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The Residual Tune Splitting due to Linear Coupling - Theory and Correction

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1. Introduction

The tune splitting due to random a_1 has been found to be large in RHIC. $|\nu_1 - \nu_2| \sim 200 \times 10^{-3}$ was found for the $\beta^* = 2$ lattice for the worse error distribution out of 10 error distributions tried. Most of this tune splitting can be corrected with a 2 family a_1 correction system,[1] but a residual tune splitting remains that is still appreciable. The residual tune splitting, $|\nu_1 - \nu_2|$, that remained in RHIC after correction with the 2 family correction system was found to be about 18×10^{-3}.

The residual $|\nu_1 - \nu_2|$ appears to be due to higher order effects of the random a_1. The theory of these effects indicates that the harmonics of a_1 near $\nu_x + \nu_y$ are the important harmonics. However, the harmonics closest to $\nu_x + \nu_y$ do not contribute much to the residual $|\nu_1 - \nu_2|$. A correction system has been proposed and simulated that appears able to reduce the residual $|\nu_1 - \nu_2|$. The lack of a dominant harmonic complicates the correction.

2. Residual Tune Splitting in RHIC – Definition and Properties

Magnitude of Tune Splitting in RHIC

Table 1: Results for the correction of the tune splitting for a $\beta^* = 2$ RHIC lattice using a 2 family tune splitting correction system set to make $\Delta\nu = 0$.

	——Uncorrected——			——Corrected——		
Error Field Dist.	ν_1	ν_2	$\lvert\nu_1 - \nu_2\rvert$ $/10^{-3}$	ν_1	ν_2	$\lvert\nu_1 - \nu_2\rvert$ $/10^{-3}$
1	0.796	0.854	59	0.828	0.823	6
2	0.707	0.935	228	0.838	0.819	19
3	0.869	0.783	86	0.825	0.829	4
4	0.772	0.883	111	0.831	0.823	7
5	0.779	0.872	93	0.836	0.820	16
6	0.848	0.805	43	0.832	0.821	11
7	0.840	0.847	7	0.852	0.834	18
8	0.742	0.895	153	0.838	0.818	20
9	0.785	0.866	81	0.828	0.823	6
10	0.749	0.891	142	0.823	0.827	5

[1] G. Parzen, AD/RHIC–AP–72 (1988).

Before Correction

Largest $|\nu_1 - \nu_2| = 228 \times 10^{-3}$. Note $\nu = 28.935$, for error field #2, is almost in $\nu = 29$ stopband. $\beta_1 \simeq 140$ shows this. Stopband danger can be avoided by going to $\beta^* = 6$, and correcting at $\beta^* = 6$.

Residual $|\nu_1 - \nu_2|$ – After Correction

Correction makes $\Delta\nu = 0$.

$$\Delta\nu = \frac{1}{4\pi\rho} \int ds \; a_1 \left(\beta_x \beta_y\right)^{1/2} \exp\left[i\bar{\nu}\left(\theta_x - \theta_y\right)\right],$$

$$\theta_x = \psi_x/\nu_x, \;\; \theta_y = \psi_y/\nu_y, \;\; \bar{\nu} = \left(\nu_x + \nu_y\right)/2.$$

Largest residual $|\nu_1 - \nu_2| = 20 \times 10^{-3}$. Note $\nu_x = 28.826$, $\nu_y = 28.821$, $|\nu_x - \nu_y| = 5 \times 10^{-3}$. Residual tune splitting is $|\nu_1 - \nu_2|$ after correction, $\Delta\nu = 0$.

We will see that after correction

$$\text{Residual } |\nu_1 - \nu_2| \sim a_1^2$$

whereas before correction

$$|\nu_1 - \nu_2| \sim a_1$$

An additional reason for having a residual tune splitting correction system, is to correct the β–functions, β_1, β_2. One finds $\beta_1 \sim 100$ m after correction with the 2 family system for $\beta^* = 2$. Correction of the residual tune splitting also corrects β_1, β_2 to a considerable extent.

Lowest Order Analytical Result for $\nu_1 - \nu_2$

$$\begin{aligned} \nu_{1,2} &= \bar{\nu} \pm \left[\left(\frac{\nu_x - \nu_y}{2}\right)^2 + |\Delta\nu|^2\right]^{\frac{1}{2}}, \\ \Delta\nu &= \frac{1}{4\pi\rho} \int ds \; a_1 \left(\beta_x \beta_y\right)^{\frac{1}{2}} \exp\left[i\bar{\nu}\left(\theta_x \theta_y\right)\right], \\ \bar{\nu} &= \frac{1}{2}\left(\nu_x + \nu_y\right), \;\; \theta_x = \psi_x/\nu_x, \;\; \theta_y = \psi_y/\nu_y \, . \end{aligned} \tag{1}$$

ν_1 is the normal mode that goes to ν_x when $a_1 \to 0$, and ν_2 is the corresponding mode for ν_y. For the $\pm$ sign, the $+$ sign is used when $\nu_x > \nu_y$ for ν_1 and the opposite sign for ν_2. Close enough to the $\nu_x = \nu_y$ resonance, $\nu_1 - \nu_x \sim |\Delta\nu|$ and is linear in a_1. Far enough from the $\nu_x = \nu_y$ resonance, $|\nu_1 - \nu_x| \simeq |\Delta\nu|^2$ and is quadratic in a_1.

$$\begin{aligned} |\nu_1 - \nu_2| &= 2\left[\left(\frac{\nu_x - \nu_y}{2}\right)^2 + |\Delta\nu|^2\right]^{\frac{1}{2}} \\ \frac{\nu_1 + \nu_2}{2} &= \frac{\nu_x + \nu_y}{2} \end{aligned} \tag{2}$$

One cannot reduce $|\nu_1 - \nu_2|$ below $2|\Delta\nu|$ using ν_x, ν_y. One can control $(\nu_1 + \nu_2)/2$ using ν_x, ν_y.

Equation (1) is useful only near the $\nu_x = \nu_y$ resonance line, as it neglects some a_1^2 terms.

Comments on Figure 1

Figure (1a) shows how well Eq. (1) describes the tune shifts due to a_1. ν_1 and ν_2 have been computed for random a_1 distribution #2, and ν_1 and ν_2 are plotted against a_1. $a_1 = 0.8$ means that the a at each element has been reduced by the same factor of 0.8. As ν_x, ν_y is on the resonance lines $\nu_x = \nu_y$, Eq. (1) predicts straight lines for ν_1 and ν_2 versus a_1, and the tune splitting agrees well with $|\nu_1 - \nu_2| = 2\Delta\nu$.

Figure (1b) shows $|\nu_1 - \nu_2|$ versus a_1 after a 2 family correction that makes $\Delta\nu = 0$. One sees that the residual $|\nu_1 - \nu_2|$ after correction is quadratic in a_1. The crosses indicate a a_1^2 curve normalized at $a_1 = 1$.

Figure (1c) shows an error distribution, #7, where the higher order a_1 terms in the ν–shift dominates even before correction.

3. Analytical Results for Residual $|\nu_1 - \nu_2|$

For the case when the tune splitting has been corrected by making $\Delta\nu = 0$ then

$$
\begin{aligned}
\nu_1 &= \nu_x + \frac{1}{2\nu_x}\Delta_x \\
\nu_2 &= \nu_y + \frac{1}{2\nu_y}\Delta_y \\
\Delta_x &= 4\nu_x\nu_y \sum_{n\neq 0} \frac{|c_n|^2}{n\left(n - \nu_x - \nu_y\right)} \\
\Delta_y &= 4\nu_x\nu_y \sum_{n\neq 0} \frac{|b_n|^2}{n\left(n - \nu_x - \nu_y\right)} \\
c_n &= \frac{1}{4\pi\rho}\int ds\ a_1 \left(\beta_x\beta_y\right)^{\frac{1}{2}} \exp\left[i\left(\nu_x\theta_x + \left(n - \nu_x\right)\theta_y\right)\right] \\
b_n &= \frac{1}{4\pi\rho}\int ds\ a_1 \left(\beta_x\beta_y\right)^{\frac{1}{2}} \exp\left[i\left(\left(n - \nu_y\right)\theta_x + \nu_y\theta_y\right)\right] \\
\theta_x &= \psi_x/\nu_x,\ \theta_y = \psi_y/\nu_y
\end{aligned}
\tag{3}
$$

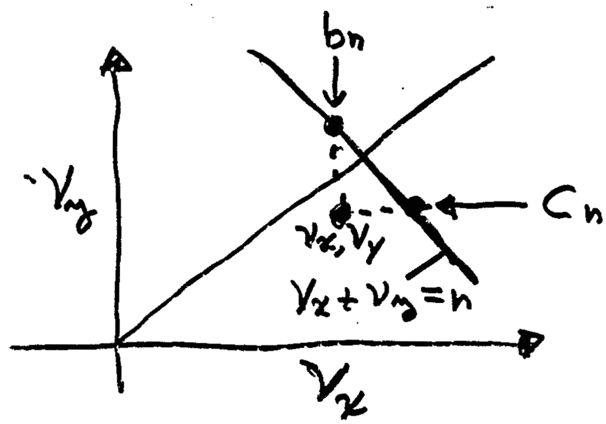
Above holds when ν_x, ν_y are close to the resonance line, $\nu_x = \nu_y$. These results will be derived in a separate paper. A numerical check of these results is given below.

Comments on Eq. (3) for ν_1, ν_2

1. In 1–dimensional case, $\nu_1 = \nu_x + 2\nu_x \sum_n |b_n|^2 / (n(n - 2\nu_x))$, when $b_0 = 0$; $b_n = (1/4\pi\rho) \int ds\ b_1 \beta \exp(in\theta)$. A similar result to Eq. (3).
2. Importance of $n \sim \nu_x + \nu_y$ harmonic because of the resonance denominator, $1/(n - \nu_x - \nu_y)$. $n \simeq 58$, for RHIC, is the harmonic closest to $\nu_x + \nu_y$.
3. b_n, c_n are similar to sum resonance stopband integrals corresponding to different ν_x, ν_y on the resonance line, $\nu_x + \nu_y = n$.

   $c_n \rightarrow \nu_x,\ n - \nu_x$

   $b_n \rightarrow n - \nu_y,\ \nu_y$



4. For n close to $\nu_x + \nu_y$, b_n and c_n are nearly equal. Thus $n = 58$ for RHIC causes the largest change in ν_1 and ν_2, but does not contribute to the residual tune splitting, $|\nu_1 - \nu_2|$ when $\Delta\nu = 0$.
5. Note that $\Delta x, \Delta y$ can be written in the integral form $\Delta x \sim \int ds\ ds'\ a_1(s)\ (\beta_x(s)\,\beta_y(s))^{\frac{1}{2}}\ a_1(s')\,(\beta_x(s')\,\beta_y(s'))^{\frac{1}{2}}\, g(s - s')$. This form may be useful in designing a correction system for the residual $|\nu_1 - \nu_2|$.

Numerical Test of Analytical Result for the Residual $|\nu_1 - \nu_2|$

The following table shows the results of a numerical check of the results in Eq. (3). In this check, the a_1 around the ring was excited according to the formula

$$a_1 = A \cos\left[n(\theta_x + \theta_y)\right],$$

$$\theta_x = \psi_x/\nu_x,\ \theta_y = \psi_y/\nu_y.$$

Only 12 high–β quadrupoles in the insertions were excited in this way. The b_n and c_n in Eq. (3) were computed and the theoretical results from Eq. (3) were compared with computed results for ν_1, ν_2.

n	Computed $(\nu_1 - \nu_x)$ $/10^{-3}$	Computed $(\nu_2 - \nu_y)$ $/10^{-3}$	Theory $(\nu_1 - \nu_x)$ $/10^{-3}$	Theory $(\nu_2 - \nu_y)$ $/10^{-3}$
59	9	7	9	9
58	32	30	33	33
57	-3	-1	-1	-1
56	25	27	26	26
55	6	7	6	6

4. Correction of the Residual Tune Splitting[2]

Optimize the 2 Family Tune Splitting Correction

Instead of the setting $\Delta\nu = 0$, set corrections to minimize $|\nu_1 - \nu_2|$. These results are shown in Table 2.

Table 2: Residual tune splitting resulting from two different methods of tune splitting correction using the 2–family tune splitting correction system.

Error Field Distribution	$|\nu_1 - \nu_2|/10^{-3}$ $\Delta\nu = 0$ Correction	$|\nu_1 - \nu_2|/10^{-3}$ minimize $|\nu_1 - \nu_2|$ Correction
	$\beta^* = 2$	
1	6	5
2	19	18
3	4	3
4	7	7
5	16	8
6	11	11
7	18	7
8	20	16
9	6	6
10	5	4

The Enlarged a_1 Correction System

For the enlarged a_1 correction system, there are 12 a_1 correctors, one at each high–β quad, Q2 or Q3. In the correction scheme that was simulated, these 12 a_1 correctors were excited according to the equation

$$a_1 = A\cos(\psi_x + \psi_y) + B\sin(\psi_x + \psi_y)$$

[2] G. Parzen, AD/RHIC–82 (1990).

Table 3: The reduction in the residual tune splitting obtained using the enlarged a_1 correction system, compared with the results obtained with the 2–family correction system, minimizing $|\nu_1 - \nu_2|$.

Error Field Distribution	$\lvert\nu_1 - \nu_2\rvert/10^{-3}$ 2 Family Correction System	$\lvert\nu_1 - \nu_2\rvert/10^{-3}$ enlarged a_1 Correction System
	$\beta^* = 2$	
1	5	5
2	16	2.8
3	3	3
4	5	5
5	5	5
6	9	3
7	14	1.5
8	10	2
9	6	3
10	1	3

The total correction field is given by

$$a_1 = A\cos(\psi_x + \psi_y) + B\sin(\psi_x + \psi_y) + C\,(a_1 \text{ due to Family 1}) + D\,(a_1 \text{ due to Family 2})$$

These 4 parameters A, B, C, D are varied to minimize $|\nu_1 - \nu_2|$. The results using the enlarged a_1 correction system are shown in Table 3.

Varying the Harmonic Number of the a_1 Correction

A further improvement in $|\nu_1 - \nu_2|$ and in the correction of β_1, β_2 is obtained by exciting the 12 correctors according to

$$a_1 = A\cos n\,(\theta_x + \theta_y) + B\sin n\,(\theta_x + \theta_y)$$

One now has 5 parameters to vary, A, B, C, D, n.

The effect of varying n is shown on the next figure, Fig. 2. This additional parameter is sometimes useful in correcting β_1, β_2 as well as ν_1, ν_2.

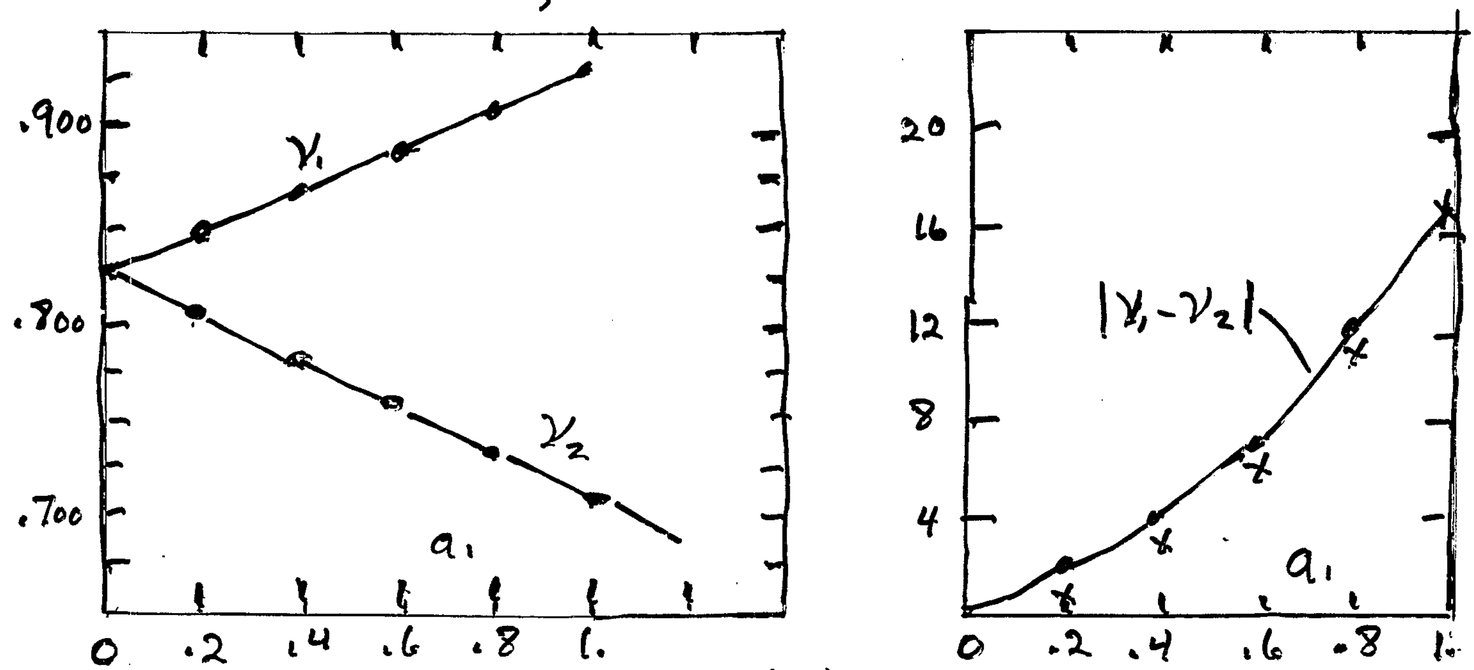
Conclusions

To correct the tune splitting, one has to reduce $\Delta\nu$, Δ_x, Δ_y. Thus a correction system with a minimum of 4 adjustable parameters is required, depending on the location of the a_1 correctors. The correction system described here is appealing because it emphasizes the important harmonics, the $|\nu_x - \nu_y|$ harmonic (0 for RHIC) and the $\nu_x + \nu_y$ harmonic (58 for RHIC). Probably, other arrangements of the a_1 correctors, with 4 or more parameters, may also work well.

The setting of the 4 or more parameters required may be difficult, partly because of the lack of a dominant harmonic. The setting of the parameters may be made easier by providing observation stations that measure β_1, β_2 or the 4 parameters of the R matrix that defines the normal mode coordinates. At the moment, this may seem like too much effort for this effect as far as RHIC is concerned.

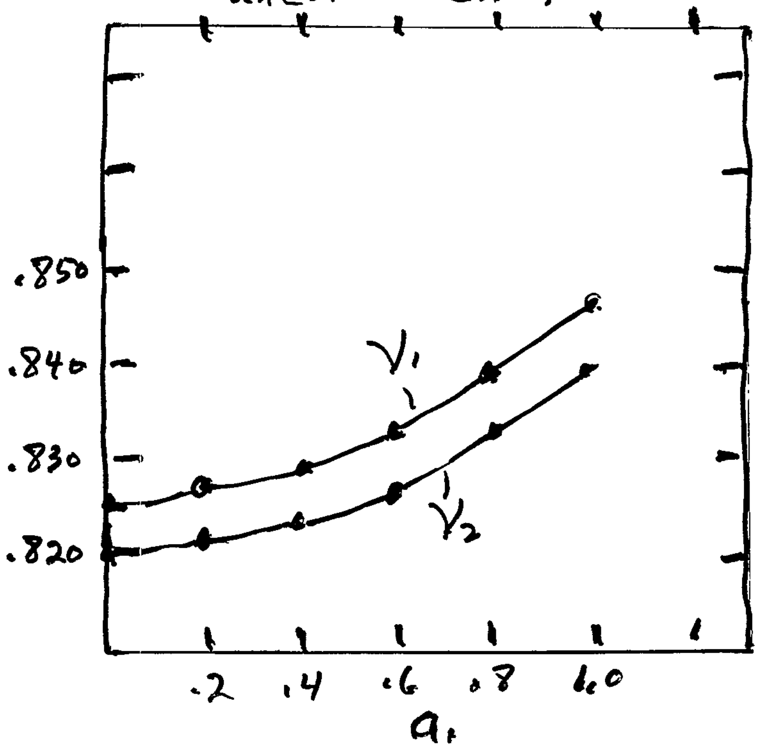
$\beta^* = 2$, Seed = 2
$b_1 = 0$, $\nu_x\ \nu_y = .824, .824$
uncorrected , $\Delta\nu = .11$

Corrected



(1a)

uncorrected , $\Delta\nu = .011$



$\beta^* = 2$, Seed = 7
$\nu_x, \nu_y = .826, .821$
(1b)

Fig. 1.

$a_1 = A \cos n(\theta_x + \theta_y) + B \sin n(\theta_x + \theta_y)$

$\beta^* = 2$, Seed = 2

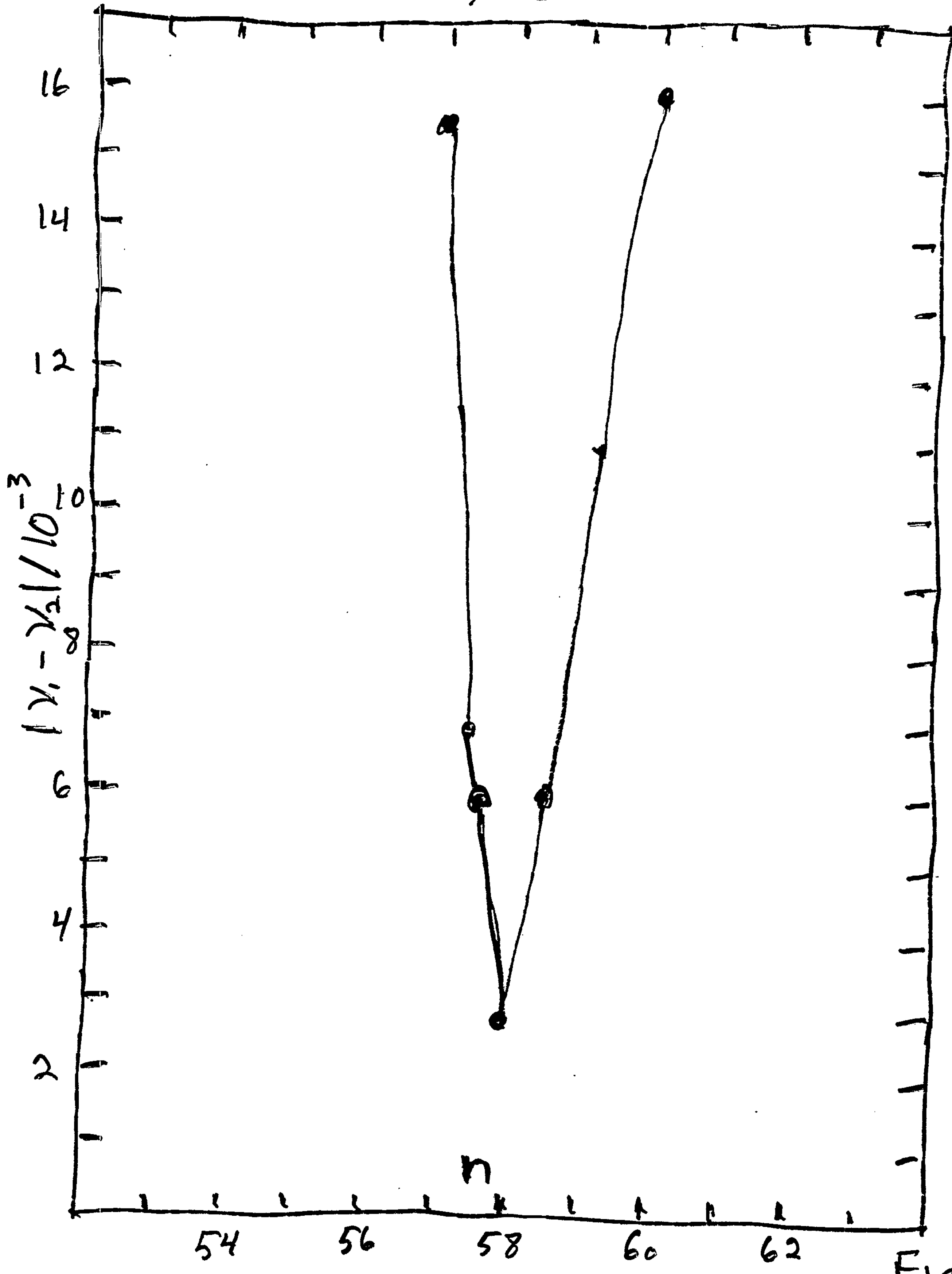


Fig. 2