

What is the Emittance of the Injected Beam?

G. Parzen

January 1988

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy

USDOE Office of Science (SC)

Notice: This technical note has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy. The publisher by accepting the technical note for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this technical note, or allow others to do so, for United States Government purposes.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

WHAT IS THE EMITTANCE OF THE INJECTED BEAM?

G. Parzen

January 10, 1988

1

The usual Statement ~~is~~ ^{is} that the ~~the~~ emittance of the injected beam is $\epsilon_x = \epsilon_y = 10$ (normalized) for heavy ions in RHIC.

Tracking studies requires a more precise statement. Tracking requires that a 4-dimensional surface be specified. ~~in~~ in x, x', y, y' space that contains 95% of the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\epsilon_T = \epsilon_x(x, x') + \epsilon_y(y, y') = C,$$

where C is a constant chosen so that this surface contains 95% of the beam. One reason for using this expression is that ϵ_T is ^{very} roughly a constant of the motion.

It will be argued below that the proper choice of C is

$$C = 10 (5/3) = 16.7$$

Corresponding to the statement that $\epsilon_x = \epsilon_y = 10$.

Assuming for the moment that $C = 16.7$ is the proper choice of C , then in tracking studies where runs ~~are~~ done with $\epsilon_x = \epsilon_y$ then the proper starting emittance is

$$\epsilon_x = \epsilon_y = 8.33$$

If runs are done with $\epsilon_y = 0$, then the starting ϵ_x is

$$\epsilon_x = 16.7$$

$$\epsilon_y = 0$$

These two points are on the surface $\epsilon_T = \text{constant}$ that contains 95% of the beam.

I assume that the statement $\epsilon_x = \epsilon_y = 10$ means that for the projection of the particles on the x, x' plane, ~~the largest~~ 95% of the particles have an ϵ_x which is smaller than $\epsilon_x = 10$, and a similar statement applies to the y, y' plane.

I assume that the distribution $p(x, x', y, y')$ is gaussian with the form

$$p(x, x', y, y') \sim \exp\left(-(\epsilon_x(x, x') + \epsilon_y(y, y'))/\bar{\epsilon}\right) \quad (1)$$

The projection on the xx' plane has the distribution

$$p(x, x') = \int dy dy' p(x, x', y, y') \sim \exp\left(-\epsilon_x(x, x')/\bar{\epsilon}\right) \quad (2)$$

In order for 95% of the particle to ~~smaller~~ have an ϵ_x which is smaller than $\epsilon_x = 10$ then

$$\bar{\epsilon} = 10/3 \quad (3)$$

The fraction of the particles that have a total emittance, $\epsilon_T = \epsilon_x + \epsilon_y$, which is smaller than $\bar{\epsilon}_T$ is given by

$$F(\epsilon_T) = 1 - \exp(-\epsilon_T/\bar{\epsilon}) (1 + \epsilon_T/\bar{\epsilon}). \quad (4)$$

This may be derived from Eq (1) for $\rho(x, x', y, y')$. The choice ϵ_T that includes 95% of the particles is $\epsilon_T = 5 \bar{\epsilon}$ (the actual answer is closer to $4.8 \bar{\epsilon}$).

Thus the ϵ_T that contains 95% of the particles is

$$\epsilon_T = 5 \bar{\epsilon}$$

$$\epsilon_T = 5 \left(\frac{10}{3} \right)$$

$$\epsilon_T = 16.67$$

I wish to thank Harald Hahn
for his suggestion ~~of~~ for the
meaning of the statement that
 $\epsilon_x = 10$.