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# Vacuum Pipe Heating in RHIC

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# Vacuum Pipe Heating

Assume M equally spaced bunches of the same shape and population with gaussian longitudinal distribution.

Let

$N$ , number of particles per bunch

$\sigma_L$ , rms bunch length

$\underline{P}$ , total power dissipated

$$\underline{P} = \frac{1}{2} \sum_{n=1}^{\infty} R_{nM} \, I_{nM}^2$$

where

$R_{nM}$ wall resistance at $nM$ harmonic

$I_{nM}$ beam current at $nM$ harmonic

$$I_{nM} = 2 I_{ave} \exp\left(-\frac{1}{2} n^2 \alpha^2\right)$$

$\alpha = M \theta_L / R$

$R$, average radius = 610.18 m

$I_{ave} = Z N e \beta c M / 2\pi R$

$Z$ charge state

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Skin Depth

$$\delta = \frac{\delta_1}{\sqrt{n}} \qquad \delta_1 = \sqrt{\frac{2 \rho R}{M Z_0}}$$

$Z_0 = 377$ ohm

| | S.S. | Copper |
|---|---|---|
| $\rho$ @ 4.2 °K | 50 $\mu\Omega \times$ cm | 0.055 $\mu\Omega \times$ cm |
| $\delta_1$, M = 57 | 0.17 mm | 0.0056 mm |
| M = 114 | 0.12 mm | 0.0040 mm |
| $\sigma / \ell$ | $1.37 \times 10^{15}$ /ohm·m$^2$ | $1.54 \times 10^{15}$ /ohm·m$^2$ |

$\ell$ , free path length

$\sigma = 1/\rho$ , conductivity

$\sigma/\ell$ is an invariant which depends on the material density

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(1) For those frequencies such that

$\ell/\delta < 1$ Normal Skin Depth

$$R_{nM} = R_1 \, n^{1/2}$$

$$R_1 = \sqrt{(Z_0 \rho R M)/2b^2}$$

(2) For those frequencies such that

$\ell/\delta > 1$ Abnormal Skin Depth

$$R_{nM} = R_1 \, n^{2/3}$$

$$R_1 = \left\{ \left(\frac{\ell}{\sigma}\right) \frac{\sqrt{3}\, Z_0^2 M^2 R}{4\pi b^3} \right\}^{1/3}$$

b, vacuum chamber radius

assuming circular geometry = 3.6 cm

Thus

$$P = 2 I_{ave}^2 \sum_{n=1}^{\infty} R_{1n} \, n^{p_n} \exp(-n^2 \alpha^2)$$

* For $n = 1$ to $n_c$

$$R_{1n} = R_{1\,normal}$$

$$p_n = 1/2$$

* For $n = (n_c+1)$ to $\infty$

$$R_{1n} = R_{1\,abnormal}$$

$$p_n = 2/3$$

There is also a cut-off due to the finite bunch length

$$n_{cut\text{-}off} \sim \frac{1}{\alpha} = \frac{R}{M \sigma_L}$$

$$n_c \sim \left(\frac{a}{\ell}\right)^2 \left(\rho\,\delta_1\right)^2$$

| | S.S. | Copper |
|---|---|---|
| $n_c$ | $1.3 \times 10^{10}$ | 22 |

$n_{cut\text{-}off}$

| M | $\sigma_L$ | |
|---|---|---|
| 57 | 0.4 | 26 |
| 114 | 0.4 | 13 |
| 57 | 0.2 | 52 |

We have summed up to $\hat{n}$ given by

$$\hat{n}\alpha = 15$$

Assuming the Beam Parameters from CDR

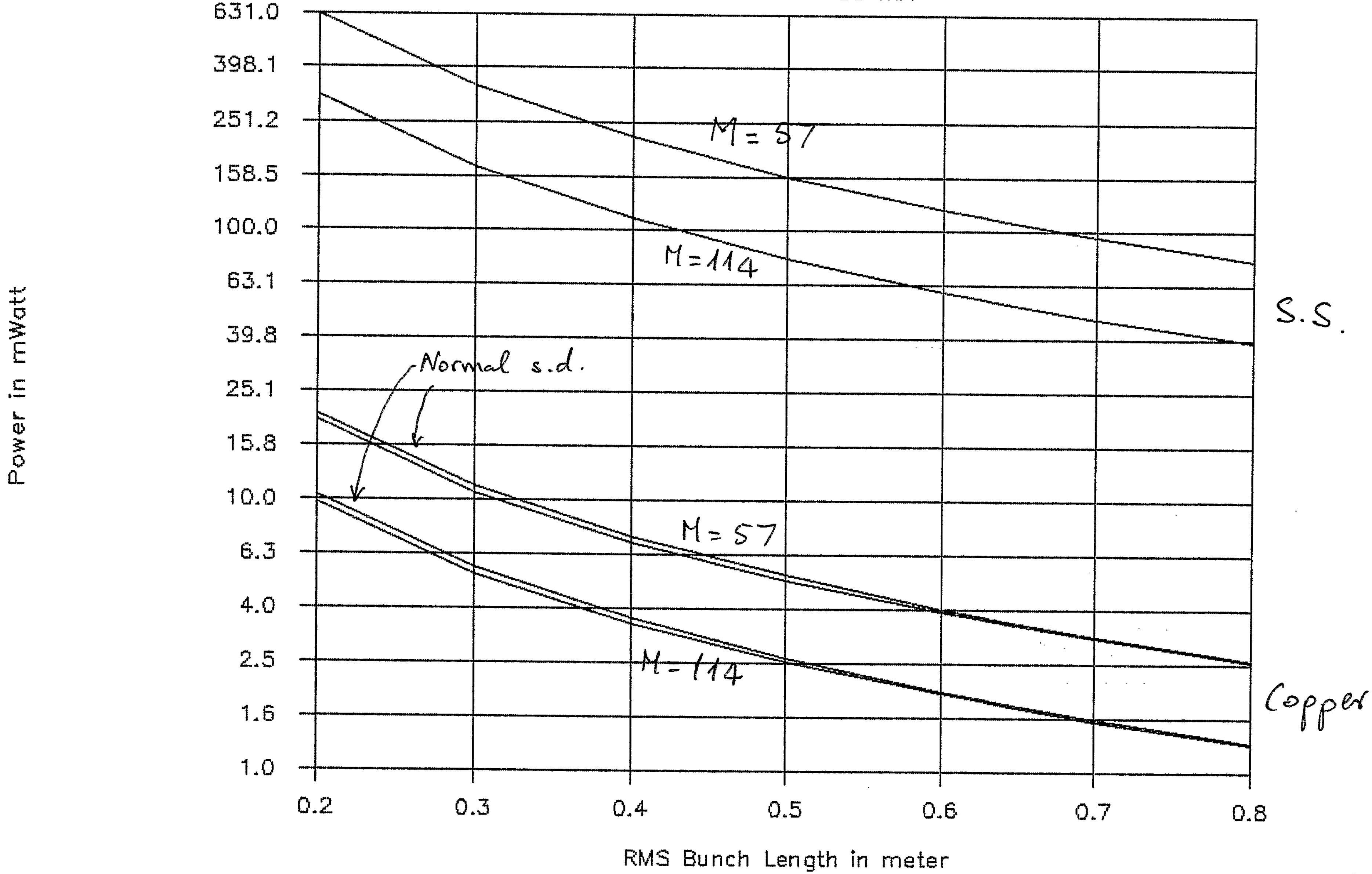
| | $Z$ | $N$ | $I_{ave}$ |
|---|---|---|---|
| proton | 1 | $100 \times 10^9$ | 71 mA (electric) |
| Carbon | 6 | 22 | 94 |
| Sulfur | 16 | 6.4 | 73 |
| Copper | 29 | 4.5 | 93 |
| Iodine | 53 | 2.6 | 98 |
| Gold | 79 | 1.1 | 62 |

with $M = 57$ bunches

As reference $\sigma_L = 40$ cm

## Power dissipated per Dipole (10m)

Ave. Curr. = 100 mA



Power Dissipated per 10m Dipole in Watts
Reference Case: Gold, Average Current = 62 mA
Stainless Steel

| M | N | | sigma-L 0.40 m | sigma-L 0.20 m |
|---|---|---|---|---|
| 57 | 1.1 | x 10^9 | 0.085 | 0.242 |
| 114 | 1.1 | | 0.170 | 0.484 |
| 57 | 2.2 | | 0.340 | 0.968 |
| 114 | 2.2 | | 0.680 | 1.936 |
| 57 | 3.3 | | 0.765 | 2.178 |
| 114 | 3.3 | | 1.530 | 4.356 |

Power Dissipated per 10m Dipole in milli-Watts
Reference Case: Gold, Average Current = 62 mA
Copper, Normal + Abnormal Skin Depth

| M | N | | sigma-L 0.40 m | sigma-L 0.20 m |
|---|---|---|---|---|
| 57 | 1.1 | x 10^9 | 2.7 | 7.6 |
| 114 | 1.1 | | 5.4 | 15.2 |
| 57 | 2.2 | | 10.8 | 30.4 |
| 114 | 2.2 | | 21.5 | 60.7 |
| 57 | 3.3 | | 24.2 | 68.3 |
| 114 | 3.3 | | 48.4 | 136.6 |

Power Dissipated per 10m Dipole in Watts
Reference Case: Protons, Average Current = 71 mA
Stainless Steel

| | | sigma-L | | |
|---|---|---|---|---|
| M | N | 0.80 m | 0.40 m | 0.20 m |
| 57 | 1 x 10^11 | 0.039 | 0.112 | 0.317 |
| 114 | 1 | 0.078 | 0.224 | 0.634 |
| 57 | 3 | 0.351 | 1.008 | 2.853 |
| 114 | 3 | 0.702 | 2.016 | 5.706 |
| 57 | 8 | 2.496 | 7.168 | 20.288 |
| 114 | 8 | 4.992 | 14.336 | 40.576 |

Power Dissipated per 10m Dipole in milli-Watts
Reference Case: Protons, Average Current = 71 mA
Copper, Normal + Abnormal Skin Depth

| | | sigma-L | | |
|---|---|---|---|---|
| M | N | 0.80 m | 0.40 m | 0.20 m |
| 57 | 1 x 10^11 | 1.3 | 3.5 | 10.0 |
| 114 | 1 | 2.6 | 7.1 | 19.9 |
| 57 | 3 | 11.7 | 31.8 | 89.6 |
| 114 | 3 | 23.4 | 63.5 | 179.1 |
| 57 | 8 | 83.2 | 225.9 | 636.8 |
| 114 | 8 | 166.4 | 451.8 | 1273.6 |