

## What is the Emittance of the Injected Beam?

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January 1988

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**U.S. Department of Energy**

USDOE Office of Science (SC)

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WHAT IS THE EMITTANCE OF THE INJECTED BEAM?

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January 10, 1988

The usual statement ~~is~~ that the ~~the~~ emittance of the injected beam is  $\epsilon_x = \epsilon_y = 10$  (normalized) for heavy ions in RHIC.

Tracking studies requires a more precise statement. Tracking requires that a 4-dimensional surface be specified ~~in~~ in  $x, x', y, y'$  space that contains 95% of the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\epsilon_T = \epsilon_x(x, x') + \epsilon_y(y, y') = C,$$

where  $C$  is a constant chosen so that this surface contains 95% of the beam. One reason for using this expression is that  $\epsilon_T$  is <sup>very</sup> roughly a constant of the motion.

It will be argued below that the proper choice of  $C$  is

$$C = 10 (5/3) = 16.7$$

Corresponding to the statement that  $\epsilon_x = \epsilon_y = 10$ .

Assuming for the moment that  $C = 16.7$  is the proper choice of  $C$ , then in tracking studies where runs ~~are~~ are done with  $\epsilon_x = \epsilon_y$  then the proper starting emittance is

$$\epsilon_x = \epsilon_y = 8.33$$

If runs are done with  $\epsilon_y = 0$ , then the starting  $\epsilon_x$  is

$$\epsilon_x = 16.7$$

$$\epsilon_y = 0$$

These two points are on the surface  $\epsilon_T = \text{constant}$  that contains 95% of the beam.

I assume that the statement  $\epsilon_x = \epsilon_y = 10$  means that for the projection of the particles on the  $x, x'$  plane, ~~the~~ 95% of the particles have an  $\epsilon_x$  which is smaller than  $\epsilon_x = 10$ , and a similar statement applies to the  $y, y'$  plane.

I assume that the distribution  $P(x, x', y, y')$  is gaussian with the form

$$P(x, x', y, y') \sim \exp\left(-(\epsilon_x(x, x') + \epsilon_y(y, y'))/\bar{\epsilon}\right) \quad (1)$$

The projection on the  $xx'$  plane has the distribution

$$P(x, x') = \int dy dy' P(x, x', y, y') \sim \exp\left(-\epsilon_x(x, x')/\bar{\epsilon}\right) \quad (2)$$

In order for 95% of the particle to ~~start~~ have an  $\epsilon_x$  which is smaller than  $\epsilon_x = 10$  then

$$\bar{\epsilon} = 10/3 \quad (3)$$

The fraction of the particles that have a total emittance,  $\epsilon_T = \epsilon_x + \epsilon_y$ , which is smaller than  $\bar{\epsilon}_T$  is given by

$$F(\epsilon_T) = 1 - \exp(-\epsilon_T/\bar{\epsilon}) (1 + \epsilon_T/\bar{\epsilon}). \quad (4)$$

This may be derived from Eq (1) for  $\rho(x, x', y, y')$ . The choice  $\epsilon_T$  that includes 95% of the particles is  $\epsilon_T = 5\bar{\epsilon}$  (the actual answer is closer to  $4.8\bar{\epsilon}$ ).

Thus the  $\epsilon_T$  that contains 95% of the particles is

$$\epsilon_T = 5\bar{\epsilon}$$

$$\epsilon_T = 5 \left( \frac{10}{3} \right)$$

$$\epsilon_T = 16.67$$

I wish to thank Harald Hahn  
for his suggestion ~~of~~ for the  
meaning of the statement that  
 $E_x = 10$ .