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Survey Control for the RHIC Transport Line

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AD/RHIC/RD-82

RHIC PROJECT
Brookhaven National Laboratory

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FOR THE
RHIC TRANSPORT LINE

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SURVEY CONTROL
FOR THE
RHIC TRANSPORT LINE

1. Introduction.

In order to position the beam components of the AGS-to-RHIC transport line properly, in conformance with the RHIC lattice specifications, during installation and alignment of beam transport elements, one needs a well-defined and unique survey control geometry along the the transport tunnel and arc bends, from the AGS extraction septum magnet to the RHIC injection points.

The scale of the transport line, about a kilometer, is large enough that one must include considerations of earth's curvature and ellipticity when adjusting transport line surveys.

Historically, the survey control in the AGS was based on a local coordinate frame where locations of AGS reference fiducials were specified vertically by gravity elevation and horizontally by local "BNL East" and "BNL North" coordinates, which were directed along the orientation of the road network of Camp Upton.

During the 1978 to 1983 construction of the RHIC tunnel, survey and alignment work was carried out from reference control monument stations well outside of the AGS machine. Coordinates of the reference stations were obtained from prior survey inform-

ation, rather than by a separate control survey tied to the control monument network of the AGS machine. While the location of the RHIC machine's primary control monument network was well determined geodetically, the link from RHIC to AGS was weak, and some uncertainty remained in the location of the AGS machine relative to RHIC.

In the spring of 1993, the RHIC injection arcs and transport line were resurveyed. Least squares adjustments to the resurvey were made, under two separate sets of assumptions: first, one assumed that the fixed reference stations for the adjustment were specified by RHIC-based least-squares-adjusted BNL coordinates of stations near the RHIC ring and injection arcs obtained from 1992 surveys of the RHIC primary monument network and the injection arcs; second, in contrast, it was assumed that the fixed reference station coordinates were AGS-based least-squares-adjusted BNL coordinates obtained from a local 1993 control survey through the AGS shield wall, which tied the AGS end of the transport line to control inside the AGS tunnel. A comparison of the two adjusted sets of coordinates for the transport line monuments, gave a connection between RHIC-based and AGS-based survey coordinate systems. For survey points at moderate height above or below the RHIC machine plane (a few meters), a simple plane coordinate transformation, i.e. a rotation plus a translation of the horizontal coordinates, was obtained, which allows one to compute horizontal survey coordinates in either system to be computed from the horizontal survey coordinates given in the other system [Appendix E] .

Consequently, it is easy to find the BNL horizontal coordinates of a survey point in the AGS-based BNL survey coordinate system from those in the RHIC-based BNL system, and vice versa. The explicit transformation equations are given in section 4 of this report.

Uncertainties relating to the accuracy of the geodetic link between the RHIC-centered BNL survey coordinate reference system and the AGS-referenced BNL coordinate system have thereby been resolved, and geodetic control of the entire transport line has been achieved.

In this note we will define several coordinate systems needed to describe different aspects of the transport line geometry, and will supply mathematical transformations which relate the sets of coordinates to one another. We will also describe computational codes used to obtain survey elevations of transport line fiducials.

In order to calculate local gravity elevations needed by survey crews for installation and alignment of magnets and other beam components, codes were developed to calculate the elevations from given RHIC-based horizontal survey coordinates and perpendicular heights of the beam components above the RHIC Machine Plane.

These codes use an ellipsoidal model of the earth's gravity equipotentials, which was used in the 1978-82 National Geodetic Survey's adjustment of the Isabelle/RHIC terrain, and which provides a good local gravity model.

2. Survey Control For The RHIC Transfer Line.

A collection of precision-machined stainless steel, hollow cylindrical bushings has been grout-cemented into the floor of the transport line, and the big bend arc tunnels. These bushings define precise locating surfaces for the kinematic mounting of optical surveyor's target balls, and of elevation reference fixtures, and provide the basic control references for survey and alignment of the transport line. The monuments have been located geodetically, by carrying out a control survey of the transport line tunnel and joining this survey to the survey control net of the RHIC ring. The monuments are of a type originally developed for CERN, and are commercially available.

To establish geodetic control, surveyors measure monument elevations by differential level measurements around a closed loop, and measure local horizontal direction changes from monument to monument, and distances between pairs of monuments. The position assigned to a monument is that of the center of a reticle surveyor's target, which has been microscopically centered in a 3.500 inch diameter, accurately spherical, steel ball, which seats upon the upper surface of the monument.

Each of the magnets to be installed in the transport line tunnels is provided with external fiducial reference marks. The external fiducials on each magnet are sufficient to define the position and orientation of that magnet in space, when the

marks are located with respect to the survey control monuments. The external fiducials are also well-determined with respect to the magnetic axis, and planes of symmetry, of each magnet. (The determination is made by surveying the external fiducials relative to internal "built-in" magnet fiducial reference targets. One observes internal and external targets by using type T-3000 theodolites having electronic readout, together with the ManCAT electronic coordinate measurement system). Tables of installation coordinates, so-called "survey coordinates", are prepared, for the external magnet fiducials (based on the required lattice location of the individual magnet, and its geometric design), to guide the installation of the magnets into the transport line.

The position and orientation of a transport line magnet is defined, fundamentally, by the RHIC lattice geometry. That is, rectangular cartesian lattice coordinates $U(FP)$, $V(FP)$, $W(FP)$ are assigned to each fiducial point, FP , of the "as-built" magnet, to define its location in the machine complex.

Subsequently, RHIC-based BNL North and East coordinates, and the gravity elevation coordinate, must be computed for each fiducial point, to allow emplacement of a magnet in the tunnel. Those coordinates are computed geodetically. The orientations of the RHIC machine lattice and the BNL laboratory coordinate systems relative to the local geographic north and east directions must be taken into account, when computing fiducial mark and survey station elevations.

A survey and alignment data file, SAGRAVEL.GEL , giving survey coordinates and gravity elevations of magnet fiducial reference survey target center points, is generated for each magnet. A separate data line is used for each of the magnet's fiducial survey points. In this file, a data line lists: the magnet's serial number; a fiducial's survey point name (which is an eight character string, whose first six characters give the magnet's installation location in the transport line, and whose last two characters indicate which fiducial on the magnet is being named); the RHIC Survey Frame "North" coordinate; the RHIC Survey Frame "East" coordinate; the gravity elevation; a reference date and time for the given data.

This file is generated from an "ideals" file: SAGRAVEL.IDL which contains the magnet fiducial reference coordinate information in terms of the horizontal survey coordinates and the perpendicular height of the fiducial reference point above the RHIC Machine Plane, instead of gravity elevation. The file SAGRAVEL.IDL is used as the input file for a coordinate transformation program, SAGRAVEL.EXE, which generates the data output file SAGRAVEL.GEL containing the elevations and horizontal survey coordinates used for magnet installation into the tunnel.

The code SAGRAVEL.EXE which computes survey point gravity elevations and prepares survey installation coordinate [.GEL] files for individual magnets and beam line elements is described in Appendix F.

A geodetic code, SATA.EXE , was developed to calculate the survey control station gravity elevations, in terms of the station normal height above the RHIC Machine Plane and the given horizontal station coordinates. The code also allows computation of the normal height of a station above the RHIC Machine Plane, given the station's horizontal coordinates and elevation. The calculations in SATA.EXE require a number of intermediate geometric reference frame coordinate transformations to be computed in order to relate the RHIC machine reference frame (which is not defined in a geodetically obvious way) to the orientation of the earth and its local gravity equipotential surfaces.

To spatially set the RHIC machine plane's orientation with respect to a geodetic coordinate system, the coordinates of the RHIC center point were, at first, specified as follows.

The center point was assumed to lie precisely at the centroid of the six beam interaction points, which were at the corners of a regular plane hexagon. Also, it was assumed that the earth's gravity equipotential was spherical and that that the interaction points were all at the common elevation of 21.082m (830.000"). The geoidal radius was assumed to be the geometric mean of the principal radii of the Clarke ellipsoid (1927 North American Datum) at the RHIC center point; that is, the osculating sphere was used as a best local approximation to the ellipsoidal gravity equipotential surface. The elevation of the machine center point was then computed as being the elevation of the center point of the base disk

of the osculating sphere cap containing the six interaction points, where the radius of the base disk circle is equal to 590.581658 Meter, which is the distance of the center point to each of the interaction points. This initially computed RHIC Center Point elevation was 21.05464 Meter (based on the above spherical gravity equipotential model).

The SATA geodetic code was configured to allow one to transform point coordinates from RHIC Survey Coordinates ($ER(P)$, $NR(P)$, $W(P)$) to Helmert coordinates (latitude $L(P)$, longitude $B(P)$, gravity elevation $H(P)$), and inversely. The code was normalized to embed the RHIC Machine Plane (the set of points $\{ P' / W(P')=0 \}$) in geographic space so that the Machine Center Point ($ER(MCP)= 30\ 230.237553$, $NR(MCP) = 32\ 284.517011$, $W(MCP) = 0.0000$ [Meter]) was placed at the previously computed gravity elevation of $H(MCP) = 21.05464$. With this embedding, the SATA code was run to compute the elevations of six points having horizontal coordinates, respectively, of the interaction points and having a common elevation of 21.082 Meter. The SATA computation gave perpendicular distances from the RHIC plane:

Point	W [Meter]
MCP	+ 0.0000000
X02, X08	+0.0000529
X04, X10	-0.0000388
X06, X12	+0.0000005

The values obtained above reflect the ellipticity of the

gravity ellipsoid, yaw of the RHIC lattice hexagon with respect to geographic north, and the paired symmetry of the interaction points with the geographic axes.

A multinomial expansion about the RHIC machine's center point was developed, to compute elevation of the survey points in terms of the differences of the RHIC Survey Frame coordinates of the survey point from those of the machine's center point. This expansion was incorporated into the code SAGRAVEL.EXE. Elevations computed using this multinomial expansion agree with those computed analytically by means of SATA.EXE (where geodetic coordinate transformation is used), to a micro-meter over the RHIC terrain and transport line, if elevation of the Machine Center Point is taken to be 21.05460 Meter and SATA-computed elevations are incremented by .0000388 Meter, to be consistent with required elevation: $H(X06) = 21.082$ Meter, while $W(X06) = 0.000000$ Meter.

Perpendicular distances computed for transport line control monuments by the two codes are given in Table III. (The heights used by SAGRAVEL are appropriate; these are the re-normalized heights appearing in the third column).

Magnet fiducial point gravity elevations can be computed from lattice-related RHIC Survey Frame Coordinates, by means of the SAGRAVEL code. After obtaining the elevation for each fiducial reference point one can, then, directly set the fiducial to its required elevation during the magnet installation.

3. Coordinate Reference Frames For Survey Control.

To specify placement of magnets in the RHIC collider the machine's designers define the magnet locations with respect to a 3-dimensional, cartesian, orthogonal coordinate system, (U,V,W), centered at the collider's center point. The W axis lies normal to the collider plane, and the U,V axes lie in the collider plane, along symmetry directions of the machine. We call the [\hat{U} , \hat{V} , \hat{W}] reference frame, attached at the collider center point, the "RHIC Machine Frame".

Coordinates specifying the collider installation location points of the "as-built" transport line magnets and collider magnets are specified initially with respect to the machine's frame. Subsequently, horizontal survey coordinates and a gravity elevation coordinate must be supplied for each installation location point, to supply the laboratory survey crews suitable local horizontal control coordinates and gravity elevations for magnet and beamline component installation.

This is done in two steps. First, the [\hat{U} , \hat{V} , \hat{W}] frame is rotated by +1.999898 degrees, clockwise, about the W axis, and translated in the machine plane, to generate a "RHIC Survey" reference frame, [\hat{E}_R , \hat{N}_R , \hat{W}], with associated coordinates (ER, NR, W). Then, the elevation of a survey point is found, by putting the length of the pedal normal from the point to an appropriate gravity reference ellipsoid.

To compute the gravity elevation, $H(P)$, of a survey point, from its (ER, NR, W) coordinates, it is necessary to employ additional, geographically related, reference frames and perform additional coordinate transformations, so that the survey frame is geodetically related to the gravity reference ellipsoid.

The computations involved are somewhat complicated in implementation. To compute the elevation $H(P)$, one needs to know the orientation of the $[\hat{ER}, \hat{NR}, \hat{W}]$ frame to some geodetically defined frame, and also a geodetically defined location for the RHIC machine's center point ($m = MCP$).

The National Geodetic Survey's Isabelle Project survey results [Appendices C, G] provide a locally valid coordinate mapping between RHIC survey coordinates $(ER(P), NR(P), H=65 \text{ feet})$ to geodetically related New York State plane grid coordinates, transferred to 65 foot grid elevation. For moderate elevation changes (up to a few tens of meters from 65 foot elevation), the perturbation of the New York State plane grid coordinates due to elevation changes is quite small, and is computed by a simple correction [Appendix C] .

Once the New York State plane grid coordinates of a point are known, one may employ a commercially available geodetic survey adjustment program, to compute the latitude and longitude of the point. We have used the STAR*Net codes [15] to generate the required latitude and longitude information. The latitude

and longitude of the RHIC machine's center point are available. Given the latitude, longitude, and elevation of the RHIC center point, the spatial placement and orientation of the RHIC machine plane are thereby determined with respect to a global geocentric reference frame. The cartesian global geocentric coordinates of an arbitrary point in space can be determined from its (ER, NR, W) coordinates, provided that the orientations of the ER and NR axes can be determined with respect to geographic east at the RHIC center point (so that the relative orientations of the geocentric and survey frame axes are well defined with respect to one another). The local geodetic coordinates of the point: geodetic latitude, geodetic longitude, and local gravity elevation can then be determined from the geocentric coordinates.

The equations specifying the location of the RHIC Survey and AGS Grid frames to the Geocentric frame are given in Appendix D.

The orientation of the AGS Grid frame to the RHIC Survey frame can be determined by these methods. However the direction of the AGS Grid frame east must be adjusted by a change in convergence in the longitude meridian between the AGS frame point and the RHIC center point, together with a small correction for survey error in joining the earlier surveys (pre-1993) of the RHIC and AGS machines via the transport line.

4. Horizontal Survey Frame Coordinate Transformation.

The 1993 BNL survey of the AGS to RHIC transport line was adjusted in two different ways. In the first procedure (File TRNY7M1R; RHIC-To-AGS; July 29,1993) a subset of the survey control monuments at the RHIC end of the transport line was least-squares-adjusted to the RHIC survey frame, via tie measurements to the RHIC ring's primary monument network. The monuments of this subset were then selected as the fixed stations for a least-squares-adjustment of the transport line. The set of surveyed control monuments for the transport line was then survey adjusted using those fixed stations. In this way, a set of RHIC-survey-frame-based coordinates were generated for the transport line control monuments.

At the other end of the transport line, another subset of the control monuments was tied by survey measurements, to the AGS survey frame, to provide AGS-frame-based fixed points for a second adjustment of the same survey. Adjustment of the transport line survey, while using AGS-based fixed stations tied to the AGS survey frame, generated a set of control monument coordinates for the transport line, with respect to the AGS coordinate frame. (File TRNY9M1R; AGS-To-RHIC; August 16, 1993).

A functional relationship between the two sets of adjusted coordinates was then searched for. A simple first degree transformation, consisting of a rotation about the AGS grid's frame

attachment point, together with a translation, was found to describe the transformation to a precision of 10 micro-meter over the length of the line.

The transformation to convert AGS-based BNL survey coordinates (EA(P), NA(P)) , of a transport line station point P, to RHIC-based survey coordinates, (ER(P), NR(P)), is:

$$NR(P) = NA(P) + [NR(a) - NA(a)] + [NA(a) - NA(P)](1 - \cos(\text{PSI})) + [EA(a) - EA(P)](\sin(\text{PSI})) ,$$

$$ER(P) = EA(P) + [ER(a) - EA(a)] + [EA(a) - EA(P)](1 - \cos(\text{PSI})) + [NA(a) - NA(P)](-\sin(\text{PSI})) .$$

Here, the AGS-based coordinates of the AGS grid frame's point of attachment, a, are:

EA(a) = 29998.8579600 Meter , NA(a) = 30830.520000 Meter,
and we get:

ER(a) - EA(a) = 0.0048694 Meter, NR(a) - NA(a) = -0.0198193 Meter
as the fitted differences of the RHIC-based and AGS-based
coordinates of a , with:

PSI = -1.782805*(10⁻⁵) Radian, as the fitted rotation angle.

The derivation of this transformation, from the two survey adjustments, is derived in a code file: CCOMP79.TRN included with this note [Appendix E]. This 2-dimensional transformation remains valid to 10 micro-meter for survey points at heights within 3 meters of the transport line beam height.

If one wishes to convert from RHIC-based survey coordinates (ER(Q), NR(Q)) of a station point Q to AGS-based BNL survey coordinates (EA(Q), NA(Q)), one uses the inverse transformation, which is easily found to be:

$$NA(Q) = NR(Q) + [NA(a) - NR(a)] + [NR(a) - NR(Q)](1 - \cos(\text{PSI})) + [ER(a) - ER(Q)](-\sin(\text{PSI})) ,$$

$$EA(Q) = ER(Q) + [EA(a) - ER(a)] + [ER(a) - ER(Q)](1 - \cos(\text{PSI})) + [NR(a) - NR(Q)](\sin(\text{PSI})) .$$

Here, the RHIC-based coordinates of the AGS grid frame's point of attachment, a , are:

$$ER(a) = 29998.8628294 \text{ Meter} , \quad NR(a) = 30830.5001807 \text{ Meter},$$

and where, as before, we have

$$ER(a) - EA(a) = 0.0048694 \text{ Meter}, \quad NR(a) - NA(a) = -0.0198193 \text{ Meter}$$

$$\text{and} \quad \text{PSI} = -1.782805 \times (10^{-5}) \text{ Radian}.$$

APPENDIX A.

Description Of The Transport Line And Injection Arcs.

The beam transport line from the AGS to RHIC is composed of the following sections in sequence:

- a. AGS Extraction and H-10 Switchyard,
- b. U-Line , which includes an 8-degree bend,
- c. W-line , which includes a 20-degree bend
and 1.4 meter level drop,
- d. A switching magnet at the bifurcation joining the
big bend arcs,
- e. X-Line, the westerly moving big bend injection
arc, to the BLUE ring, towards 7 o'clock,
- f. Y-Line, the easterly moving big bend injection
arc, to the YELLOW ring, towards 5 o'clock.

Survey penetration pipes exist through the earth berm at the bifurcation, at one location along the X-Line, at one location along the Y-line, and near the two injection points. Control monuments below these penetrations are used to tie the outside survey of the RHIC rings to the inside survey of the injection arcs and beam transport line.

The AGS-To-RHIC Transport Line

X06 (6-O'Clock Interaction Point)

X-Line

Y-Line

Big Bend Arcs

Switching Magnet

Use RHIC-based
BNL Survey
Coordinates

Use Transform Equations
To Link Coordinate Systems

W-LINE

U-LINE

AGS

Use AGS-based BNL Survey
Coordinates in AGS Machine
and in V-LINE to G-2 Exp't

Preliminary descriptions of the transport line geometry and magnets are given by Claus and Foelsche[3], and by Thern [11,12]. Parameters describing the line's geometry are given by Tsoupas [N. Tsoupas, Informal Communication].

It is a basic requirement, that the two injection arcs to six o'clock, the X-Line and the Y-Line, have to meet the collider rings in a mirror-symmetric manner. Furthermore, the "switch point" at the beginning of the arcs must be approached from AGS along a straight line (the W-Line) which bisects RHIC at the 6 o'clock and 12 o'clock crossing points.

As a consequence of these requirements, the injection arcs, as well as the straight part of the W-Line ahead of the switch point, are, by definition, specified and surveyed with respect to the RHIC-centered BNL frame, irrespective of whatever inaccuracies may still exist in the linkage to the AGS.

The RHIC machine plane is to pass through the 6 o'clock crossing point at a local gravity elevation of 830.000 inches [≈ 21.082 meter; $2.54\text{cm}=1''$] above mean sea level.

The 6-fold symmetry of RHIC about its center point, and the condition that the machine plane lies normal to local gravity at the machine center point imply that the crossing points will be at the same elevation, to the extent that the local gravity equipotential surface is well-approximated by a sphere. There is a variation of elevation, about their mean elevation, of $\pm 0.039\text{mm}$ due to the non-sphericity of the gravity field.

The transport line is to approach RHIC on a path which is parallel to the RHIC machine plane and is 1.9009 inches above that plane, before its final vertical bending. (The distance from the transport line final approach path to the RHIC machine plane was decreased, from the 40 cm reported in [3], because of changes made in the machine's lattice).

The X- and Y-Lines, as well as their extensions backwards into the W-Line, could be set into a plane 1.9009" above, and parallel to, the machine plane. But as a practical matter, one prefers to use "local gravity leveling", and to install those lines so that the design reference orbit curve of the lines runs at constant local elevation, in order to smooth out small effects due to curvature of the earth, and to simplify magnet installation surveying.

Upstream, in the W-line, there is a change of the local beam elevation, from the AGS beam elevation, to 1.9009 inches above the RHIC machine plane, approaching the collider injection points, while the beam also bends 20 degrees horizontally.

Adjustments in the beam transport line from AGS to RHIC are made in the W-line, in the 20 degree bend region, to eliminate any discontinuity or lack of smoothness in the line.

If local gravity levelling is used in the vertical plane; magnet heights can be set according to local gravity elevation. Geodetic computation codes (SATA, SAGRAVEL) were developed, to carry out calculations of perpendicular distances of transport line fiducial markers to the RHIC machine plane, as a function of horizontal and elevation survey coordinates, or, inversely, to provide fiducial marker elevations, when given the three cartesian lattice coordinates of the markers.

The ideal geometry of the transport line is generated by a "reference design curve". This is a continuous spatial curve, with continuous tangent. (It also may be sectionally straight). At each point along this curve a local moving rectangular coordinate frame is specified. The points of the curve are given in RHIC Survey Frame coordinates, or equivalently in "global coordinates", as parametric functions of arc length along the curve measured from an appropriate start point. That is, global coordinates $X(s)$, $Y(s)$, $Z(s)$ define points of the reference design curve, where s is arc length.

The local frame is described in Euclidean 3-space by a right trihedron of unit vectors $\hat{x}(s)$, $\hat{y}(s)$, $\hat{z}(s)$. The vector $\hat{z}(s)$ is defined to be tangent to the reference design curve. While it would be convenient to define the vectors $\hat{y}(s)$ and $\hat{x}(s)$ to lie along the principal normal and binormal respectively, when the curvature was nonzero, and to continue in parallel translation along straight sections of

the curve, this will not be demanded. Rather, the local unit frame vectors will be specified explicitly in terms of their projections on the global frame vectors $\hat{e}_R \equiv \hat{z}$, $\hat{e}_N \equiv \hat{x}$, $\hat{e}_W \equiv \hat{y}$, for each point of the curve. That is, the local moving trihedron will be specified by explicitly defining $\hat{x}(s)$, $\hat{y}(s)$, and $\hat{z}(s)$ in terms of the global RHIC Survey Frame unit base vectors \hat{e}_R , \hat{e}_N , and \hat{e}_W .

[The local coordinates: $x(s)$, $y(s)$, $z(s)$ in the moving frame can be used to describe the trajectory of a beam particle along the transport line, when performing particle beam-optical calculations. The reference orbit curve itself describes an ideal trajectory desired for the beam particles.]

TABLE I.

STAR*NET-Adjusted Geodetic Coordinates
Of RHIC Distinguished Points

POINT	Latitude (B)			Longitude (L)		
	[Deg-Min-Sec]			[Deg-Min-Sec]		
MCP	40	53	02.2336894	-72	52	34.4210626
X02	40	53	06.3031316	-72	52	09.7722420
X04	40	52	48.0670924	-72	52	17.4536523
X06	40	52	43.9971907	-72	52	42.1004630
X08	40	52	58.1627831	-72	52	59.0690443
X10	40	53	16.3995830	-72	52	51.3904829
X12	40	53	20.4700298	-72	52	26.7404910
AGSNOMCP	40	52	26.8569707	-72	52	49.9850993
AGSCPTRA	40	52	26.8564552	-72	52	49.9853156
AGSNOMFP	40	52	18.8947073	-72	53	00.7611465
AGSFPTRA	40	52	18.8940461	-72	53	00.7611757

These latitudes and longitudes are from the August18,
1993 run of the adjustment model code NYS_XPT.DAT .

TABLE Ib.

Computed RHIC Survey Frame Coordinates
Of RHIC Interaction Points

$$E = E(\text{MCP}) + R * \cos(A - \theta)$$

$$N = N(\text{MCP}) + R * \sin(A - \theta)$$

$$R = 590.581658 \text{ Meter}$$

$$\theta = 1.999898 \text{ Degree}$$

$$E(\text{MCP}) = 30 \ 230.237553 \text{ Meter}$$

$$N(\text{MCP}) = 32 \ 284.517011 \text{ Meter}$$

POINT	ER [Meter]	NR [Meter]	A = Angle To Machine's U Axis (Degree)
-----	-----	-----	-----
X02	30 751.689713	32 561.779233	030
X04	30 731.079761	31 971.557305	330
X06	30 209.627601	31 694.295082	270
X08	29 708.785393	32 007.254788	210
X10	29 729.395345	32 597.476717	150
X12	30 250.847505	32 874.738939	090

TABLE II.

RHIC Survey Frame Coordinates Of
Transport Line Survey Control Monuments

Output From STAR*NET Adjustment, RHIC To AGS,
Of The 1993 RHIC Transport Line Survey

Ellipsoidal Earth's Gravity Model using
Lambert NAD '27 System
NY Long Island Zone 3104

MONUMENT STATION	NR	ER	
	[Meter]	[Meter]	
SPLIT M1	31591.657879	30206.865420	! !
TRN007M1	31222.666452	30273.508270	
TRN049M1	31251.389577	30259.513385	
TRN067M1	31268.550071	30251.999735	
TRN074M1	31279.089664	30246.728778	
TRN092M1	31291.438595	30241.499131	
TRN108M1	31306.546329	30236.784468	
TRN129M1	31326.345490	30230.667537	
TRN148M1	31344.019417	30224.838997	
TRN170M1	31364.958065	30218.036390	
TRN190M1	31383.804495	30210.805594	
TRN221M1	31414.544318	30207.684838	
TRN252M1	31445.138262	30202.100271	
TRN280M1	31472.846257	30203.102031	
TRN320M1	31512.445606	30204.489993	
TRN360M1	31552.050799	30205.879905	
WST150M1	31614.044770	30203.404090	! !
WST200M1	31625.542120	30200.823210	! !
EST200M1	31610.906720	30209.604330	! !
EST250M1	31622.302900	30212.629820	! !

The symbol (!) denotes fixed stations in the adjustment.

Table III.

Elevations (H) and Computed Normal Heights (W)
Of Transport Line Survey Control Stations
Above the RHIC Machine Plane MCP

Transport Line Monument	Survey Adjusted Elevation H [Meter]	(SATA) Computed Height Wsata [Meter]	Renormalized Height W = Wsata+.000039 [Meter]	(SAGRAVEL) Computed Elevation H(Sagravel) [Meter]
WST150M1	19.85841	-1.231600	-1.231561	19.85841
WST200M1	19.84856	-1.240260	-1.240221	19.84856
EST200M1	19.85105	-1.239270	-1.239231	19.85105
EST250M1	19.84956	-1.239550	-1.239511	19.84956
SPLIT M1	19.83453	-1.257860	-1.257821	19.83453
TRN360M1	19.83608	-1.260750	-1.260711	19.83608
TRN320M1	20.0012	-1.100320	-1.100281	20.0012
TRN280M1	21.52887	0.422419	0.422458	21.52887
TRN252M1	21.64332	0.533271	0.533310	21.64332
TRN221M1	21.67346	0.559325	0.559364	21.67346
TRN190M1	21.6585	0.540098	0.540137	21.65850
TRN170M1	21.66814	0.547060	0.547099	21.66814
TRN148M1	21.65976	0.535629	0.535668	21.65976
TRN129M1	21.65746	0.530694	0.530733	21.65746
TRN108M1	21.36305	0.233268	0.233307	21.36305
TRN092M1	21.37599	0.243862	0.243901	21.37599
TRN074M1	21.37502	0.240940	0.240979	21.37502
TRN067M1	21.3649	0.229130	0.229169	21.36490
TRN049M1	21.34212	0.203556	0.203595	21.34212
TRN007M1	21.35059	0.207217	0.207256	21.35059

Station height is taken at center of a 3.500" diameter ball
target resting on the station survey monument (CERN bushing).

Point	Assumed Code Input Elevation H [Meter]	(SATA) Computed Height Wsata [Meter]	Assumed Height W [Meter]	(SAGRAVEL) Computed Elevation H(Sagravel) [Meter]
(MCP)	(21.054640)	0.000000	0.000039	(21.054640)
MCP	21.054601	(-0.000039)	0.000000	21.054601
X06	21.082000	-0.000039	0.000000	21.082000
X02	21.081908	-0.000039	0.000000	21.081908
X04	21.081962	-0.000039	0.000000	21.081962
X08	21.081908	-0.000039	0.000000	21.081908
X10	21.081962	-0.000039	0.000000	21.081962
X12	21.082000	-0.000039	0.000000	21.082000
AGSNOMCP	22.85873	1.633866	1.633905	22.85873

APPENDIX B.

Description Of Coordinate Frames And Coordinates Used To Define RHIC Transport Line Control.

Frame System #1

Name Of Frame: Global Geocentric

Unit Orthonormal Base Vectors: \hat{X}_g , \hat{Y}_g , \hat{Z}_g

Frame Attachment Point: Center Of Earth Gravity Ellipsoid
(1927 NAD, Clarke 1866)

Generic Point Coordinates: $X_g(P)$, $Y_g(P)$, $Z_g(P)$

Coordinates Of AGS Center Point (ac) : $X_g(ac)$, $Y_g(ac)$, $Z_g(ac)$

Coordinates Of RHIC Center Point ($m = MCP$): $X_g(m)$, $Y_g(m)$, $Z_g(m)$

Comments:

These are rectangular cartesian right handed coordinates, whose origin is at the center of the gravity reference ellipsoid. They are variables appearing in the code SATA.BAS, in a sequence of coordinate transformations used to convert AGS-based survey coordinates to RHIC-based survey coordinates.

Frame System #2

Name Of Frame: Local Geodetic (Helmert Ellipsoidal)

Unit Orthonormal Base Vectors:

$\hat{B}(P)$ Unit Tangent Vector To Meridian Circle

$\hat{L}(P)$ Unit Tangent Vector To Latitude Circle

$\hat{H}(P)$ Unit Local Upward Vertical Vector at survey point P

Frame Attachment Point: Survey Point P
[This frame is comoving with P]

Generic Point Coordinates: $B(P)$ Geodetic Latitude
 $L(P)$ Geodetic Longitude
 $H(P)$ Local Gravity Elevation
of P above the gravity
reference ellipsoid

Coordinates Of AGS Grid Frame Origin (a) : $B(a)$, $L(a)$, $H(a)$

Coordinates Of RHIC Center Point ($m = MCP$): $B(m)$, $L(m)$, $H(m)$

$B(m) = 40^\circ 53' 2.2336894''$, $L(m) = -72^\circ 52' 34.4210626''$,
 $H(m) = 21.05425$ [Meter]

Comments:

Local geodetic coordinates of the RHIC center point and AGS center point are calculated by using STAR*NET survey adjustment codes, starting from the RHIC-based BNL North and East survey coordinates and sea-level elevation of these points, and are tabulated in this Note.

Transformations mapping BNL survey coordinates and local geodetic coordinates to one another are used to convert elevations of survey control monuments and survey fiducial points to perpendicular distances to the RHIC machine plane. A code, SATA.BAS, was generated in order to perform these coordinate transformations. By use of a multinomial power series expansion of the SATA results, a stand-alone executable code, SAGRAVEL, was generated to compute magnet fiducial elevations.

The Global Geocentric coordinates of P are obtained from the Local Geodetic coordinates of P by the relations:

$$X_g(P) = \{ N(P) + H(P) \} * \cos L(P) * \cos B(P)$$

$$Y_g(P) = \{ N(P) + H(P) \} * \sin L(P) * \cos B(P)$$

$$Z_g(P) = \{ [N(P) * \sqrt{(1 - e^2)}] + H(P) \} * \sin B(P) ,$$

where

$$N(P) = A / \{ \sqrt{[1 - e^2 \sin^2 B(P)]} \}$$

is the meridian radius of curvature, at P, of the gravity reference ellipsoid, and "A" and "e" are the semi-major axis and eccentricity, respectively. (North Latitudes are positive and West Longitudes are negative, in this calculation.)

Frame System #3

Name Of Frame: RHIC Survey Frame

Unit Orthonormal Base Vectors: \hat{ER} , \hat{NR} , \hat{W}

Frame Attachment Point: RHIC Center Point MCP (=m)

Generic Point Coordinates: $ER(P)$, $NR(P)$, $W(P)$

These are the RHIC-based BNL coordinates of P. For magnet installation, the gravity elevation $H(P)$ must be computed.

AGS-Frame-Point Coordinates: $ER(a)$, $NR(a)$, $W(a)$

Coordinates of RHIC Center Point ($m = MCP$):

$ER(m) = 30230.237553$, $NR(m) = 32284.517011$, $W(m) = 0.00$.
[Meter] [Meter] [Meter]

Comments:

These coordinates are the same as the "global coordinates" Z , X , Y in RHIC Accelerator Physics particle optics computation codes [7]:

$$\begin{vmatrix} ER(P) \\ NR(P) \\ W(P) \end{vmatrix} = \begin{vmatrix} Z(P) \\ X(P) \\ Y(P) \end{vmatrix} .$$

In those codes, one uses the non-local basis vectors:

$$\hat{X} \equiv \hat{NR} , \quad \hat{Y} \equiv \hat{W} , \quad \hat{Z} \equiv \hat{NE} .$$

The coordinates $ER(P)$, $NR(P)$ are used to define survey monument horizontal control, and set magnet locations in the RHIC tunnel.

Frame System #4

Name Of Frame: RHIC Geographic Oriented

Unit Orthonormal Base Vectors: $\hat{E}_g(m)$, $\hat{N}_g(m)$, \hat{W}

Point Of Attachment Of Frame: RHIC Center Point MCP (=m)

Generic Point Coordinates: $E_g(P)$, $N_g(P)$, $W(P)$

AGS Center Point Coordinates: $E_g(ac)$, $N_g(ac)$, $W(ac)$

Coordinates OF RHIC Center Point (MCP): 0.0, 0.0, 0.0

Comments:

The coordinates in this frame are rectangular Cartesian coordinates. The base vectors of this frame point along the geodetic East, geodetic North, and local gravity vertical at the RHIC Center Point.

This frame is used as an intermediate frame, suitable for geodetic computations, to allow coordinate conversions to Helmert's coordinates.

[The survey coordinate frames employed for survey control at Brookhaven National Laboratory originate historically from the Camp Upton road grid, and are not convenient for geodetic computations. They are rotated about the gravity vertical at the frame attachment points, to give geodetic orientation, to make geodetic computations tractable] .

Frame System #5

Name Of Frame: RHIC Machine Frame

Unit Orthonormal Base Vectors: \hat{U} , \hat{V} , \hat{W}

Point Of Attachment Of Frame: RHIC Center Point MCP

Generic Point Coordinates: $U(P)$, $V(P)$, $W(P)$

AGS Machine Center Point Coordinates: $U(ac)$, $V(ac)$, $W(ac)$

Coordinates OF RHIC Center Point (MCP): 0.0, 0.0, 0.0

Comments:

The U , V , W coordinates are used to describe beam device fiducials' locations with respect to the RHIC machine lattice. For surveys, and for aligning of magnets and other beam devices, these locations will be computed and data-base tabulated in RHIC Survey Frame coordinates.

The base vector \hat{V} is directed from the Machine Center Point, MCP, to the 12 o'clock intersection point, X12 .

The base vector \hat{W} is the unit upward gravity vertical at the machine center point MCP . The base vector \hat{U} is defined by the relation $\hat{U} = \hat{V} \times \hat{W}$.

Frame System #6

Name Of Frame: AGS Grid Frame

Unit Orthonormal Base Vectors: \hat{I} , \hat{J} , \hat{K}

Point Of Attachment Of Frame: AGS Frame Point (=a)

Generic Point Coordinates: Two coordinate systems are useful:

(1) AGS Grid Coordinates: $I(P)$, $J(P)$, $K(P)$

(2) AGS-based BNL Survey Coordinates and Elevation: $EA(P)$, $NA(P)$, $H(P)$

AGS Grid Coordinates: $I(a) = J(a) = K(a) = 0.0$ [Meter]

These are rectangular cartesian coordinates, having their origin at the frame attachment point a .

$EA(a) = 29998.857960$, $NA(a) = 30830.52$, $H(a) = 19.812$ [Meter]

AGS Machine Center Point Coordinates:

$EA(ac) = 30175.2$, $NA(ac) = 31135.32$, $H(ac) = 22.85873$ [Meter]

$I(ac) = 176.34204$, $J(ac) = 304.8$ [Meter]

RHIC Center Point Coordinates:

$I(m)$, $J(m)$, $K(m)$ [AGS Grid coordinates]

$EA(m)$, $NA(m)$, $H(m)$ [AGS-based BNL Survey coordinates, elevation]

The horizontal coordinates $EA(m)$, $NA(m)$ are computed by using the transformation derived from the 1993 transport line survey adjustment, given in Section 4.

Comments:

The AGS frame attachment point, a , is defined to have an elevation 65 [Feet], the elevation of the New York State map grid used in the NGS 1978-1982 survey of the RHIC terrain.

Note that the point a is not located at the center of the AGS machine.

$EA(a)$, $NA(a)$, and $H(a)$ are, respectively, the horizontal AGS-based BNL Survey coordinates, and elevation, of the AGS Frame Point.

The AGS grid coordinates ($I(P)$, $J(P)$, $K(P)$) of a generic survey point, P , are related to the AGS-based BNL survey coordinates by a translation, given by the relations:

$$\begin{aligned}EA(P) &= I(P) + EA(a) \\ NA(P) &= J(P) + NA(a) .\end{aligned}$$

A third relation, giving the normal distance, $K(P)$, to the local horizontal plane (the I-J plane) passing through the point a , in terms of: $EA(P)$, $NA(P)$, and $H(P)$ is complicated, and is obtained by means of an analytic approximation. (The calculation, which is due to Bowring (1976), is outlined in [4], pp. 31-37).

Frame System #7

Name Of Frame: Transport Line Local Reference Frame

Unit Orthonormal Base Vectors: $\hat{x}(s)$, $\hat{y}(s)$, $\hat{z}(s)$

Point Of Attachment Of Frame:

The local transport frame is a moving reference frame, whose point of attachment lies upon the design reference orbit curve of the transport line. The base vector \hat{z} is the unit tangent vector along the reference orbit curve, directed in the sense from AGS towards RHIC. At points of the design reference curve whose principal curvature is non-zero, either the vector $(\pm) \hat{x}$ will be a principal normal to the curve (at points where the bend is primarily horizontal) or $(\pm) \hat{y}$ will be a principal normal (where bending is primarily vertical). The orientations of the local frame axes to the global RHIC survey frame axes vary continuously along the reference orbit curve, with the exception of a specified few points of zero curvature and torsion where the \hat{x} and \hat{y} unit vectors will undergo finite rotation in the normal plane to the curve.

Generic Point Coordinates: $x(P(s))$, $y(P(s))$, $z(P(s))$

These coordinates can be used to specify locations of beam particles or beamline magnet fiducial markers, with respect to particular points on the reference orbit. The parameter s is the length along the reference orbit, and $P(s)$ is the frame-attachment point on this reference curve. [The point described by the local x , y , z coordinates could, for example, be a fiducial reference mark of a beam transport magnet, which is to be positioned in space with respect to the reference curve, so that the magnet center lies at the position $P(s)$, and the magnet's axis points along the reference curve's tangent \hat{z} direction.]

Specification Of Local Frame Orientation:

The orientation of the local transport frame with respect to the global RHIC survey frame is specified by pitch, azimuth, and roll angles.

Given the local frame, $[\hat{x}(s), \hat{y}(s), \hat{z}(s)]$, which moves along the design orbit of the transport line, one can compute the pitch, azimuth (yaw), and roll angles of this frame with respect to the global RHIC Survey frame $[\hat{ER}, \hat{NR}, \hat{W}]$.

In the local frame, if the unit tangent vector pointing along the direction of the design reference orbit is given as:

$$\hat{s} \equiv \hat{z} = (s_X)\hat{X} + (s_Y)\hat{Y} + (s_Z)\hat{Z} ,$$

in terms of the global unit base vectors

$$\hat{X} \equiv \hat{NR} , \quad \hat{Y} \equiv \hat{W} , \quad \hat{Z} \equiv \hat{ER} ,$$

and if the local unit frame vector \hat{y} is given as:

$$\hat{y} = (\hat{y} \cdot \hat{X})\hat{X} + (\hat{y} \cdot \hat{Y})\hat{Y} + (\hat{y} \cdot \hat{Z})\hat{Z} ,$$

then the pitch, azimuth, and roll angles are given respectively by:

$$\text{Pitch Angle} = \Phi = \text{Arcsin}(s_Y) ,$$

$$\text{Azimuth Angle} = \Theta = \text{Arcsin} \left[\frac{s_X}{\sqrt{s_X^2 + s_Z^2}} \right] ,$$

$$\text{Roll Angle} = \Psi = \text{Arcsin} \left\{ \frac{(s_X)(\hat{Z} \cdot \hat{y}) - (s_Z)(\hat{X} \cdot \hat{y})}{\sqrt{s_X^2 + s_Z^2}} \right\} ,$$

where $s_X = \hat{s} \cdot \hat{X}$, $s_Y = \hat{s} \cdot \hat{Y}$, $s_Z = \hat{s} \cdot \hat{Z}$.

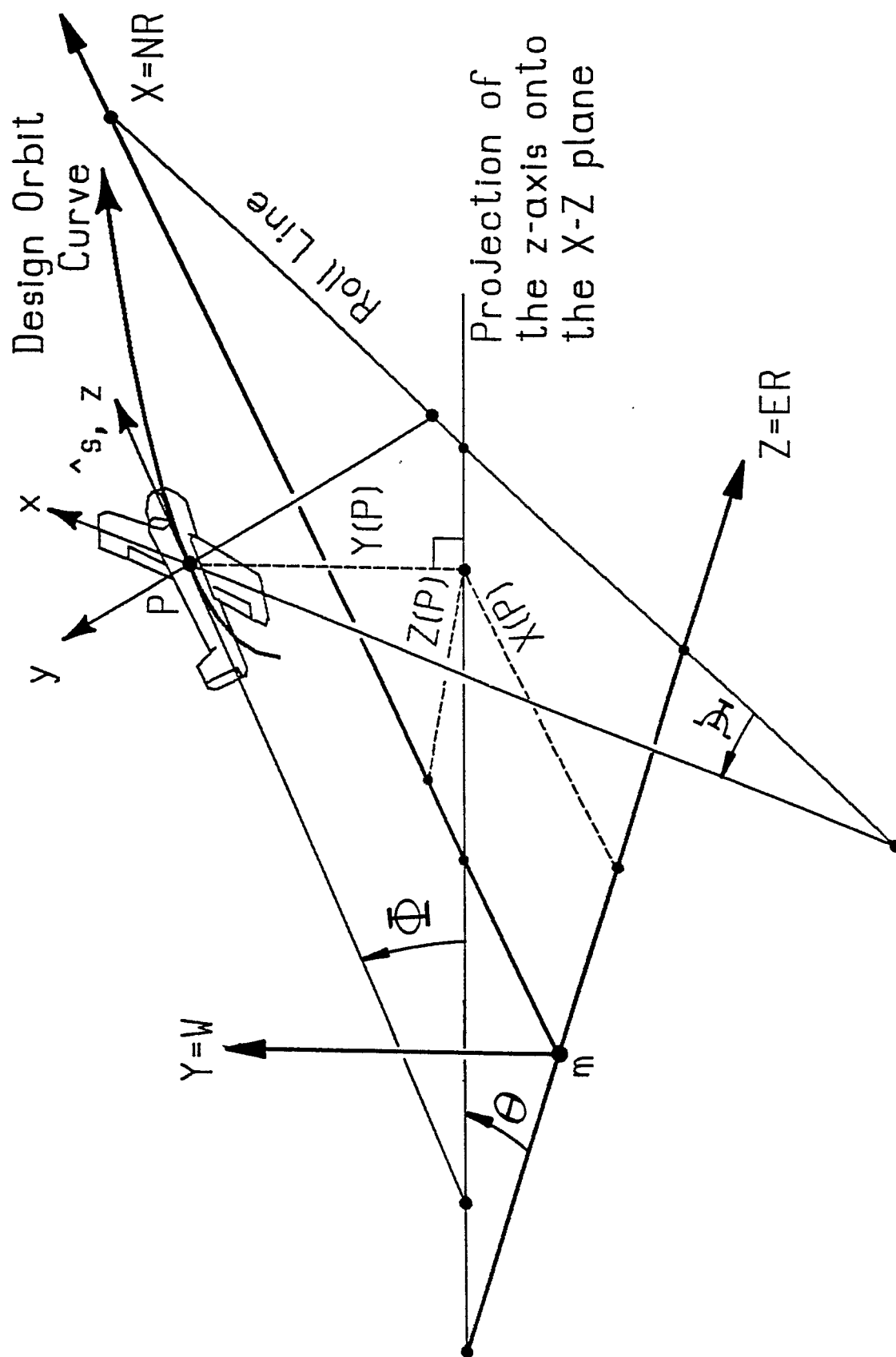
The pitch angle, Φ , is the angle of inclination of the local design orbit curve's tangent vector, \hat{s} , to the RHIC Machine Plane.

The azimuth angle, Θ , is the angle between the \hat{ER} -axis and the projection vector of \hat{s} onto the RHIC Machine Plane.

The roll angle, Ψ , is the angle of inclination of the local \hat{x} -axis to the line of intersection of the x - y plane with the RHIC Machine Plane (X - Z plane).

Note that the pitch, azimuth, and roll angles are denoted in uppercase Greek letters.

Orientation Of The Local Frame To The RHIC Survey Frame



APPENDIX C.

The New York State Plane Coordinate System, Long Island Zone

Coordinate system:

New York State Plane Grid, Long Island Zone (3104),
Transferred To Sixty-Five Foot Elevation

Unit Orthonormal Base Vectors: $\hat{NYS_X}$, $\hat{NYS_Y}$ in the grid plane

Point Of Attachment Of Grid:

Meridian $-74^{\circ} 00' 00''$
Latitude $+40^{\circ} 10' 00''$
Elevation 65 [Feet]

AGS-Frame-Point Coordinates: $NYS_X(a)$, $NYS_Y(a)$, $H(a)$

Generic Point Coordinates: $NYS_X(P)$, $NYS_Y(P)$, $H(P)$

Coordinates OF RHIC Center Point ($m = MCP$):

$NYS_X(m) = 704315.090774$	$NYS_Y(m) = 73724.214868$
[Meter]	[Meter]

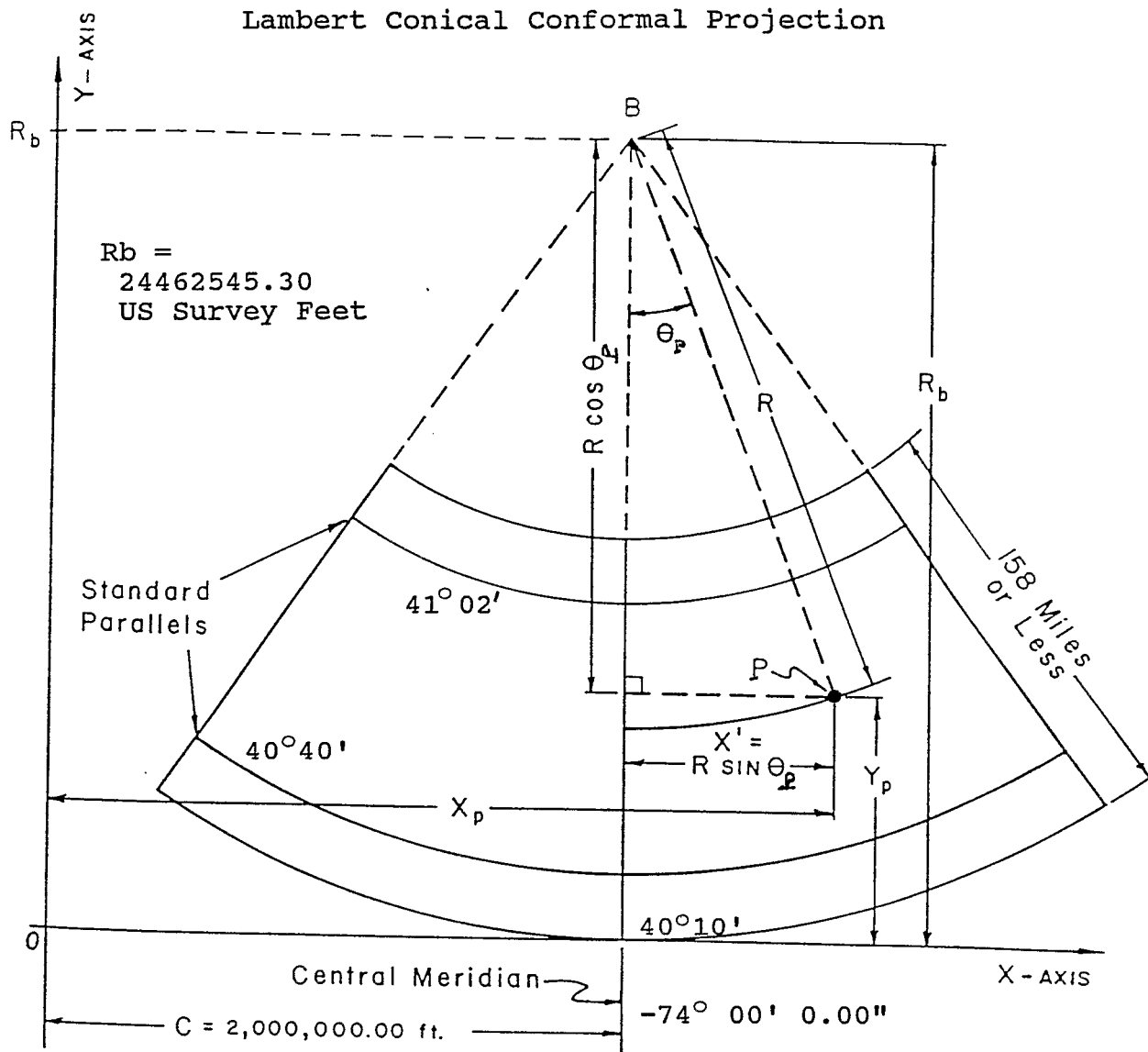
Comments:

This coordinate system is not tied to a frame of three orthogonal base vectors. The New York State grid is defined in a rather complicated way by a conformal Lambert conical secant mapping from the gravity reference ellipsoid. Local horizontal position is described by New York State (Long Island Zone) Y and X coordinates; the third spatial coordinate is specified by the local elevation above mean sea level. The mapping from the reference ellipsoid to the developed cone sector is locally isometric at the parallels of intersection of the cone and the reference ellipsoid.

If one wishes to perform 3-dimensional survey adjustments of transport line survey points, with an ellipsoidal gravity model, using the STAR*NET survey adjustment codes, one requires input survey point coordinates in the New York State grid system, along with local elevations.

The 1982 NGS survey adjustment includes a coordinate mapping to transform BNL RHIC-based horizontal coordinates to New York State plane grid coordinates (scaled to 65 foot elevation). This is a simple translation in the grid plane, followed by a rotation in the grid plane about the NGS survey station point "10 DEGREE POINT 3 FT. OFF 1979" [5].

New York State - Long Island Zone
Lambert Conical Conformal Projection



Notes:

1. This projection is not a perspective projection. It is defined by the conformality requirement.
2. Point P represents the NY State Grid projection of a survey station.
3. The central RHIC tower monument (13 16281 CENTER POINT ISA) has Helmert coordinates: Lmon = $-72^{\circ} 52' 34.35778''$ (Longitude)
Bmon = $+40^{\circ} 53' 2.24156''$ (Latitude) .
4. The angle θ_p for the tower monument is $(74^{\circ} + \text{Lmon}) / \lambda$
where $\lambda = 0.65408209$.

The final adjustment of the 1982 survey of the RHIC terrain by the National Geodetic Survey provided a local transformation between RHIC Survey Frame coordinates and the New York State grid plane coordinates.

The given transformation is, in the notation in the present document:

$$ER(P) = Eo + [(NYS_X(P) - Xo) * (Cos(Alpha)) + (NYS_Y(P) - Yo) * (Sin(Alpha))] / S ,$$

$$NR(P) = No - [(NYS_X(P) - Xo) * (Sin(Alpha)) - (NYS_Y(P) - Yo) * (Cos(Alpha))] / S ,$$

where

$$Eo = 99120.567 \text{ feet,}$$

$$No = 102962.035 \text{ feet,}$$

$$Xo = 2309921.896 \text{ feet,}$$

$$Yo = 239035.269 \text{ feet,}$$

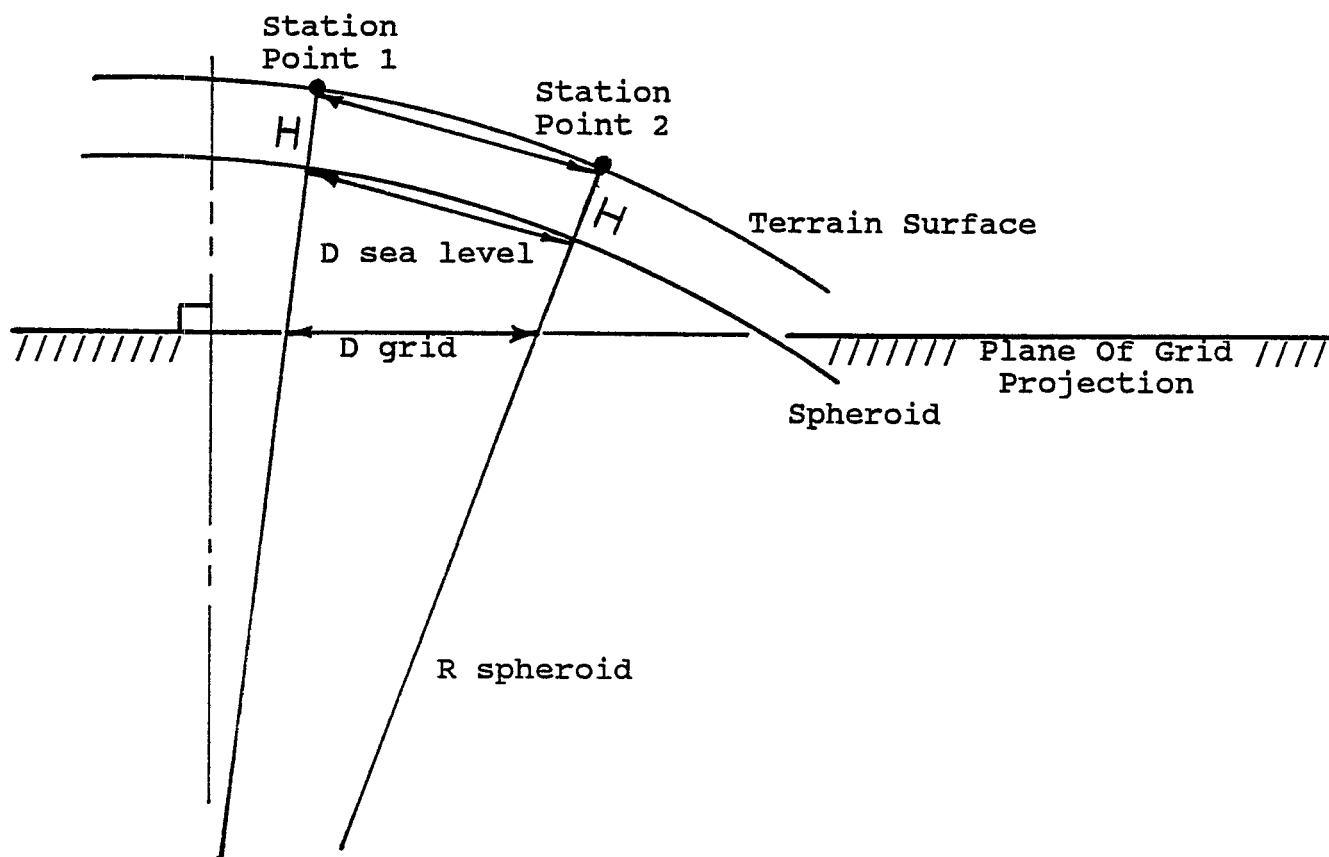
$$Alpha = 345^\circ 00' 33.55'' = 360^\circ - 14^\circ 59' 26.45'' ,$$

$$S = 0.999991959 .$$

The final adjustment sheet did not specify which foot unit was referred to, but a check of the meter reduced lengths in the detailed Trav10 survey adjustment program showed that the values provided are in International Feet.

[To convert International Feet to meters, multiply the International Feet by 0.3048 . To convert U.S. Survey Feet to meters, multiply the U.S.Survey Feet by 12.0/39.37].

Relation Of Grid To Ground Distance



A distance measured on the earth's surface is first reduced to an equivalent distance on the spheroid (that is the reference gravity ellipsoid), and then reduced to an equivalent distance on the grid projection plane.

$$\begin{array}{l} \text{Ground Distance Between} \\ \text{Two Stations At} \\ \text{Orthometric Height } H \end{array} = \frac{\text{Grid Distance}}{\text{Grid Factor}}, \text{ where the}$$

$$\text{Grid Factor} = \text{Elevation Factor} \times \text{Scale Factor} .$$

For a Lambert conformal conical projection, the scale factor is a tabulated function of latitude, and can be found in the state plane coordinate projection tables [8].

$$\text{Elevation Factor} = \frac{\text{Sea Level Distance}}{\text{Ground Distance}} = \frac{R \text{ spheroid}}{R \text{ spheroid} + H} .$$

For RHIC,

Floor height = 65 feet = 19.812 meter ,

Geoidal height = 0.0 meter +/- 1 meter .

R spheroid = 6 375 069.914 meter
(Gaussian osculating radius)

at

Geodetic latitude 40deg 53min 2.24156sec ,

of center tower survey monument.

Interpolated from page 32 of the New York S.P.C. 1954 Projection Tables, the Scale Factor for this latitude is 0.99999511 . A better value, from the NGS final survey adjustment tabulation for CENTER POINT ISA, for the Scale Factor is 0.9999950743 .

The Elevation Factor is equal to

$$1 - H/(R \text{ spheroid} + H) = 1 - 19.812/6275069.914 .$$

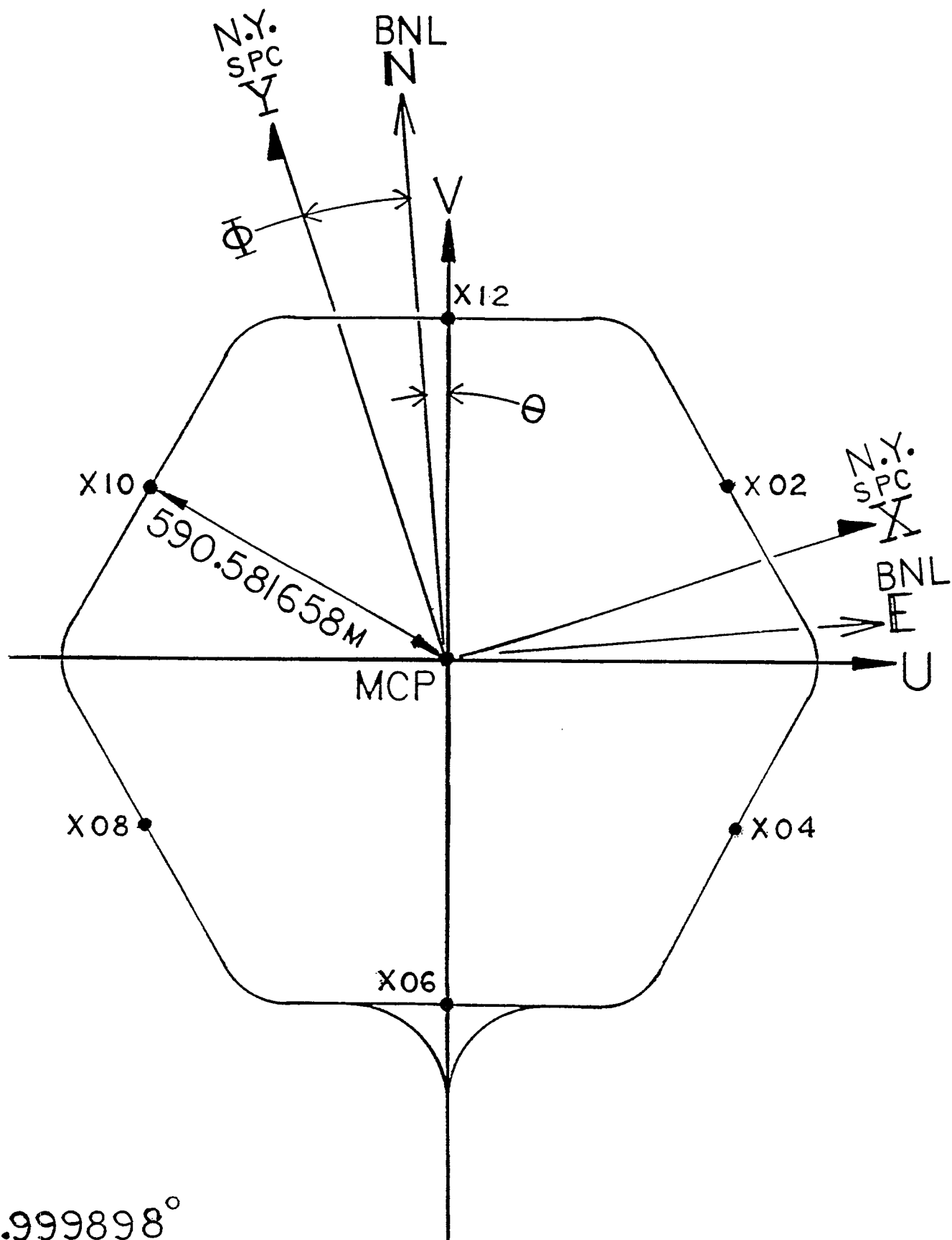
The product of the latter two values gives

$$\text{Grid Factor} = 0.999991967 .$$

The mean Grid Factor (cited for the entire ring) for the 1982 NGS survey adjustment is

$$S = 0.999991959 .$$

These values agree to 0.8 part in 10^8 .



$$\theta = 1.999898^\circ$$

$$\Phi = 14^\circ 59' 26.45''$$

APPENDIX D. Description Of Computation Codes For Coordinate Transformations.

In this appendix, the coordinate transformations relating the various coordinate frames describing the RHIC machine and transport line geometry are developed.

The unit frame basis vectors \hat{e}_L , \hat{e}_B , \hat{e}_n of the Local Geodetic (Helmert) Frame are related to the unit basis vectors of the Geocentric Frame by the relations:

$$\hat{e}_L = \hat{e}_{Xg}(-\sin L) + \hat{e}_{Yg}(\cos L)$$

$$\hat{e}_B = \hat{e}_\rho(-\sin B) + \hat{e}_{Zg}(\cos B)$$

$$\hat{e}_\rho = \hat{e}_{Xg}(\cos L) + \hat{e}_{Yg}(\sin L)$$

$$\hat{e}_n = \hat{e}_L \times \hat{e}_B \quad ,$$

which give

$$\hat{e}_B = \hat{e}_{Xg}(-\sin B)(\cos L) + \hat{e}_{Yg}(-\sin B)(\sin L) + \hat{e}_{Zg}(\cos B)$$

and

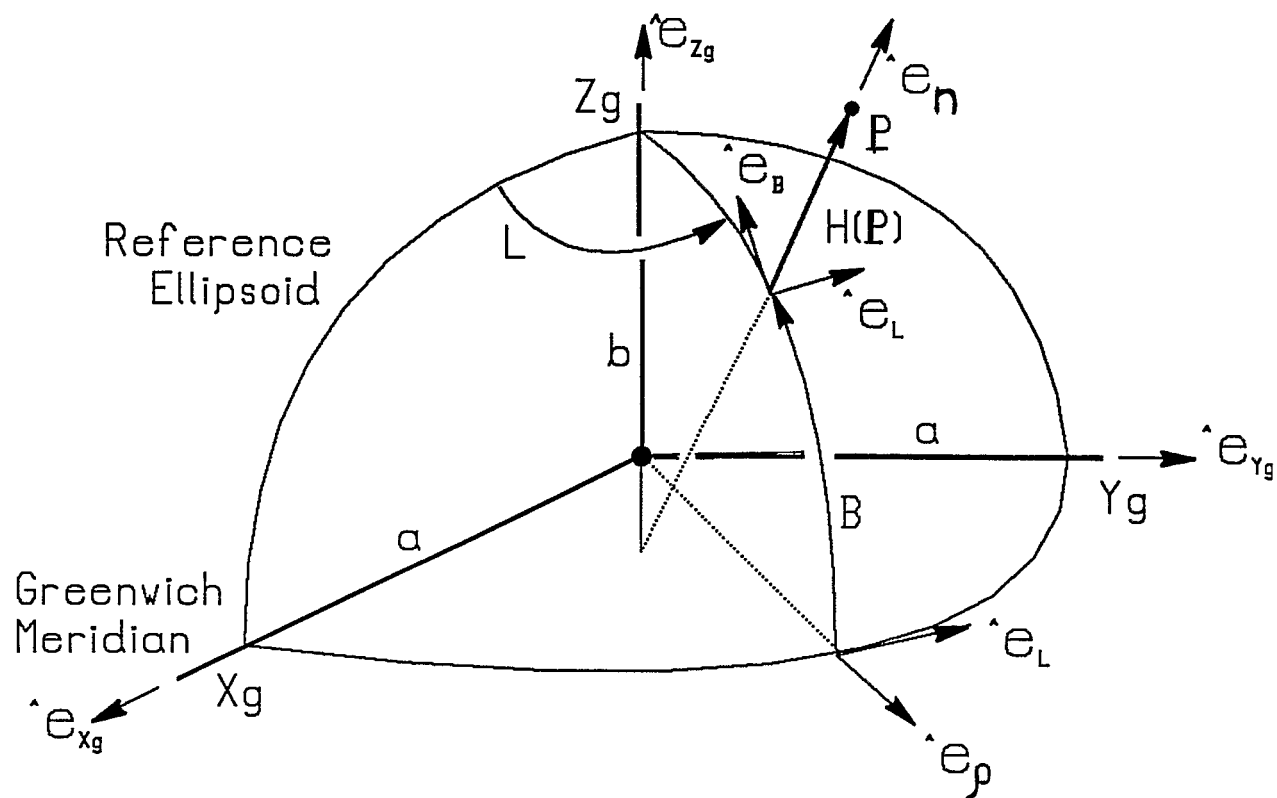
$$\hat{e}_n = \hat{e}_{Xg}(\cos B)(\cos L) + \hat{e}_{Yg}(\cos B)(\sin L) + \hat{e}_{Zg}(\sin B) \quad .$$

At the AGS frame's point of attachment, a , let B_a and L_a be the latitude and longitude coordinates, respectively. Then:

$$\hat{e}_{na} = \hat{e}_{Xg}(\cos B_a)(\cos L_a) + \hat{e}_{Yg}(\cos B_a)(\sin L_a) + \hat{e}_{Zg}(\sin B_a)$$

$$\hat{e}_{La} = \hat{e}_{Xg}(-\sin L_a) + \hat{e}_{Yg}(\cos L_a)$$

$$\hat{e}_{Ba} = \hat{e}_{Xg}(-\sin B_a)(\cos L_a) + \hat{e}_{Yg}(-\sin B_a)(\sin L_a) + \hat{e}_{Zg}(\cos B_a) \quad .$$



At the RHIC machine center point m ($= MCP$) let B_m and L_m be the latitude and longitude coordinates, respectively. Then:

$$\hat{e}_{nm} = \hat{e}_{Xg}(\cos B_m)(\cos L_m) + \hat{e}_{Yg}(\cos B_m)(\sin L_m) + \hat{e}_{Zg}(\sin B_m)$$

$$\hat{e}_{Lm} = \hat{e}_{Xg}(-\sin L_m) + \hat{e}_{Yg}(\cos L_m)$$

$$\hat{e}_{Bm} = \hat{e}_{Xg}(-\sin B_m)(\cos L_m) + \hat{e}_{Yg}(-\sin B_m)(\sin L_m) + \hat{e}_{Zg}(\cos B_m) \quad .$$

The orientations of the local geodetic frames based at the RHIC center point m and at the AGS frame-attachment-point a , with respect to one another, are specified by the set of components of projection of the unit basis vectors of each frame onto those of the other. The resulting orthonormal matrix of direction cosines provides the linear transformation of displacement vector coordinates from one frame to the other. The projection components are:

$$\hat{e}_{La} \cdot \hat{e}_{Lm} = \hat{e}_{Lm} \cdot \hat{e}_{La} = (\sin L_a)(\sin L_m) + (\cos L_a)(\cos L_m) = \cos(L_a - L_m)$$

$$\hat{e}_{La} \cdot \hat{e}_{Bm} = (\sin L_a)(\sin B_m)(\cos L_m) + (-\cos L_a)(\sin B_m)(\sin L_m) = (\sin B_m)(\sin(L_a - L_m))$$

$$\hat{e}_{Ba} \cdot \hat{e}_{Lm} = \hat{e}_{Lm} \cdot \hat{e}_{Ba} = (\sin B_a)(\sin(L_m - L_a))$$

$$\hat{e}_{Ba} \cdot \hat{e}_{Bm} = (\sin B_a)(\sin B_m)(\cos(L_a - L_m)) + (\cos B_a)(\cos B_m)$$

$$\hat{e}_{Ba} \cdot \hat{e}_{nm} = (-\sin B_a)(\cos B_m)(\cos(L_a - L_m)) + (\cos B_a)(\cos B_m)$$

$$\hat{e}_{na} \cdot \hat{e}_{Lm} = (\cos B_a)(\sin(L_a - L_m))$$

$$\hat{e}_{na} \cdot \hat{e}_{Bm} = (-\cos B_a)(\sin B_m)(\cos(L_a - L_m)) + (\sin B_a)(\cos B_m)$$

$$\hat{e}_{na} \cdot \hat{e}_{nm} = (\cos B_a)(\cos B_m)(\cos(L_a - L_m)) + (\sin B_a)(\sin B_m) \quad .$$

At the RHIC center point the local geodetic frame base vectors are:

$$\begin{aligned}\hat{e}_{nm} &= \hat{W} \quad (\text{local vertical at } m) \\ \hat{e}_{Lm} &= \hat{E}_{gm} \quad (\text{local geographic East at } m) \\ \hat{e}_{Bm} &= \hat{N}_{gm} \quad (\text{local geographic North at } m) .\end{aligned}$$

At the AGS frame-attachment-point the local geodetic frame base vectors are:

$$\begin{aligned}\hat{e}_{na} &= \hat{K} \quad (\text{local vertical at } a) \\ \hat{e}_{La} &= \hat{E}_{ga} \quad (\text{local geographic East at } a) \\ \hat{e}_{Ba} &= \hat{N}_{ga} \quad (\text{local geographic North at } a) .\end{aligned}$$

There is an offset angle, let us call it η , between the unit vector along the BNL RHIC survey east $\hat{ER}(m)$ and geographic east \hat{E}_{gm} at the RHIC center point. This is also the offset angle between BNL RHIC survey north $\hat{NR}(m)$ and geographic north \hat{N}_{gm} at this point.

We have:

$$\begin{aligned}\hat{e}_{Lm} &= \hat{E}_{gm} = \hat{ER}(m)(\cos\eta) + \hat{NR}(m)(\sin\eta) \\ \hat{e}_{Bm} &= \hat{N}_{gm} = \hat{ER}(m)(-\sin\eta) + \hat{NR}(m)(\cos\eta) \\ \hat{e}_{nm} &= \hat{W} .\end{aligned}$$

There is also an offset angle, let us call it ν , between the AGS-based BNL survey east $\hat{EA}(a)$ and the AGS frame-point-based local geographic east \hat{E}_{ga} unit vectors. The angles η and ν are not quite identical because of small shifts of latitude and longitude, from m to a , which gives a small computable meridian convergence contribution, but also because of- possible error in the 1978-1982 survey of the transport line between the RHIC and AGS machines. We do not assume that $\eta = \nu$! A best value of ν will be selected based upon an analysis of the 1993 transport line survey.

We have:

$$\begin{aligned}\hat{e}_{La} &= \hat{E}_{ga} = \hat{EA}(a)(\cos\nu) + \hat{NA}(a)(\sin\nu) \\ \hat{e}_{Ba} &= \hat{N}_{ga} = \hat{EA}(a)(-\sin\nu) + \hat{NA}(a)(\cos\nu) .\end{aligned}$$

The local frame at the AGS frame-attachment point a is specified by an orthogonal triple of unit basis vectors:

$$\begin{aligned}\hat{I} &= \hat{EA}(a) = \hat{e}_{La}(a)(\cos \nu) + \hat{e}_{Ba}(a)(-\sin \nu) \\ \hat{J} &= \hat{NA}(a) = \hat{e}_{La}(a)(\sin \nu) + \hat{e}_{Ba}(a)(\cos \nu) \\ \hat{K} &= \hat{e}_{na}\end{aligned}$$

The "AGS Grid coordinate system" is a cartesian coordinate system whose origin is at point a and whose positive I, J, K-axes pass through a and point in the directions of \hat{I} , \hat{J} , \hat{K} , respectively. The coordinates of a generic point, P, are I(P), J(P), K(P).

The "AGS-based BNL Survey coordinate system" is a mixed coordinate system. It uses, as horizontal coordinates of a generic point P, coordinates EA(P) and NA(P), in the I-J plane (which passes through the point a). The EA- and NA-axes are parallel to the \hat{I} and \hat{J} vectors. The EA and NA coordinates are translations of the I and J coordinates of the AGS Grid system, and are given by relations:

$$\begin{aligned}EA(P) &= I(P) + EA(a) \\ NA(P) &= J(P) + NA(a)\end{aligned}$$

The "vertical" coordinate is the gravity elevation H(P).

The matrix of direction cosines describing the orientations of the AGS Grid Frame and the RHIC Survey Frame to one another is:

$$\begin{aligned}\hat{W} \cdot \hat{I} &= (\cos \nu)(\hat{e}_{Ba} \cdot \hat{e}_{nm}) + (-\sin \nu)(\hat{e}_{La} \cdot \hat{e}_{nm}) \\ \hat{W} \cdot \hat{J} &= (\sin \nu)(\hat{e}_{Ba} \cdot \hat{e}_{nm}) + (\cos \nu)(\hat{e}_{La} \cdot \hat{e}_{nm}) \\ \hat{W} \cdot \hat{K} &= \hat{e}_{na} \cdot \hat{e}_{nm} \\ \hat{ER} \cdot \hat{I} &= (\cos \nu)(\cos \eta)(\hat{e}_{Ba} \cdot \hat{e}_{Bm}) + (\sin \nu)(\sin \eta)(\hat{e}_{La} \cdot \hat{e}_{Lm}) \\ \hat{ER} \cdot \hat{J} &= (\sin \nu)(\cos \eta)(\hat{e}_{Ba} \cdot \hat{e}_{Bm}) + (-\sin \nu)(\sin \eta)(\hat{e}_{La} \cdot \hat{e}_{Lm}) \\ \hat{ER} \cdot \hat{K} &= (\cos \eta)(\hat{e}_{na} \cdot \hat{e}_{Bm}) \\ \hat{NR} \cdot \hat{I} &= (\cos \nu)(\sin \eta)(\hat{e}_{Ba} \cdot \hat{e}_{Bm}) + (-\sin \nu)(\cos \eta)(\hat{e}_{La} \cdot \hat{e}_{Lm}) \\ \hat{NR} \cdot \hat{J} &= (\sin \nu)(\sin \eta)(\hat{e}_{Ba} \cdot \hat{e}_{Bm}) + (\cos \nu)(\cos \eta)(\hat{e}_{La} \cdot \hat{e}_{Lm}) \\ \hat{NR} \cdot \hat{K} &= (\sin \eta)(\hat{e}_{na} \cdot \hat{e}_{Bm}) + (\cos \eta)(\hat{e}_{na} \cdot \hat{e}_{Lm})\end{aligned}$$

Table IV. Projections Of Coordinate Frame Unit Vectors
Onto One Another.

The matrix of direction cosines describing the orientations of the AGS Grid Frame and the RHIC Survey frame to one another is given numerically below.

$$\begin{aligned}
 \hat{ER} \cdot \hat{I} &= (1.0) & \hat{ER} \cdot \hat{J} &= -1.7876 \times 10^{-5} & \hat{ER} \cdot \hat{K} &= -3.6021 \times 10^{-5} \\
 \hat{NR} \cdot \hat{I} &= 1.7786 \times 10^{-5} & \hat{NR} \cdot \hat{J} &= (1.0) & \hat{NR} \cdot \hat{K} &= -2.2842 \times 10^{-4} \\
 \hat{W} \cdot \hat{I} &= 3.6025 \times 10^{-5} & \hat{W} \cdot \hat{J} &= 2.2842 \times 10^{-4} & \hat{W} \cdot \hat{K} &= (1.0) .
 \end{aligned}$$

The projections of the unit gravity normal vector at the AGS machine's center, \hat{n}_{ac} , onto the unit base vectors \hat{ER} , \hat{NR} , and \hat{W} of the RHIC Survey Frame, and onto the unit base vectors \hat{I} , \hat{J} , \hat{K} of the AGS Grid Frame are given below.

$$\begin{aligned}
 \hat{n}_{ac} \cdot \hat{ER} &= -8.4365 \times 10^{-6}, & \hat{n}_{ac} \cdot \hat{NR} &= -1.8056 \times 10^{-4}, & \hat{n}_{ac} \cdot \hat{W} &= (1.0) . \\
 \hat{n}_{ac} \cdot \hat{I} &= 2.7586 \times 10^{-5}, & \hat{n}_{ac} \cdot \hat{J} &= 4.7859 \times 10^{-5}, & \hat{n}_{ac} \cdot \hat{K} &= (1.0) .
 \end{aligned}$$

The values above are computed by means of the SATA code. The direction cosines squared of a projected vector summed over the frame unit vectors add to unity; where a direction cosine is just slightly less than unity, it is reported above as 1.0, and can be calculated more precisely using the sum relation.

APPENDIX E

A Transformation For Conversion Of Coordinates From The BNL RHIC Survey Frame To The BNL AGS Survey Frame

FILE: CCOMP79.TRN

M.A. Goldman

August 18, 1993

This file is a calculation of a proposed conversion from AGS-based BNL Coordinates to RHIC-based BNL Coordinates, using results from the transport line survey, which indicate that the transform is a translation plus a rotation, to close approximation. It uses output from STAR*NET Ellipsoidal Earth adjustments TRNY7M1R (RHIC-To-AGS) of July 29, 1993 and TRNY9M1R (AGS-To-RHIC) of August 16, 1993.

All distances are reported in [Meter].

TRNY7M1R
STAR*NET-ADJUSTED
COORDINATES

(RHIC-Based)

TRNY9M1R
STAR*NET-ADJUSTED
COORDINATES

(AGS-Based)

MONUMENT	RHIC-To-AGS		AGS-To-RHIC	
	ER [Meter]	NR [Meter]	EA [Meter]	NA [Meter]
TRN007M1	30273.5082700	31222.6664520	30273.5104000	31222.6813600
TRN049M1	30259.5133850	31251.3895770	30259.5160200	31251.4047500
TRN067M1	30251.9997350	31268.5500710	30252.0026760	31268.5653780
TRN074M1	30246.7287780	31279.0896640	30246.7319070	31279.1050650
TRN092M1	30241.4991310	31291.4385950	30241.5024800	31291.4540900
TRN108M1	30236.7844680	31306.5463290	30236.7880860	31306.5619070
TRN129M1	30230.6675370	31326.3454900	30230.6715080	31326.3611770
TRN148M1	30224.8389970	31344.0194170	30224.8432830	31344.0352090
TRN170M1	30218.0363900	31364.9580650	30218.0410490	31364.9739780
TRN190M1	30210.8055940	31383.8044950	30210.8105890	31383.8205370
TRN221M1	30207.6848380	31414.5443180	30207.6903770	31414.5604120
TRN252M1	30202.1002710	31445.1382620	30202.1063610	31445.1544590
TRN280M1	30203.1020310	31472.8462570	30203.1086130	31472.8624370
TRN320M1	30204.4899930	31512.4456060	30204.4972800	31512.4617600
TRN360M1	30205.8799050	31552.0507990	30205.8878950	31552.0669290
SPLIT_M1	30206.8654200	31591.6578790	30206.8741160	31591.6739910

RHIC-Based BNL Coordinates

AGS-Based BNL Coordinates

RHIC Machine Center Point:

ER(m) = NR(m) =
MCP 30230.2375530 32284.5170110

EA(m) NA(m)

AGS Grid Frame Attachment Point:

AGSFP ER(a) NR(a) EA(a) = NA(a) =
29998.8579600 30830.5200000

AGS Machine Center Point:

AGSCP ER(ac) NR(ac) EA(ac) = NA(ac) =
30175.2000000 31135.3200000

The transformation proposed, to convert AGS-based BNL survey coordinates to RHIC-based BNL coordinates is a rotation plus a translation. The rotation angle (PSI) is obtained by a direct comparison of the station pair azimuths in the two adjustments. The translation parameters are calculated below, by comparing coordinates in the mean, for the two adjustments.

$$ER(a) = \langle ER \rangle + [EA(a) - \langle EA \rangle] (\cos(\text{PSI})) + [NA(a) - \langle NA \rangle] (\sin(\text{PSI}))$$

$$ER(a) - EA(a) = \langle ER - EA \rangle + [NA(a) - \langle NA \rangle] (\sin(\text{PSI})) + O(10^{-7})$$

$$NR(a) = \langle NR \rangle + [NA(a) - \langle NA \rangle] (\cos(\text{PSI})) - [EA(a) - \langle EA \rangle] (\sin(\text{PSI}))$$

$$NR(a) - NA(a) = \langle NR - NA \rangle - [EA(a) - \langle EA \rangle] (\sin(\text{PSI})) + O(10^{-7})$$

MONUMENT	EA - ER	NA - NR	Averages <EA - ER> 0.0048686	Averages <NA - NR> 0.0157602
TRN007M1	0.0021300	0.0149080		
TRN049M1	0.0026350	0.0151730		
TRN067M1	0.0029410	0.0153070	<ER> 30226.5315464	<NR> 31376.7182048
TRN074M1	0.0031290	0.0154010		
TRN092M1	0.0033490	0.0154950		
TRN108M1	0.0036180	0.0155780	<ER - EA> -0.0048686	<NR - NA> -0.0157602
TRN129M1	0.0039710	0.0156870		
TRN148M1	0.0042860	0.0157920		
TRN170M1	0.0046590	0.0159130		
TRN190M1	0.0049950	0.0160420	<EA> 30226.5364150	<NA> 31376.7339649
TRN221M1	0.0055390	0.0160940		
TRN252M1	0.0060900	0.0161970		
TRN280M1	0.0065820	0.0161800		
TRN320M1	0.0072870	0.0161540	EA(a) = 29998.8579600	NA(a) = 30830.5200000
TRN360M1	0.0079900	0.0161300		
SPLIT_M1	0.0086960	0.0161120		

RHIC Survey Frame
Coordinates of
AGS Frame Point
Derived from
Transport Line
Survey

AGSFPTRA

ER(a) - EA(a) = 0.0048694
[Meter]

Computed ER(a)
= 29998.8628294
[Meter]

NR(a) - NA(a) = -0.0198193
[Meter]

Computed NR(a)
= 30830.5001807
[Meter]

The proposed transform to convert AGS-based BNL survey coordinates, (EA(P), NA(P)) , of a transport line station point P, to RHIC-based survey coordinates, (ER(P), NR(P)), is:

$$NR(P) - NA(P) = [NR(a) - NA(a)] + [NA(a) - NA(P)](1 - \cos(\text{PSI})) + [EA(a) - EA(P)](\sin(\text{PSI}))$$

$$ER(P) - EA(P) = [ER(a) - EA(a)] + [EA(a) - EA(P)](1 - \cos(\text{psi})) + [NA(a) - NA(P)](-\sin(\text{PSI}))$$

Use transport line AGS-based survey coordinates EA, NA and revised AGS origin in transform for calculated fit.

LINE MONUMENT	SURVEY ADJUSTMENT VALUE [ER] [Meter]	SURVEY ADJUSTMENT VALUE [NR] [Meter]	FITTED VALUE USING FORMULA [ER] [Meter]	FITTED VALUE USING FORMULA [NR] [Meter]
TRN007M1	30273.5082700	31222.6664520	30273.5082779	31222.6664372
TRN049M1	30259.5133850	31251.3895770	30259.5133858	31251.3895777
TRN067M1	30251.9997350	31268.5500710	30251.9997358	31268.5500718
TRN074M1	30246.7287780	31279.0896640	30246.7287789	31279.0896648
TRN092M1	30241.4991310	31291.4385950	30241.4991318	31291.4385966
TRN108M1	30236.7844680	31306.5463290	30236.7844684	31306.5463295
TRN129M1	30230.6675370	31326.3454900	30230.6675374	31326.3454905
TRN148M1	30224.8389970	31344.0194170	30224.8389974	31344.0194185
TRN170M1	30218.0363900	31364.9580650	30218.0363901	31364.9580663
TRN190M1	30210.8055940	31383.8044950	30210.8055941	31383.8044964
TRN221M1	30207.6848380	31414.5443180	30207.6848340	31414.5443157
TRN252M1	30202.1002710	31445.1382620	30202.1002726	31445.1382632
TRN280M1	30203.1020310	31472.8462570	30203.1020306	31472.8462590
TRN320M1	30204.4899930	31512.4456060	30204.4899916	31512.4456068
TRN360M1	30205.8799050	31552.0507990	30205.8799006	31552.0508006
SPLIT_M1	30206.8654200	31591.6578790	30206.8654154	31591.6578802

Fitted Value Fitted Value
EA(ac) = NA(ac) =
30175.2000000 31135.3200000

AGS Center Point, computed from transport line survey comparison:

Computed Value:		AGSCPTRA Computed Coordinates:
ER(ac) - EA(ac) =	-0.0005647	ER(ac) = 30175.1994353
NR(ac) - NA(ac) =	-0.0166755	NR(ac) = 31135.3033245

FORMULA ESTIMATE MINUS ADJUSTED VALUE:

	DIFFERENCE [ER] [Meter]	DIFFERENCE [NR] [Meter]
TRN007M1	0.0000079	-0.0000148
TRN049M1	0.0000008	0.0000007
TRN067M1	0.0000008	0.0000008
TRN074M1	0.0000009	0.0000008
TRN092M1	0.0000008	0.0000016
TRN108M1	0.0000004	0.0000005
TRN129M1	0.0000004	0.0000005
TRN148M1	0.0000004	0.0000015
TRN170M1	0.0000001	0.0000013
TRN190M1	0.0000001	0.0000014
TRN221M1	-0.0000040	-0.0000023
TRN252M1	0.0000016	0.0000012
TRN280M1	-0.0000004	0.0000020
TRN320M1	-0.0000014	0.0000008
TRN360M1	-0.0000044	0.0000016
SPLIT_M1	-0.0000046	0.0000012

$PSI = (3.6773/3600.0) * (-1) * (Pi/180)$ [Radian]

$(10^5) * PSI = -1.782805$ [Radian]

$(10^5) * SIN(PSI) = -1.7828050$

$(10^9) * (1 - COS(PSI)) = 0.1589197$

[Note: Here, PSI is the azimuth angle shift between the RHIC-To_AGS transport line adjustment and the AGS-To-RHIC adjustment, for the ellipsoidal earth gravity model used in the TRNY7M1R.LST and TRNY8M1R.LST adjustments.]

APPENDIX F. Multinomial Expansion Parameters For Elevation
In RHIC Survey Frame Coordinates.

The elevation, H , in meters, of a survey station, P , has been expanded in a multinomial series, to terms of fourth order, in the variables:

$$\begin{aligned} n &= (NR(P) - NR(MCP))/1000 , \\ e &= (ER(P) - ER(MCP))/1000 , \text{ and} \\ W &, \text{ where} \end{aligned}$$

$NR(P)$, $ER(P)$ are the RHIC Survey Frame horizontal coordinates of P and $NR(MCP)$, $ER(MCP)$ are the corresponding coordinates of the RHIC machine's center point, MCP . W is the perpendicular height of the station P above the RHIC machine plane. Coordinate values are in [Meter].

This series expansion about the machine center point has been incorporated into the standalone code *SAGRAVEL.EXE* which is used to compute fiducial point elevations.

The expansion is:

$$H(P) = \sum_{a,b,c=0}^{a+b+c \leq 4} A_{abc} n^a e^b W^c .$$

The constant term is: $A_{000} = H(MCP)$.

The expansion coefficients are given in the following table.

Expansion Coefficients For Elevation.

$$\begin{aligned}
A_{000} &= 21.05460119 \\
A_{100} &= 1.041567734 \times 10^{-09} \\
A_{010} &= 2.491716802 \times 10^{-08} \\
A_{001} &= -4.015375869 \times 10^{-08} \\
A_{110} &= -1.592834280 \times 10^{-06} \\
A_{101} &= -2.354152670 \times 10^{-08} \\
A_{011} &= 3.062840655 \times 10^{-08} \\
A_{200} &= 7.856045823 \times 10^{-04} \\
A_{020} &= 7.830048405 \times 10^{-04} \\
A_{002} &= 1.408636158 \times 10^{-08} \\
A_{111} &= -2.019687804 \times 10^{-09} \\
A_{210} &= -2.596763780 \times 10^{-09} \\
A_{201} &= -2.503168948 \times 10^{-09} \\
A_{120} &= 3.844514744 \times 10^{-10} \\
A_{021} &= 8.108702219 \times 10^{-09} \\
A_{102} &= 5.504938138 \times 10^{-09} \\
A_{012} &= -9.916240694 \times 10^{-09} \\
A_{300} &= -1.770666474 \times 10^{-11} \\
A_{030} &= -4.314201806 \times 10^{-10} \\
A_{003} &= 3.336042594 \times 10^{-10} \\
A_{211} &= -9.609673053 \times 10^{-10} \\
A_{121} &= 3.675419377 \times 10^{-09} \\
A_{112} &= -2.584872756 \times 10^{-09} \\
A_{220} &= 4.638482729 \times 10^{-10} \\
A_{202} &= 6.021038954 \times 10^{-10} \\
A_{022} &= -6.095390866 \times 10^{-10} \\
A_{301} &= -1.871967141 \times 10^{-11} \\
A_{310} &= -2.557224611 \times 10^{-10} \\
A_{130} &= 9.742894539 \times 10^{-11} \\
A_{031} &= -3.127436423 \times 10^{-09} \\
A_{103} &= 5.246448116 \times 10^{-10} \\
A_{013} &= -3.358851151 \times 10^{-09} \\
A_{400} &= 4.934792935 \times 10^{-12} \\
A_{040} &= -1.576058990 \times 10^{-10} \\
A_{004} &= -1.930032621 \times 10^{-10}
\end{aligned}$$

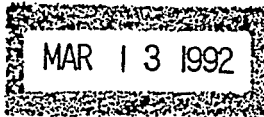
APPENDIX G.

Documents Relating To The Final Survey Adjustment Of The National Geodetic Survey Organiztion's 1981 ISABELLE Project Observations

The following three pages are a summary of the results of the 1981 ISABELLE Project survey results, from the National Geodetic Survey organization, National Oceanic and Atmospheric Administration, United States Department Of Commerce. The detailed TRAV10 code survey analysis printout is available at the Brookhaven National Laboratory Survey and Alignment Group.



UNITED STATES DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
NATIONAL OCEAN SERVICE
Coast and Geodetic Survey
Rockville, Maryland 20852



Mr. Frank Karl
Brookhaven National Laboratory
Bldg #923
Upton, Long Island, New York 11973-5000

Dear Mr. Karl:

As you requested, enclosed is the final adjustment of the project which NGS observed in 1981. I believe it is the printout referred to in the letter to Captain Baker from Chuck Fronczek. I have marked page 43 with the accuracies since the page numbers start over several times with each subsequent program run. I am also enclosing a copy of the coordinates which is probably a copy of the one you have so you can compare them.

We would still like the printout returned as soon as possible. I made a copy, but it is poor and I hope you get better results. My address is the same as Ms. Sollers, except refer to the Horizontal Network Branch, Room 429B.

Please call if you have any further questions or requests.

Sincerely,

Maralyn L. Vorhauer

Maralyn L. Vorhauer
Technical Assistant
Horizontal Network Branch
National Geodetic Survey

Enclosures



STATION NAME	STATE PLANE COORDINATES		B.N.L. COORDINATES*	
	NEW YORK - LONG ISLAND ZONE		(E)	(N)
	(X)	(Y)		
10 DEGREE POINT 3 FT. OFF 1979	2 309 921.896	239 035.269	99 120.567	102 962.035
6 ISA 1979	2 310 019.228	239.837.655	99 007.039	103 762.297
CENTER POINT ISA 1979	2 310 745.277	241 877.727	99 180.692	105 920.760
1A ISA	2 311 840.633	243 821.129	99 736.091	105 081.368
1B ISA	2 312 693.647	242 815.438	100 820.218	107 330.539
3A ISA	2 312 888.009	241 885.146	101 248.600	106 482.174
3B ISA	2 312 527.865	240 661.947	101 217.107	105 207.437
5A ISA	2 311 807.946	239 990.863	100 695.268	104 372.970
5B ISA	2 310 584.284	239 725.820	99 581.798	103 800.427
7A ISA	2 309 676.138	240 003.613	98 632.696	103 833.864
7B ISA	2 308 801.698	240 941.066	97 545.522	104 513.234
9A ISA	2 308 588.069	241 866.427	97 099.804	105 351.852
9B ISA	2 308 791.337	243 130.431	96 969.204	106 625.428
11A ISA	2 309 656.806	243 739.464	97 647.691	107 437.604
11B ISA	2 310 906.309	244 028.447	98 779.931	108 039.956
12RQ 7 OUT	2 311 755.043	243 591.400	99 712.836	107 837.317
2LQ 7 OUT	2 312 525.031	242 766.055	100 670.113	107 239.222
2RQ 7 OUT	2 312 734.491	241 859.527	101 106.932	106 417.717
4LQ 7 OUT	2 312 404.642	240 780.143	101 067.504	105 289.739
4RQ 7 OUT	2 311 724.521	240 145.592	100 574.659	104 500.855
5IQ F OUT	2 310 752.108	239 867.174	99 707.348	103 980.382
8LQ 7 OUT	2 308 964.939	240 988.574	97 690.921	104 601.351

*B.N.L. Coordinates computed using the following:

$$E_i = E_o + [(X_i - X_o) * \cos \alpha + (Y_i - Y_o) * \sin \alpha] / S$$

$$N_i = N_o - [(X_i - X_o) * \sin \alpha - (Y_i - Y_o) * \cos \alpha] / S$$

Where: $E_o = 99120.567$ ft.

$N_o = 102\ 962.035$ ft.

$X_o = 2\ 309\ 921.896$ ft.

$Y_o = 239\ 035.269$ ft.

$\alpha = 345^\circ\ 00'\ 33''55$

$S = 0.99999\ 1959$



UNITED STATES DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
NATIONAL OCEAN SURVEY
Rockville, Md. 20852

June 9, 1982

OA/C17:CJF

Captain Leonard S. Baker (Retired)
Brookhaven National Laboratory
Building 902A
P.O. Box 301
Upton, New York 11973

Dear Sam,

I am sorry for the long delay in delivering the results for the Brookhaven National Laboratory's (B.N.L.) ISABELLE Survey. However, I now feel we have a reliable product. I am enclosing with this letter the following:

o A list of the adjusted State Plane Coordinates (S.P.C.) (New York/Long Island Zone) and the B.N.L. Coordinates for the stations on the ISABELLE project.

o A copy of the adjustment from which the adjusted data came. This copy includes all software program changes and data rejections. You will probably be most interested in the relative accuracy estimates beginning on page 43.

o Magnetic program cards for program PCTTRANS. This program was used on the H.P. 41CV (with printer) to convert State Plane Coordinates to B.N.L. Coordinates.

o A listing of the program GPPCGP. This program was used to convert the geographic positions to S.P.C.

There are two points to note concerning the B.N.L. grid.

1) The orientation between A.G.S. and ISABELLE is determined by the line 10 DEGREE POINT 3FT OFF 1979 - 6 ISA 1979. From the 1979 survey I found the rotation angle over this line between the B.N.L. Coordinates and S.P.C. Coordinates to be $-14^{\circ} 59' 26''.45$. This compares favorably with the original rotation of $-14^{\circ} 59' 11''.56$ (per conversations with Frank Atkinson).

2) The S.P.C. Coordinates have been scaled by using the average scale factor from the S.P.C. computation and were then raised to the 65-foot level.

Sincerely yours,

Charles J. Fronczek
Geodesist
National Geodetic Survey

Enclosures



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