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Emittance and 4 Dimensional Beam Surfaces in RHIC

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The particle transverse motion is described by 4 coordinates x, x', y, y'. A quantitative study of the behavior of the beam requires a description of the 4-dimensional space occupied by the beam, and how this 4-dimensional space changes with time.

The 4-dimensional space occupied by the beam is bounded by a surface in 4-dimensional space. This note reviews the various 4-dimensional surfaces that arise in studying particle motion in RHIC. These surfaces include

1) The injected beam surface
2) The beam surface after 10 hrs
3) The 6σ beam surface (Safety Surface)
4) The stability surface
5) The beam abort surface

Injected Beam Surface

The usual statement is that the emittance of the injected beam is $\varepsilon_x = \varepsilon_y = 10$ (normalized) for heavy ions in RHIC.

Tracking studies require a more precise statement. Tracking requires that a 4-dimensional surface be specified in x, x', y, y' space that contains the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\varepsilon_T = \varepsilon_x(x, x') + \varepsilon_y(y, y') = C,$$

where C is a constant chosen so that this surface contains the beam. One reason for using this expression is that ε_T is very roughly a constant of the motion.

The 95% Surface for the Injected Beam

This surface contains 95% of the particles. It is given by (to be shown below)

$$\mathcal{E}_x + \mathcal{E}_y = 16 \quad \text{(normalized)}$$

under the following assumptions

1) $\mathcal{E}_t = \mathcal{E}_x + \mathcal{E}_y$ is approximately a constant of the motion.

2) The beam distribution is gaussian of the form

$$\rho(x, x', y, y') \simeq \exp\left(-\left(\mathcal{E}_x(x, x') + \mathcal{E}_y(y, y')\right)/\bar{\mathcal{E}}\right)$$

$$\mathcal{E}_x(x, x') = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$$

$$\mathcal{E}_y(y, y') = \gamma_y y^2 + 2\alpha_y y y' + \beta_y y'^2$$

I assume that the statement $\varepsilon_x = \varepsilon_y = 10$ means that for the projection of the particles on the x, x' plane, 95% of the particles have an ε_x which is smaller than $\varepsilon_x = 10$, and a similar statement applys to the y, y' plane.

I assume that the distribution $\rho(x, x', y, y')$ is gaussian with the form

$$\rho(x, x', y, y') \sim \exp\left(-\left(\varepsilon_x(x, x') + \varepsilon_y(y, y')\right)/\bar{\varepsilon}\right) \quad (1)$$

The projection on the $x x'$ plane has the distribation

$$\rho(x, x') = \int dy\, dy'\, \rho(x, x', y, y')$$

$$\sim \exp\left(-\varepsilon_x(x, x^3)/\bar{\varepsilon}\right). \quad (2)$$

In order for 95% of the particle to have an ε_x which is smaller then $\varepsilon_x = 10$ then

$$\bar{\varepsilon} = 10/3 \quad (3)$$

The fraction of the particles that have a total emittance, $\varepsilon_T = \varepsilon_x + \varepsilon_y$, which is smaller than ε_T is given by

$$F(\varepsilon_T) = 1 - \exp(-\varepsilon_T/\bar{\varepsilon})(1 + \varepsilon_T/\bar{\varepsilon}). \quad (4)$$

This may be derived from Eq (1) for $\rho(x, x', y, y')$. The choice ε_T that includes 95% of the particles is $\varepsilon_T = 5\bar{\varepsilon}$ (the actual answer is closer to $4.8\bar{\varepsilon}$).

Thus the ε_T that contains 95% of the particles is

$$\varepsilon_T = 4.8\bar{\varepsilon}$$
$$\varepsilon_T = 4.8\left(\frac{10}{3}\right)$$
$$\varepsilon_T = 16.60$$

~~For $\varepsilon_x = \varepsilon_y$, and $x' = 0$, then $x = x_{max}$ at a Q.F. is on this surface~~

Application to the γ_t Jump Lattice

Tracking studies show that the stability surface with the tuning quads present is

$$\epsilon_x + \epsilon_y = 2 \quad \text{(unnormalized)}$$

at $\Delta p/p = .005$

$$\epsilon_x + \epsilon_y = 50 \quad (\text{normalized}, \gamma = 25)$$

Compare this with the 95% surface of the injected beam

$$\epsilon_x + \epsilon_y = 16$$

Beam Emittance after 10 hrs

Intra Beam Scattering Results for Au.

Old Hybrid Result

$$\varepsilon_{x,95} = 33 = \varepsilon_{y,95} \quad \text{at } \gamma = 30$$

after 10 hrs (normalized)

What is the $\varepsilon_{t,95}$ surface?

$$\bar{\varepsilon} = \bar{\varepsilon}_x = \bar{\varepsilon}_y = 11$$

$$\varepsilon_{T,95} = 4.8\,\bar{\varepsilon} = 53$$

New Results

~~Result depends on coupling assumed.~~

~~No coupling~~

~~$\varepsilon_{x,95} = 33$, $\varepsilon_{y,95} = 21$ at $\gamma = 30$.~~

~~$\bar{\varepsilon}_x = 11$, $\bar{\varepsilon}_y = 7$, $\bar{\varepsilon}_t = 18$~~

Complete Coupling

$$\bar{\varepsilon}_t = 18, \quad \bar{\varepsilon} = \bar{\varepsilon}_x = \bar{\varepsilon}_y = 9$$ ~~same as no coupling result~~,

$$\varepsilon_{t,95} = 2.4\,\bar{\varepsilon}_t = 43$$

Beam Surface for 95% of beam after 10 hours is

$$\varepsilon_t = \varepsilon_x + \varepsilon_y = 43$$

Compare this with

$\varepsilon_t = \varepsilon_x + \varepsilon_y = 16$, 95% Beam Surface at injection.

6σ Beam Surface

Stability is required inside the 6σ Beam Surface

Point on surface is $x = 6\sigma$ $y = y' = x' = 0$

$$\mathcal{E}_T = \mathcal{E}_x + \mathcal{E}_y = \frac{(6\sigma)^2}{\beta_x} = \frac{36\,\sigma_x^2}{\beta_x}$$

For complete coupling, $\sigma_x = 2.7$ mm
and $\overline{\mathcal{E}_t} = 4\sigma_x^2/\beta_x = 18$ at $\gamma = 30$.

$\mathcal{E}_t = \mathcal{E}_x + \mathcal{E}_y = 162$ 6σ Surface (normalized)

Compare this with

$\mathcal{E}_x + \mathcal{E}_y = 43$, 95% Beam surface after 10 hours

$\mathcal{E}_x + \mathcal{E}_y = 16$, 95% Beam Surface at injection

Stability Surface (Dynamic Aperture Surface)

Particles started outside stability surface are unstable

Tracking gives Stability limit of initial $x = 17$ mm when $\varepsilon_x = \varepsilon_y$ and $x' = y' = 0$
This is one point on the stability surface -

assuming stability surface is given by

$$\varepsilon_x + \varepsilon_y = \text{constant}$$

then $\varepsilon_t = \varepsilon_x + \varepsilon_y = 2\,(17\times10^{-3})^2/50$

$$\varepsilon_x + \varepsilon_y = 11.6$$ Stability Surface (unnormalized)

at $\gamma = 30$,

$$\varepsilon_x + \varepsilon_y = 350$$ Stability Surface (normalized)

Compare this with 6σ surface

$\varepsilon_x + \varepsilon_y = 160$ normalized, $\gamma = 30$

$\varepsilon_x + \varepsilon_y = 5.3$ unnormalized, $\gamma = 30$

Beam Abort Surface

Particles with $\varepsilon_y \le 6$ (unnormalized) will be ejected.

Assuming complete coupling, particles with $\varepsilon_t \le 6$ will be ejected.

What fraction of beam has $\varepsilon_t > 6$?

after 10 hours, at $\gamma = 30$,

$\varepsilon_{t,95} = 43 \quad \bar{\varepsilon}_t = 18, \quad \bar{\varepsilon} = 9,$

and $\varepsilon_t > 6$ (unnormalized) $\rightarrow \varepsilon_t > 180$ normalized

Using Eq (4), 4×10^{-8} of the particles have $\varepsilon_t > 180$.

Note

$$F(\varepsilon_t) = 1 - \exp(-\varepsilon_t/\bar{\varepsilon})(1 + \varepsilon_t/\bar{\varepsilon})$$

and $\varepsilon_t/\bar{\varepsilon} = 20$