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Lattice Alternative For RHIC

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DISCLAIMER

RHIC-PG-26

LATTICE ALTERNATIVES FOR RHIC

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ASSUMPTIONS

EQUAL IBS DIFFUSION RATES

$$\tau_H^{-1} = \tau_E^{-1} = 0.5\ h^{-1}\ (= \tau_V^{-1})$$

$$\tau_L^{-1} \approx 1\ h^{-1}$$

BEAM-BEAM TUNE SHIFT

$$\Delta\nu_{BB} = 3 \times 10^{-3}$$

BUNCHED BEAM OPERATION

$$h_B = 57$$

$$\sigma_L = 1\ m$$

EXISTENCE OF BOOSTER

NO MOMENTUM STACKING

BUT DESIGN OF COLLIDER ALLOWS FOR FUTURE INJECTOR IMPROVEMENTS

$$N_B \geq 1 \times 10^9\ \text{ions/bunch}$$

ONLY Au-IONS STUDIED

DEFINITIONS

rms beam radius	$\sigma_{H,V} = \left(\frac{\beta_{H,V}}{6\gamma}\frac{\varepsilon}{\pi}\right)^{1/2}$
beam size maximum in arc	$= 2\sqrt{6}\,(\sigma_H + X_p\,\delta_E)$
rms diamond length	$\sigma_{diam} = \dfrac{\sigma_H\,\sigma_L}{2\sigma_H + \frac{1}{2}\alpha\,\sigma_L}$
rms bunch length	$= \sigma_L$
bunch area	$A_B = 6\pi\,\sigma_L\,\delta_E\gamma\,E_o/c$

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LATTICE

ψ = phase shift/cell

L_{HC} = length of half cell

θ_{HC} = deflection/half cell

N_{CS} = number of cells/sextant

$$\langle\beta\rangle = \frac{2\ L_{HC}}{\sin\psi}$$

$$\beta_{max} = \langle\beta\rangle\ (1 + \sin\tfrac{1}{2}\psi)$$

$$\langle X_p\rangle = L_{HC}\ \theta_{HC}\ \frac{1}{\sin^2\frac{1}{2}\psi}$$

$$\gamma_{tr} = \frac{\langle\beta\rangle}{\langle X_p\rangle}\ \cos\tfrac{1}{2}\psi\ (\times\ 1.37\quad)$$

$$X_{p\ max} = \langle X_p\rangle\ (\ 1 + \tfrac{1}{2}\sin\tfrac{1}{2}\psi)$$

$$\theta_{HC} = \frac{\pi}{6\ N_{cs}}$$

$$\int_{HC} B\ d\ell = \frac{388}{N_{cs}}\ \text{Tesla} \cdot \text{meter}$$

$$\int_{quad} G\ d\ell = 9.61\ N_{cs}\ \sin\tfrac{1}{2}\psi\quad \text{Tesla}$$

LUMINOSITY

$$L = \frac{f_o}{4\pi} h_B N_B^2 \frac{1}{\sigma_V^* \sigma_{Heff}}$$

$$f_o = \frac{c}{2\pi R}$$

$$\sigma_{Heff} = \left(\sigma_H^{*2} + (\tfrac{1}{2} \alpha \sigma_L)^2\right)^{1/2}$$

BEAM-BEAM TUNE SHIFT ($\Delta\nu_{BB} \leq 3 \times 10^{-3}$)

$$\Delta\nu_{BB} = \frac{r_p}{2\pi} \frac{Q^2}{A} \frac{N_B}{\gamma} \frac{\beta_V^*}{\sigma_V \; (\sigma_V + \sigma_{Heff})}$$

TOLERABLE COUPLING IMPEDANCE

$$\frac{Z}{n} = 4 \frac{A}{Q^2} \frac{\gamma E_o}{eI_p} |n| \delta_E^2$$

$$n = \gamma^{-2} - \gamma_{tr}^{-2}$$

$$I_p = \frac{c}{\sqrt{2\pi}} \frac{N_B}{\sigma_L}$$

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INTRA BEAM SCATTERING
(Courant, Bjorken & Mtingwa)

Diffusion rate ($\gamma >> \gamma_{tr}$)

$$\tau^{-1} = \tau_H^{-1} + \tau_V^{-1} + \tau_E^{-1}$$

$$\tau_V^{-1} \approx 0$$

$$\tau_E^{-1} = \left(\frac{\sigma_H}{\langle X_p \rangle \delta_E}\right)^2 \tau_H^{-1}$$

$$\tau_H^{-1} = \frac{9}{4} L_g \, r_p^{\,2} \, c \left(\frac{Q^2}{A}\right)^2 \frac{N_B}{\sigma_L} \frac{1}{\gamma \delta_E} \frac{\pi^2}{\varepsilon_H \varepsilon_V} \frac{\langle X_p \rangle}{\langle \beta \rangle} \left[1 + \left(\frac{\sigma_H}{\langle X_p \rangle \delta_E}\right)^2\right]^{-1/2}$$

$$-\frac{1}{L}\frac{dL}{dt} = \tau_V^{-1} + \frac{\sigma_H}{\sigma_{Heff}} \tau_H^{-1} + \frac{\frac{1}{2}\alpha \, \sigma_L}{\sigma_{Heff}} \tau_E^{-1}$$

with $L_g \approx 20$
$r_p = 1.5347 \times 10^{-18}$ m
N_B = number of ions/bunch
σ_L = bunch length

EQUIPARTITION

$$\tau_E = \tau_H$$

$$\delta_E = \frac{\sigma_H}{\langle X_p \rangle}$$

GOAL: $\tau_H = \tau_E = 2$ hours

DESIGN CONSTRAINTS: $\varepsilon = \varepsilon_H = \varepsilon_V \propto (\tau_H \ldots)^{1/2 \cdot 5}$

$$\delta_E \propto (\tau_H \ldots)^{1/5}$$

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LATTICE ALTERNATIVES

Lattice	L_{HC} (m)	$<X_p>$ (m)	$X_{p\ max}$ (m)	$<\beta>$ (m)	β_{max} (m)	γ_{tr}	$\int_{HC} B\ dl$ (Txm)	$\int_{quad} G\ dl$ (T)
9/90°	19.8	2.05	2.77	39.5	67.5	19	2x4.94x4.36	61.2
9/105°	19.8	1.63	2.27	40.9	73.4	21	2x4.94x4.36	68.7
9/120°	19.8	1.37	1.96	45.6	85.2	23	2x4.94x4.36	74.9
12/90°	14.8	1.15	1.56	29.7	50.6	25	2x3.3 x4.9	81.6
15/120°	11.9	0.49	0.70	27.4	51.1	38	3.3 x7.83	124.9

NO CELLS/SEXTANT= 9 PHI/CELL= 90.0 SIGMAL= 1.0

L/HCELL= 19.76 HL= 41.42 GL= 54.87
XP= 2.05 XPMAX= 2.77 BETA= 39.53 BETAMAX= 67.49 BETA*V= 3.0 BETA*H= 3.0
GAMMATR= 10.3

GAMMA= 100.0 A= 197.0 Z= 79.0
HB= 57.0

ALPHA	PART/BUNCH	EMITTANCE/PI	DELTA E RMS	EV.SEC/BUNCH	LUMINOSITY	NU SHIFT	DIAM RMS	BEAM SIZE	Z/N
0.	1.00E+09	1.35E-05	4.61E-04	2.72E+00	5.25E+26	1.72E-03	5.00E-01	1.23E-02	3.4E+02
0.	2.00E+09	1.78E-05	5.29E-04	3.12E+00	1.59E+27	2.60E-03	5.00E-01	1.41E-02	2.2E+02
0.	3.00E+09	2.10E-05	5.74E-04	3.38E+00	3.04E+27	3.32E-03	5.00E-01	1.53E-02	1.8E+02
0.	4.00E+09	2.35E-05	6.08E-04	3.59E+00	4.82E+27	3.94E-03	5.00E-01	1.62E-02	1.5E+02
0.	5.00E+09	2.57E-05	6.35E-04	3.75E+00	6.89E+27	4.51E-03	5.00E-01	1.70E-02	1.3E+02
0.	6.00E+09	2.77E-05	6.59E-04	3.89E+00	9.22E+27	5.03E-03	5.00E-01	1.76E-02	1.2E+02
0.	7.00E+09	2.94E-05	6.80E-04	4.01E+00	1.18E+28	5.52E-03	5.00E-01	1.82E-02	1.1E+02
0.	8.00E+09	3.11E-05	6.98E-04	4.12E+00	1.46E+28	5.98E-03	5.00E-01	1.86E-02	9.7E+01
0.	9.00E+09	3.26E-05	7.15E-04	4.22E+00	1.76E+28	6.42E-03	5.00E-01	1.91E-02	9.1E+01
2.00E-03	1.00E+09	1.35E-05	4.61E-04	2.72E+00	1.[illegible]2E+26	6.90E-04	1.71E-01	1.23E-02	3.4E+02
2.00E-03	2.00E+09	1.78E-05	5.29E-04	3.12E+00	4.55E+26	1.16E-03	1.87E-01	1.41E-02	2.2E+02
2.00E-03	3.00E+09	2.10E-05	5.74E-04	3.38E+00	9.38E+26	1.56E-03	1.97E-01	1.53E-02	1.8E+02
2.00E-03	4.00E+09	2.35E-05	6.08E-04	3.59E+00	1.56E+27	1.93E-03	2.03E-01	1.62E-02	1.5E+02
2.00E-03	5.00E+09	2.57E-05	6.35E-04	3.75E+00	2.33E+27	2.28E-03	2.09E-01	1.70E-02	1.3E+02
2.00E-03	6.00E+09	2.77E-05	6.59E-04	3.89E+00	3.22E+27	2.60E-03	2.13E-01	1.76E-02	1.2E+02
2.00E-03	7.00E+09	2.94E-05	6.80E-04	4.01E+00	4.23E+27	2.91E-03	2.17E-01	1.82E-02	1.1E+02
2.00E-03	8.00E+09	3.11E-05	6.98E-04	4.12E+00	5.36E+27	3.21E-03	2.20E-01	1.86E-02	9.7E+01
2.00E-03	9.00E+09	3.26E-05	7.15E-04	4.22E+00	6.60E+27	3.49E-03	2.23E-01	1.91E-02	9.1E+01
1.00E-02	1.00E+09	1.35E-05	4.61E-04	2.72E+00	2.72E+25	1.70E-04	4.71E-02	1.23E-02	3.4E+02
1.00E-02	2.00E+09	1.78E-05	5.29E-04	3.12E+00	9.48E+25	2.93E-04	5.34E-02	1.41E-02	2.2E+02
1.00E-02	3.00E+09	2.10E-05	5.74E-04	3.38E+00	1.97E+26	4.03E-04	5.73E-02	1.53E-02	1.8E+02
1.00E-02	4.00E+09	2.35E-05	6.08E-04	3.59E+00	3.30E+26	5.05E-04	6.03E-02	1.62E-02	1.5E+02
1.00E-02	5.00E+09	2.57E-05	6.35E-04	3.75E+00	4.93E+26	6.02E-04	6.27E-02	1.70E-02	1.3E+02
1.00E-02	6.00E+09	2.77E-05	6.59E-04	3.89E+00	6.85E+26	6.95E-04	6.48E-02	1.76E-02	1.2E+02
1.00E-02	7.00E+09	2.94E-05	6.80E-04	4.01E+00	9.03E+26	7.85E-04	6.65E-02	1.82E-02	1.1E+02
1.00E-02	8.00E+09	3.11E-05	6.98E-04	4.12E+00	1.15E+27	8.71E-04	6.81E-02	1.86E-02	9.7E+01
1.00E-02	9.00E+09	3.26E-05	7.15E-04	4.22E+00	1.42E+27	9.55E-04	6.95E-02	1.91E-02	9.1E+01

RHIC PERFORMANCE ESTIMATES - Au

$N_B = 1 \times 10^9$
$\beta_V^* = \beta_H^* = 3$ m
$\alpha = 0$
$\sigma_L = 1$ m
$h_B = 57$

Lattice	ε/π (m)	δ_E	H•size (cm)	L ($cm^{-2} sec^{-1}$)	$\Delta\nu_{BB}$
	$\times 10^{-6}$	$\times 10^{-3}$		$\times 10^{26}$	$\times 10^{-3}$
		$\gamma = 100$			
9/90°	13.5	0.46	1.23	5.25	1.72
9/105°	11.0	0.53	1.16	6.44	2.11
9/120°	9.0	0.60	1.13	7.91	2.59
12/90°	10.1	0.61	0.92	7.00	2.29
15/120°	5.4	1.0	0.68	13.2	4.32
		$\gamma = 10$ (to be verified)			
9/90°	21.4	1.83	4.90	0.33	1.08
9/105°	17.5	2.12	4.62	0.41	1.33
9/120°	14.2	2.41	4.51	0.50	1.63
12/90°	16.1	2.44	3.67	0.44	1.44
15/120°	8.5	4.01	2.71	0.83	2.72

RHIC PERFORMANCE POTENTIAL - Au

$\gamma = 100$; $\Delta\nu_{BB} = 3 \times 10^{-3}$
$\beta^*_V = 2$ $\beta^*_H = 4.5$ m
$\sigma_L = 1$ m $h_B = 57$

Lattice	α	N_B	ε/π (m)	δ_E	L ($cm^{-2}sec^{-1}$)	Diamond (cm rms)
	$\times 10^{-3}$	$\times 10^{9}$	$\times 10^{-6}$	$\times 10^{-3}$	$\times 10^{27}$	
9/90°	0	3.7	22.8	0.60	4.26	50
	2	9.3	33.0	0.72	8.28	24.9
9/105°	0	2.6	16.1	0.65	2.97	50
	2	7.7	24.9	0.80	6.70	23.2
9/120°	0	1.9	11.6	0.69	2.21	50
	2	6.4	18.8	0.88	5.43	21.5
12/90°	0	2.3	14.1	0.73	2.65	50
	2	7.1	22.2	0.91	6.08	22.5
15/120°	0	0.8	4.9	0.96	0.92	50
	2	4.2	9.6	1.34	3.39	17.4

CONCLUSIONS

- ACCURACY MAY BE QUESTIONABLE BUT TREND IS CORRECT

- "WEAK" FOCUSSING - LARGE APERTURE HIGHEST LUMINOSITY POTENTIAL REQUIRES BETTER INJECTOR

- STUDY & COST IN MORE DETAIL 2 OPTIMUM-COMPROMISE LATTICES

  ($\gamma_{tr} \leq 30$)

  9/120° CBA MAGNETS

  12/90° NEW MAGNET 3.3T
  ~3 in. coil id. (bent)
  ~3½ in. coil id. (straight)

- INJECTOR DESIGN

  $N_B = 2 \times 10^9$ (Au)

  BOOSTER ENERGY ?