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Brookhaven National Laboratory
Upton, NY 11973

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J. Scott Berg[a,1]

[a]*Brookhaven National Laboratory; Building 901A; PO Box 5000; Upton, NY 11973-5000; USA*

Abstract

I estimate FFAG parameters for a muon collider with a 70 mm longitudinal emittance. I do not discuss the lower emittance beam for a Higgs factory. I produce some example designs, giving only parameters relevant to estimating cost and performance. The designs would not track well, but the parameters of a good design will be close to those described. I compare these cost estimates to those for a fast-ramping synchrotron and a recirculating linear accelerator. I conclude that FFAGs do not appear to be cost-effective for the large longitudinal emittance in a high-energy muon collider.

Frequency (MHz)	Gradient (MV/m)	Energy Gain MeV	Cells/cavity
325	20	9.22	3
650	25	5.77	5
975	30	4.61	7
1300	35	4.04	9

Table 1: Assumed RF gradients and maximum energy gain per half cell. Cells per cavity is guessed from 2-cell limit for 201 MHz, 5-cell cavities for ESS 704 MHz [1] and the SPL [2, 3], and 9-cell cavities for ILC [4]. Gradients are consistent with the 704 MHz SPL design [2, 3] and the achieved ILC cavity gradients [5].

1. FFAG Energy Ranges

Based on choosing some approximate waypoints and guessing where RLAs are no longer most efficient, I consider FFAGs for the following ranges

1. 10 GeV to 25 GeV (325 MHz)
2. 25 GeV to 63 GeV (650 MHz)
3. 63 GeV to 173 GeV (650 MHz)
4. 173 GeV to 375 GeV (975 MHz)

The RF frequencies are guesses for the optimal operating frequency, assuming multiples of 325 MHz. Cavities are assumed to have the gradients given in Table 1.

2. Cost Model

I will modify the cost model given in [6]. First, I will not use the f_S function, replacing it with 1. This is based on the observation that it is difficult to take advantage of the lower current density required for a combined-function magnet [7].

Second, I need to include a frequency dependence in my scaling. Cavity surface area is proportional to f^{-2}, where f is the frequency. Peak power costs appear to be proportional to f^{-1}. For a fixed Q, required power is proportional to f^{-1}. Thus, my cost formula becomes

$$V\left(\frac{f_0}{f}\right)^2\left(C_C\frac{G_0}{G}+C_P\frac{G}{G_0}\right) \tag{1}$$

where now $f_0 = 201.25$ MHz.

3. Design Parameters

The FFAGs should transmit a normalized longitudinal emittance of 70 mm. Using the procedure in [8], we find design parameters that will limit the growth of this emittance or the growth in ellipse distortion at 3σ to 3%. The results are given in Table 2.

I can make an approximate scaling of one FFAG design from another. First, start with the relations

$$a = \frac{V}{\omega\Delta T\Delta E} \tag{2}$$

$$\Delta E \approx nV \tag{3}$$

$$V \approx NU \tag{4}$$

$$\Delta T \propto \frac{L}{N}\eta^2 \tag{5}$$

$$\eta = \frac{r-1}{r+1} \tag{6}$$

Here V is the energy gain per turn, ω is 2π times the RF frequency, ΔE is the difference between the minimum and maximum energy, n is the number of turns, N is the number of cells, L is the cell length, U is the energy gain per cell, and r is the ratio of the minimum to the maximum energy. Assuming maximum quad and dipole fields stay about the same,

$$L \propto \sqrt{E/\eta} \tag{7}$$

$$L_Q \propto \sqrt{E\eta} \tag{8}$$

Email address: jsberg@bnl.gov (J. Scott Berg)

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Table 2: Parameters for FFAG designs. Number of turns are an estimate based on scaling laws.

Minimum Energy (GeV)	Maximum Energy (GeV)	Frequency (MHz)	Scaled Emittance	a Emittance	Ellipse	Turns Emittance	Ellipse
10	25	325	3.36×10^{-3}	1.114	0.260	2.5	4.5
25	63	325	1.33×10^{-3}	0.344	0.127	3.5	6.5
25	63	650	2.65×10^{-3}	0.825	0.208	1.5	3.5
63	173	325	0.46×10^{-3}	0.134	0.079	5.5	7.5
63	173	650	0.92×10^{-3}	0.232	0.104	2.5	4.5
63	173	975	1.37×10^{-3}	0.357	0.129	1.5	2.5
63	173	1300	1.83×10^{-3}	0.512	0.156	1.5	2.5
173	375	325	0.25×10^{-3}	0.097	0.067	7.5	8.5
173	375	650	0.50×10^{-3}	0.141	0.082	3.5	5.5
173	375	975	0.75×10^{-3}	0.192	0.095	2.5	3.5
173	375	1300	1.00×10^{-3}	0.252	0.109	1.5	2.5

Table 3: Parameters determining the relative cost and performance of FFAG designs accelerating from 63 to 173 GeV.

RF Frequency (MHz)	Cavities per Cell	Turns	Decay %	Cost (A.U.)
650	1	8.5	5.6	262
650	2	5.3	3.8	350
650	3	3.9	3.1	440
975	1	6.4	4.6	225
975	2	4.0	3.0	283
1300	1	5.0	3.9	219
1300	2	3.1	2.5	262

where L_Q is the quadrupole length and E is the central energy. Simplistically, this tells us that approximately,

$$U \propto G\sqrt{E} \tag{9}$$

where G is the RF gradient. Putting all this together,

$$n \approx n_0 \left(\frac{f}{f_0}\frac{a}{a_0}\frac{G}{G_0}\right)^{-1/2} \left(\frac{\eta}{\eta_0}\right)^{-1/4} \tag{10}$$

where f is the RF frequency and the 0 subscript indicates the parameters for a known design. We have a neutrino factory design accelerating from 12.6 to 25 GeV. It has $a = 0.077$, $n = 11.5$, $f = 200$ MHz, and $G = 17$ MV/m. The results of scaling are given in Table 2.

These results certainly indicate that FFAGs do not look very favorable. This is partly caused by the large longitudinal emittance we are trying to transport and the resulting unfavorable values for a. However, I think my assumptions for how the cell length increases are also partly to blame.

I will therefore produce optimized designs for the more favorable-looking energy ranges: 63–173 GeV and 173–375 GeV.

Table 4: Parameters determining the relative cost and performance of FFAG designs accelerating from 173 to 375 GeV.

RF Frequency (MHz)	Cavities per Cell	Turns	Decay %	Cost (A.U.)
650	1	16.9	5.7	298
650	2	11.2	3.7	359
650	3	8.5	3.0	433
650	4	6.8	2.7	510
975	1	12.6	4.6	263
975	2	8.5	3.0	300
975	3	6.5	2.4	345
1300	1	10.0	3.9	255
1300	2	6.7	2.5	279

I use the the ellipse distortion bound. I try various RF frequencies and numbers of cavities per cell. For the long drift, in addition to the length of the cavity cells, I assume an additional 1.5 RF wavelengths per cavity, plus an additional 1.5 m. Tables 3 and 4 show the results.

Assuming a 70% decay allowance, I estimate a required average RF gradient of 5 MV/m. For a larger allowance of 50%, I estimate 2.5 MV/m. For the 63–173 GeV machine, these gradients correspond to 3.2% and 6.3% decay, respectively. For the 173–375 GeV machine, the decay allowances are 2.5% and 4.8%. The designs given can meet these specifications for either average gradient.

The higher-frequency designs have a cost advantage for both energy ranges and both decay allowances, despite the smaller number of turns achievable for those designs. This primarily arises from the quadratic dependence of the RF cost on frequency. Other costs drop as well at the higher frequencies since the decay requirements can be achieved with shorter cells at higher frequencies.

4. Non-FFAG Design Options

To determine if FFAGs are a good choice for muon acceleration, we must compare them to other design alternatives.

Throughout these calculations, I am assuming 1.3 GHz RF, though it is unclear whether we have sufficient bucket area. The costs are generally dominated by the linear costs, so the optimum is unlikely to change significantly with a lower frequency RF. I will only include linear and RF costs in these calculations: from the FFAG calculations, the magnet costs for the optimum designs made only a modest contribution to the cost.

4.1. Fast Ramping Synchrotron

With a maximum dipole field of 1.5 T, a synchrotron from 63 to 375 GeV must have a circumference of 8733 m with a 60% packing fraction, taking around 7 turns for 5 MV/m and 14 turns for 2.5 MV/m. Using 1.3 GHz RF, and ignoring the need to run off-crest, the RF and linear costs come out to 295 for the 5 MV/m case, and 256 for the 2.5 MV/m case. Magnet costs make a small contribution to the FFAG costs, and presumably the same will hold true for the synchrotron. Thus, this seems to be a better option from the point of view of cost. Pulse times are 0.2 ms for the 5 MV/m case, and 0.4 ms for the 2.5 MV/m case.

Instead consider a hybrid design. Considering only the dipoles,

$$L_C = \frac{\pi}{qB_C}(p_+ + p_-) \qquad L_W = \frac{\pi}{qB_W}(p_+ - p_-) \tag{11}$$

where L_C is the total length of the fixed field (cold) dipoles, B_C is the field in that dipole, the W subscripted values are the corrresponding values for the ramped (warm) dipole, p_- is the injection momentum, and p_+ is the extraction momentum.

The design described in [9, 10] has $B_C = 8$ T, $B_W = 1.8$ T, $p_- = 375\,\text{GeV}/c$, $p_+ = 750\,\text{GeV}/c$, and a circumference of 6294.5 m. The required dipole length is 3656.8 m. Thus, computing the required dipole length, we just need to divide by 0.58 to get the circumference.

Using $B_C = 10$ T and $B_W = 1.5$ T, we end up with circumferences of 1751 m for the 63–173 GeV accelerator, and 3423 m for the 173–375 GeV accelerator. At 5 MV/m, this is 12.5 turns for the 63–173 GeV machine, and 11.5 turns in the 173–375 GeV machine. The RF plus linear costs for the machines are 59 for the 63–173 GeV machine, and 116 for the 173–375 GeV machine. Totalling 175, this easily beats the corresponding non-hybrid synchrotron. The pulsing times become even more challenging, however: for the 63–173 GeV machine, 0.073 ms for 5 MV/m, 0.143 ms for 2.5 MV/m; for the 173–375 GeV machine, 0.13 ms for 5 MV/m, 0.26 ms for 2.5 MV/m. Furthermore, the shorter circumferences will likely require the use of shorter cavities to be able to replace the stored energy, reducing somewhat the packing fraction and cost-effectiveness of these designs.

4.2. Recirculating Linear Accelerator

For a dogbone RLA design, I assume 10 T dipoles with a 70% packing fraction in the arcs, with a 45 degree initial bend out of the dipole. For the 1.3 GHz linac, I assume 5-cell cavities due to the short time for replacing the power (I'm assuming 1.2 MW per cavity, one coupler per cavity), 3-cell gaps between cavities, and a 90% packing fraction on top of that. The resulting cost optimum is 249 for RF plus linear costs, optimal at 7 passes.

A racetrack design is likely to have a number of problems. The two advantages are sharing a tunnel and a smaller footprint in one dimension. However, for the same number of turns, the beamline spacing at the switchyard is significantly denser than for the dogbone case. In addition, the low energy beamlines are needlessly long, resulting in increased decays and magnet costs.

5. Conclusion

FFAGs do not appear to be a cost-effective solution for accelerating muons for a muon collider, at least with a 70 mm longitudinal emittance. A direct comparison to other options is difficult due to the inability to directly compare magnet costs, but in the FFAG designs, and presumably the others, the magnet costs are not a large fraction of the total cost.

The most cost-effective option for accelerating from 63 to 375 GeV appears to be a pair of hybrid synchrotrons. However, it is not clear whether we can achieve the fast ramp rates required for these scenarios.

As for the next-most cost-effective option, a dogbone recirculating accelerator and a non-hybrid ramped synchrotron appear to be comparable. For the RLA, this conclusion on cost-effectiveness assumes the use of the dogbone geometry and high-field magnets in the arcs. It presents two more challenges. First, that the switchyard would need to support 8 beamlines at each end (4 in each direction, which includes injection and extraction lines). However, the cost penalty in switch to 5 passes would be only 15 cost units, though with fewer passes the fit would be tight on the Fermilab site. Second, the linac focusing would need to work with a factor of 6 in energy range. If the energy range is a problem but not far from being manageable, one could consider extending the low end of the range of the next hybrid synchrotron stage. Using two stages of dogbone RLAs would likely make the dogbone RLA much less cost-effective.

As to the non-hybrid synchrotron, the magnet ramping times are comparable to the 375–750 GeV design, but with the advantage that they are effectively slower because the ramp is not bipolar. Furthermore, a careful choice of energy breakpoint to allow a shared tunnel with the next stage may make this option more attractive.

The above assumed the use of 1300 MHz RF cavities in the non-FFAG designs. I have not checked whether there is sufficient bucket area for the longitudinal emittance, though from the results reported in [10], it appears to be possible.

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