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?-Splitting Due to Random Skew Quadrupole Fields

G. Parzen

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Collider Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

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Random a_1 and ν-splitting

Random a_1 in magnets causes coupling and ν-splitting. ν-splitting is important beause ν-values are required to lie between .8000 and .8333 (the box beteen the 4/5 and 5/6 resonances which is 33×10^{-3} wide) (see Fig 1B)

The coupling introduces 2 modes of oscillation with the ν-values ν_1 and ν_2. Both ν-values are present in the x and y oscillations.

ν_1 and ν_2 are given by

$$\nu_1 = \nu_x + |\Delta\nu| ,$$

$$\nu_2 = \nu_y - |\Delta\nu| ,$$

$$\Delta\nu = \frac{1}{4\pi}\int ds\, (\beta_x \beta_y)^{1/2}\, \frac{a_1}{\rho}\, e^{i(\psi_x - \psi_y)} ,$$

provided $\nu_x \simeq \nu_y$, and correct to first order in a_1; and $|\Delta\nu| >> |\nu_x - \nu_y|$.

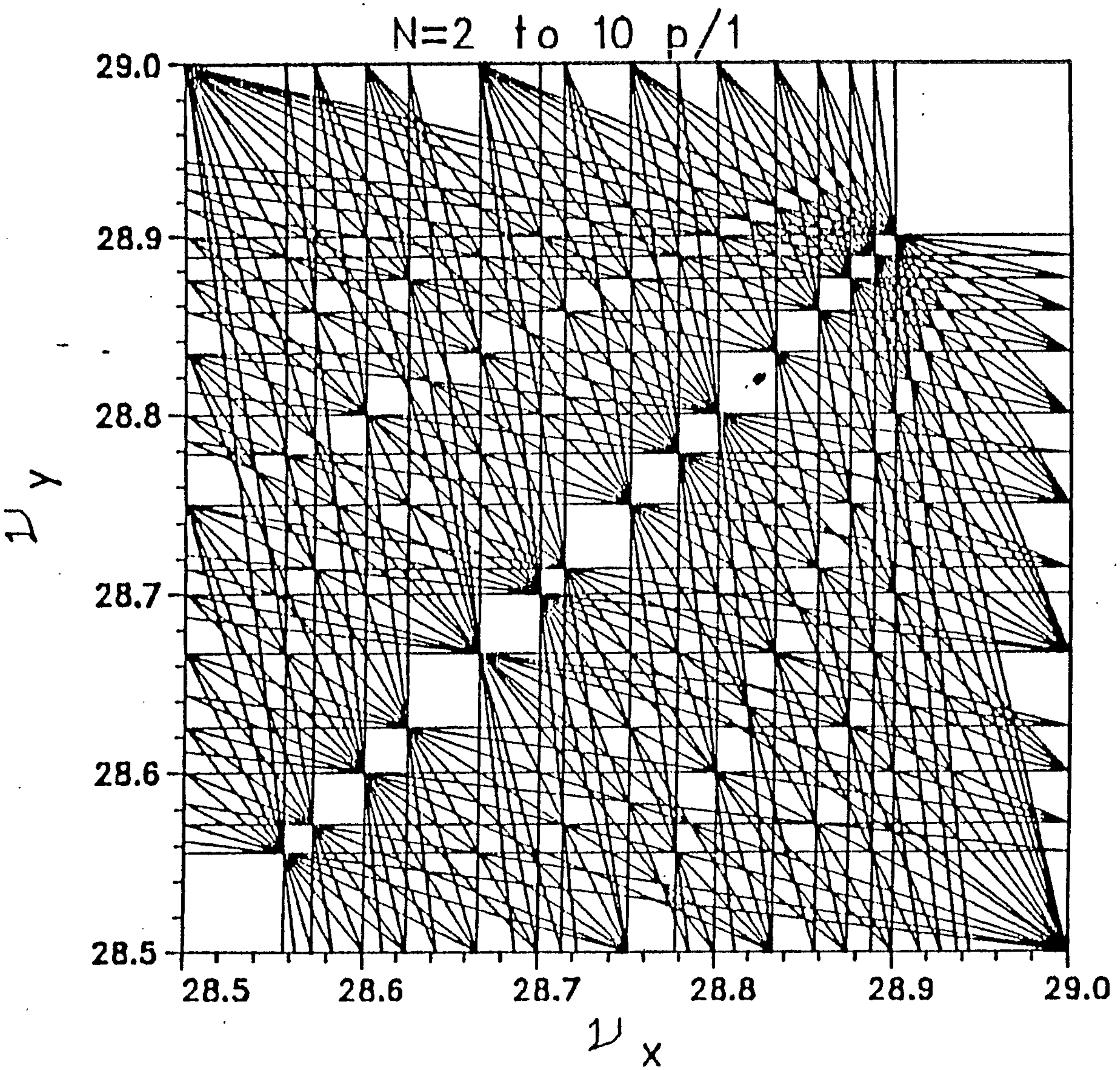
For $a_1 = 17 \times 10^{-5}$ /cm in dipoles rms

$a_1 = 40 \times 10^{-5}$ /cm in quads rms ($\Delta\theta = 1$ mr rotation error)

tracking runs for 10 different set of random errors gives

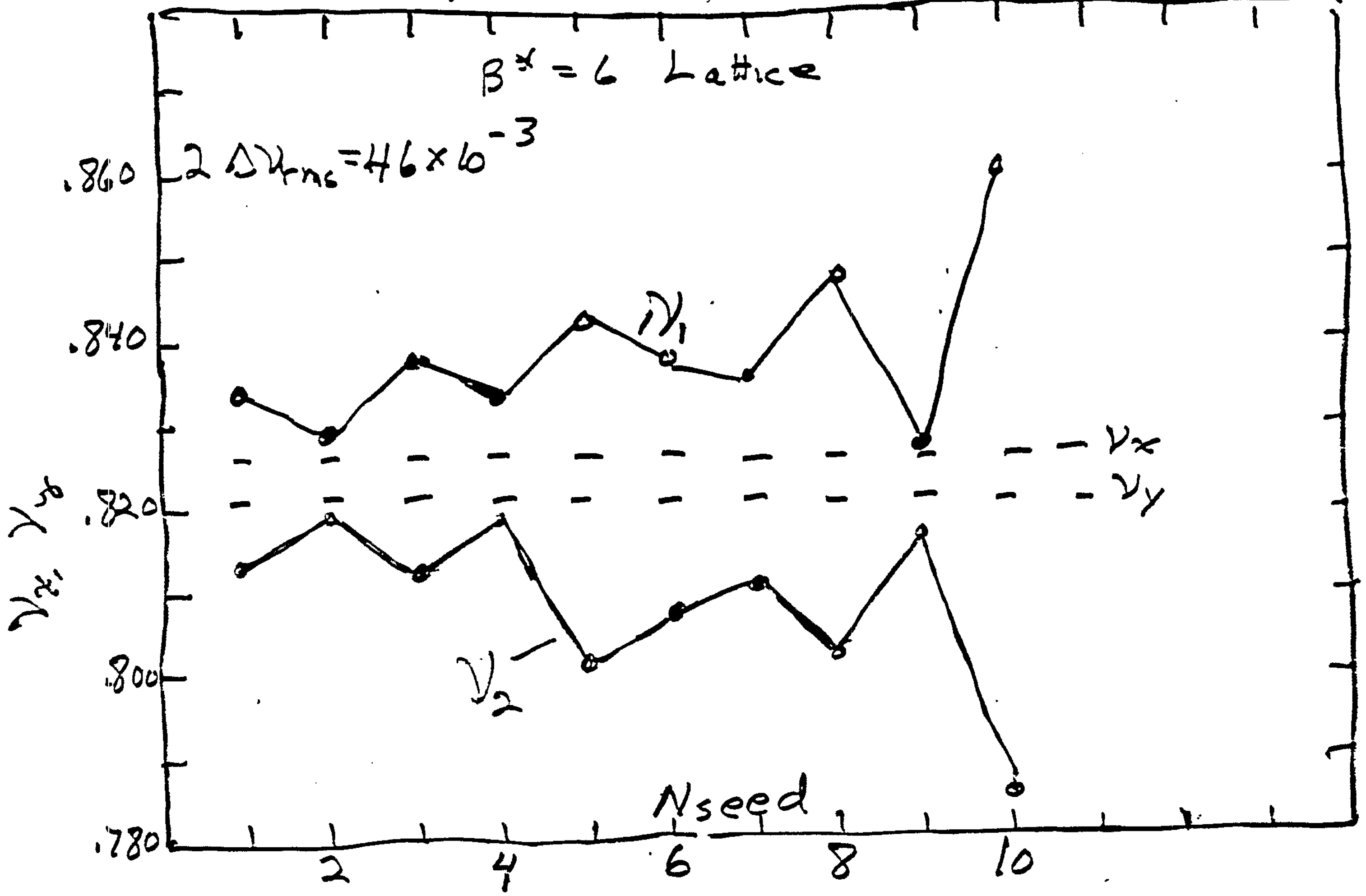
$$(\Delta\nu_{splitting})_{max} = (\nu_1 - \nu_2)_{max} = 97 \times 10^{-3} \qquad \beta^* = 6$$

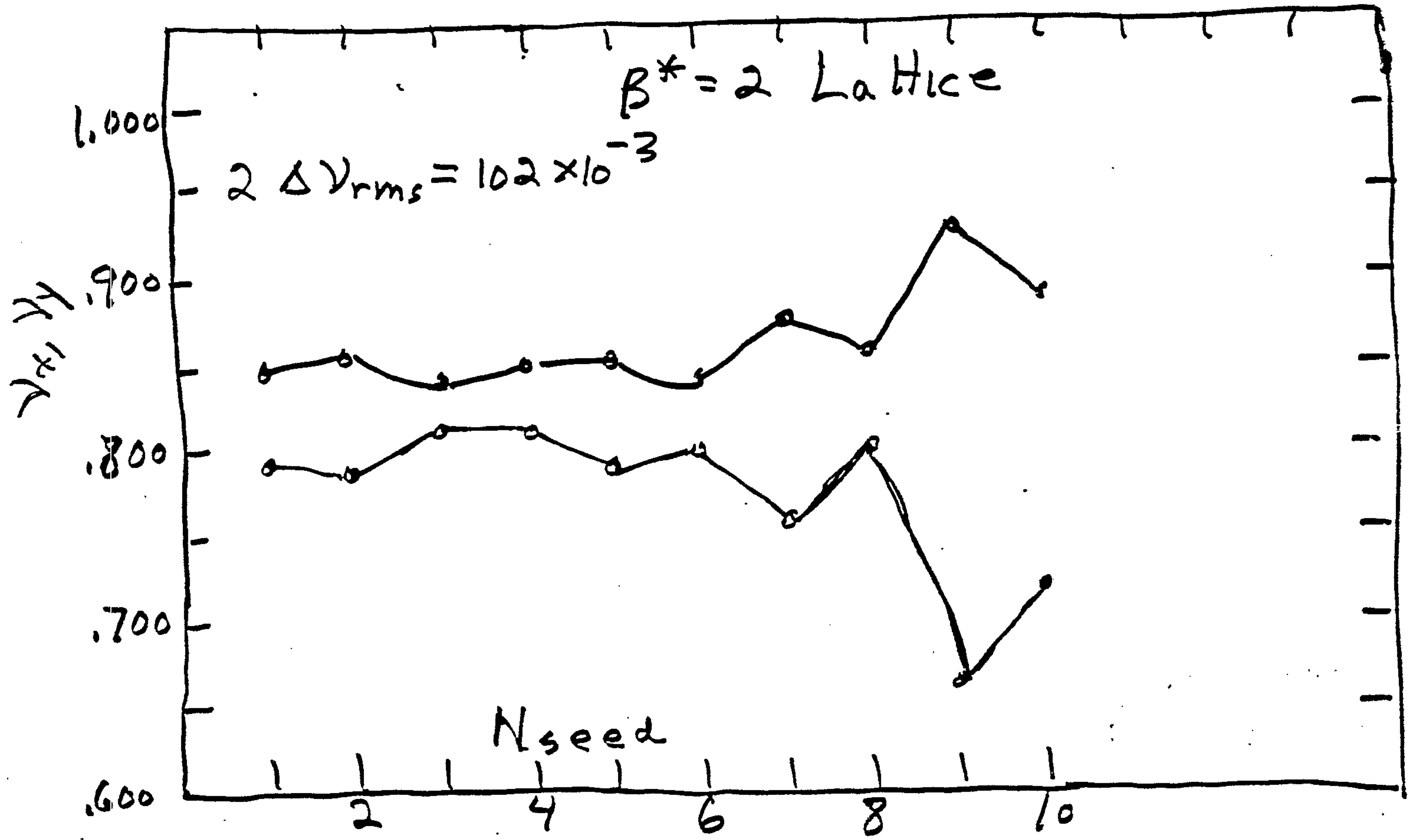
$$= 260 \times 10^{-3} \qquad \beta^* = 2$$



N=2 to 10 p/1
29.0
28.9
28.8
28.7
28.6
28.5
28.5
28.6
28.7
28.8
28.9
29.0
ν y
ν x

ν-splitting for 10 different Sets of Random a_i





Magnet sources of ν-splitting

$2 \cdot \Delta\nu_{rms}$

	$\beta^* = 6$	$\beta^* = 2$
B	.028	.028
QF	.006	.006
QD	.006	.006
Q3I	.008	.025
Q2I	.018	.052
Q1I	.009	.025
Q1O	.009	.025
Q2O	.018	.052
Q3O	.008	.025
Total $2\Delta\nu_{rms}$	.046	.102
$\Delta\nu_{splitting\ max}$	97×10^{-3}	260×10^{-3}

alligning Q2I, Q2O more accurately than $\Delta\theta = 1$ mr rms would help.

Correction of ν-splitting

ν-splitting determined by $\Delta\nu$

$$\Delta\nu = \frac{1}{4\pi}\int ds\,(\beta_x \beta_y)^{1/2}\,\frac{a_1}{\rho}\,e^{i(\psi_x-\psi_y)}$$

At the quads, QD and QF in the arcs,

$\psi_x - \psi_y \simeq$ constant or $\psi_x - \psi_y \simeq 0$

$$\Delta(\psi_x - \psi_y) = \int ds\left(\frac{1}{\beta_x} - \frac{1}{\beta_y}\right) \cong 0 \quad \text{between quads.}$$

If the quadrupoles were the only source of a_1, then $\Delta\nu$ is real, and a 1-power supply correction system to make $\langle a_1\rangle = 0$ would be sufficient.

For the dipoles in the arcs

$$\psi_x - \psi_y \sim \pm .03(2\pi) \simeq \pm .2 \text{ rad}.$$

For Q2 when $\beta^* = 2$

$$\psi_x - \psi_y \simeq \pm .18\,(2\pi) \simeq \pm 1 \text{ rad}$$

$\Delta\nu$ is complex with an appreciable imaginary part. 2 family power supply correction system is required.

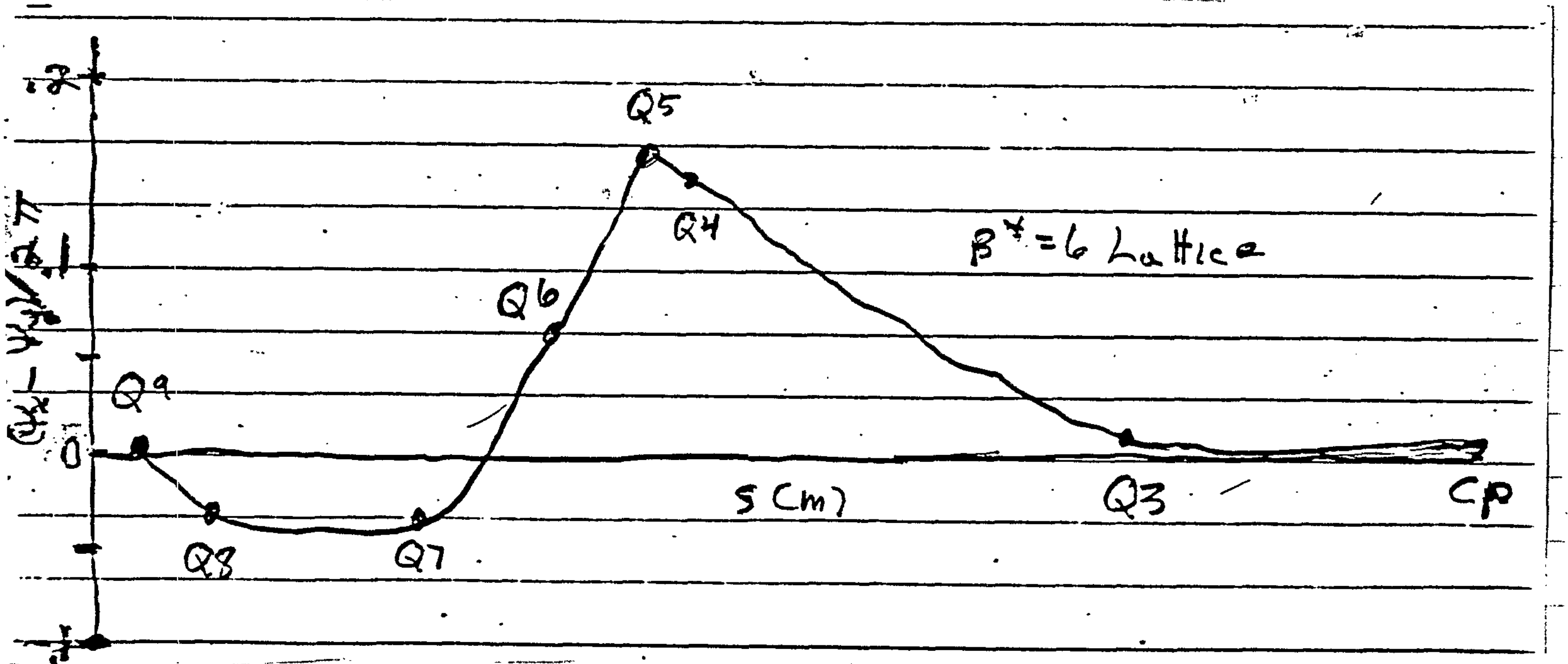
Where to put a_1 Correctors

At Q2, Q3 where β_x, β_y are large

But, two a_1 families need to have different values of $\psi_x - \psi_y$.

$\beta^* = 6$ Lattice

Near Q2, Q3, $\psi_x - \psi_y \simeq 5\times10^{-4}(2\pi) \simeq .003$
only 1 family can be near Q2, Q3



Near Q5, $\psi_x - \psi_y \simeq \pm .16(2\pi)$
Second family can be near Q5.

a_1 Correctors Location (Continued)

$\beta^* = 2$ Lattice

$\psi_x - \psi_y \simeq \pm .18\,(2\pi)$ near Q2
alternates between crossing points

1 family near Q2 at inner arc to outer arc crossing points.

Second family near Q2 at outer arc to inner arc crossing points.

Coupling Correction – $\beta^* = 6$ lattice

N_{seed}	Un Corrected $\Delta\nu/10^{-2}$	ν_1		ν_2	$\lvert\nu_1-\nu_2\rvert/10^{-3}$	Corrected a1L(1)	a1L(2)	ν_1	ν_2	$\lvert\nu_1-\nu_2\rvert/10^{-3}$
1	-1.1	-.03	.834	.813	16	.004	.085	.826	.821	5
2	-.58	.04	.829	.819	5	-.028	.047	.826	.821	5
3	1.50	.41	.838	.813	20	-.213	-.10	.829	.821	8
4	-.51	-.26	.833	.819	9	.144	.029	.829	.822	7
5	-2.22	.41	.843	.801	37	-.262	.192	.822	.817	5
6	1.24	-.24	.838	.808	25	.152	-.108	.825	.819	6
7	1.39	-.15	.836	.811	20	.104	-.116	.826	.820	6
8	-2.6	-.21	.848	.802	41	.083	.194	.828	.819	9
9	.45	.35	.827	.817	5	-.193	-.021	.824	.819	5
10	3.91	-.33	.861	.815	70	.242	-.321	.821	.814	7

$$a1L = \int ds\, a_1 = A1L / B_0 , \quad A1L = \int ds\, B_0 a_1$$

Maximum Correction, $A1L = .32 \times (97/70) \times 34.5$ KG
$= 15$ KG

ν-splitting, $|\nu_1 - \nu_2| \leq 70 \times 10^{-3}$

Corrected to $|\nu_1 - \nu_2| \leq 9 \times 10^{-3}$

Loss of 4×10^{-3} of box width $= 33 \times 10^{-3}$

ν-values $(a_1 = 0)$ .826, .821 $|\nu_1 - \nu_2| = 5 \times 10^{-3}$

Correctors at Q2I, Q5I of inner arc to outer arc in sections

Coupling Correction $\beta^* = 2$ lattice

	Uncorrected				Corrected				
N_{seed}	$\Delta\nu/10^{-2}$	ν_1	ν_2	$\lvert\nu_1-\nu_2\rvert/10^{-3}$	$a_{1L}(1)$	$a_{1L}(2)$	ν_1	ν_2	$\lvert\nu_1-\nu_2\rvert/10^{-3}$
1	-1.29, 2.21	.850	.796	49	.071	.006	.828	.820	8
2	-.19, 3.25	.860	.789	66	.053	-.043	.828	.821	7
3	1.19, .59	.840	.819	16	-.027	-.046	.833	.827	6
4	-1.16, .73	.851	.818	28	.046	.025	.842	.828	14
5	-2.52, 2.19	.856	.791	60	.108	.045	.826	.819	7
6	1.72, .56	.842	.802	35	-.044	-.062	.825	.816	9
7	1.46, -5.16	.878	.759	114	-.120	.033	.825	.819	6
8	-2.78, -.48	.857	.805	47	.076	.093	.836	.827	9
9	.71, -11.3	.930	.666	259	-.187	.149	.827	.820	7
10	1.09, -5.14	.890	.770	115	-.108	.044	.839	.828	11

Maximum Correction, $A1L = (.19)(34.5) = 6.6$ KG

ν-splitting, $\lvert\nu_1 - \nu_2\rvert \le 259 \times 10^{-3}$

Corrected to $\lvert\nu_1 - \nu_2\rvert \le 14 \times 10^{-3}$

Loss of 9×10^{-3} of the box width 33×10^{-3}

ν-Values ($a_1 = 0$) .826, .821 $\lvert\nu_1 - \nu_2\rvert = 5 \times 10^{-3}$

Correctors at Q2I of inner arc to outer arc insertion

Q2O of outer arc to inner arc insertion

Using Q2, Q5 correctors increases A1L required by factor 12.

Suggested Conclusions

1) 3 families of a_1 correctors
with 3 seperate power supplies.

2) family one, before Q2I in inner arc
to Outer arc insertions

3) family two before Q20 in outer arc
to Inner arc insertions

4) family three before Q50 in outer arc
to inner arc insertions (to avoid
Q5I at 6 oclock (J. Claus))

5) Required strenth A1L = 15 KG

$$A1L = \int ds\, B_0\, a_1$$