

## Intrabeam Scattering with Coupling

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# Intra beam Scattering with Coupling

At high  $\gamma$ , vertical growth <sup>rate</sup> is almost zero.  
Coupling might reduce growth rates  
by as much as a factor of 2.

However, smaller growth rates  
need not cause large changes in  
final state of beam after 10 hours.

Growth rate  $\sim \frac{1}{\sigma} \frac{d\sigma}{dt} \sim N_b / 6\text{-dimensional phase space}$

$$\frac{1}{\sigma} \frac{d\sigma}{dt} \sim N_b / \sigma^6$$

Factor 2 in growth rate can be  
compensated for by changing  $\sigma$  by  $2^{1/6}$   
which is a 12% change in  $\sigma$ .

~~It~~ It was observed previously, that  
when  $N_b$  is changed by factor 2,  $\sigma$   
only changes by about 12%.

(2)

## How to do Intra beam Scattering with Coupling

### Review of IBS theory

It is assumed that, in the absence of Intra beam scattering ~~scattering~~

$$P(x, x', y, y') \cong \exp \left[ - \left( \frac{x^2}{2\sigma_x^2} + \frac{x'^2}{2\sigma_{x'}^2} + \frac{y^2}{2\sigma_y^2} + \frac{y'^2}{2\sigma_{y'}^2} \right) \right]$$

at elements where  $d=0$ , and

$$P(x, x', y, y') \cong \exp \left[ - \left( \epsilon_x(x, x') / \bar{\epsilon}_x + \epsilon_y(y, y') / \bar{\epsilon}_y \right) \right]$$

$$\bar{\epsilon}_x = 2\sigma_x^2 / \beta_x \quad \bar{\epsilon}_y = 2\sigma_y^2 / \beta_y$$

Note that

$$\frac{dP}{dt} = 0 \quad \left( \text{if no linear coupling} \right)$$

in the absence of Intra beam Scattering.

# Review of IBS Theory (Continued)

In the presence of Intra-beam Scattering it is assumed that  $P(x, x', y, y')$  keeps the same form

$$P(x, x', y, y') \approx \exp \left[ - \left( \epsilon_x(x, x') / \bar{\epsilon}_x + \epsilon_y(y, y') / \bar{\epsilon}_y \right) \right]$$

but now  $\bar{\epsilon}_x = \bar{\epsilon}_x(t)$ ,  $\bar{\epsilon}_y = \bar{\epsilon}_y(t)$  and grow slowly with time.

Using the result ~~for~~

$$\bar{\epsilon}_x = \langle \epsilon_x \rangle = \int dx dx' dy dy' P(x, x', y, y') \epsilon_x(x, x')$$

and the collision cross-section  $\sigma(\theta)$ , one can then find expressions for

$$\frac{1}{\bar{\epsilon}_x} \frac{d \bar{\epsilon}_x}{dt}, \quad \frac{1}{\bar{\epsilon}_y} \frac{d \bar{\epsilon}_y}{dt} \quad \text{etc}$$

(4)

## Intra beam Scattering with Linear Coupling

The following procedure could be used

There are now two invariants

$$I_1(x, x', y, y') \quad I_2(x, x', y, y')$$

Assume for  $P(x, x', y, y')$ , in the absence of Intra beam scattering,

$$P(x, x', y, y') \cong \exp \left[ - \left( \frac{I_1}{\bar{I}_1} + \frac{I_2}{\bar{I}_2} \right) \right]$$

$\bar{I}_1$  and  $\bar{I}_2$  are constants and can probably be related to  $\langle I_1 \rangle$  and  $\langle I_2 \rangle$ .

Note

$$\frac{dP}{dt} = 0$$

in the absence of Intra beam Scattering

~~In the~~

In the presence of Intrabeam Scattering,  
 it is assumed that  $\bar{I}_1 = \bar{I}_1(t)$ ,  $\bar{I}_2 = \bar{I}_2(t)$   
 and grow slowly with time.

Using the relationship between  
 $\bar{I}_1$ ,  $\bar{I}_2$  and  $\langle I_1 \rangle$  and  $\langle I_2 \rangle$  one  
 can then find expressions for

$$\frac{1}{\bar{I}_1} \frac{d\bar{I}_1}{dt} \quad \text{and} \quad \frac{1}{\bar{I}_2} \frac{d\bar{I}_2}{dt}$$

Above procedure would be difficult  
 to carry out. <sup>The</sup> Expressions for  $\bar{I}_1$  and  $\bar{I}_2$   
 are complicated.



Intrabeam Scattering ~~with Complete Coupling~~ with Complete Coupling

It is assumed that

$$\epsilon_t = \epsilon_x(x, x') + \epsilon_y(y, y')$$

a constant of the motion - one assumes for  $P(x, x', y, y')$

$$P(x, x', y, y') \approx \exp[-\epsilon_t(x, x', y, y') / \bar{\epsilon}]$$

where ~~it grows slowly~~  $\bar{\epsilon} = \bar{\epsilon}(t)$  and grows slowly with time

$$\bar{\epsilon} = \frac{1}{2} \langle \epsilon_t \rangle = \frac{1}{2} \int dx dx' dy dy' P(x, x', y, y') \epsilon_t(x, x', y, y')$$

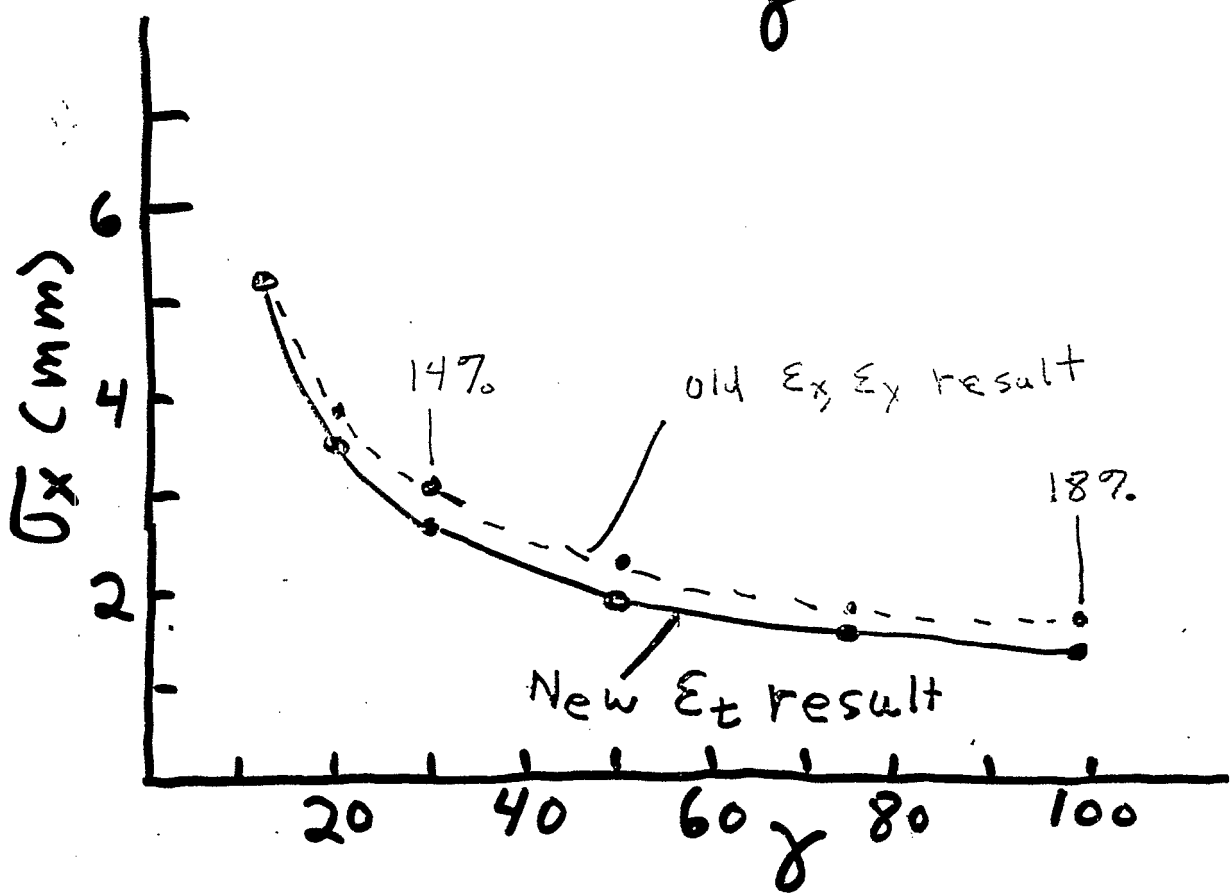
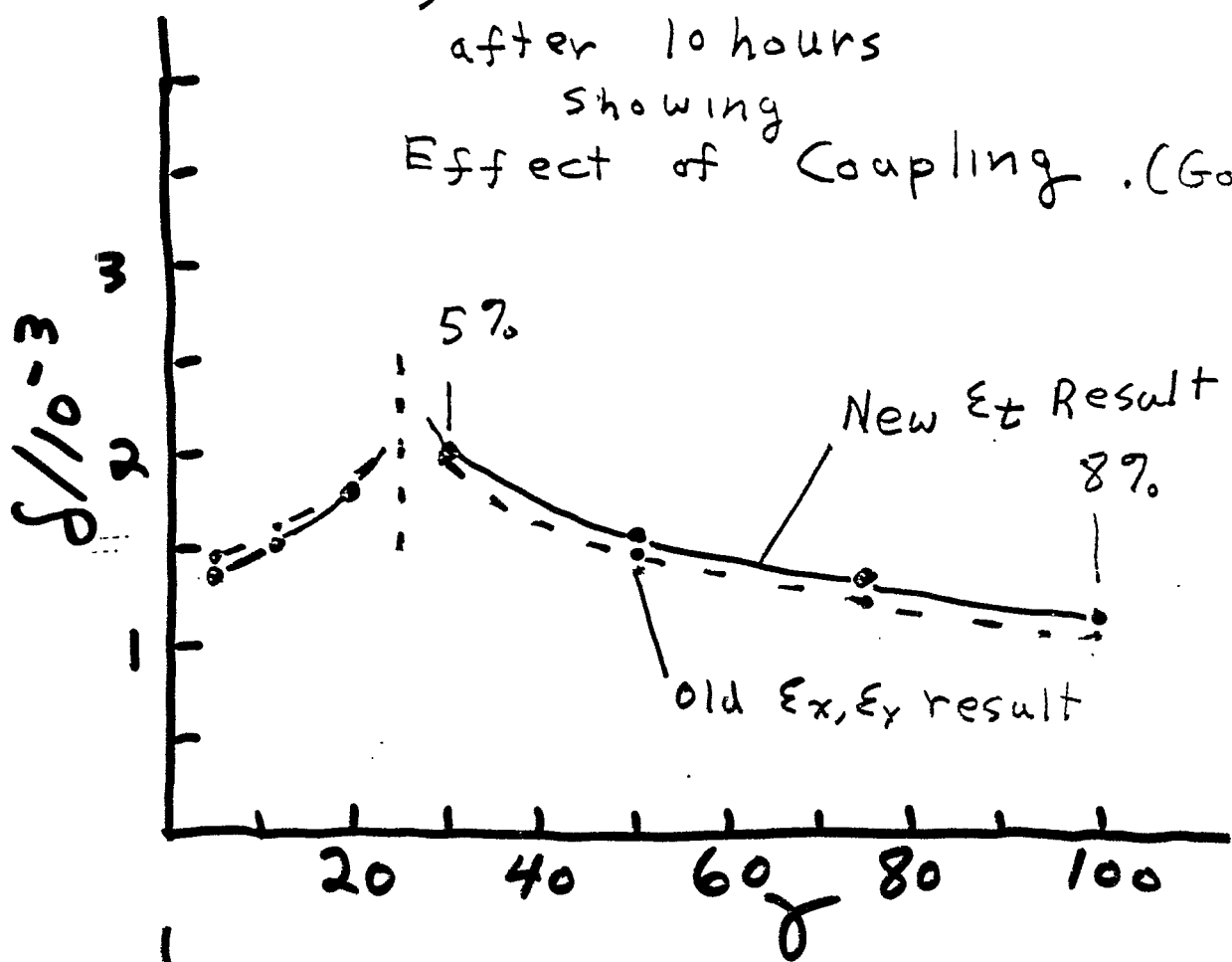
one can then find an expression for

$$\frac{1}{\langle \epsilon_t \rangle} \frac{d \langle \epsilon_t \rangle}{dt}$$

At high  $\delta$ , starting at  $t=0$   
 with equal  $E_x$  and  $E_y$ , the  
 growth rate for  $E_t$  is  $1/2$   
 the growth rate for  $E_x$   
 in the absence of linear coupling

The present program  
 for computing Intrabeam Scattering  
 can be modified to compute  
 the beam growth with the above  
 assumption for  $p(x, x', y, y')$   
 The program then computes  
 the growth of  $E_t$

$\sigma_x, \delta$  versus  $\gamma$   
after 10 hours  
showing  
Effect of Coupling .(Gold)



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Why does  $\delta$  grow?

In the old  $\epsilon_x, \epsilon_y$  theory, at high  $\delta$ , there exists the time invariant

$$\sigma_E^2 - \sigma_x^2 = \text{const}$$

$$\sigma_E = X_p \delta$$

and  $\sigma_E \sim \sigma_x$  at large  $t$

In the new  $\epsilon_t$  theory, the invariant becomes

$$\sigma_E^2 - 2\sigma_x^2 = \text{const.}$$

and  $\sigma_E \rightarrow 1.4 \sigma_x$  at large  $t$

$\sigma_E^2$  versus  $\sigma_x^2$

over 10 hours

at  $\delta = 100$  for Gold

$$\sigma_E = X_p \delta$$

New  $\epsilon_t$  Result

