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Analysis and Correction of the Closed Orbit Distortion for RHIC

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Brookhaven National Laboratory

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Summary

This is a review of the RHIC sensitivity to closed orbit distortion and of the errors that are the cause. There are four sections. In the first we review the analytical calculation of the expectation values of the distortion and we compare the results with those obtained with numerical calculation. The expected uncorrected closed orbit distortion is so large to cause some concern for day–one operation of RHIC which is discussed in the second section where methods are proposed for investigation. A summary of all possible known correction methods is given in the third section; as one can see there is still work to be done. Finally the side effects of non–vanishing closed orbit deviations in the sextupole magnets are reviewed and discussed in the last section.

Estimate of Closed Orbit Distortion

Closed orbit distortions are expected to be caused by random errors of the integrated dipole field and of the actual direction of the dipole field from the reference medium plane. Moreover there are installation errors which will introduce random displacement of the quadrupole axis from the reference orbit, either horizontally or vertically, as well as rotation of the dipole magnets around their longitudinal axis. The analysis of the consequences of these errors has been done assuming a gaussian distribution of the random errors with standard deviations given below.

An analysis of the magnet imperfections has shown that the following rms widths of the errors can be expected:

Axial rotation around the reference orbit of the dipole field, including both installation and actual direction of the field in the magnet	1 mrad
Integrated field error in the dipole magnets $\Delta(B\ell)/B\ell$	0.5×10^{-3}
Displacement of the axis of the quadrupoles from the reference orbit (horizontally and vertically)	0.25 mm

These values have been used in the analysis that follows.

We first give an analytical estimate of the distortion of the closed orbit in the presence of the errors so specified. We assume that no other magnet imperfections and/or installation errors are present and that there is no restriction to the physical aperture. Moreover we assume no momentum spread in the beam; that is all the particles have the same momentum value which corresponds to the reference orbit and that the lattice is otherwise perfect. We also neglect initially the presence of the sextupole magnets. We shall consider them later. This analysis is standard procedure and is used to determine the sensitivity of the lattice chosen to the installation and magnet errors.

Though in principle not required, but to simplify the calculations, we assume also that the errors are lumped as "kicks" in the center of dipole and quadrupole magnets. This is a good approximation for the errors in the quadrupoles, but it is somewhat weaker for those in the dipoles, since these magnets have a considerable length when compared to the cell length.

If we denote with z either the horizontal (x) or the vertical (y) displacement of the closed orbit distortion we have, at a location where the curvilinear coordinate is s, the amplitude lattice function $\beta(s)$ and the phase advance $\phi(s)$, (increasing by 2π every turn)

$$z(s) = \frac{\sum_i \sqrt{\beta(s)\beta_i}\ \theta_i \cos\nu(\phi - \phi_i + \pi)}{2\sin\pi\nu} \tag{1}$$

where the sum is over all the errors, the i–th of which is at the location where the amplitude–lattice function is β_i and the phase advance is ϕ_i. The amplitude θ_i of the error represents the amount of the effective "kick". In Eq. (1) the origin of the phase advance ought to be chosen so that $\phi - \phi_i > 0$ for any i. The same equation also shows that one can expect the closed orbit distortion at one location to be proportional to the squared root of the local β–function. Similarly, the errors which give the largest contribution are those located in places with the largest β–values. Finally, the choice of the betatron tune determines the overall magnitude of the distortion which increases as the tune approaches integral values. For the RHIC case $\nu = 28.82$ and $\sin\pi\nu = 0.54$, so that the distortion is expected to be two times larger when compared to the case the tune is closer to the half–integer value.

We estimate the expectation value of the closed orbit on the horizontal plane in the middle of the QF quadrupoles, and that on the vertical plane in the middle of the QD quadrupoles. In both of these locations $\beta = 50$ m.

If we ignore the contribution of the few special dipoles, BS1, BS2, BC1 and BC2, all dipole magnets are located in locations where $\beta_i \sim 30$ m, so that their contribution to the distortion of the closed orbit is

$$z = (36m) \sum_i \theta_i \cos \nu(\phi - \phi_i + \pi) \tag{2}$$

where $\theta_i = \alpha_B (\frac{\Delta B\ell}{B\ell})_i$ is the "kick" due to the error on the integrated field, or $\theta_i = \alpha_B \alpha_i$ is the "kick" due to the axial rotation α_i of the dipole field. The bending angle per dipole is $\alpha_B = 38.8$ mrad. Assuming random errors, uncorrelated to each other, we can estimate the expectation value of the closed orbit distortion as

$$\langle z \rangle = (36m) \sqrt{M \frac{\langle \theta^2 \rangle}{2}} \tag{3}$$

where $M = 162$ is the number of dipole magnets (increased to include also BS1 and BS2). We obtain in particular a closed orbit distortion in the horizontal plane, due to the integrated dipole field errors, of $\langle x \rangle = 6.3$ mm and one in the vertical plane, due to the dipole rotation errors, of $\langle y \rangle = 12.6$ mm.

Let us turn now to the contribution from the quadrupoles. These magnets are of two types: those in the regular arcs (QF and QD) and those in the insertions (Q1 to Q9). There are 23 quadrupoles in each arc and 18 in each insertion. We shall consider first the contribution to the closed orbit distortion of the arc quadrupoles, those in the insertion are expected to give at most a similar contribution. In the arcs the quadrupoles are also divided in two groups: QF and QD. In each group the value of the β– function is the same, either 50 m or 10 m, so that we can write

$$\begin{aligned} z = (46m) \sum_i^{(1)} \theta_i \cos\,(\phi - \phi_i + \pi) \, + \\ + (21m) \sum_i^{(2)} \theta_i \cos\,(\phi - \phi_i + \pi) \; . \end{aligned} \tag{4}$$

This expression is valid for both horizontal and vertical plane; the first sum is over all the quadrupoles at large β–value (50 m), the second sum is over those at small value (10 m). In either case

$$\theta_i = \left(\frac{B'\ell}{B\rho} d \right)_i$$

where B' is the quadrupole gradient and d the lateral displacement of the magnet axis. Independently of the low–β insertion

$$|B'\ell/B\rho| = 0.097\ m^{-1}$$

for both QF and QD. Using statistical arguments again, we can estimate the expectation value of the closed orbit distortion. From Eq. (4), this is $\langle z \rangle = 10.2$ mm in either horizontal or vertical plane. We can add to this the contribution from the quadrupoles in the insertion which we assume to be of the same amount. Since to obtain the expectation value is done adding quadratically all the contributions, one can expect a total $\langle z \rangle = 14.4$ mm, for either plane.

In conclusion, adding quadratically all the contributions, the expectation value of the closed orbit distortion on the horizontal plane estimated in the center of the QF quadrupoles is $\langle x \rangle \sim 16$ mm, and the value of the same on the vertical plane in the center of the QD quadrupoles is $\langle y \rangle \sim 19$ mm.

The computer code PATRIS has been used[1] to estimate the closed orbit distortion using the same approximations we have used in the analytical calculation. The only difference, and a fundamental one, is that the errors for each type of magnet are generated randomly at the computer (with cuts at $\pm 2.5\sigma$) and the actual closed orbit estimated at every quadrupole location directly, without recurring to the statistical manipulation for one given set of random errors. The operation is then repeated a large number of times, and all cases are then statistically compared. The results of the local expectation value of the closed orbit in each plane are given in Figure 1, when 21 cases have been generated on the computer and statistically compared. It is obvious that the distortion varies with the local value of the β–function, and that in the arcs it peaks in correspondence of the large β–value. We see that the computer results are in agreement with those we have derived analytically. The difference, (if there is any), can be explained by the fact that the phase advance per cell in the arcs is about 90^o, for which value there is actually a partial cancellation of the errors that we have neglected in our analysis. This can be seen very easily when all the errors have the same magnitude (systematic errors). For instance in the case of the dipole magnets in one arc, all their contributions cancel exactly with each other since there are 24 dipoles in one arc separated by about 45^o from each other. The same is true for the quadrupoles (QF or QD) in one arc because, again, there are 12 of one

[1] J. Milutinovic and A.G. Ruggiero, "Closed Orbit Correction for RHIC," AD/RHIC–AP–78, February 1989.

type in one arc all separated by each other by 90^o. Thus it results that the fortunate combinations of a 90^o phase advance cell and a number of cells per arc (12) which is multiple of four cancel automatically the average errors and reduces somewhat the magnitudes of the purely random errors.

Nevertheless the results shown in Fig. 1 are still significantly large, essentially as a consequence of the magnitude of the distribution of errors assumed. To observe that the contribution to the closed orbit distortion is larger for the dipole rotation. Though the number of quadrupoles in the ring almost double that of the dipole, nevertheless, each quadrupole magnet can produce a "kick" of smaller magnitude of that of a dipole. Also to observe that the contribution from the integral field errors is the smallest one. This result is in agreement with those obtained with the computer exercise. Actually PATRIS is also capable to perform "realistic" closed orbit calculations, by using 7×7 matrix notation (like SYNCH), and physically rotating or displacing magnets. The results, also with this more refined method of calculation, are still in agreement[1] with those shown in Fig. 1.

Injection and Day–One Operation

In absence of the actual list of errors we can only estimate, as we have done, the expectation values of the closed orbit distortions. These are shown in Fig. 1, to which, by definition, we can associate a confidence level of about 63% that the actual distortion will be less or at most equal to. A higher confidence level can be obtained by multiplying the values of Fig. 1 by a safety factor. A factor of 3 will increase the confidence to 95% that the actual closed orbit will be smaller than the value so determined.

The physical aperture, due the vacuum chamber, at QF and QD in the arcs is 36 mm. Thus, there is also about a 85% confidence level that on day one of the RHIC operation, the actual closed orbit distortion, before correction, is contained with the physical aperture. To increase the probability, to say 98%, to establish an original and uncorrected closed orbit within the vacuum chamber we need to reduce the magnitude of the magnet and installation errors by a factor of 2. The situation is made worse by the presence of the nonlinear magnet imperfections which reduce the available aperture. Thus, there is a good chance that the orbit will hit the aperture available somewhere with consequent loss of the beam itself. To observe that the design procedure adopted for RHIC is not consistent with the traditional methods used for other conventional accelerators, like the AGS–Booster, where the magnitude of the magnet and installation errors are determined as tolerances to provide a high confidence level (close to unit) that the uncorrected closed orbit already from start will be entirely within the available aperture.

It is important for the RHIC project, to determine a strategy to achieve operationally a "first-turn" around. The beam is injected, observed with beam position monitors as far as it goes, and determined where it is lost. In this mode of operation low intensity and small dimensions are required. Also, the beam position monitors are to be capable to observe a single beam pass, though absolute sensitivity and linearity are of lesser importance. According to the observed orbit displacements, the beam is steered past the point where the lost is observed by acting on the steering dipoles. This procedure, which still requires a verification by computer simulation, will be repeated until the beam completes one full turn around, and then eventually few more. The orbit will not necessarily close turn after turn but likely will oscillate around a new closed orbit, partially corrected, which will now be entirely contained within the available aperture.

Before proceeding with the correction of the residual distortion of the closed orbit, it is important to damp the free betatron oscillations with an active feedback system. If there is a spread $\Delta\nu$ of betatron tunes, the beam emittance can dilute considerably over a number of turns roughly given by $1/\Delta\nu$ which we expect to be as low as a hundred. The free betatron oscillations are to be damped also over the same period of time to avoid the emittance dilution. The initial amplitude of the betatron oscillations are to be kept to a reasonable small value to avoid exceedingly large rf power for the feedback system. Similarly, injection errors, position and angle, are also to be kept to a minimum since they also contribute to the initial amplitude of the betatron oscillations. Furthermore there will be a momentum error at injection that will cause beam bunches to oscillate in their buckets. Since the amount of error may vary from bunch to bunch a large bandwidth longitudinal phase damper will be required to act on and damp the oscillations of each bunch.

For the time being all these considerations remains as such, that is "considerations". It is important to note that the injection method, the establishment of the first-turn, and the damping of the coherent oscillations, both transverse and longitudinal, are to be studied in more details and followed with a computer simulation. This phase is very important and it has to precede the final correction of the closed orbit. At the end of this phase indeed we can still expect a considerable closed orbit distortion, because of the large errors involved; but the method will have ensured that now the orbit is entirely within the stable available aperture.

Correction of the Closed Orbit

The closed orbit correction system for RHIC is made of steering dipoles and beam position monitors located next to each quadrupole, in the arcs and in the insertions. The correctors are divided in two families: the one acting on the horizontal plane next to quadrupoles horizontally focusing (QF), and the one acting on the vertical plane next to quadrupoles vertically focusing (QD). The beam position monitors are made of pairs of striplines, 20 cm long, terminated at one end to 50 ohm, which is also the characteristic impedance of each plate to ground, and shorted to the other end. The sensitivity is 0.1 mm and the linearity over several millimeters. The steering elements are superconducting dipole correctors, 51 cm long, with a maximum integrated field of 0.3 T·m.

There are several methods to correct a closed orbit distortions, notably:

- Fourier analysis
- Quadrupole movement
- Least square method
- Sequence of local bumps.

(i) The first method is based on the fact that we can represent exactly the closed orbit distortions with a Fourier expansion

$$x \text{ or } y = \nu^2 \sqrt{\beta} \sum_n \frac{f_n e^{in\phi}}{\nu^2 - n^2} \tag{5}$$

where

$$f_n = \frac{1}{2\pi\nu} \oint \beta^{1/2} \left(\frac{\Delta B}{B\rho} \right) e^{-in\phi}\, ds$$

is the Fourier harmonic of the error distribution around the ring. Since we expect the errors are not correlated to each other, the spectrum given by f_n is more or less flat with a cut–off for $|n| \sim 400$, the total number of magnets (M). For larger values of $|n|$, f_n drops quickly to zero. Statistically, we can make an estimate of the expectation value of f_n

$$\langle f_n \rangle \sim \frac{1}{2\pi\nu} \sum_s \beta_s^{1/2} \langle \theta_s \rangle \sqrt{M_s}$$

where the summation is over the four different types of errors. It is seen then from Eq. (5) that the harmonics $n \sim \pm\nu$ give the largest contribution so that

$$x \text{ or } y \sim \nu^2 \sqrt{\beta}\, \frac{f_n e^{in\phi} + f_{-n} e^{-in\phi}}{|\nu^2 - n^2|}$$

with the expectation value

$$
\begin{aligned}
\langle x \text{ or } y\rangle &\sim \frac{\nu^2\sqrt{\beta}\langle f_n\rangle}{|\nu^2-n^2|} \\
&\sim \sum_s \frac{\nu^2\sqrt{\beta\beta_s}}{2\pi\nu|\nu^2-n^2|}\sqrt{M_s}\langle\theta_s\rangle
\end{aligned}
\tag{6}
$$

which is indeed about the expectation value we can obtain from Eq. (1). Thus the correction of the harmonic $n = 29$ already should give a considerable reduction of the closed orbit distortion and we recommend to leave the correction of this harmonic in place during the injection process to help establish the first–turn around. We leave to another time the exercise to find the arrangement of the steering magnets to provide the required harmonic correction with the amplitude as given by Eq. (6) and with a tunable phase.

(ii) The method by quadrupole movement is less effective than the method of Fourier analysis. In the past it was usually used to correct an unusually large closed orbit distortion in one particular location due mostly to bad quadrupole misalignments that were causing also a physical aperture reduction. The method was used for the Main Ring at Fermilab during the early years, and also for the SPS in CERN. Computer programs are available which make a selection of the quadrupoles that ought to be moved, once their number has been assigned, in order to minimize the peak–to–peak variation of the closed orbit distortion.

We believe that this method is rather difficult to be used with superconducting magnets and therefore we do not recommend it for RHIC.

(iii)&(iv) The last two methods to correct the closed orbit distortions are based on the same principle. RHIC will be provided with one set of N_{pu} beam position monitors and one set of N_s steering dipoles for each of the planes of oscillations. The two methods rely on the observation of the closed orbit at the location of the beam position monitors and act with the steering dipoles to obtain desired values of displacement at the PU locations. From the operation point of view the two methods will otherwise ignore the displacement in other places, like for instance inside magnets.

The correctors are of short length (75 cm) and thus the thin lens approximation is valid. The algorithm used for determining the strength of the correctors is the same as specified above: that is the lattice is assumed otherwise ideal, with no momentum spread in the beam, and without sextupoles and nonlinear imperfections in magnets. Once the steering elements are turned on, the new closed orbit distortion is given by

$$z(s) = \frac{\sum_i \sqrt{\beta(s)\beta_i}\theta_i \cos\nu(\phi - \phi_i + \pi) + \sum_\ell \sqrt{\beta(s)\beta_\ell}\theta_\ell \cos\nu(\phi - \phi_\ell + \pi)}{2\sin\pi\nu} \tag{7}$$

where again z stands for either x and y. The first sum is over the errors proper and the second sum is over all the steering elements. Since we are allowed to determine the displacements only at the PU locations we can re–write Eq. (7) for the j–th PU

$$z_j = \overline{z}_j + \frac{\sum_\ell \sqrt{\beta_j\beta_\ell}\theta_\ell \cos\nu(\phi_j - \phi_\ell + \pi)}{2\sin\pi\nu} \tag{8}$$

z_j is the desired displacement and $\overline{z}_j$ is the distortion due to the errors alone. Eq. (8) is written for $j = 1, 2, \ldots N_{pu}$ and thus represents a linear system of N_{pu} equation in N_s variables $(\theta_1, \theta_2, \ldots, \theta_{N_s})$. $\overline{z}_j$ is a vector of observed displacements and z_j is a vector of desired values.

In the special case the number N_{pu} of beam position monitors equal the number N_s of steering dipoles there is always one solution. In particular it is possible to choose $z_j = 0$. In the case $N_s = N_{PU}$ this always works. The method is very simple and requires only a matrix inversion. Years ago, when computer techniques were still limited and the inversion of a matrix of large order was difficult, the problem was avoided by showing the equivalence of the system (8) with the sequence of local bumps which grouped the steering dipoles 3 at a time in a chained fashion. The method is also described in Ref. 1 and called the Fermilab method because extensively used at Fermilab, though the method was probably originated at Cornell. This method has been fully investigated for the RHIC project and it works according to expectation,[1] though it is essential that the number of pick–ups equals the number of correctors, which is indeed the case for RHIC where actually, to simplify the methods, beam position monitors and steering elements are located in neighboring positions. The method once applied indeed provides zero closed orbit distortion at any observed location with a maximum required strength of the correctors about half of what is available.[1]

There is nevertheless one observation to be made. Consider, for instance, the horizontal plane. In the arcs, steering magnets and position monitors are located next to QF at 90^o apart. At these location the resulting closed orbit is indeed zero, but this is not true at the locations of the QD quads which are half–way. Here the orbit is also corrected but only partially; there is still a residual error that could be as large as half a millimeter, but the corresponding β–value is also lower. The same is true for the QF quads when the orbit is corrected on the vertical plane. In particular, this residual displacement is also significant at the location of the sextupole magnets. This problem can be made to disappear by inserting correctors and position monitors also in the alternate lattice locations.

Let us investigate now the case the number of beam position monitors is different from that of steering elements. Let us assume first that there are fewer position monitors as in the case when few PU's in RHIC are not working. In this case the system (8) is made of fewer equations than variables. Since we have fewer beam position readings we have a redundance in the number of correctors. We can intentionally disregard few correctors, for example those next to the beam position monitors that happened to be off, and one can still solve the system (8) requiring that the closed orbit distortion is fully corrected at the location of all the active position monitors. This has been indeed verified on the computer. Unfortunately the method cannot perform any correction at the location of the malfunctioning monitors. Actually because these locations are exactly at $\pm 90^o$ from the neighboring active element, they are by–passed by the system and the distortion is <u>not</u> corrected at all. At these locations the distortion retains the original uncorrected value and could be large. This is a point of utter importance. It is very important that all position monitors are functioning in RHIC at the same time.

The opposite case, when there are fewer correctors than position monitors, can be resolved with the last method of "least square". In this case, the system (8) gives more equations than variables and thus it is not possible in general to find a solution with $z_j = 0$ at every monitor position. Acting on all the available correctors, the method tries to minimize the magnitude of the distortion globally. We introduce the sum of the error squares

$$S = \sum_j \left\{ \bar{z}_j + \frac{\sum_\ell \sqrt{\beta_j \beta_\ell}\,\theta_\ell \cos\nu(\phi - \phi_\ell + \pi)}{2\sin\pi\nu} \right\}^2$$

which is the function we want to minimize. We obtain

$$\frac{\partial S}{\partial \theta_k} = 2\sum_j \frac{\sqrt{\beta_j \beta_k}\cos\nu(\phi_j - \phi_k + \pi)}{2\sin\pi\nu} \left\{ \bar{z}_j + \frac{\sum_\ell \sqrt{\beta_j \beta_\ell}\,\theta_\ell \cos\nu(\phi_j - \phi_\ell + \pi)}{2\sin\pi\nu} \right\} = 0$$

which is a system of N_s linear equations ($k = 1, 2, \ldots, N_s$) in N_s variables $\theta_\ell (\ell = 1, 2, \ldots, N_s)$ which can again be solved with the usual method of matrix inversion. Once the solution has been found, this is inserted back in the original equation (8) to determine the new, partially corrected, closed orbit distortion.

There are several computer codes that perform the operations described here. One is known as MIKADO which is included in the SYNCH program. If this method is applied to the case $N_s = N_{pu}$, then the obvious result is that the last two methods give the same identical answer with the closed orbit distortion vanishing exactly at all position monitor locations.

The "least square method" would be very useful for RHIC in those instances when several correctors are not functioning at the same time. This method deserves good consideration and it is still waiting to be investigated more in details for the RHIC application.

Closed Orbit and Sextupoles

A major complication in dealing with the closed orbit distortion and its correction arises from the presence of sextupole magnets for the chromaticity correction. If the beam is displaced from the axis of the sextupoles, the equivalent of quadrupole gradient errors are introduced. If the displacement is horizontal a type b_1–error is generated; if the displacement is vertical, one has an equivalent a_1–type error. These errors can be of very large magnitude as it is shown in the following table.

	Length	a_1 ($\times 10^{-4}$cm^{-1})	b_1 ($\times 10^{-4}$cm^{-1})	number per arc
arc dipole (B)	9.45 m	1.6	0.8	24
arc quadrupole (QF,QD)	1.13	2.0	4.0	2×12
quadr. axial rotation (1 mrad rms)	1.13	4.2	—	2×12
SF sextupoles	0.75	6.8	6.8	12
SD sextupoles	0.75	13.3	13.3	12

We compared several error sources. The values shown are the expectation rms widths of the random errors. The contribution of the sextupoles has been determined as

$$\langle a_1 \text{ or } b_1 \rangle = \frac{B''}{B} \langle y_c \text{ or } x_c \rangle$$

for the case of $\beta^* = 2$m and a local rms displacement of 1 mm, either horizontal or vertical.

The effects of the sextupoles on the uncorrected closed orbit have already been assessed earlier[1] (with the use of PATRIS). The lattice functions are distorted, betatron tune–shifts and half–integral stopbands are introduced and often the lattice is unstable, that is the working point is shifted directly inside one of the stopbands. Typically out of 21 simulations with different random number seeds for the closed orbit determination, 10 runs have produced an unstable lattice. For the remaining 11, the tune–shift can be as large as 0.12 in the horizontal plane and 0.08 in the vertical one. If one estimates the distortion of the β–function in the middle of the RHIC lattice, one finds a $\Delta\beta/\beta$ as large as 150% in the horizontal plane and 120% in the vertical plane. All this is also indicative of a large enhancement of the neighboring half–integral stopbands.

In the following we give an analytical estimate of the tune–shift due to these types of errors. The tune–shift in one plane (x or y) is given by the contributions of all the sextupoles, which are short enough so that their effects can be represented as "kicks",

$$\Delta\nu = \frac{1}{4\pi}\sum_{\ell}\beta_\ell S_\ell z_\ell \tag{9}$$

where z_ℓ stays for either the horizontal x_ℓ or vertical y_ℓ beam displacement from the axis of the ℓ–th sextupole, and $S_\ell = (B''\ell/B\rho)_\ell$ is the strength of the ℓ–th sextupole located in a place where the β–function takes the value β_ℓ.

We shall consider only the case with two sextupole families: SF and SD, located respectively next to QF and QD in the arcs. We have

$$S_F = 0.21\ \mathrm{m}^{-2} \quad \text{and} \quad S_D = -0.41\ \mathrm{m}^{-2}$$

for $\beta^* = 2$m. Then Eq. (9) can be written also as follows

$$\Delta\nu = M\frac{\beta_F S_F}{4\pi}\overline{z}_F + M\frac{\beta_D S_D}{4\pi}\overline{z}_D$$

where $\overline{z}_F$ and $\overline{z}_D$ is the average of the closed orbit deviations at the sextupole locations, respectively SF and SD. The number of sextupoles per family is M=72.

If the beam displacements at the various locations are not correlated with each other, but have a random distribution of rms width $\langle z\rangle$, then the rms expectation value of the averages is

$$\overline{z} = \frac{\langle z\rangle}{\sqrt{M}} \tag{10}$$

and the rms expectation value of the resulting tune–shifts is

$$\langle\delta\nu\rangle = \sqrt{M}\frac{\beta_F S_F}{4\pi}\langle z_F\rangle + \sqrt{M}\frac{\beta_D S_D}{4\pi}\langle z_D\rangle \ .$$

For the case of uncorrected closed orbit, using the results obtained in the previous sections, we have

$$\langle\delta\nu\rangle_H = 0.13 \quad \text{and} \quad \langle\delta\nu\rangle_V = 0.27$$

which are large values indeed. Nevertheless one might argue that the beam position errors are not random and that therefore the effect could be considerably smaller. Indeed we have seen that the closed orbit distortion can be represented with a Fourier expansion as given by Eq. (5). Nevertheless, it is not easy to determine analytically what fraction of the value (10) can be really considered random. This can certainly be determined with the computer.

The same analysis can be applied to determine the width of the stopbands at $2\nu = 57$ or 58. The rms expectation value of the full width is just equal to that of the tune–shift. Since the unperturbed tune $\nu = 28.82$, and because the shift can be of either sign, there is a 50% chance of the motion being unstable if $\delta\nu \geq 0.09$ by landing on the $2\nu = 58$ stopband, and a 50% chance of the motion being unstable by landing on the $2\nu = 57$ stopband if $\delta\nu \geq 0.16$. Thus there is a 100% chance of instability in any case if $\delta\nu \geq 0.16$.

In order to provide a 95% confidence level that either stopband is avoided during the early operation of RHIC the rms expectation value of the tune shift should not exceed 0.03 in either plane. If this is caused by the closed orbit displacement in the sextupoles then the closed orbit is to be corrected by an order of magnitude, down to about 2 mm rms before the sextupole magnets can be safely turned on.

A more severe effect encountered when correcting the initial closed orbit with sextupoles on, is the introduction of linear coupling and a large contribution being added to the $\nu_x - \nu_y = 0$ resonance. This effect can be enhanced by the random contribution of the closed orbit and it is not easily treatable analytically but with computer tracking. In conclusion, the suggestion is to correct initially the closed orbit with the sextupoles turned off. It was indeed proven that after the initial correction, a converging and fast iteration method *à la* Fermilab can be adopted turning the sextupoles on subsequently.

Finally we have examined the question: what happens if one beam position monitor is not working? Clearly then we cannot infer what is the beam position at that location. All the methods of correcting closed orbit described above unfortunately will not be able to correct the deviation at the blind spot. To complicate matters, the neighboring PU's are at $\pm 90^o$, so that requiring that the closed orbit deviation vanishes at both of these locations would still leave the deviation unchecked and uncorrected in the middle. Since now sextupoles cannot be turned off, if even only one location exists where a PU does not work, the unchecked closed orbit in the neighboring sextupole can cause a tune shift with an rms expectation value of 0.03. This is too large and one should find operational methods to correct blindly the error (probably by faking the actual error) until the beam stability is recovered.

During the operation of RHIC, a very useful method to explore the actual physical or dynamic aperture in one particular location of the ring is to move locally the beam laterally as far as it can go measuring at the same time beam parameters like betatron tunes. To do this the beam can be moved with the help of 3–magnet bumps, which the closed orbit correction system is capable to do effectively at least over a range of $\pm$ 20 mm at locations where $\beta = 50$ m in either plane. This type of experiment is useful to determine also the effects on non–linear magnet imperfections and the chromatic behavior of the collider in which case sextupoles are to be left on. Unfortunately RHIC cannot withstand a large orbit distortion, even if only locally, and the sextupoles being on at the same time. Actually, the 3–magnet bump operation has been common practice at Fermilab and CERN and no problems were encountered with sextupoles and closed orbit distortions. The dipole packing factor in RHIC is only 40% against 75% in Tevatron and Sp$\bar{\text{p}}$S. Each straight section is longer than an arc and there are fewer and stronger sextupoles to correct chromaticity.

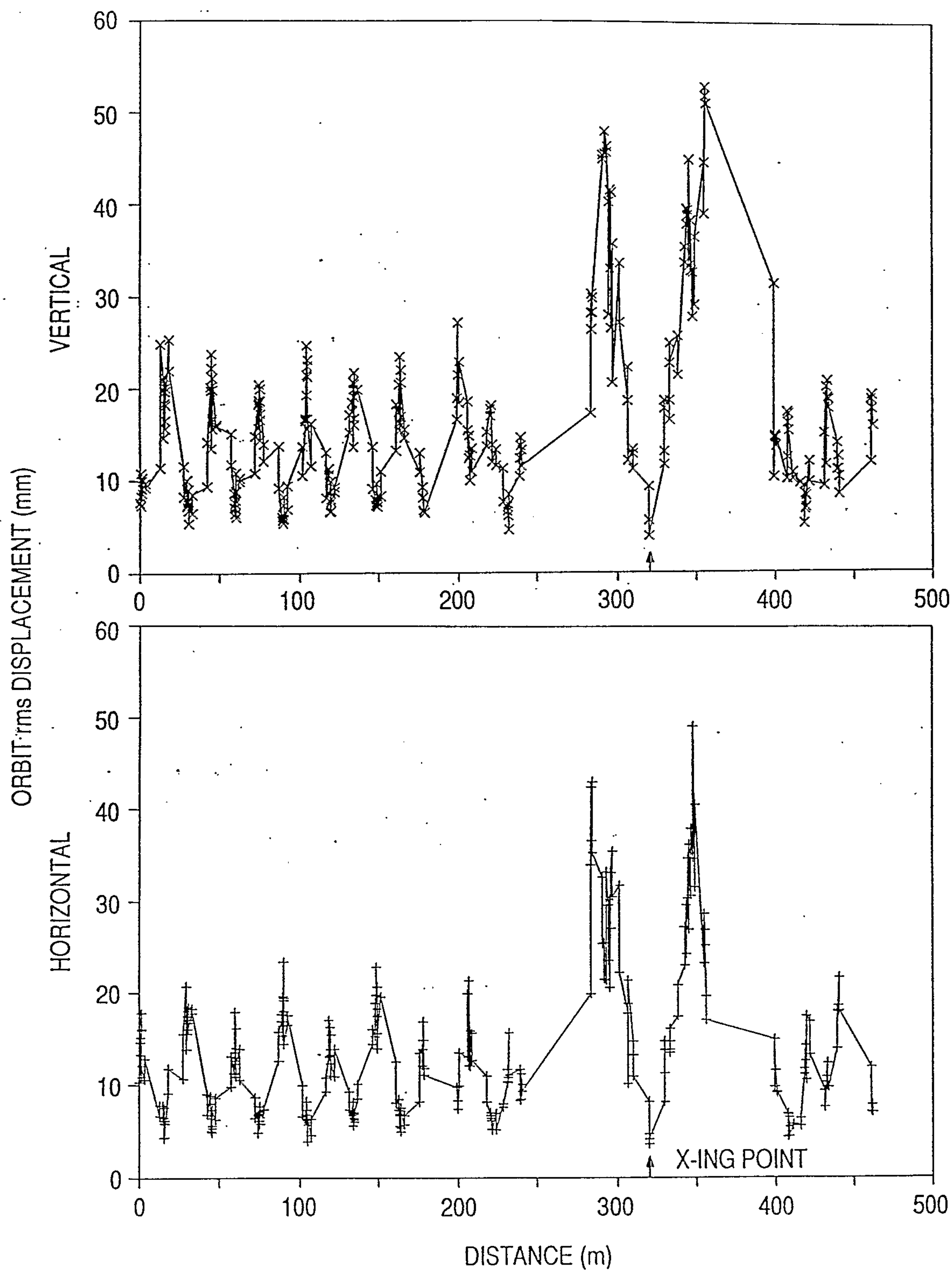

Fig. 1 Horizontal and vertical rms closed orbit distortion based on 21 random error distribution.