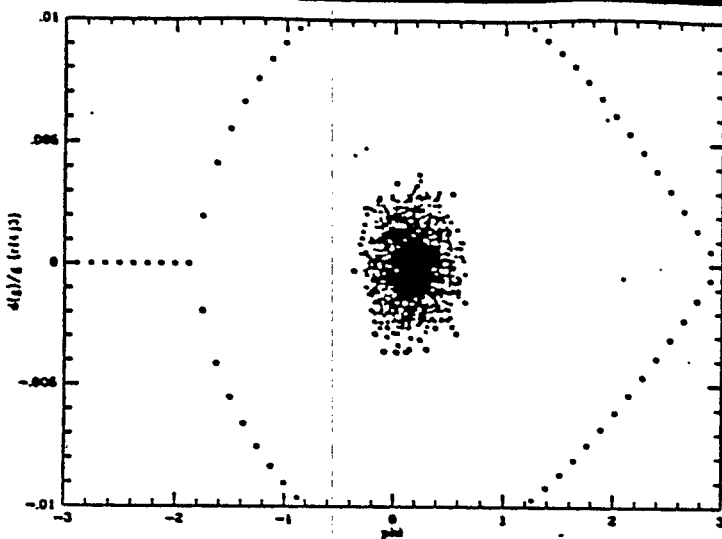


- $\gamma$  increase of  $\Delta\gamma = 0.38$ . in 40 ms
- then transfer to 160 MHz RF system  
on  $\gamma = \gamma_{\text{transfer}}$

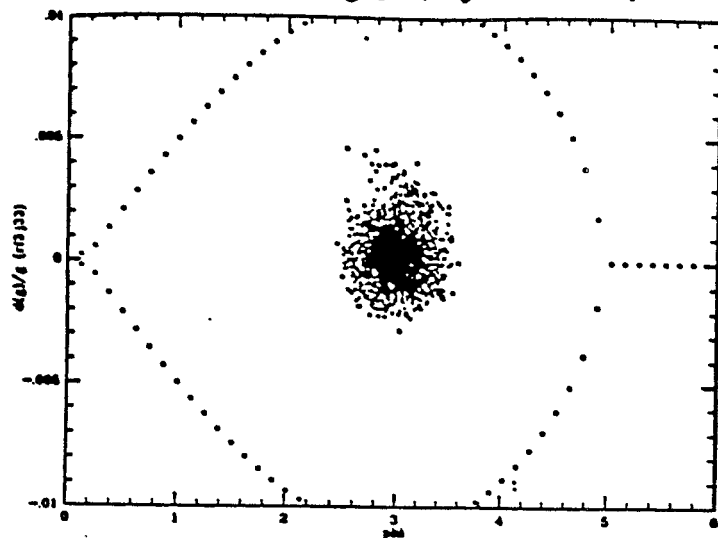
$\gamma_t$  jump,  $\hat{V}_t = 300$  kV

$\Delta\gamma_t = 0.6$   
in 60 ms

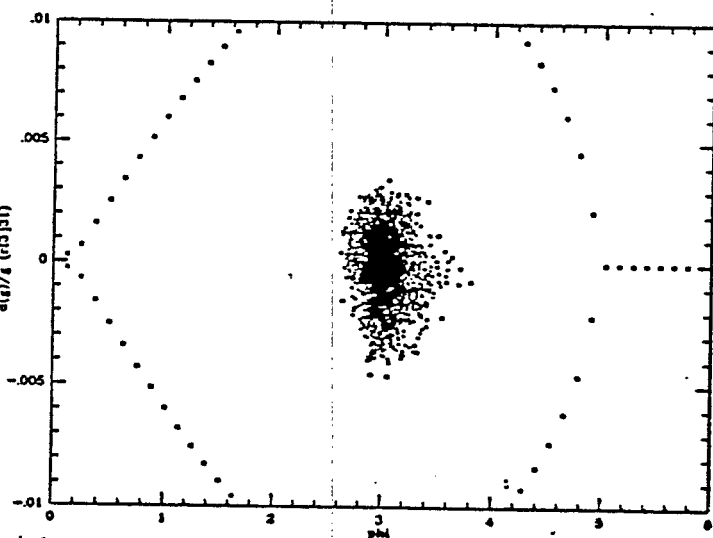
Au<sup>217</sup>  
RHIC



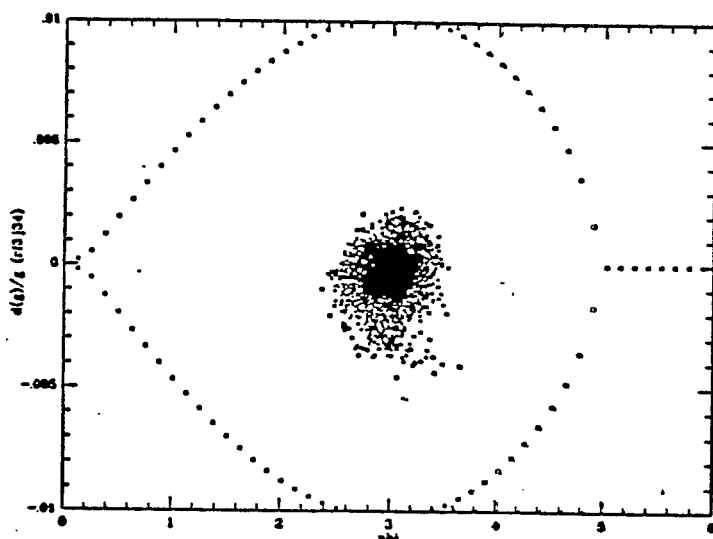
-80 ms



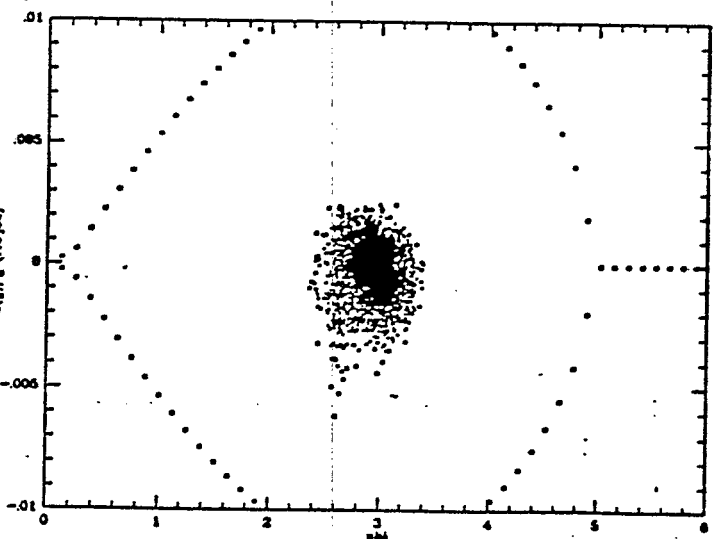
+125 ms



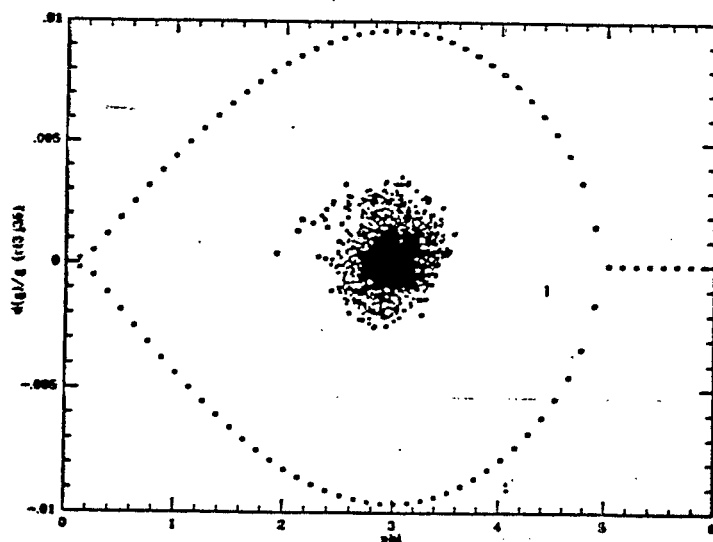
+25 ms



+175 ms



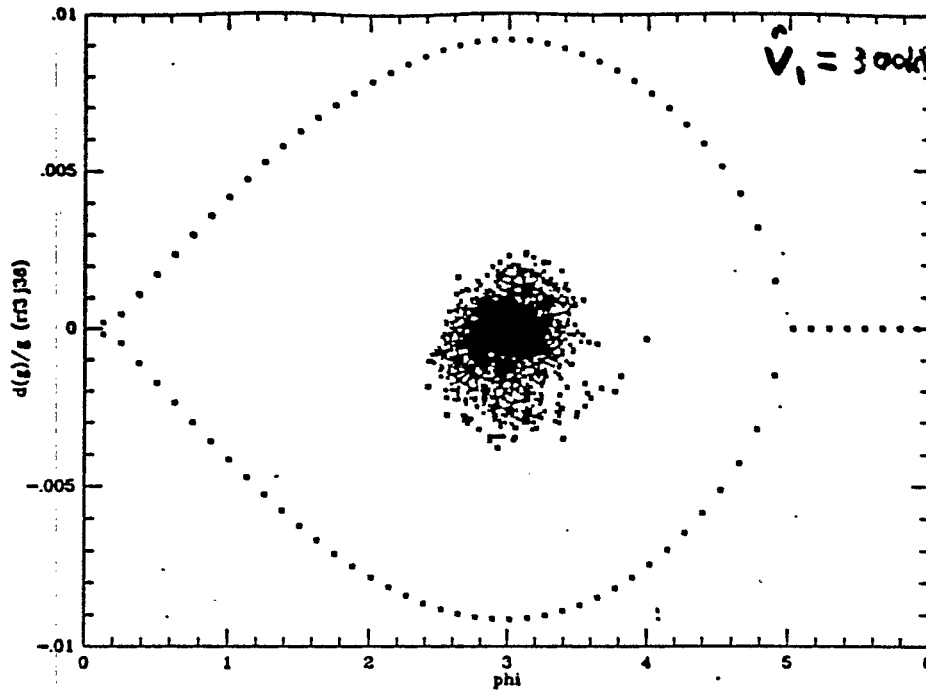
+75 ms



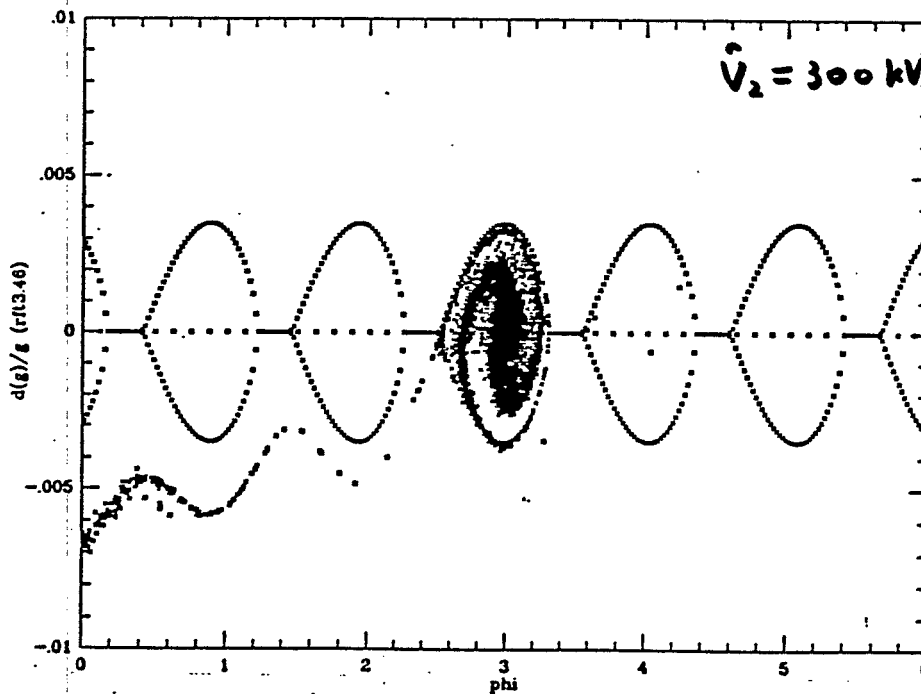
+225 ms No Loss

# Switch over near transition (after a $V_t$ jump)

$A_u^{+29}$   
RHIC



+ 275 ms

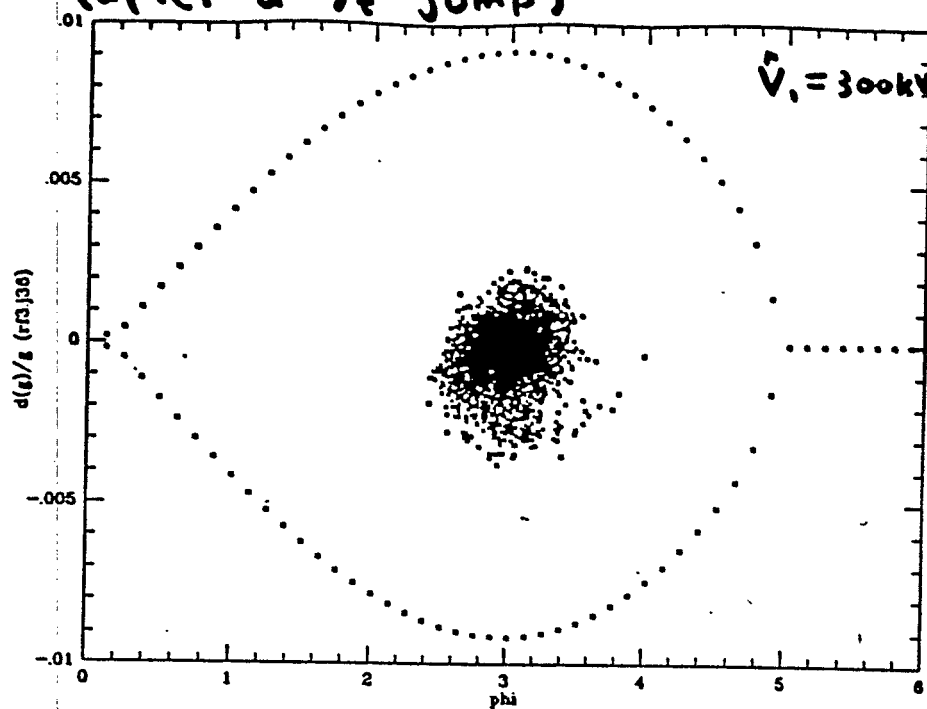


+ 340 ms

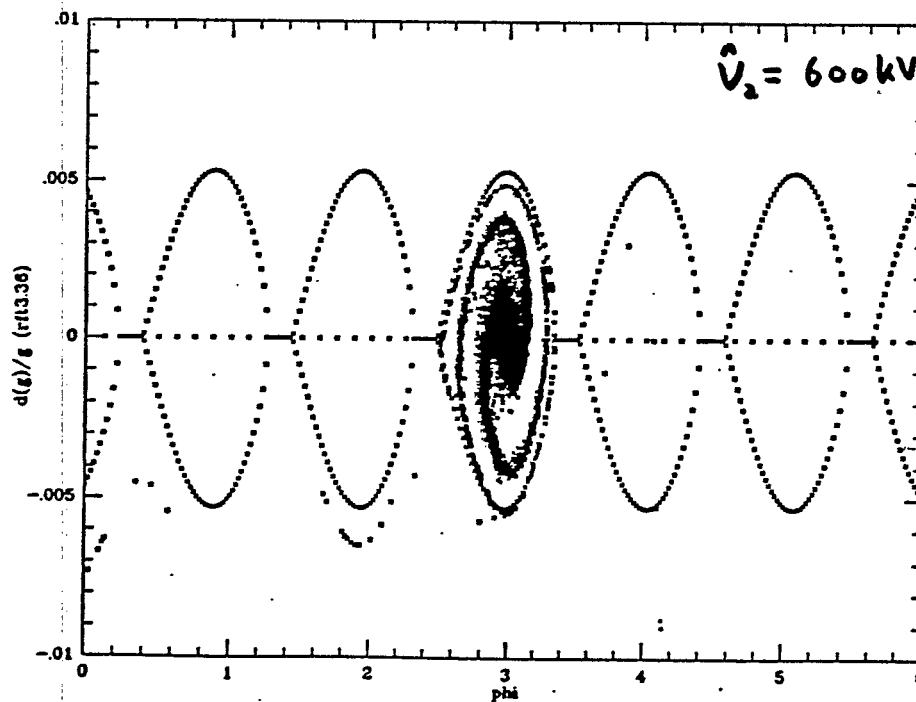
92% surv.  
in 0.6 ev. s/c  
bucket

# Switch over near transition (after a $V_t$ jump)

$A_u^{+39}$   
RHIC



+ 275 ms

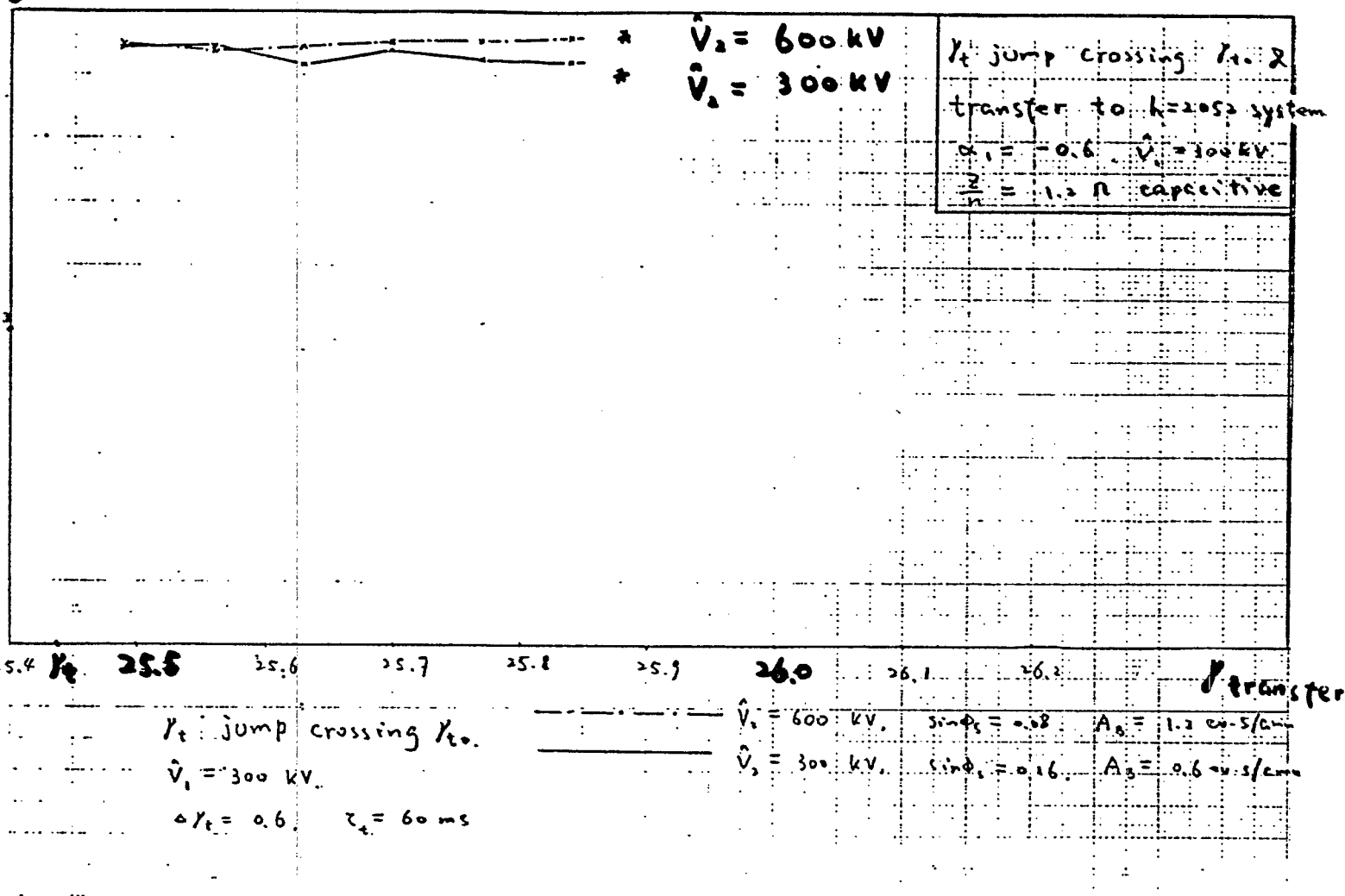


+ 340 ms

96% surv.  
in 1.2 eV.s/a  
bucket



# Transfer efficiency



- $V_t$  jump of  $\Delta V_t = 0.6$  in  $60 \text{ ms}$ ,  $V_1 = 300 \text{ kV}$
- then transfer to  $160 \text{ MHz}$  RF System
- on  $V = V_{\text{transfer}}$

## Summary on the previous study:

- \* A  $\gamma_t$  jump, or a  $\dot{\gamma}$  increase near transition is very helpful
- \* A r.f. voltage of 300 kV for 26.7 MHz system helps in achieving  $\dot{\gamma}$  increase, rotation and providing sufficient bucket area at low energy
- \* A second system of 160 MHz is comfortable for r.f. system transfer.