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What is the Emittance of the Injected Beam?

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The usual Statement is that the emittance of the Injected beam is $\varepsilon_x = \varepsilon_y = 10$ (normalized) for heavy ions in RHIC.

Tracking studies requires a more precise statement. Tracking requires that a 4-dimenional surface be specified in x, x', y, y' space that contains 95% of the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\varepsilon_T = \varepsilon_x(x, x') + \varepsilon_y(y, y') = C,$$

where C is a constant chosen so that this surface contains 95% of the beam. One reason for using this expression is that ε_T is very roughly a constent of the motion.

It will be argued below that the proper choice of C is

$$C = 10\,(5/3) = 16.7$$

corresponding to the statement that $\varepsilon_x = \varepsilon_y = 10$.

Assuming for the moment that $C = 16.7$ is the proper choice of C, then in tracking studies where runs are ~~are~~ done with $\varepsilon_x = \varepsilon_y$ then the proper starting emittance is

$$\varepsilon_x = \varepsilon_x = 8.33$$

If runs are done with $\varepsilon_y = 0$, then the starting ε_x is

$$\varepsilon_x = 16.7$$
$$\varepsilon_y = 0$$

These two points are on the surface $\varepsilon_T = \text{constant}$ that contains 95% of the beam.

I assume that the statement $\varepsilon_x = \varepsilon_y = 10$ means that for the projection of the particles on the x, x' plane, 95% of the particles have an ε_x which is smaller than $\varepsilon_x = 10$, and a similar statement applys to the y, y' plane.

I assume that the distribution $\rho(x, x', y, y')$ is gaussian with the form

$$\rho(x, x', y, y') \sim \exp\left(-\left(\varepsilon_x(x, x') + \varepsilon_y(y, y')\right)/\bar{\varepsilon}\right) \quad (1)$$

The projection on the $x x'$ plane has the distribution

$$\rho(x, x') = \int dy\, dy'\, \rho(x, x', y, y')$$

$$\sim \exp\left(-\varepsilon_x(x, x')/\bar{\varepsilon}\right). \quad (2)$$

In order for 95% of the particle to have an ε_x which is smaller then $\varepsilon_x = 10$ then

$$\bar{\varepsilon} = 10/3 \quad (3)$$

The fraction of the particles that have a total emittance, $\varepsilon_T = \varepsilon_x + \varepsilon_y$, which is smaller than ε_T is given by

$$F(\varepsilon_T) = 1 - \exp(-\varepsilon_T/\bar{\varepsilon})(1 + \varepsilon_T/\bar{\varepsilon}). \qquad (4)$$

This may be derived from Eq (1) for $\rho(x, x', y, y')$. The choice ε_T that includes 95 % of the particles is $\varepsilon_T = 5\bar{\varepsilon}$ (the actual answer is closer to $4.8\,\bar{\varepsilon}$).

Thus the ε_T that contains 95% of the particles is

$$\varepsilon_T = 5\,\bar{\varepsilon}$$

$$\varepsilon_T = 5\left(\frac{10}{3}\right)$$

$$\varepsilon_T = 16.67$$

I wish to thank Harald Hahn for his suggestion ~~of~~ for the meaning of the statement that $\varepsilon_x = 10$.