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Intrabeam Scattering with Coupling

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Intrabeam Scattering with Coupling

At high γ, vertical growth rate is almost zero. Coupling might reduce growth rates by as much as a factor of 2.

However, smaller growth rates need not cause large changes in final state of beam after 10 hours.

Growth rate $\sim \frac{1}{\sigma}\frac{d\sigma}{dt} \sim N_b /$ 6-dimensional phase space

$$\frac{1}{\sigma}\frac{d\sigma}{dt} \sim N_b / \sigma^6$$

Factor 2 in growth rate can be compensated for by changing σ by $2^{1/6}$ which is a 12% change in σ.

It was observed previously, that when N_b is changed by factor 2, σ only changes by about 12%

How to do Intrabeam Scattering with Coupling

Review of IBS theory

It is assumed that, in the absence of Intrabeam scattering ~~the~~

$$\rho(x, x', y, y') \cong \exp\left[-\left(x^2/2\sigma_x^2 + x'^2/2\sigma_{x'}^2 + y^2/2\sigma_y^2 + y'^2/2\sigma_{y'}^2\right)\right]$$

at elements where $\alpha = 0$, and

$$\rho(x, x', y, y') \simeq \exp\left[-\left(\varepsilon_x(x, x')/\bar{\varepsilon}_x + \varepsilon_y(y, y')/\bar{\varepsilon}_y\right)\right]$$

$$\bar{\varepsilon}_x = 2\sigma_x^2/\beta_x \qquad \bar{\varepsilon}_y = 2\sigma_y^2/\beta_y$$

Note that

$$\frac{d\rho}{dt} = 0 \quad \text{(if no linear coupling)}$$

in the absence of Intrabeam Scattering.

Review of IBS Theory (continued)

In the presence of Intrabeam Scattering it is assumed that $\rho(x, x' y y')$ Keeps the same form

$$\rho(x, x' y' y') \simeq \exp\left[-\left(\varepsilon_x(x,x')/\bar{\varepsilon}_x + \varepsilon_y(y,y')/\bar{\varepsilon}_y\right)\right],$$

but now $\bar{\varepsilon}_x = \bar{\varepsilon}_x(t)$, $\bar{\varepsilon}_y = \bar{\varepsilon}_y(t)$ and grow slowly with time.

Using the result

$$\bar{\varepsilon}_x = \langle \varepsilon_x \rangle = \int dx\,dx'\,dy\,dy'\; \rho(x, x', y, y')\; \varepsilon_x(x, x'),$$

and the collision cross-section $\sigma(\theta)$, one can then find expressions for

$$\frac{1}{\bar{\varepsilon}_x}\frac{d\bar{\varepsilon}_x}{dt}, \quad \frac{1}{\bar{\varepsilon}_y}\frac{d\bar{\varepsilon}_y}{dt} \quad \text{etc}$$

Intrabeam Scattering with Linear Coupling

The following procedure could be used

There are now two invarients
$I_1(x, x' y, y')$ $I_2(x\ x'\ y\ y')$

assume for $\rho(x, x' y\ y')$, in the absence of Intrabeam scattering,

$$\rho(x, x'\ y\ y') \cong \exp\left[-\left(I_1/\bar{I}_1 + I_2/\bar{I}_2\right)\right].$$

$\bar{I}_1$ and $\bar{I}_2$ are constants and can probably be related to $\langle I_1 \rangle$ and $\langle I_2 \rangle$,

Note

$$\frac{d\rho}{dt} = 0$$

in the absence of Intrabeam Scattering

~~In the~~

In the presence of Intrabeam Scattering, it is asssumed that $\bar{I}_1 = \bar{I}_1(t)$, $\bar{I}_2 = \bar{I}_2(t)$ and grow slowly with time.
Using the relationship between $\bar{I}_1$, $\bar{I}_2$ and $\langle I_1 \rangle$ and $\langle I_2 \rangle$ one can then find expressions for

$$\frac{1}{\bar{I}_1} \frac{d\bar{I}_1}{dt} \quad \text{and} \quad \frac{1}{\bar{I}_2} \frac{d\bar{I}_2}{dt}$$

Above procedure would be difficult to carry out. The Epressions for I_1 and I_2 are complicated.

Intrabeam Scattering with Complete Coupling

It is assumed that

$$\mathcal{E}_t = \mathcal{E}_x(x, x') + \mathcal{E}_y(y, y') \text{ is}$$

a constant of the motion. One assumes for $\rho(x, x' y, y')$

$$\rho(x, x' y, y') \simeq \exp\left[-\mathcal{E}_t(x, x' y y')/\bar{\mathcal{E}}\right],$$

where $\bar{\mathcal{E}} = \bar{\mathcal{E}}(t)$ and grows slowly with time

$$\bar{\mathcal{E}} = \frac{1}{2}\langle \mathcal{E}_t \rangle = \frac{1}{2}\int dx\,dx'\,dy\,dy'\,\rho(x x' y y')\,\mathcal{E}_t(x x' y y')$$

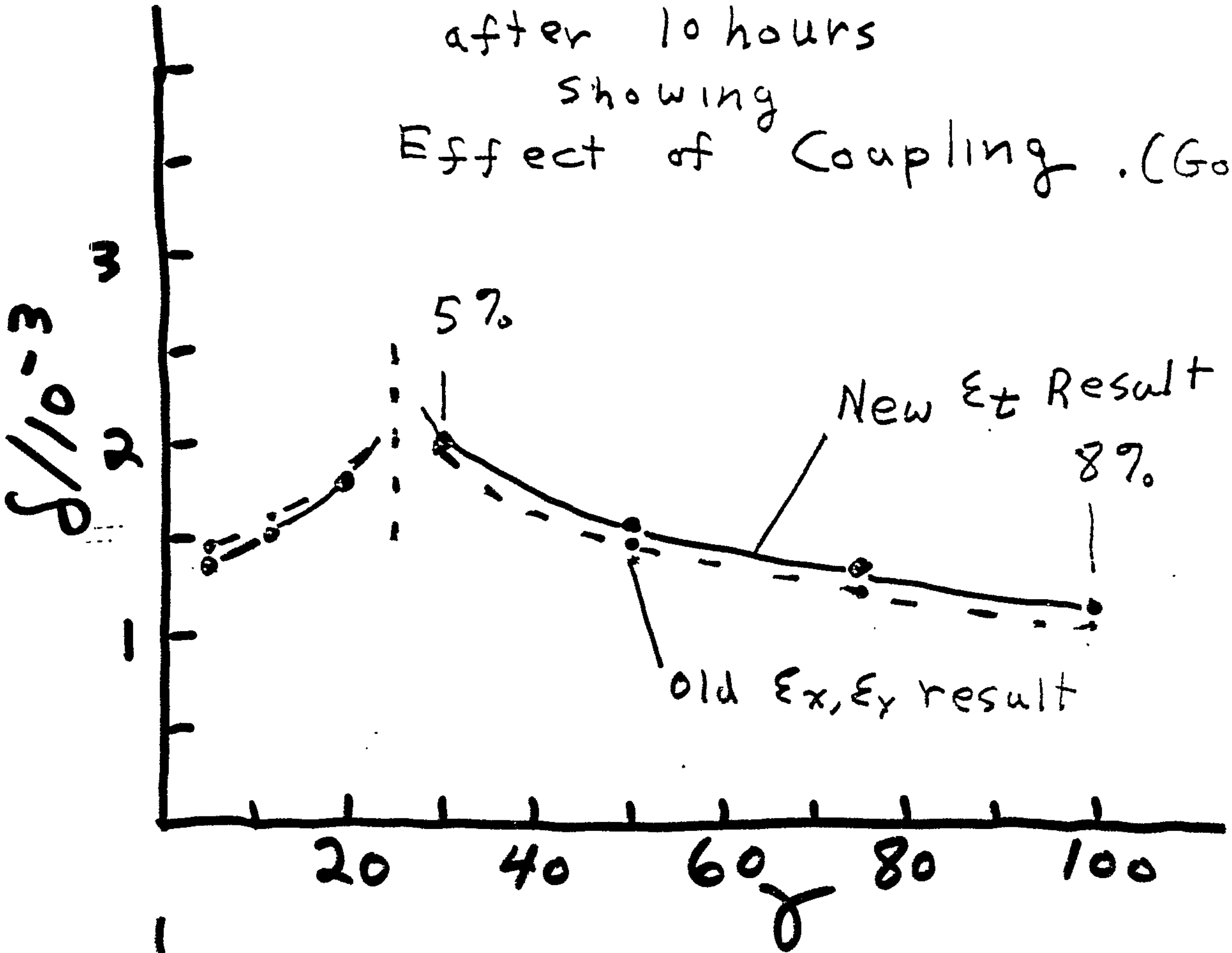
One can then find an expression for

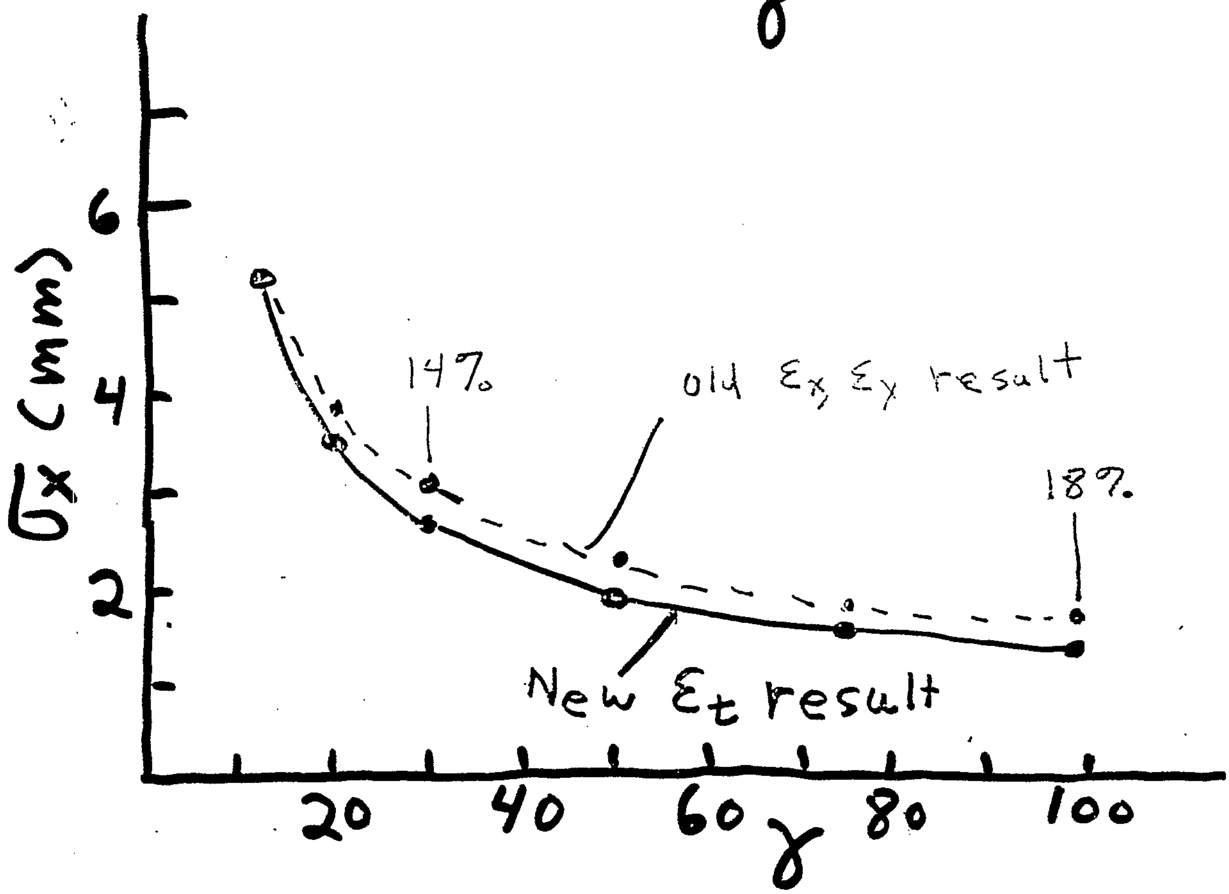
$$\frac{1}{\langle \mathcal{E}_t \rangle}\frac{d\langle \mathcal{E}_t \rangle}{dt}$$

At high γ, starting at $t=0$
with equal ε_x and ε_y, the
growth rate for ε_t is 1/2
the growth rate for ε_x
in th absence of linear coupling

The present program
for computing Intrabeam Scattering
can be modified to compute
the beam growth with the above
assumption for $\rho(x, x', y\, y')$
The program then computes
the growth of ε_t

σ_x, δ versus γ after 10 hours showing Effect of Coupling. (Gold)





Why does δ grow?

In the old $\varepsilon_x, \varepsilon_y$ theory, at high δ, thee exists the time invariant

$$\sigma_E^2 - \sigma_x^2 = \text{Const}$$

$$\sigma_E = X_p \delta$$

and $\sigma_E \sim \sigma_x$ at large t

In the new ε_t theory, the invariant becomes

$$\sigma_E^2 - 2\sigma_x^2 = \text{Const}.$$

and $\sigma_E \rightarrow 1.4\, \sigma_x$ at large t

σ_E^2 versus σ_x^2
over 10 hours
at $\gamma = 100$ for Gold
$\sigma_E = X_p \delta$