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Space Charge Effects in the RHIC

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ABSTRACT

Space charge tune shift for the RHIC operation is evaluated. The limitation on the high intensity operation may be important to the planning of the future improvment on RHIC. The transverse instability due to the resistive wall is also evaluated.

1. Introduction

Injection energy of RHIC is limited by the maximum operational energy of the AGS. It is useful therefore to evaluated the limit of the intensity in RHIC due to the space charge. This is important because the luminosity of the collider will be upgraded in the future. Space charge may impose certain limit on the intensity of the beam.

The charge density of the bunched beam is given by,

$$\rho = \frac{qeN_b}{\sqrt{2\pi}\ \sigma_l S}$$

where q, N_b, σ_l and S are charge state of the particles, number of particle per bunch, bunch length and the beam cross section area respectively. The linear magnetic force acts on a particle in the bunch is given by,

$$\delta F_r = \frac{qe\rho}{2\varepsilon_o\gamma^2}\ r$$

The magnetic force produce a defocusing force on the particle according to,

$$\delta F_r = \gamma m_o A\ \delta\left(\frac{d^2r}{dt^2}\right) = \gamma m_o A\ (\beta c)^2\ \delta\left(\frac{d^2r}{ds^2}\right)$$

$$\delta\left(\frac{d^2r}{ds^2}\right) = \frac{r_o 2\pi\rho q^2}{A\beta^2\gamma^3 e}\ r$$

where r_o=1.535 $\cdot 10^{-18}$m, A is the mass number of the ion. The defocusing force induce a tune shift $\Delta\nu$,

$$\Delta\nu = \frac{r_o 2\pi\rho q^2}{2A\beta^2\gamma^3\nu e}\ R^2 = \frac{r_o q^2 N_b\sqrt{2\pi}}{2A\beta^2\gamma^3\nu\ \sigma_l S}\ R^2$$

Note here that the space charge tune shift is proportional to the number of particles in the bunch and inverse proportional to the volume of the bunch.

For a low to medium energy accelerator, the tolerence of the space charge tune shift is rather large. This is because the accelerator has fewer important resonances. AGS and CPS are believed to be able to accept and accelerate particles with space charge tune shift of the order of 0.7. The number of important resonances becomes large for higher energy storage rings. This paper aimed to evaluate the space charge tune shift and the effect on the particle dynamics. Section 2 discusses the numerical result of the space charge tune shifts. Section 3 discusses the transverse instability limit and compared with the resistive wall impedance.

2. Space charge tune shift in the COLLIDER.

Table 1 lists the parameters for the normal RHIC operation. At the normal operation intensity, the space charge tune shift is most important only for the heavy ion, where the injection energy is low due to the small charge to mass ratio. The space charge tune shift of 0.02 may be important to the particles in the bunch. It is useful to note that particles suffer large tune shift are particles with smaller emittances. The nonlinear effect becomes less important. The operational window of the high energy machine is also small, i.e. about .033. It is therefore important to take good care of the machine tune to obtain large tolerance for the space charge.

Table 1. Parameters at injection and the space charge tune shift.

particle species	P	D	C	S	Cu	I	Au
A	1	2	12	32	63	127	197
$\langle Q \rangle$	1	1	6	16	29	53	79
$N_b (10^9)$	100	100	22	6.4	4.5	2.6	1.1
$B\rho_{inj}$(Tm)	98	98	98	98	98	98	98
β_{inj}	.99949	.99799	.99799	.99799	.99763	.99712	.99689
γ_{inj}	31.57	15.81	15.81	15.81	14.56	13.21	12.69
ε_N (πmm-mrad)	20	20	10	10	10	10	10
η	0.00047	-0.0025	-0.0025	-0.0025	-0.0032	-0.0042	-0.0047
$\Delta\nu_{sc}$	0.0036	0.00725	0.0189	0.0147	0.0203	0.0236	0.0155

There has been discussion on the ultimate performance of RHIC machine[3]. It is noted that the intensity of proton can be increased by a factor of 10 and the intensity of Au ion is increased by a factor of 5. To maintain the tune shift within the operational window of the tune space, it is necessary to injected the beam with a minimum normalized emittance of 24 and 26 π-mm-mrad for Proton and Gold respectively, where the limiting space charge tune shift is taken to be 0.03. At these large emittance at the injection, it is important also to evaluate the dyanmical aperature for the beam. Tracking calculations are needed.

3. Intensity limitation due to the microwave instabilities.

Besides the space charge tune shift, the beam intensity limitation is given be the microwave instability. The instability is damped by the tune spread in the beam. The limiting impedance is given by,

$$|Z_{\parallel}/n| \le \frac{2\pi|\eta|EA}{qeI_p}(\Delta p/p)^2$$

$$|Z_{\perp}| \le \frac{4\sqrt{2\pi}|\eta|EA}{qeI_p\langle\beta\rangle}(\Delta p/p)$$

Based on the scenario of the rf acceleration voltage, we calculate the longitudinal bunch length and the momentum spread in table 2. The limiting impedance is given in the last row of table 2. Since the impedance is inverse proportional to the peak current, or the number of particles per bunch. we expect that the collective instability is also given a limit to the intensity of the bunch. We observe that the longitudinal microwave instability may be not important at the injection. It will become more important at the transition energy[4] where the Landau damping is not effective. Heavy ions have to be accelerated through the transition energy. Careful evaluation has been previously addressed. On the other hand, serveral experiments in the FNAL booster have been carried out for the proton bunch transition crossing. It is observed that the space charge force is the main cause in the longitudinal phase space dilution. Simulation may be important to the RHIC as well for the heavy ion operation. The transverse instability limit is observed to be 33MΩ/m for the proton. It may sound rather large. However, in the high intensity operation at 10^{12} particles per bunch, the limiting impedance becomes 3.3 MΩ/m. The resistive wall of the stainless steel will contribute 1.8MΩ/m. Thus the transverse instability may pose intensity limitation to the proton operation.

On the other hand the heavy ion intensity is not limited by the instability. At the five times of intensity of Au ion, the limiting impedance is still rather large. The real limit resides on the transition crossing, where the Landau damping vanishes, i.e. $|\eta|=0$. At the transition the growth rate is proportional to resistive wall impedance component. Table 3 lists the growth rate of transverse instability for stainless steel and for copper plated beam pipe. The growth rate for the Copper plated pipe is far better for the transverse instability especially when the heavy ions have to cross the transition energy. The stainless steel contribute about 1.8MΩ/m to the transverse impedance, while the copper plated beam pipe has only 0.2 MΩ/m of impedance. At the transition energy, these effect will be important. Naturally, it is also important to find other possible scheme for the transition crossing.

Table 2. LIMITING IMPEDANCE due to the microwave instability

particle species	P	D	C	S	Cu	I	Au
τ_{rev} (μs)	12.79	12.81	12.81	12.81	12.81	12.82	12.82
ω_{rev} (rad/s)	491068	490331	490331	490331	490154	489904	489788
α(evs/amu)	17.68	3.84	3.84	3.84	3.12	2.47	2.25
ω_s (rad/s)	505.15	1161.6	1161.6	1161.6	1316.0	1508.1	1589.5
Δp/p(^-3)	5.82	2.53	2.53	2.53	2.23	1.95	1.85
σ_L (m)	1.64	1.64	1.64	1.64	1.64	1.64	1.64
I(peak)(A)	1.16	1.16	1.53	1.19	1.52	1.60	1.01
I(av.)(A)	0.43	0.43	0.56	0.44	0.56	0.59	0.37
limiting impedances for microwave $\langle n \rangle_{micro.wav}$ = 105181.							
$Z_{\parallel}/n$ (Ω)	2558	5140	23364	80315	124207	237485	584573
$Z_{\perp}/n$ (MΩ/m)	33	153	695	2391	4190	9181	23819

Table 3. Growth rate for the transverse instability+

particle species	P	D	C	S	Cu	I	Au
V(Steel)	3.62	3.61	4.76	3.87	4.92	6.12	6.26
V(Cu)	0.42	0.42	0.55	0.45	0.57	0.70	0.72

+ Note here that the growth rate is proportional to the intensity of the bunch. At higher intensity, it is important also to have copper plated beam pipe.

Table 4. space charge and resistive wall impedances

particle species	P	D	C	S	Cu	I	Au
U=	-1.07	-4.28	-5.65	-4.38	-6.59	-8.46	-5.77
Z_{sc}/n (Ω)	.00001	.00005	.00005	.00005	.00006	.00007	.00008
$Z_{\perp}$ s.c (MΩ)	0.54	2.16	2.16	2.16	2.54	3.09	3.35
$Z_{\parallel}$ (Ω)	56.92	56.88	56.88	56.88	56.87	56.85	56.85
$Z_{\perp}$ (MΩ/m) (stainless steel)	1.81	1.82	1.82	1.82	1.82	1.82	1.82

Table 4 lists also the impedance for the space charge and the resistive wall. They do not impose much constraint except at the transition crossing.

4. Conclusion

We have estimate the effect of the space charge and the resistive wall impedance on the intensity of the RHIC operation. At the norminal operation of RHIC, the space charge is of the order of 0.02. It is suggested that careful evaluations,such as tracking etc., may be needed to understand the implication. Due to the limiting operational window of the tune space. We estimate the minimum normalized emittance for the high intensity operation.

The transverse instability may also poses limit to the intensity of the bunch. We find that the norminal operation does not require copper plated beam pipe. With copper plated pipe, the growth rate is highly reduced. This effect may be important to the transition crossing, where the transverse effect has not been considered.

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