

Emittance and 4 Dimensional Beam Surfaces in RHIC

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The particle transverse motion is described by 4 coordinates x, x', y, y' . A quantitative study of the behavior of the beam requires a description of the 4-dimensional space occupied by the beam, and how this 4-dimensional space changes with time.

The ~~existing~~ 4-dimensional space occupied by the beam is bounded by a surface in 4-dimensional space.

This note reviews the various 4-dimensional surfaces ~~that~~ arise in studying ~~the~~ particle motion in RHIC. These surfaces include

- 1) The injected beam surface
- 2) The beam surface after 10 hrs
- 3) The 6 σ beam surface (Safety Surface)
- 4) The stability surface
- 5) The beam abort surface

Surface Injected Beam

The usual Statement is that the ~~the~~ emittance of the injected beam is $\epsilon_x = \epsilon_y = 10$ (normalized) for heavy ions in RHIC.

Tracking studies requires a more precise statement, Tracking requires that a 4-dimensional surface be specified in x, x', y, y' space that contains ~~the~~ the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\epsilon_T = \epsilon_x(x, x') + \epsilon_y(y, y') = C,$$

where C is a constant chosen so that this surface contains ~~the~~ the beam. One reason for using this expression is that ϵ_T is ^{very} roughly a constant of the motion.

The 95% Surface for the Injected Beam

This surface contains 95% of the particles. It is given by (to be shown below)

$$\epsilon_x + \epsilon_y = 16 \quad (\text{normalized})$$

under the following assumptions

1) $\epsilon_t = \epsilon_x + \epsilon_y$ is approximately a constant of the motion.

2) The beam distribution is gaussian of the form

$$\rho(x, x', y, y') \approx \exp(-(\epsilon_x(x, x') + \epsilon_y(y, y'))/\bar{\epsilon})$$

$$\epsilon_x(x, x') = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$$

$$\epsilon_y(y, y') = \gamma_y y^2 + 2\alpha_y y y' + \beta_y y'^2$$

4.

I assume that the statement $\epsilon_x = \epsilon_y = 10$ means that for the projection of the particles on the x, x' plane, ~~the~~ 95% of the particles have an ϵ_x which is smaller than $\epsilon_x = 10$, and a similar statement applies to the y, y' plane.

I assume that the distribution $p(x, x', y, y')$ is gaussian with the form

$$p(x, x', y, y') \sim \exp\left(-(\epsilon_x(x, x') + \epsilon_y(y, y'))/\bar{\epsilon}\right) \quad (1)$$

The projection on the xx' plane has the distribution

$$p(x, x') = \int dy dy' p(x, x', y, y') \sim \exp\left(-\epsilon_x(x, x')/\bar{\epsilon}\right) \quad (2)$$

In order for 95% of the particle to ~~have~~ have an ϵ_x which is smaller than $\epsilon_x = 10$ then

$$\bar{\epsilon} = 10/3 \quad (3)$$

The fraction of the particles that have a total emittance, $\epsilon_T = \epsilon_x + \epsilon_y$, which is smaller than $\bar{\epsilon}_T$ is given by

$$F(\epsilon_T) = 1 - \exp(-\epsilon_T/\bar{\epsilon}) (1 + \epsilon_T/\bar{\epsilon}). \quad (4)$$

This may be derived from Eq (1) for $\rho(x, x', y, y')$. The choice ϵ_T that includes 95% of the particles is $\epsilon_T = 5 \bar{\epsilon}$ (the actual answer is closer to $4.8 \bar{\epsilon}$).

Thus the ϵ_T that contains 95% of the particles is

$$\epsilon_T = 4.8 \bar{\epsilon}$$

$$\epsilon_T = 4.8 \left(\frac{10}{3} \right)$$

$$\epsilon_T = 16.60$$

~~For $\epsilon_x = \epsilon_y$ and $x' = y' = 0$ then $x = y$ at a QF is on this surface~~

Application to the \sqrt{t} Jump Lattice

Tracking studies show that the stability surface with the tuning quads present is

$$\epsilon_x + \epsilon_y = 2 \quad (\text{unnormalized})$$

at $\Delta p/p = 0.005$

$$\epsilon_x + \epsilon_y = 50 \quad (\text{normalized}, \gamma = 25)$$

Compare this with the 95% surface of the injected beam

$$\epsilon_x + \epsilon_y = 16$$

Beam Emittance after 10 hrs

Intra Beam Scattering Results for Au.

Old Hybrid Result

$$\epsilon_{x,95} = 33 = \epsilon_{y,95} \quad \text{at } \delta = 30$$

after 10 hrs (normalized)

What is the $\epsilon_{t,95}$ surface?

~~$$\bar{\epsilon} = \bar{\epsilon}_x = \bar{\epsilon}_y = 11$$~~

$$\epsilon_{t,95} = 4.8 \bar{\epsilon} = 53$$

New Results

~~Result depends on coupling assumed.~~

~~No coupling~~

$$\epsilon_{x,95} = 33, \epsilon_{y,95} = 21 \quad \text{at } \delta = 30.$$

$$\bar{\epsilon}_x = 11, \bar{\epsilon}_y = 7, \bar{\epsilon}_t = 18$$

Complete Coupling

$$\bar{\epsilon} = \bar{\epsilon}_x = \bar{\epsilon}_y = 9$$

$$\bar{\epsilon}_t = 1.8, \quad \text{same as no coupling result,}$$

$$\epsilon_{t,95} = 2.4 \bar{\epsilon}_t = 43$$

Beam Surface for 95% of beam after 10 hours is

$$\epsilon_t = \epsilon_x + \epsilon_y = 43$$

Compare this with

$$\epsilon_t = \epsilon_x + \epsilon_y = 16, \quad 95\% \text{ Beam Surface at injection.}$$

6σ Beam Surface

7

Stability is required inside the
6σ Beam Surface

Point on surface is $x=6\sigma$ $y=y'=x'=0$

$$\epsilon_T = \epsilon_x + \epsilon_y = \frac{(6\sigma)^2}{\beta_x} = \frac{36\sigma_x^2}{\beta_x}$$

For complete coupling, $\sigma_x = 2.7 \text{ mm}$

and $\bar{\epsilon}_t = 4\sigma_x^2/\beta_x = 18$ at $\delta = 30$.

$$\epsilon_t = \epsilon_x + \epsilon_y = 162 \quad \begin{array}{l} \text{6}\sigma \text{ Surface} \\ \text{(normalized)} \end{array}$$

Compare this with

$\epsilon_x + \epsilon_y = 43$, 95% Beam surface after 10 hours

$\epsilon_x + \epsilon_y = 16$, 95% Beam surface at injection

Stability Surface (Dynamic Aperture Surface)

started
 Particles outside stability surface
 are unstable

Tracking gives ~~the~~ stability limit
 of ^{initial} $x = 17 \text{ mm}$ when $\epsilon_x = \epsilon_y$ and $x' = y' = 0$
 This is one point on the stability surface -

Assuming stability surface is given by

$$\epsilon_x + \epsilon_y = \text{Constant}$$

then $\epsilon_t = \epsilon_x + \epsilon_y = 2 (17 \times 10^{-3})^2 / 50$

$$\epsilon_x + \epsilon_y = 11.6 \quad \text{Stability Surface (unnormalized)}$$

at $\gamma = 30$,

$$\epsilon_x + \epsilon_y = 350 \quad \text{Stability Surface (normalized)}$$

Compare this with 6σ surface

$$\epsilon_x + \epsilon_y = 160 \quad \text{normalized, } \gamma = 30$$

$$\epsilon_x + \epsilon_y = 5.3 \quad \text{unnormalized, } \gamma = 30$$

Beam Abort Surface

Particles with $\epsilon_y \leq 6$ (unnormalized)
will be ejected.

Assuming complete coupling, particles
with $\epsilon_t \leq 6$ will be ejected.

What fraction of beam has $\epsilon_t > 6$?

after 10 hours, at $\delta = 30$,

$$\epsilon_{t, 95} = 43 \quad \bar{\epsilon}_t = 18, \quad \bar{\epsilon} = 9,$$

and $\epsilon_t > 6$ (unnormalized) $\rightarrow \epsilon_t > 180$ normalized

Using Eq (4), 4×10^{-8} of the particles
have $\epsilon_t > 180$.

Note

$$F(\epsilon_t) = 1 - \exp(-\epsilon_t/\bar{\epsilon}) (1 + \epsilon_t/\bar{\epsilon})$$

$$\text{and } \epsilon_t/\bar{\epsilon} = 20$$