BNL-101564-2014-TECH
RHIC/PG/21;BNL-101564-2013-IR

Luminosity Loss Rate Due To Intrabeam Scattering

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December 1983

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RHIC-PG-21

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(BNL, December 11, 1983)

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In the limit of short interaction region the luminosity can be very well represented by

$$L = \frac{N^2 B \, f_{rev}}{4\pi \, \sigma_V^* \, \sigma_H^* \, f}$$

where, for identical colliding beams,

N, number of particles per bunch
B, number of bunches per beam
f_{rev}, revolution frequency
σ_V^*, vertical rms beam size at the collision point
σ_H^*, horizontal rms beam size at the collision point

assuming crossing ~~at~~ at an angle on the horizontal plane

$$f = \sqrt{1 + p^2}$$

and

$$p = \frac{\alpha \, \sigma_e}{2 \, \sigma_H^*}$$

α, total crossing angle
σ_ℓ, rms bunch length.

Intrabeam scattering has the effect of having σ_V^*, σ_H^* and σ_E, the r.m.s energy spread to vary, generally by growing.
~~Obviously~~ Define the luminosity loss rate

$$\tau_L^{-1} = -\frac{1}{L}\frac{dL}{dt}$$

Obviously:

$$\tau_L^{-1} = \frac{1}{\sigma_V^*}\frac{d\sigma_V^*}{dt} + \frac{1}{\sigma_H^*}\frac{d\sigma_H^*}{dt} + \frac{1}{f}\frac{df}{dt}$$

One can work out that

$$\frac{1}{f}\frac{df}{dt} = \frac{p^2}{f^2}\left[\frac{1}{\sigma_\ell}\frac{d\sigma_\ell}{dt} - \frac{1}{\sigma_H^*}\frac{d\sigma_H^*}{dt}\right]$$

Assumptions:

1. The bunch length increases (or decreases) at the same rate the energy spread does, that is

$$\frac{1}{\sigma_e}\frac{d\sigma_e}{dt} = \tau_E^{-1}$$

2. We assume that the two modes of oscillations (H and V) are fully coupled to each other on a time shorter than the intrabeam scattering diffusion times, that is

$$\frac{1}{\sigma_V^\alpha}\frac{d\sigma_V^\alpha}{dt} = \frac{1}{\sigma_H^\alpha}\frac{d\sigma_H^\alpha}{dt} = \tau_H^{-1} + \tau_V^{-1}$$

Here τ_E^{-1}, τ_H^{-1} and τ_V^{-1} are the diffusion (or damping rates) from intrabeam scattering respectively in energy, horizontal (betatron) size and vertical (betatron) size.

In conclusion

$$\tau_L^{-1} = \frac{p^2}{f^2}\tau_E^{-1} + \left(2 - \frac{p^2}{f^2}\right)\left(\tau_H^{-1} + \tau_V^{-1}\right)$$

For head-on collision $p=0$, $f=1$ and

$$\tau_L^{-1} = 2\left(\tau_H^{-1} + \tau_V^{-1}\right)$$