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RHIC PROJECT

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Populating 114 Buckets by Bunch Shifting
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Abstract

Cottingham proposed a scheme to populate 114, instead of 57, rf buckets at the RHIC injection, which employes a low-Q rf system of maximum 40 kV in addition to the original rf system to shift the particle bunches azimuthally along the ring.

The first part of this report studies the adiabatic condition required by this scheme. It is shown that the entire process can be accomplished during a time shorter than the cycling period of the AGS. Results of computer simulation, which are presented in the second part, indicate that particle loss is negligible during the manipulation of the 0.3 eV·s/u $^{197}Au^{79+}$ bunches.

I. Introduction

According to the Conception Design Manual, 57 of the 342 buckets produced by the rf accelerating system in the RHIC are to be populated[1] by the particle bunches injected from the AGS. Recently, several schemes have been proposed to increase this number from 57 to 114.

One of the schemes, which is proposed by Cottingham[2-4], employes a low-Q rf system of maximum 40 kV in addition to the original $h = 342$ high-Q system. Every time after 3 bunches are injected into the RHIC, this low-Q system is switched on which, combined with the original system, provides moving buckets that shift the injected bunches azimuthally along the ring. Using this method, the distance between successive bunches can be reduced from 6 to 3 bucket widths.

This report studies the theoretical aspects of this scheme. The adiabatic condition required for the bunch shifting is derived in section II. Results of computer simulation is presented in section III.

II. Condition of Adiabaticity

Let ϕ and $W = \Delta E/h\omega_s$ denote the deviations in rf phase and energy of the particles from the synchronous values relative to the high-Q rf system, where ω_s is the synchronous revolution frequency, E is the total energy of the particle, and h is the rf harmonic number. During each revolution, the voltage of the high-Q system experienced by the particle is

$$V_1 = \hat{V} \sin \phi, \tag{1}$$

where $\hat{V}$ is the peak voltage.

The low-Q voltage V_2 applies only on the bunches that is required to be shifted. The proposed voltage to achieve azimuthally shifting rf buckets is

$$V_2 = -2\hat{V} \sin \frac{\phi_d}{2} \sin(\phi + \frac{\pi}{2} - \frac{\phi_d}{2}), \tag{2}$$

where ϕ_d is linear in time t,

$$\phi_d = -v_d t, \tag{3}$$

with v_d a constant. The timing of this low-Q rf system has to be controlled in accordance with the locations of the bunches inside the ring.

For particles of the bunches that are to be shifted, the total voltage experienced each revolution is

$$V = V_1 + V_2 = \hat{V} \sin(\phi - \phi_d). \tag{4}$$

This voltage provides buckets that move azimuthally in ϕ with a speed v_d. The longitudinal motion of the particles under this voltage can be described by

$$\begin{cases} \dot{W} &= C_\phi \sin(\phi - \phi_d) \\ \dot{\phi} &= C_W W, \end{cases} \tag{5}$$

where

$$C_\phi = \frac{qe\hat{V}}{2\pi h}, \quad \text{and} \quad C_W = \frac{h^2 \omega_s^2 \eta}{E\beta^2} . \tag{6}$$

The solution to Eq. 5 is a superposition of a linear shift and that of the harmonic oscillator, which satisfies

$$\ddot{\phi} = \Omega_s^2 \sin(\phi + v_d t), \tag{7}$$

where

$$\Omega_s = \sqrt{C_\phi C_W} \tag{8}$$

is the linear synchrotron-oscillation frequency.

The crucial condition for the particles to be confined by the moving buckets and shifted azimuthally is the adiabaticity. According to the Liouville theorem, particles follow the evolution of the rf buckets with an invariant phase-space area if the relative changes in the buckets are small in one synchrotron-oscillation period. If this condition is satisfied, particles near the original synchronous point

$$\phi = \phi_s \quad \text{and} \quad W = 0, \quad \text{at} \quad t = 0, \tag{9}$$

shift with approximately the speed v_d in ϕ. The solution to Eq. 7 is then obtained as

$$\phi = -v_d t + \hat{\phi} \sin(\Omega_s t + \phi^0), \tag{10}$$

where $\hat{\phi}$ and ϕ^0 are constants determined by the initial conditions.

For instance, the motion of the original synchronous particle satisfying Eq. 9 is obtained from Eq. 10 as

$$\phi_s(t) = -v_d t + \frac{v_d}{\Omega_s} \sin \Omega_s t. \tag{11}$$

The second term at the right hand side of Eq. 11 indicates the deviation from a pure shift (the first term). In order to minimize this deviation, the process must be accomplished adiabatically,

$$N_d = \frac{\Omega_s}{v_d} \gg 1, \tag{12}$$

i.e. the number N_d of synchrotron oscillations that corresponds to a shift of 2π radian in ϕ must be much larger than 1.

III. Results of Computer Simulation

The program TIBETAN has been[5] modified to simulate the process of azimuthal shifting of the $^{197}Au^{79+}$ ion bunches. Assume that the initial total bunch area is 0.3 eV·s/u. With the peak voltage $\hat{V}$ =40 kV, the phase-space area of the shifting bucket is about 0.4 eV·s/u. The simulation shows that to achieve a reasonable efficiency, the time to shift the bunch by 2π radian in phase should be more than 3 times the synchrotron-oscillation period, i.e. $N_d \geq 3$.

Fig. 1 shows the phase-space diagram of a matched Gaussian-like bunch before the low-Q system is switched on. The low-Q system is controlled such that $N_d = 3$, i.e.

$$v_d = \frac{\Omega_s}{3} \approx 1.6 \times 10^2 \ \mathrm{s}^{-1}. \tag{13}$$

Fig. 2 shows the corresponding diagram after 9 synchrotron-oscillation period (about 0.11s). The bunch is shifted through a distance of 3 rf bucket widths. The particle loss is negligible.

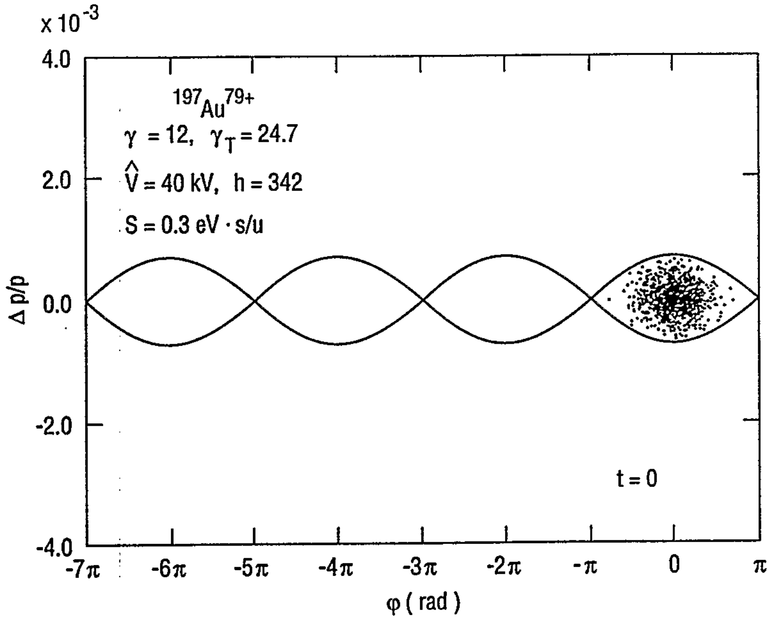

Figure 1: Longitudinal phase-space diagram of a bunch of $^{197}Au^{79+}$ ions with an area 0.3 eV·s/u, matched to the $h = 342$ rf bucket of 40 kV peak voltage.

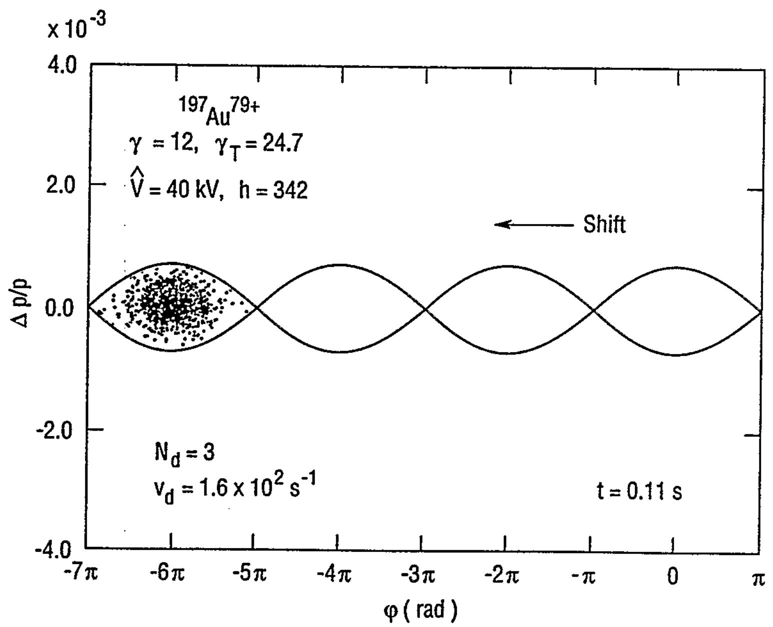

Figure 2: Longitudinal phase-space diagram of the $^{197}Au^{79+}$ bunch 0.11 s after the low-Q rf system is switched on. The bunch is shifted in phase for a distance 3 times that of the bucket width.

IV. Discussion

With an rf voltage of 40 kV, particles of a 0.3 eV·s/u bunch occupies about 3/4 of the bucket area. Therefore, the bunch-shifting process should be performed sufficiently slowly to avoid particle loss. It is indicated that the process of shifting the three injected bunches of $^{197}Au^{79+}$ to the desired locations along the ring can be accomplished in about 0.5 s, which is shorter than the cycling period of the AGS.

In order to avoid particle loss and bunch area growth, the bunches injected from the AGS should be captured by the high-Q system at a peak voltage of about 220 kV which provides buckets matched to the injected bunches. The voltage of this high-Q system is then adiabatically reduced to 40 kV before the low-Q system is switched on.

Acknowledgment

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